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# LOW-FREQUENCY Ly$\alpha$ POWER SPECTRA OBSERVED BY UVCS IN A POLAR CORONAL HOLE

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## ABSTRACT

The occurrence of $f^{-1}$ noise in interplanetary magnetic fields (in the $1 \times 10^{-5}$ to $1 \times 10^{-4}$ Hz band) and other plasma parameters has now been known for about 20 years and has been recently identified also in the photospheric magnetic fields. However, the relationship between interplanetary and solar fluctuation spectra and the identification of their sources at the Sun are problems that still need to be addressed. Moreover, interplanetary density and magnetic field power spectra show a $f^{-2}$ interval at frequencies smaller that $\sim 6 \times 10^{-4}$ Hz whose source on the Sun is at present not fully understood. In this work we report on the first study of low-frequency density fluctuations in the solar corona at 2.1 $R_\odot$. In 2006 June the Ultraviolet Coronagraph Spectrometer (*SOHO* UVCS) observed over a period of about 9.2 days H Ly$\alpha$ intensity fluctuations at 2.1 $R_\odot$ over a polar coronal hole. The Ly$\alpha$ intensity power spectra $S(f)$ (related mainly to density fluctuations) showed a $S(f) \propto f^{-2}$ frequency interval between $2.6 \times 10^{-6}$ and $3.0 \times 10^{-5}$ Hz and a $S(f) \propto f^{-1}$ frequency interval between $3.0 \times 10^{-5}$ and $1.3 \times 10^{-4}$ Hz. The detection of a $f^{-2}$ interval, in agreement with interplanetary density and magnetic field power spectra, has been also predicted in solar wind models as a consequence of phase-mixing mechanisms of waves propagating in coronal holes. High-latitude power spectra show a $f^{-1}$ band approximately in the same frequency interval where $f^{-1}$ noise has been detected in interplanetary densities, and interplanetary and photospheric magnetic fields, providing a connection between photospheric, coronal, and interplanetary $f^{-1}$ noises.

*Subject headings:* Sun: corona — Sun: oscillations — Sun: UV radiation — turbulence

## 1. INTRODUCTION

In the last decades the study of in situ data acquired by various spacecraft (such as *Ulysses*, *Voyager 1* and 2, and *Helios*) demonstrated the presence of waves propagating through the interplanetary medium (see review by Bruno & Carbone 2005 and references therein) from analyses of power spectra $S(f)$ of magnetic field $\delta B$, outflow velocity $\delta v$, and plasma density $\delta\rho$ fluctuations. Matthaeus & Goldstein (1986) suggested that the well-known $1/f$ noise detected in the interplanetary magnetic field over a frequency range of $\sim 2.7 \times 10^{-6}$ to $8.0 \times 10^{-5}$ Hz (Matthaeus & Goldstein 1986; Ruzmaikin et al. 1996; Goldstein et al. 1995) could emerge from a cascade of scale-invariant reconnection processes occurring in the lower solar atmosphere able to transfer energy from small to large spatial scales and to generate a power spectrum $S(f) \propto 1/f$. However, the details of this process are unknown, and the question of whether the $1/f$ noise originates in the corona, or elsewhere, for example in the photosphere, is still open. Moreover, during the same years, it has been demonstrated that power spectra of interplanetary in situ densities (see, e.g., Marsch & Tu 1990; Bellamy et al. 2005) and magnetic field fluctuations (see, e.g., Roberts et al. 1990) show also the presence of a $S(f) \propto 1/f^2$ and a $S(f) \propto 1/f^{5/3}$ band at low frequencies with a break point between the $f^{-2}$ and the $f^{-5/3}$ region moving to lower frequency with increasing heliocentric distance. However, the origin of these phenomena is still unclear and the properties of density power spectra in the low corona are so far unknown.

