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SCALING LAWS AND LUMINOSITY SEGREGATION

LORIS P. L. COLOMBO[1] AND SILVIO A. BONOMETTO[2,3]

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ABSTRACT

We debate how the scaling properties of the angular correlation function $w(\theta)$ depend on luminosity segregation. Under the approximation that there is no deviation from Euclidean geometry and no evolution, we find that the scaling with catalog depth (D_*) is the same both for a luminosity (L) independent clustering length (r_0) and for a generic dependence of r_0 on L. Recent angular data, however, extend to depths where the above approximation is unsuitable and the simple scaling $w \propto D_*^{-\gamma}$ should be modified. We find that such modifications depend on the shape of the L–dependence of r_0 and are indeed different depending on whether luminosity segregation is or is not considered. In particular, we find that a luminosity segregation as observed at $z = 0$ causes effects of the same order as varying the rate of clustering evolution. For the sake of example, we apply our expressions to available angular galaxy data in the B– and R–bands and show that significant constraints on the evolution of clustering can already be found with public data.

Subject headings: cosmology: theory — dark matter — large-scale structure of universe

1. INTRODUCTION

A great deal of effort has been devoted by Peebles and collaborators to measuring galaxy clustering properties (see Peebles 1980, and references therein). The observational data available at the time amounted to angular catalogs of different depth D_* (a precise definition will be reported in the next section). Using such data, they computed the two-point angular function and found that it *scaled* with $D_*^{-\gamma}$, as predicted by the Limber equation (Limber 1953), if the spatial two-point function reads $\xi(r) = (r_0/r)^\gamma$. This allowed them to confirm that the clustering length r_0 had general, sample-independent significance. In turn, this was an evidence in favor of the gravitational instability picture.

In the following years, angular measurements and the Limber equation played a key role as a test of cosmological and structure formation models. Simultaneously, however, the Automated Plate Measuring Facility (APM) survey (Maddox et al. 1990a, 1990b) provided one of the first evidences that a significant luminosity segregation exists. This was confirmed by inspections based on large three-dimensional catalogs (Dominguez-Tenreiro & Martinez 1989; Valotto & Lambas 1997; Willmer, da Costa, & Pellegrini 1998; Guzzo et al. 2000). How such dependence may interfere with the scaling predicted by the Limber equation is a problem that was essentially overlooked, trusting that an *average* correlation length might be used, however, without causing discernible errors. In this paper we examine this problem in detail and find that, while the neglect of luminosity segregation could not cause appreciable errors in early estimates, currently available angular data should be analyzed taking it into account. In particular, we show that conclusions regarding clustering evolution obtained neglecting luminosity segregation can be misleading.

It should also be stressed that, even in the era of large three-dimensional surveys, the use of the Limber equation to study galaxy and galaxy cluster clustering properties from angular catalogs is far from being obsolete. At any time, two-dimensional samples will tend to be wider than three-dimensional samples, the compilation of which requires an extra effort dedicated to each object (e.g., a spectral analysis). At large distances, spectroscopic redshifts are good distance indicators, being mildly affected by peculiar-motion distortions. Obtaining them, however, is a complex task. The information obtainable from photometric redshifts, on the other hand, bears only a statistical meaning and must be handled with caution. Comparing results obtainable with or without them is, at least, an indirect way to make sure that no systematic error is caused by their use.

In a previous paper (Gardini, Bonometto, & Macciò 1999) it was shown that, in the Euclidean regime and in the absence of evolution, the scaling law of the angular function valid for constant r_0 also holds if it is assumed that $r_0 \propto D_L$ (where D_L is the mean distance of objects whose luminosity exceeds L). Here, first of all, we show that such a conclusion can be extended to a generic dependence of r_0 on L, still provided that there is no evolution and Euclidean geometry is assumed. This point will be made in detail in § 2.

Galaxy data, extending to a low (apparent) luminosity, are now available. They include distant galaxy systems, the analysis of which now requires us to abandon the above limitation. In § 3, we see that, if evolution and non–Euclidean geometry are now taken into account, deviations from the scaling $w \propto D_*^{-\gamma}$ are indeed different depending on whether r_0 does or does not depend on L.

In order to perform a quantitative evaluation of such effects, in § 4 we review data on luminosity segregation. A tentative fit to data will be proposed, which will be used in successive evaluations.

Section 5 presents a comparison between observational behavior and theoretical predictions. Data in the B band and the R band will be considered. As we shall see, available data are not yet so stringent as to fully constrain the rate of clustering evolution. On the contrary, it will be evident that working out clustering evolution without considering luminosity segregation is surely misleading. Section 6 will be devoted to make this point in detail and to illustrating some consequences of our results.

[1] Physics Department, Università degli Studi di Milano, Via Celoria 16, I-20133 Milano, Italy.
[2] Istituto Nazionale di Fisica Nucleare, Via Celoria 16, I-20133 Milano, Italy.
[3] Physics Department G. Occhialini, Università degli Studi di Milano-Bicocca, Milano, Italy.

2. SCALING LAW OF THE ANGULAR FUNCTION IN THE EUCLIDEAN LIMIT

The aim of this section is to work out the scaling of the angular two-point correlation function, dropping the assumption that the correlation length r_0 is luminosity independent, neglecting, however, departures from Euclidean geometry or galaxy evolution. We first reproduce the procedure leading to the *usual* Limber equation, in a way similar to Peebles (1980). The changes made here only aim to allow an easier passage to the case of luminosity-dependent correlations; in particular, we systematically refer to luminosities instead of magnitudes. We also use symbols similar to Peebles, unless this causes confusion when an extension to the case of luminosity segregation is performed.

2.1. *Spatial Correlations*

Let then $\Phi(L)$ be the galaxy luminosity function, such that

$$n(>L) = \int_L^\infty dL' \, \Phi(L') \quad (1)$$

is the cumulative galaxy number density and

$$D_L = n^{-1/3}(>L) \quad (2)$$

is the average separation of galaxies more luminous than L. Equations (1) and (2) set a correspondence between L and D_L; note, in particular, that

$$dL \, \Phi(L) = 3dD_L/D_L^4 \, . \quad (3)$$

Let us then assume that the spatial two-point function for galaxies with luminosity greater than L reads

$$\xi_{>L}(r) = B(D_L) r^{-\gamma} \, . \quad (4)$$

The dependence on D_L of the coefficient B allows us to take into account the observed luminosity segregation. No L dependence of γ will be considered. The L dependence of the *cumulative* $\xi_{>L}$ is obtainable if the *differential* two-point function reads

$$\xi(L_1, L_2, r_{12}) = b(D_{L_1}) b(D_{L_2}) r_{12}^{-\gamma} \quad (5)$$

(r_{12} is the separation between the points considered, where objects of luminosities L_1 and L_2 are set), and

$$B^{1/2}(D_L) = 3D_L^3 \int_{D_L}^\infty dD \, D^{-4} b(D) \, . \quad (6)$$

This can be shown by taking into account equation (3) and replacing L_1 and L_2 by D_{L_1} and D_{L_2}, as integration variables, in the relation

$$\xi_{>L}(r_{12}) = \frac{1}{n^2(>L)} \int_L^\infty dL_1 \, \Phi(L_1) b(D_{L_1}) \int_L^\infty dL_2 \, \Phi(L_2) b(D_{L_2}) r_{12}^{-\gamma} \, . \quad (7)$$

This is equivalent to equation (49.3) in Peebles (1980), after the right-hand side of his equation (51.1) has been replaced in it, just using for ξ the expression given by our equation (5).

Among possible laws, it is worth considering the power law $b(D) = b_* D^{\beta/2}$, where b_* is a constant. According to equation (6), in this case it is $B(D_L) \propto D_L^\beta$. For galaxy clusters, as discussed below, the case $B(D_L) = (aD_L)^\gamma$ may be significant, and it is soon obtained by setting $\beta = \gamma$. The case of an L-independent clustering length can also be recovered by setting $\beta = 0$.

2.2. *Angular Correlations*

Let L_* be the luminosity of a galaxy at the turning point of the luminosity function; for a Schechter parametrization, $L_* \simeq 10^{10} \, h^{-2} \, L_\odot$ (where h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}). Then let l_m be the apparent luminosity limit defining an angular galaxy sample and let D_* be the distance at which a galaxy of luminosity L_* has an apparent luminosity l_m. Assuming an Euclidean geometry and neglecting the cosmic expansion, $D_* = (L_*/4\pi l_m)^{1/2}$; in more general cases, one must take into account the appropriate relation between flux and luminosity (see § 3.1 below).

When D_* is given by the above expression, the angular number density of galaxies, $\mathcal{N}(D_*)$, scales as D_*^3; obviously, this is quite independent of the scaling of two-point functions and is a well known *volume* effect, which does not persist, e.g., if the galaxy distribution is a fractal (see, e.g., Murante et al. 1997 and references therein).

For a sample characterized by a depth D_*, the two-point angular function can be obtained by integrating $\xi(L_1, L_2, r_{12})$ over the radii r_1 and r_2 and the luminosities L_1 and L_2, starting from $L_*(r_1/D_*)^2$ and $L_*(r_2/D_*)^2$, respectively, so as to include those galaxies whose apparent luminosity exceeds l_m,

$$w(\vartheta) = \frac{1}{\mathcal{N}^2(D_*)} \int_0^\infty dr_1 \, r_1^2 \int_{L_*(r_1/D_*)^2}^\infty dL_1 \, \Phi(L_1) \times \int_0^\infty dr_2 \, r_2^2 \int_{L_*(r_2/D_*)^2}^\infty dL_2 \, \Phi(L_2) \xi(L_1, L_2, r_{12}) \, . \quad (8)$$

This equation is equivalent to equation (51.2) in Peebles (1980), where, however, the change of variables we make in equation (12) has already been performed.