To address these questions, a survey of density fluctuations through measurements of the Ly$\alpha$ intensity with the Ultraviolet Coronagraph Spectrometer (UVCS; Kohl et al. 1995) aboard the *Solar and Heliospheric Observatory* (*SOHO*) was carried out in 2006 June for about 9.2 days, in conjunction with a *SOHO*-Sun-*Ulysses* quadrature (Poletto et al. 2002). This will give us the opportunity to carry out for the first time a direct comparison of density power spectra of the *same plasma* observed in the low corona and later on in the interplanetary medium and to study the evolution of density spectra with heliocentric distance during the plasma propagation (see Bruno et al. [2005] for a review on the problem of turbulence evolution with the solar wind expansion). In this Letter we focus on the analysis of the UVCS data only: after a description of the data set and of the coronal morphology during these observations (§ 2), we derive the properties of Ly$\alpha$ power spectra in a very low frequency band (§ 3) and demonstrate the presence of spectral intervals characterized by $1/f^2$ and $1/f$ noises. The implications of these results are discussed in the last section (§ 4).

## 2. UVCS Ly$\alpha$ OBSERVATIONS

In 2006 June the UVCS instrument acquired spectra of the hydrogen Ly$\alpha$ $\lambda$1215.67 line (for a detailed description of the UVCS instrument see Kohl et al. 1995). Data were acquired continuously for about 4.4 days, from 13:27 on June 10 to 23:50 UT on June 14. After a gap of 1.7 days, these were followed by 3.1 days of observations from 17:11 on June 16 to 18:26 UT on June 19. The Ly$\alpha$ data were obtained with a spectral binning of 0.14 Å bin$^{-1}$, a spatial binning of 42″ bin$^{-1}$, and a temporal resolution of 300 s (exposure time). The UVCS slit, 0.3 mm wide, was centered at a latitude of 70° southeast (see Fig. 1) and an heliocentric distance of 2.14 $R_\odot$.

[1] Work partially made while at the Arcetri Astrophysical Observatory.

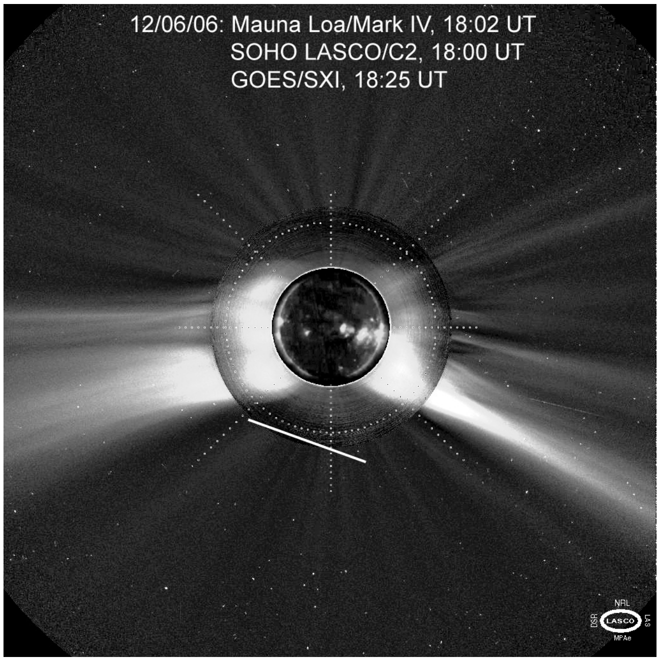


FIG. 1.—Composite image made by a superposition of *SOHO* LASCO C2 (from ∼2 to ∼6 $R_\odot$), Mauna Loa Mark IV (from ∼1.12 to ∼2 $R_\odot$), and *GOES* SXI observations for 2006 June 12. This image shows the coronal morphology at an intermediate time during the UVCS observations; the white straight solid line shows the position of the UVCS slit centered at 2.14 $R_\odot$ in the southern hemisphere.