Using D_L instead of L and equation (5) for $\xi(L_1, L_2, r_{12})$, equation (8) yields

$$w(\vartheta) = \frac{1}{\mathcal{N}^2(D_*)} \int_0^\infty dr_1 \, r_1^2 \int_{D_{L_*(r_1/D_*)^2}}^\infty dD_1 \, \frac{3b(D_1)}{D_1^4} \times \int_0^\infty dr_2 \, r_2^2 \int_{D_{L_*(r_2/D_*)^2}}^\infty dD_2 \, \frac{3b(D_2)}{D_2^4} \, r_{12}^{-\gamma} \, . \quad (9)$$

This can be simplified using equation (6), obtaining

$$w(\vartheta) = \frac{D_*^{6-\gamma}}{\mathcal{N}^2(D_*)} \int_0^\infty dq_1 \, q_1^2 \, D_{L_* q_1^2}^{-3} B^{1/2}(D_{L_* q_1^2}) \times \int_0^\infty dq_2 \, q_2^2 \, D_{L_* q_2^2}^{-3} B^{1/2}(D_{L_* q_2^2}) q_{12}^{-\gamma} \quad (10)$$

(where $q_i = r_i/D_*$) and, when $B(D_L)$ is a power law, it becomes

$$w(\vartheta) = \frac{D_*^{6-\gamma}}{\mathcal{N}^2(D_*)} \left(\frac{b_*}{1-\beta/6} \right)^2 \int_0^\infty dq_1 \, q_1^2 \, D_{L_* q_1^2}^{\beta/2-3} \times \int_0^\infty dq_2 \, q_2^2 \, D_{L_* q_2^2}^{\beta/2-3} \, q_{12}^{-\gamma} \, . \quad (11)$$

Owing to the scaling $\mathcal{N} \propto D_*^3$, equations (10) and (11) already show that $w \propto D_*^{-\gamma}$, provided that no further dependence on D_* is conveyed by D_L. Note that no assumption about the L dependence of $B(D_L)$ is made in equation (10). Hence, the above scaling property is generic. It depends only on the universality of the function $B(D_L)$, not on its shape. Such scaling is true, in particular, when this function is constant, but this is not required in order that $w \propto D_*^{-\gamma}$.

Let us then perform the change of variables $q_1 + q_2 = 2q$, $r_2 - r_1 = u$, yielding

$$q_{12} \simeq [q^2 \vartheta^2 + (u/D_*)^2]^{1/2} ; \quad (12)$$

then equation (10) yields

$$w(\vartheta) = \tilde{A}_\gamma \left(\frac{\tilde{r}_0}{D_*}\right)^\gamma \vartheta^{1-\gamma} , \quad (13)$$

where

$$\tilde{A}_\gamma \tilde{r}_0^\gamma = 2c_\gamma L_*^{\gamma/2} \frac{\int_o^\infty dL\, L^{2-\gamma/2} D_L^{-6} B(D_L)}{(\int_o^\infty dL\, L^{1/2} D_L^{-3})^2} , \quad (14)$$

with

$$c_\gamma = \int_{-\infty}^{+\infty} dt\,(1 + t^2)^{-\gamma/2} = \frac{\Gamma(1/2)\Gamma((\gamma-1)/2)}{\Gamma(\gamma/2)} . \quad (15)$$

Note that Peebles (1980) denotes by A the whole product $\tilde{A}_\gamma(\tilde{r}_0/D_*)^\gamma$. Equation (14), obtained by turning the integration over q into an integration on $L = L_* q^2$, extends Peebles' result to the case of luminosity segregation. In the case $B(D_L) \equiv r_0^\gamma$ (L-independent r_0), we can set $\tilde{r}_0 \equiv r_0$, and equation (14) returns Peebles' expression for A.

On the other hand, if we forcibly interpret angular data as being due to a constant r_0, even though the physical correlation length is luminosity dependent, we obtain an *apparent* clustering length expression

$$r_0^\gamma = \frac{\int_0^\infty dL\, L^{2-\gamma/2} D_L^{-6} B(D_L)}{\int_0^\infty dL\, L^{2-\gamma/2} D_L^{-6}} . \quad (16)$$

In the case of a power law, with $\beta = \gamma$, equation (16) becomes

$$r_0^\gamma = \left(\frac{b_*}{1-\gamma/6}\right)^2 \frac{\int_0^\infty dL\, L^{2-\gamma/2} D_L^{-6+\gamma}}{\int_0^\infty dL\, L^{2-\gamma/2} D_L^{-6}} , \quad (17)$$

as was shown by Gardini et al. (1999).

The conclusions of this section can be summarized as follows:

1. The presence of a scaling

$$\mathcal{N} \propto D_*^3 , \quad w(\vartheta) \propto D_*^{-\gamma} \quad (18)$$

of number density and a two-point angular function is not sufficient to determine that a luminosity-independent correlation length exists.

2. If, however, data are interpreted as originating in a *constant* correlation length, the value given by equation (17) can be worked out. A visual inspection of equation (17) shows that the value of r_0 it provides, being a suitably weighted D_L value, will not differ much from the average separation of galaxies in the sample.

All of the above assumptions hold in the Euclidean limit and in the absence of evolution. Real sample data, however, are noisy, and departures from Euclidean geometry, as well as evolution, could hardly be detected in samples limited to redshifts $z \lesssim 0.2$–0.3.

3. RELATIVISTIC CORRECTIONS AND EVOLUTION

In this section we debate the effects of evolution and relativistic corrections. Evolutionary effects will be considered within the Press & Schechter (1974) approach; of course, this can be much improved, but this simple model already allows calculation of the impact of evolution on the clustering scale length. Relativistic corrections depend on the cosmological model. Here we consider two models only, both spatially flat. In the first (SCDM), the matter density parameter $\Omega_m = 1$. In the second (LCDM), $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. We need not make any assumption about H_0, Ω_b, etc.; however we take $a_0 = 1$, a being the cosmological scale factor, to simplify notation.

3.1. *General Properties and Number Densities*

The evolution of the luminosity function, in the Press & Schechter approach, depends on the shape of the mass (M) dependence of the rms fluctuation amplitude, $\sigma_0(M)$, on the assumed density contrast threshold, δ_c (~ 1.7 for spherical symmetry), and on the assumed M dependence of the M/L ratio. The chosen cosmological model determines the redshift dependence of the linear growth factor $\Delta \equiv \Delta(z, \Omega_m)$, and the luminosity function is therefore given by

$$\Phi(L, z) = \frac{\Phi_*}{L_*}\left(\frac{L}{L_*}\right)^{\alpha-2} \frac{1}{\Delta} \exp\left[-\left(\frac{L}{L_*}\right)^{2\alpha} \frac{1}{\Delta^2}\right] . \quad (19)$$

Here we assume that

$$\sigma_0(M) = (\tilde{M}/M)^\alpha , \quad M_* = (2/\delta_c)^{1/2\alpha} \tilde{M} ,$$

$$\Phi_* = \frac{2\alpha\rho_m}{\sqrt{\pi} M_*} ,$$

while M/L is M- and z-independent, and L_* is the luminosity of an object of mass M_*. For $\alpha = 0.5$, equation (19) becomes a usual Schechter function. From equation (19) it is easy to evaluate the comoving number density of objects with intrinsic luminosity greater than a given threshold $\bar{L}$:

$$n(>\bar{L}, z) \equiv \int_{\bar{L}}^\infty dL\, \Phi(L, Z)$$
$$= \frac{\Phi_*}{\Delta} \int_{\bar{L}/L_*}^\infty d\lambda\, \lambda^{\alpha-2} \exp\left(-\frac{\lambda^{2\alpha}}{\Delta^2}\right) . \quad (20)$$

Using the incomplete Gamma function

$$\Gamma(x, y) \equiv \int_y^\infty dt\, e^{-t} t^{x-1} ,$$

and defining

$$P(\bar{L}, \Delta) \equiv \left(\frac{\bar{L}}{L_*}\right)^{2\alpha} \frac{1}{\Delta^2} ,$$

equation (20) can be given the simpler form

$$n(>\bar{L}, z) = \frac{\Phi_*}{2\alpha} \Delta^{-1/\alpha} \Gamma(\alpha_1, P(\bar{L}, \Delta)) , \quad (21)$$

where $\alpha_1 = (\alpha - 1)/2\alpha$, which may be useful for computational purposes. As in the Euclidean limit, let us finally define a (z-dependent) mean galaxy separation

$$D_L(z) = n^{-1/3}(>L, z) . \quad (22)$$

Relativistic geometry, on the other hand, enters the problem when the apparent luminosity l of a source of intrinsic luminosity L is to be evaluated. Let z_r be the redshift of a source of line-of-sight comoving distance r; then

$$l = \frac{10^{-(2/5)k(z_r)}L}{[4\pi r^2(1+z_r)^2]}\,, \tag{23}$$

which is just the usual relation between apparent and absolute magnitude (see, e.g., Peebles 1993) expressed in terms of flux and luminosity. It may be useful to recall that, in a spatially flat model, the relation between redshift and comoving distance r can be obtained by solving the equation (see, e.g., Peebles 1993)

$$H_0 r = \int_0^z \frac{c\,dz'}{\sqrt{\Omega_m(1+z')^3 + (1-\Omega_m)}}\,,$$

while the k-correction is given by (see Oke & Sandage 1968)

$$k(z) = -2.5 \log \frac{\int_0^\infty d\lambda\, F_0(\lambda/(1+z))S(\lambda)}{(1+z)\int_0^\infty d\lambda\, F_0(\lambda)S(\lambda)}\,,$$

where $F_0(\lambda)$ is the spectral energy distribution of the source in its rest frame, and $S(\lambda)$ is the relevant filter response function.

According to equation (23), a source of line-of-sight comoving distance r will be in the sample if its luminosity $L > L_m(r)$, where

$$L_m(r) = L_*\left(\frac{r}{D_*}\frac{1+z_r}{1+z_*}\right)^2 10^{(2/5)[k(z_r)-k(z_*)]} \tag{24}$$

and $z_* = z(D_*)$ (note that the distance D_*, at which an object of luminosity L_* appears at the sample limiting luminosity, must now be obtained by taking into account the non-Euclidean relation between flux and luminosity, eq. [23]), and angular number density for such a sample will read

$$\mathscr{N}(D_*) = \int_0^\infty dr\, r^2 \int_{L_m(r)}^\infty dL\, \Phi(L, z_r)\,. \tag{25}$$