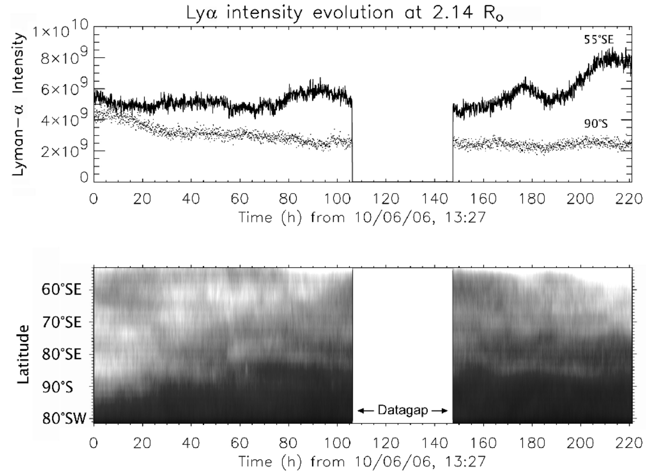


FIG. 2.—*Top*: Evolution with time of the Lyα line intensity at a latitude of 55° southeast (*solid line*) and 90° south (*dotted line*). *Bottom*: Evolution with time of the Lyα line intensity at all latitudes along the UVCS slit; the gray scale ranges from 0 (*black*) to 6 × $10^9$ photons $cm^{-2}$ $s^{-1}$ $sr^{-1}$ (*white*).

As shown in the example of Figure 1, taken during the middle of the first observing sequence, no bright coronal structures, such as streamers, are visible at the latitudes covered by the UVCS slit field of view. Hence, a significant fraction of the plasma sampled by UVCS originated from the south polar coronal hole. After calibrating the data with the standard UVCS analysis software package (DAS40), we obtained time series of the Lyα intensity $I$(Lyα) (photons $cm^{-2}$ $s^{-1}$ $sr^{-1}$) at different latitudes by integrating the Lyα line profile over the wavelength intervals where the emission falls to 1/100 of the peak value in each spatial bin along the slit and each exposure. The $I$(Lyα) time evolution (Fig. 2) shows significant changes over our observing interval; this result, in apparent contradiction with the LASCO images where only faint structures appear at the UVCS latitudes, is probably due to both differences in the adopted color scales and in the instrumental sensitivities. The amplitude of the $I$(Lyα) fluctuations (∼7%–10%) is slightly larger than the amplitude of statistical uncertainties (∼5%–6%), but this has no consequences for our study because, as we verified, the superposition of a Poissonian noise over a signal whose power spectrum has a slope $\alpha$ flattens only the high-frequency part of the spectrum, preserving its lower frequency part. Also, as already discussed by Morgan et al. (2004) the contributions of UVCS instrumental effects to the shape of power spectra are negligible; we conclude that the observed Lyα fluctuations are representative of real changes in the physical properties of the emitting plasma crossing the slit field of view.

In typical coronal conditions, the emission of the Lyα line is mainly due to excitation of H atoms via absorption of photons emitted from the underlying levels (radiative excitation), followed by spontaneous emission. The fluctuations of the total line intensity $\langle \delta I(\mathrm{Ly}\alpha)^2 \rangle$ are in principle related to fluctuations in the electron temperature $T_e$ (affecting the neutral fraction of H atoms), electron density $n_e$ (the radiative component is roughly proportional to $n_e$), and outflow speed $v_{\mathrm{out}}$ (via Doppler dimming effect). However, fluctuations of $T_e$ are expected to be smaller than those of $n_e$ and $v_{\mathrm{out}}$; moreover, because to first approximation $I(\mathrm{Ly}\alpha) \propto n_e$, it turns out that a 25% variation in $n_e$ implies a corresponding $I$(Lyα) variation by 25%, while, using Doppler dimming factors from Kohl et al. (1997), we conclude that a 25% variation in $v_{\mathrm{out}}$ implies a corresponding $I$(Lyα) variation by less than 10%. Hence, analogously to what was assumed also by Morgan et al. (2004), we conclude here that the observed fluctuations of Lyα intensity are mainly associated with density fluctuations. The spectral analysis of the observed intensity versus time profiles is described in the next section.