If we set

$$p(q) = \left[\frac{L_m(qD_*)}{L_*}\right]^{2\alpha} \frac{1}{\Delta^2(z_q, \Omega_m)}\,, \tag{26}$$

where $z_q = z(qD_*)$, the angular number density can be given the expression

$$\mathscr{N}(D_*) = \frac{D_*^3\Phi_*}{2\alpha}\int_0^\infty dq\, q^2\Delta^{-1/\alpha}\Gamma(\alpha_1, p(q))\,, \tag{27}$$

which, in the case of a SCDM model, for which the z-dependence of Δ is simple, further simplifies into

$$\mathscr{N}(D_*) = \frac{D_*^3\Phi_*}{2\alpha}\int_0^\infty dq\, \frac{q^2\Gamma(\alpha_1, p(q))}{(1-\varepsilon q)^{2/\alpha}} \tag{28}$$

where $\varepsilon = D_*/2r_H$ ($r_H = c/H_0$) and $1 + z_q = (1-\varepsilon q)^{-2}$, while equation (26) for $p(q)$ explicitly becomes

$$p(q) = \frac{q^{4\alpha}(1-\varepsilon)^{8\alpha}}{(1-\varepsilon q)^{8\alpha+4}10^{(2/5)[k(1/(1-\varepsilon q)2)-k(1/(1-\varepsilon)2)]}}\,. \tag{29}$$

3.2. *Luminosity-Independent Correlations*

In this subsection we formulate the scheme leading to angular functions, in the standard case of L-independent r_0, in a way suitable to performing an easy transition to the case of nonconstant r_0.

As seen in § 2, the angular two-point function for a magnitude-limited sample can be obtained by integrating the joint distribution in luminosity and position along the lines of sight and over the luminosities of the two points. Assuming, as in the previous section, that the joint distribution can be written as the product of the (z-dependent) luminosity functions and differential correlation function, we have

$$\begin{aligned} w(\vartheta) = \frac{1}{\mathscr{N}^2(D_*)}&\int_0^\infty dr_1\, r_1^2 \int_{L_m(r_1)}^\infty dL_1\,\Phi(L_1, z_1)\\ &\times \int_0^\infty dr_2\, r_2^2 \int_{L_m(r_2)}^\infty dL_2\,\Phi(L_2, z_2)\\ &\times \xi(L_1, L_2; z_1, z_2; r_{12})\,. \end{aligned} \tag{30}$$

Here $\xi(L_1, L_2; z_1, z_2; r_{12})$ plays the same role as $\xi(L_1, L_2, r_{12})$ in equation (5), except for the dependence on redshift, which needs to be enclosed. The above expression simplifies, however, if we assume, as is usually done, that correlations vanish over scales implying a nonnegligible redshift variation. If superclustering is to be taken into account, this assumption may have to be suitably improved. Here we also introduce the usual parametrization performed in order to analyze clustering evolution (Groth & Peebles 1977), by assuming an L-independent comoving scale r_0 and writing

$$\xi(L_1, L_2; z_1, z_2; r_{12}) = \left(\frac{r_0}{r_{12}}\right)^\gamma (1 + z_{\bar{r}_{12}})^{-(3+\epsilon-\gamma)}\,, \tag{31}$$

where $\bar{r}_{12} = (r_1 + r_2)/2$. In the absence of z-dependence, r_0 is the usual correlation length; note that it is assumed that the slope γ is z-independent, so that correlation evolution is fully parametrized by the exponent ϵ. For $\epsilon = 0$, clustering is fixed in proper coordinates; for $\epsilon = \gamma - 3$ we have fixed clustering in comoving coordinates, while $\epsilon = \gamma - 1$ corresponds to a linear growth of clustering.

By replacing equation (31) in equation (30) and using the definition of $D_L(z)$ (eq. [22]), we obtain

$$\begin{aligned} w(\vartheta) = \frac{r_0^\gamma}{\mathscr{N}^2(D_*)}&\int_0^\infty dr_1\, r_1^2\, D_{L(r_1)}^{-3}(z_1)\\ &\times \int_0^\infty dr_2\, r_2^2\, D_{L(r_1)}^{-3}(z_2) r_{12}^{-\gamma}(1 + z_{\bar{r}_{12}})^{-(3+\epsilon-\gamma)}\,. \end{aligned} \tag{32}$$

Changing integration variables as in the Euclidean case, so that $r_{12} = [q^2(\vartheta_{12}D_*)^2 + u^2]^{1/2}$, we find that

$$w(\vartheta) = A_\gamma(D_*)\left(\frac{r_0}{D_*}\right)^\gamma \vartheta^{1-\gamma}\,, \tag{33}$$

all complications being hidden in the expression of

$$A_\gamma(D_*) = c_\gamma \times \frac{\int_0^\infty dq\, q^{5-\gamma}(1+z_q)^{-(3+\epsilon-\gamma)}[\Delta^{-1/\alpha}\Gamma(\alpha_1, p(q))]^2}{[\int_0^\infty dq\, q^2\Delta^{-1/\alpha}\Gamma(\alpha_1, p(q))]^2}\,, \tag{34}$$

where c_γ and $p(q)$ are defined in equations (15) and (26), respectively. In contrast to the Euclidean case, the coefficient A_γ here depends on the sample depth D_*; such dependence becomes gradually stronger for deeper samples. Let us also note that equation (34) can be significantly simplified for SCDM and then reads

$$A_\gamma(D_*) = c_\gamma \frac{\int_0^\infty dq\, q^{5-\gamma}(1+z_q)^\zeta \Gamma^2(\alpha_1, p(q))}{[\int_0^\infty dq\, q^2(1+z_q)^{1/\alpha}\Gamma(\alpha_1, p(q))]^2}, \quad (35)$$

where $\zeta = 2/\alpha + \gamma - 3 - \epsilon$.