## 3. THE Lyα POWER SPECTRA

Starting from the Lyα intensity time series, we derived the line intensity power spectra $S(f)$ (photons$^2$ $cm^{-4}$ $s^{-2}$ $sr^{-2}$ $Hz^{-1}$) in each bin along the slit; this has been done by taking the modulus squared of the Lyα intensity Fourier transform (FT). In order to find possible time localization of various frequency components, $S(f)$ have been also computed with a wavelet transform (WT) analysis transform provided by Torrence & Compo (1998). However, the analysis via FT and WT requires that there are no data gaps. Because in our case we have approximately 1.7 days of gap in the middle of our observations (see Fig. 2), we computed at each bin the $S(f)$ only over the longest time interval of continuous data coverage. Moreover, in order to increase the significance of our results, we also analyzed the Lyα time series with the Lomb-Scargle transform (LS) (Scargle 1982), which can be applied to data sets with gaps, as in our case. Before computing the $S(f)$ with FT, WT, and LS, we subtracted from each time series its average value; as we verified, a further subtraction of a polynomial fit turns out only to decrease the power at very low frequencies, where uncertainties are larger. Data leakage has been reduced by filtering the Lyα time series with a standard Hanning windowing.

Time series detected in situ strongly depend on the solar rotation because the spacecraft intercepts different radial paths at different times, scanning different spatial scales on the Sun (see discussion in Ruzmaikin et al. 1996). The Ly$\alpha$ intensity evolution observed over a period of ∼9.2 days may also be affected by solar rotation. The interpretation of fluctuations detected by remote sensing techniques in the coronal UV line intensities is much more complicated, because of the integration along the line of sight (LOS). For instance, by assuming $n_e(r)$, $v_{\rm out}(r)$, and $T_e(r)$ coronal hole profiles from Cranmer et al. (1999), Doppler dimming factors from Kohl et al. (1997), and the H ionization equilibrium from Arnaud & Rothenflug (1985), it turns out that at 2.1 $R_\odot$ 90% of the observed Ly$\alpha$ intensity arises from an integration along the LOS over $L \approx 3\ R_\odot$ long centered on the plane of the sky. However, by dividing the LOS into $n$ elements and assuming each element along the LOS to have a power spectrum with a given slope $\alpha$, we numerically verified that the spectrum from the superposition of these $n$ signals has the same slope $\alpha$. This means that the solar rotation does not play a role in the shape of the power spectra, opposite to what happens for in situ data, because in our case the observed fluctuations do not reflect different spatial scales on the Sun, but an average over intensity time series in flux tubes aligned along the LOS and rooted on the Sun in regions at different latitudes and longitudes. In this average the evidence for periodic signals emitted from the Sun cancel out, because the waves propagating in different flux tubes will not be in phase, but the slope of the power spectra will be preserved. We add that, in any case, changes related to the solar rotation will occur on timescales of 1 or more days, while in this work we are interested in fluctuations occurring on much shorter timescales (from hours to minutes); for instance, after 1 day the Sun has rotated by ∼13°, while ∼80% of the observed Ly$\alpha$ intensity (due to an integration along the LOS over a coronal sector ∼110° wide) is due to the same coronal sector. Moreover, the eventual effects of solar rotation are minimal at bins located above the pole.

Because of the integration along the LOS, the Ly$\alpha$ intensity observed at a given projected latitude along the UVCS slit is due to the emission of plasma located at different latitudes and longitudes in the corona. For instance, by assuming a LOS depth of $L \approx 3\ R_\odot$, it turns out that the intensity observed along the slit above the pole represents an average over plasma aligned along the LOS and located in the corona in a latitude band between 90° and ∼60° south, while the intensity observed in the bin at 50° south is due to plasma located in a latitude band between 50° and ∼40° south. Hence, spectra at different bins along the slit represent on average plasma originating over different latitude bands. In order to emphasize variations with latitude of the $S(f)$ curves, we averaged the spectra $S(f)$ obtained over different angular regions. In particular, Figure 3 (*top panels*) shows a comparison between power spectra $S(f)$ we obtained from FT, WT, and LS analyses averaged over the south pole (*left*), where the effects of solar rotation, if any, are minimal, and at lower latitudes (*right*). The top panels of Figure 3 show frequency intervals where power spectra have different slopes; moreover, a comparison between the top panels of this figure indicates that spectra at high and low latitude are different, in particular at frequencies below $\approx 10^{-4}$ Hz.