3.3. *Luminosity-Dependent Correlations*

As we did in the Euclidean case, we now consider the possibility that correlations depend on luminosity; let us then write the cumulative spatial correlation function as

$$\xi(r_{12}, > L, z) = \frac{B(D_L, z_{\bar r_{12}})}{r_{12}^\gamma}(1 + z_{\bar r_{12}})^{-(3+\tilde\epsilon-\gamma)}, \quad (36)$$

outlining also the dependence on z. Again, we assume that correlations vanish on scales far shorter than the sample's depth, D_*. Let us consider, in particular, the case in which

$$B(D_L, z) = [r_a + aD_L(z)]^\gamma$$

(here a is a proportionality constant), which we call "LP" (see below). If $r_a = 0$ and $a = 0.4$, LP reduces to the Bahcall & West (1992) conjecture (BW). As in the Euclidean case, the integral law (eq. [36]) can be derived by a differential expression

$$\xi(r_{12}, L_1, L_2, z_{\bar r_{12}}) = \frac{b(D_{L_1}, z_{\bar r_{12}})b(D_{L_2}, z_{\bar r_{12}})}{r_{12}^\gamma} \times (1 + z_{\bar r_{12}})^{-(3+\tilde\epsilon-\gamma)}, \quad (37)$$

where B and b are related by

$$B^{1/2}(D_L, z) = 3D_L^3(z)\int_{D_L(z)}^\infty dD\, \frac{b(D, z)}{D^4}. \quad (38)$$

By substituting equation (36) into equation (30) and using equation (38), we work out

$$w(\vartheta) = \frac{1}{\mathcal{N}^2}\int_0^\infty dr_1\, r_1^2 \frac{[B(D_{L(r_1)}, z_{r_1})]^{1/2}}{D_{L(r_1)}^3} \times \int_0^\infty dr_2\, r_2^2 \frac{[B(D_{L(r_2)}, z_{r_2})]^{1/2}}{D_{L(r_2)}^3} \times r_{12}^{-\gamma}(1 + z_{\bar r_{12}})^{-(3+\tilde\epsilon-\gamma)}. \quad (39)$$

Performing the same change of variables as in § 3.2, we can write the angular correlation function as

$$w(\vartheta) = \tilde A_\gamma(D_*)\left(\frac{\tilde r_0}{D_*}\right)^\gamma \vartheta^{1-\gamma}, \quad (40)$$

provided that we define

$$\tilde A_\gamma(D_*)\tilde r_0^\gamma = c_\gamma \frac{\int_0^\infty dq\, q^{5-\gamma}(1+z_q)^{-(3+\tilde\epsilon-\gamma)}B(D_L, z_q)D_L^{-6}(z_q)}{[\int_0^\infty dq\, q^2 D_L^{-3}(z_q)]^2}, \quad (41)$$

where c_γ is given by equation (15). Again, the coefficient $\tilde A_\gamma(D_*)\tilde r_0^\gamma$ depends on D_*, but in a different way from $A_\gamma(D_*)$ defined in equation (34). Therefore, in principle, by studying the scaling of the angular function with the sample's depth, it should be possible to determine the relationship between correlations and luminosity. In the case of a SCMD model with LP correlations, equation (41) becomes

$$\tilde A_\gamma(D_*)\tilde r_0^\gamma = c_\gamma \frac{\int_0^\infty dq\, q^{5-\gamma}(1+z_q)^{-(3+\tilde\epsilon-\gamma)}[r_a + aD_L(z_q)]^\gamma D_L^{-6}}{(\int_0^\infty dq\, q^2 D_L^{-3})^2}. \quad (42)$$

The apparent correlation length found by interpreting a luminosity-dependent correlation as being due to a luminosity-independent r_0 is given by

$$r_0^\gamma = \frac{\int_0^\infty dq\, q^{5-\gamma}(1+z_q)^{-(3+\tilde\epsilon-\gamma)}B(D_L, z_q)D_L^{-6}}{\int_0^\infty dq\, q^{5-\gamma}(1+z_q)^{-(3+\epsilon-\gamma)}D_L^{-6}}. \quad (43)$$

For luminosity-independent r_0, a possible redshift dependence of clustering can be interpreted as yielding $r_0^\gamma \propto (1+z)^{\gamma-(3+\epsilon)}$. Equation (43) makes it clear that, in general, the *apparent* r_0 has a different z dependence. On the other hand, if data are forcibly interpreted to work out an r_0 value through equation (43), this has misleading effects on the study of clustering evolution. In particular, the evolutionary exponent ϵ worked out if data are treated in such a way does not yield the rate of evolution of the actual clustering.

4. LUMINOSITY SEGREGATION IN THREE-DIMENSIONAL SAMPLES

According to Peebles (1980), the scaling relation given in equation (18) played an important role in testing that the angular correlations of galaxies in the catalogs do reflect the presence of a uniform spatial galaxy clustering, rather than something else, e.g., systematic errors due to patchy obscuration in the Milky Way. Hauser & Peebles (1973), in their seminal work on cluster clustering, made use of the scaling relation given in equation (18) as a test for the cluster correlation length, $r_c \sim 30\ h^{-1}$ Mpc, they obtained for Abell clusters. The results found here are not in contradiction with such statements, although different spatial clustering laws, with L-dependent clustering lengths, can give rise to the same scaling properties. The actual luminosity dependence of the clustering length can then be found by either using three-dimensional samples or, if we keep to two-dimensional samples, studying the deviations from the Euclidean behavior in deep catalogs.