Curves in the top panels of Figure 3 also show differences between FT spectra $S_{\rm FT}(f)$ (*dotted lines*), WT global spectra $S_{\rm WT}(f)$ (i.e., averaged over the whole observation interval; *solid lines*), and LS spectra $S_{\rm LS}(f)$ (*dashed lines*). However, FT and WT spectra have been obtained from an analysis performed only over the first time interval of continuous data, while the LS spectra have been made using data taken over the whole observation interval. Hence, in the following we

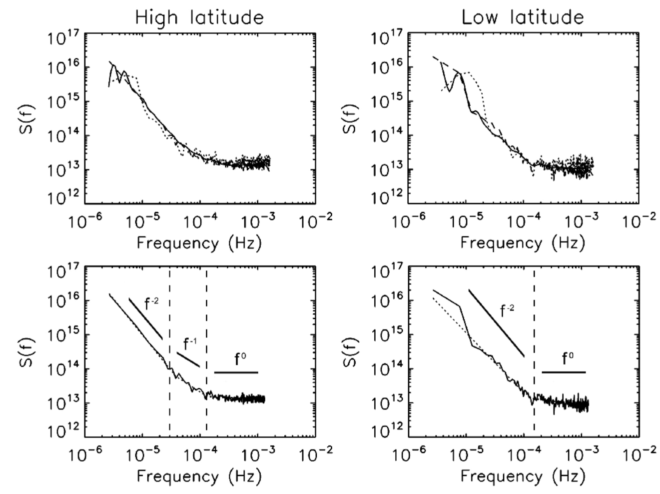


FIG. 3.—*Top*: Ly$\alpha$ power spectra $S(f)$ (photons$^2$ cm$^{-4}$ s$^{-2}$ sr$^{-2}$) from FT (*dotted lines*), WT (*solid lines*), and LS (*dashed lines*) analyses averaged over a 10° latitude interval around the south pole (*left*) and around a latitude of 60° southeast (*right*) in order to show latitudinal differences in the spectral range extents of $1/f$ interval (see text). *Bottom*: Ly$\alpha$ power spectra from LS analysis (*solid lines*) averaged over the same latitude intervals as in the top panels and the corresponding fitting functions (*dotted lines*); reference solid lines show the $f^{-2}$, $f^{-1}$, and $f^0$ slopes.

concentrate only on the study of the $S_{\rm LS}(f)$ curves. These spectra have been analyzed in both high- and low-latitude intervals by defining a fitting function with 5 free parameters (3 different slopes and 2 knee frequencies), estimating each parameter with the standard $\chi^2$ method. It turns out that the high-latitude Ly$\alpha$ spectrum has slopes $\alpha = 2.1 \pm 0.1$ between $2.6 \times 10^{-6}$ Hz (our frequency lower limit) and $(3 \pm 1) \times 10^{-5}$ Hz, $\beta = 1.1 \pm 0.2$ between $(3 \pm 1) \times 10^{-5}$ and $(1.3 \pm 0.4) \times 10^{-4}$ Hz, and $\gamma = 0.1 \pm 0.2$ between $(1.3 \pm 0.4) \times 10^{-4}$ and $1.6 \times 10^{-3}$ Hz (our frequency upper limit). On the contrary, the low-latitude $S_{\rm LS}(f)$ curve shows only two frequency intervals with slopes $\alpha = 1.9 \pm 0.3$ and $\gamma = 0.2 \pm 0.4$ and a frequency break around $(1.6 \pm 0.1) \times 10^{-4}$ Hz (see bottom panels of Fig. 3). In order to verify whether the flat power spectrum has to be expected from the average Poissonian noise by 5%–6% present in our data, we made a simulation superposing a Poissonian distributed noise to a synthetic signal with power spectrum $S(f) \propto 1/f$. The outcome of our simulation reproduces the observed flat spectra at frequencies larger than $\approx(1–2) \times 10^{-4}$ Hz, which has to be ascribed entirely to Poissonian noise.