The former task can be rather easily achieved using existing data on galaxy and cluster two-point functions. In Figure 1 we show a compilation of several analyses of three-dimensional samples. For galaxies, the Perseus-Pisces Survey (PPS), Southern Sky Redshift Survey 2 (SSRS2), and ESO Slice Project (ESP) surveys were considered (see, Giovanelli & Haynes 1991; da Costa et al. 1994; Willmer et al. 1998; Guzzo et al. 2000, and references therein). Luminosity segregation has already been studied in these samples, finding the dependence of the clustering length on the minimum intrinsic luminosity L_m of fixed subsamples. From L_m and sample data, we can easily turn this into a dependence on the mean separation D_L. In this way we obtain r_0 versus D_L, which we plot in Figure 1.

Plotting r_0 versus D_L is usual for cluster data, and helps to overcome different definitions used for different samples. In Figure 1, r_0 estimates for Abell clusters (Abell 1958; Abell, Corwin, & Olowin 1989) of various richness and for clusters worked out from the APM survey (Maddox et al. 1990a, 1990b) are plotted. The latter values were obtained by Dalton et al. (1992) and Croft et al. (1997), but were later

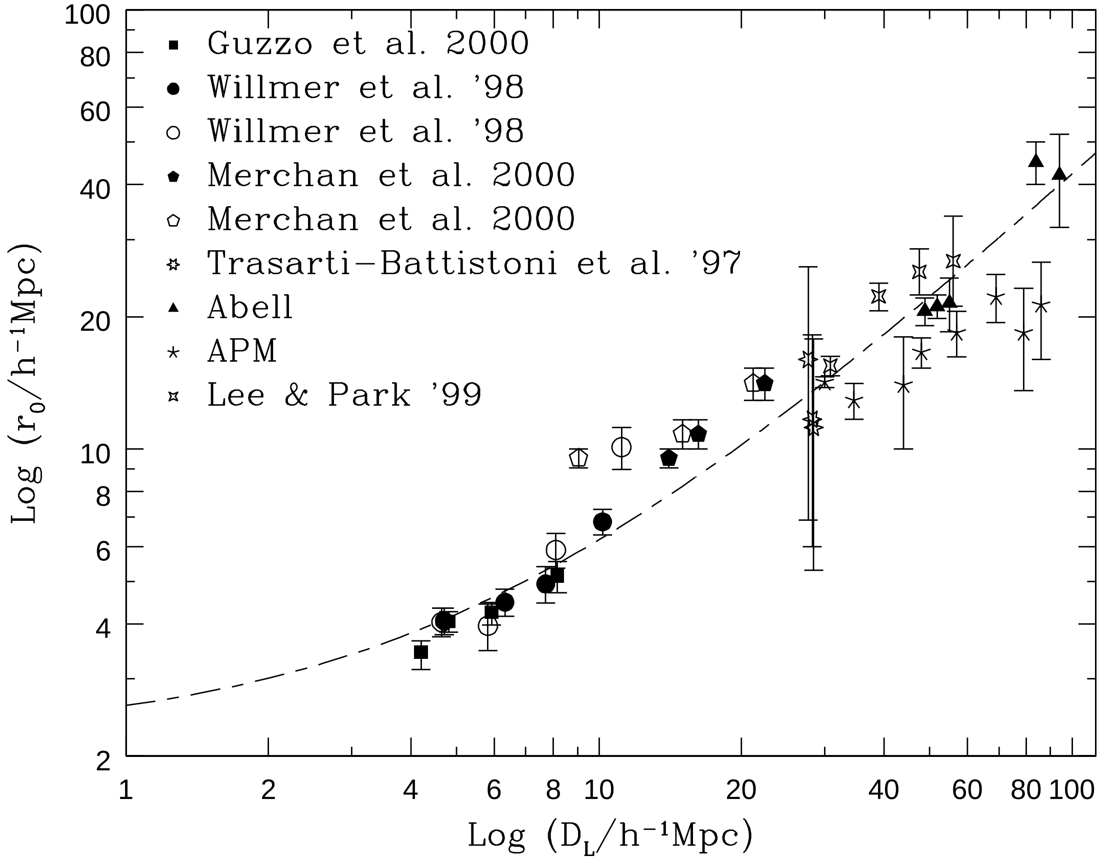

FIG. 1.—Correlation length, r_0, as a function of the mean separation, D_L. See the original works for details. In particular, the Abell data are from Peacock & West (1992) and Postman, Huchra, & Geller (1992), while the APM points are from Dalton et al. (1992), Nichol et al. (1992), and Croft et al. (1997). The dashed line represents the LP expression $r_0 = 0.4D_L + 2.2$; this is not a best fit, but is plotted here to give a qualitative example of the impact of luminosity segregation on galaxy scale.

criticized by Lee & Park (1999), who, from the same observational material, obtained significantly greater values of r_0. In Figure 1 we report the Dalton et al. (1992), Nichol et al. (1992), and Croft et al. (1997) results, as well as the Lee & Park (1999) results.