## 4. DISCUSSION AND CONCLUSIONS

Previous studies of oscillations in the UV Ly$\alpha$ line intensity (Morgan et al. 2004) and the polarized brightness (Ofman et al. 1997, 2000) fluctuations focused only on data sets with a short duration in time (less than ∼10 hr). In particular, Morgan et al. (2004) found, from UVCS Ly$\alpha$ observations over a polar coronal hole at altitudes ranging between 1.5 and 2.0 $R_\odot$, evidence for oscillations with periods of 7–8 minutes, while Ofman et al. (2000) demonstrated, from UVCS pB data acquired over a coronal hole at 1.9 and 2.1 $R_\odot$, the presence of oscillations with periods of 6–10 minutes and a coherence time of 30 minutes. These authors concluded, from observations in the $\sim 3 \times 10^{-5}$ to $9 \times 10^{-3}$ Hz (Ofman et al. 2000) and $\sim 4 \times 10^{-4}$ to $4 \times 10^{-3}$ Hz (Ofman et al. 2000) frequency bands, that the observed density oscillations were a signature of fast or slow magnetosonic waves.

In this work we analyzed Ly$\alpha$ oscillations (representative of electron density oscillations) in a band (∼$2.6 \times 10^{-6}$ to $1.6 \times 10^{-3}$ Hz) that does not include frequencies large enough to verify the presence of the 6–10 minute (∼$1.7 \times 10^{-3}$ to $2.8 \times 10^{-3}$ Hz) oscillations, but we had the opportunity to extend these studies to the low-frequency regime. Results shown here demonstrate (to our knowledge for the first time) that high-latitude density power spectra $S(f)$ acquired around 2.1 $R_{\odot}$ show two frequency intervals with different power slopes: a $S(f) \propto f^{-2}$ band ($2.6 \times 10^{-6}$ to $3 \times 10^{-5}$ Hz) and a $S(f) \propto f^{-1}$ band ($3 \times 10^{-5}$ to $1.3 \times 10^{-4}$ Hz); as we discussed, this result is independent of the solar rotation. A comparison between the high- and low-latitude density spectra (Fig. 3, *top panels*) points toward a latitudinal dependence of the $f^{-1}$ range. In particular, $1/f$ noise seems to be more typical of higher latitudes (Fig. 3, *left panels*), while closer to the equator the $f^{-2}$ range possibly extends to higher frequencies hiding the $f^{-1}$ band (Fig. 3, *right panels*).

Recently, Matthaeus et al. (2007) showed that the $1/f$ noise is present also in in situ density spectra (above ∼$4 \times 10^{-5}$ Hz) and in high-latitude photospheric line-of-sight magnetic field spectra from *SOHO* MDI (in the $10^{-5}$ to $10^{-4}$ Hz range). Even if, as discussed in § 3, power spectra from in situ data and remote sensing observations are intrinsically different, interestingly we found the $1/f$ band approximately in the same frequency interval where $1/f$ noise is observed in the interplanetary density spectra (Matthaeus et al. 2007), in magnetic field fluctuations (Matthaeus & Goldstein 1986), and in the photospheric magnetic fields (Nakagawa & Levine 1974; Matthaeus et al. 2007); this implies that $1/f$ noise is a persistent feature in the solar corona. Moreover, consistently with our results, interplanetary and photospheric magnetic field fluctuations show a latitudinal dependence, with a $1/f$ band more evident at higher latitudes (Matthaeus et al. 2007). The above similarities suggest that the scale-invariant processes responsible for the $1/f$ noise might be either entirely or partially photospheric, but also raise the question of how this signal is processed, transmitted, or perhaps enhanced in the corona. In principle it is possible that the corona not only transmits the $1/f$ noise, as we demonstrated here, but also enhances it. First, observations using *Helios* data (see Bruno & Carbone 2005) show that magnetic spectra in the inner solar wind admit a $1/f$ range in fast wind periods (likely originating at high latitudes) that extends to higher frequencies than what we have seen at 1 AU and in the *Ulysses* data sets; this higher frequency $1/f$ signal may originate in the corona. Second, from a theoretical perspective, a recent survey (Dmitruk & Matthaeus 2007) of fluid and MHD models has shown that only those systems of equations that admit both a direct and an inverse cascade spontaneously generate $1/f$ magnetic noise. Standard linear and direct-cascade related phenomena are not able to explain $1/f$ signals, while Dmitruk & Matthaeus (2007) showed that a $1/f$ range is associated mainly with the longest wavelength modes that are involved, via inverse cascade, in a back-transfer of energy from low to high frequencies connecting widely separated scales. Such processes may occur in the corona leading to an enhancement of $1/f$ noise possibly generated in the photosphere.

The presence of a $f^{-2}$ band is also in qualitative agreement with in situ density and magnetic field power spectra. For instance Bellamy et al. (2005) found, from *Voyager 2* data, that density spectra have a $f^{-2}$ band at frequencies smaller than ∼$6 \times 10^{-4}$ Hz, consistent with data acquired inside 1 AU (see, e.g., Marsch & Tu 1990). The $f^{-2}$ part of density spectra is in general interpreted as a signature of abrupt discontinuities occurring on the Sun (such as jets or flaring bright points) able to generate compressions and rarefactions propagating within the solar wind. Similar processes have been suggested as a possible explanation for the plasma pileup and compression microstreams observed in high-latitude *Ulysses* data (see Neugebauer et al. 1995), although at present there is no good physical model to relate such impulsive events to the observed microstreams. Interestingly, MHD models of fast solar wind predict that the initial $S(f) \propto f^{-1}$ magnetic field power spectrum of Alfvén waves injected at 1 $R_{\odot}$ steepens during the propagation into the solar corona into a $S(f) \propto f^{-2}$ spectrum at larger altitudes (18 $R_{\odot}$) because of phase-mixing processes that dissipate higher frequency waves (see Ofman 2004, 2005). However, models of MHD turbulence developed so far concentrate mainly on nearly incompressible MHD waves where density turbulences are dynamically unimportant (see discussion in Bellamy et al. 2005). Results shown here from remote sensing data and previous studies from in situ data (see, e.g., Marsch & Tu 1990; Bellamy et al. 2005; Matthaeus et al. 2007) suggest that density fluctuations may play a role in the propagation of MHD turbulence.

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REFERENCES

Arnaud, M., & Rothenflug, R. 1985, A&AS, 60, 425
Bellamy, B. R., et al. 2005, J. Geophys. Res., 110, A10104
Bruno, R., & Carbone, V. 2005, Living Rev. Sol. Phys., 2, 4
Bruno, R., et al. 2005, Adv. Space Res., 35, 939
Cranmer, S. R., et al. 1999, ApJ, 511, 481
Dmitruk, P., & Matthaeus, W. H. 2007, Phys. Rev. E, 76, 036305
Goldstein, M. L., et al. 1995, ARA&A, 33, 283
Kohl, J. L., et al. 1995, Sol. Phys., 162, 313
———. 1997, Sol. Phys., 175, 613
Marsch, E., & Tu, C.-Y. 1990, J. Geophys. Res., 95, 11945
Matthaeus, W. H., & Goldstein, M. L. 1986, Phys. Rev. Lett., 57, 495
Matthaeus, W. H., et al. 2007, ApJ, 657, L121
Morgan, H., et al. 2004, ApJ, 605, 521
Nakagawa, Y., & Levine, R. H. 1974, ApJ, 190, 441
Neugebauer, M., et al. 1995, J. Geophys. Res., 100, 23389
Ofman, L. 2004, J. Geophys. Res., 109, A07102
———. 2005, Space Sci. Rev., 120, 67
Ofman, L., et al. 1997, ApJ, 491, L111
———. 2000, ApJ, 529, 592
Poletto, G., et al. 2002, J. Geophys. Res., 107(A10), SSH 9-1
Roberts, D. A., et al. 1990, J. Geophys. Res., 95, 4203
Ruzmaikin, A., et al. 1996, in AIP Conf. Proc. 382, Solar Wind Eight, ed. D. Winterhalter et al. (Woodbury: AIP), 225
Scargle, J. D. 1982, ApJ, 263, 835
Torrence, C., & Compo, G. P. 1998, Bull. Am. Meteorol. Soc., 76, 61