For the sake of completeness, we also report r_0 estimates for loose groups (Trasarti-Battistoni, Invernizzi, & Bonometto 1997; Merchán, Maia, & Lambas 2000). Also for completeness sake, we point out a study of luminosity segregation in the galaxies of the Las Campanas Redshift Survey (LCRS; see Shectman et al. 1996) by Valotto & Lambas (1997). Although not directly comparable with the data we report here, their results suggest an increase of clustering strength with source luminosity.

Szalay & Schramm (1985) proposed a single law,

$$r_0 = aD_L , \tag{44}$$

to fit both cluster and galaxy data. When more detailed cluster clustering data became available, Bahcall & West (1992) used such an expression for clusters only.

After reevaluating the clustering lengths for APM cluster subsamples, Lee & Park (1999) suggested correcting the BW conjecture by adding a constant term

$$r_0 = aD_L + r_a ; \tag{45}$$

this certainly improves the fit with the data they had been using. In Figure 1 we plot an LP expression with $a = 0.4$ and $r_a = 2.2\ h^{-1}$ Mpc, which meets both galaxy and cluster error bars in a reasonable way. We wish to stress that this is not a best fit, which would hardly make sense in the presence of data obtained in such nonuniform ways. Rather, what we want to stress is that the luminosity segregation for galaxies, although not so strong as for clusters, is still significant and may need to be taken into account when angular samples are used. As a matter of fact, the LP law seems to be close to the data, except for the SSRS2 point at $11.2\ h^{-1}$ Mpc, which would indicate an even stronger L dependency.

We make use of the LP conjecture as an alternative hypothesis, in respect to L-independent r_0, to test the deviations from the scaling law (eq. [18]).

5. COMPARISON WITH DATA

When deep data samples are considered, substantial deviations from the scaling law (eq. [18]) may be expected. For the sake of example, in this section we consider recent observational results in the B and R-bands (Brunner, Szalay, & Connolly 2000 and Villumsen, Freudling, & da Costa 1997, respectively; see also references therein) in light of the results of § 3.

In both bands, we compare data on the amplitude of the angular two-point function, $A_w = w(\vartheta)/\vartheta^{1-\gamma}$, with theoretical predictions obtained assuming that either (1) $r_0 = 4\ h^{-1}$ Mpc is luminosity independent, or (2) r_0 is given by the LP expression discussed in § 4. In both cases, we take $\gamma = 1.7$ and $\alpha = 0.5$; k-corrections are calculated through analytical expressions that provide a good fit of Coleman, Wu, & Weedman (1980) data and also maintain a regular behavior at redshifts greater than those spanned by the data; finally, we adopt the nonevolving morphological type mix given in Brunner, Connolly, & Szalay (1999).

Our main aim here amounts to exploring the impact of luminosity segregation on the scaling of A_w, and more sophisticated assumptions would not affect our conclusions. If full quantitative conclusions regarding clustering evolution are sought, a more advanced discussion on the evolution of the morphological mix is certainly required. However, the main uncertainty may come from k-corrections. Here data extend up to redshift $z \simeq 1.5$ and $z \simeq 2$ in the B and R bands, respectively. Clustered galaxies, instead, have redshifts up to $z \simeq 1.2$ and $z \simeq 3$ in the two bands, respectively. Hence, while k-corrections are reasonably safe in the B band, they involve nonnegligible extrapolations in R.

In Figure 2 the amplitude of the angular function versus limiting B magnitude is shown; data obtained by Brunner et al. (2000) are given, together with previous results, in both panels. The superimposed curves in Figures 2*a* and 2*b* are obtained for SCDM and LCDM, respectively. They show the expected behavior of $w(\vartheta)$ for three different ϵ values, corresponding to three different clustering evolution rates. For each ϵ value we have two curves: the upper one refers to case 1, while the lower one relates to case 2. As a general feature, we note that cases 1 and 2 also have different slopes; present data, however, are certainly insufficient to distinguish between slopes. Remember that any difference between the two sets of curves disappears in the absence of deviations from the Euclidean geometry and/or evolution of galaxy numbers.

Figure 2*a* shows that the curve taking luminosity segregation into account is in good agreement with data for SCDM, for all ϵ values, while L-independent correlations tend to favor a linear growth of clustering ($\epsilon = 0.7$). In principle, agreement could be recovered taking either $r_0 \ll 4$ h^{-1} Mpc or $\gamma < 1.62$; however, r_0-γ pairs compatible with $z = 0$ clustering data do not permit such a fit. In an LCDM model, L-independent correlations are in a better agreement with data; here too, higher ϵ values provide the best fits. From Figure 2*b* it is also evident that in an LCDM model, the LP expression lies below the data; however, increasing γ to 1.75 presents a picture similar to the SCDM case.

In Figure 3, we plot data taken from Villumsen et al. (1997) for the R band and curves representing the expected behavior of the angular amplitude. Data can be divided in three sets: (1) for R magnitudes up to $\sim$23, we have ground-based data from various authors; (2) for R magnitudes between $\sim$23 and $\sim$25.5 there are four estimates obtained by Brainerd, Smail, & Mould (1995), which seem to indicate a rapid decrease of A_w; and (3) above R magnitude $\simeq$26 there are data obtained from the Hubble Deep Field–North (HDF-N; Clements & Couch 1996).

If luminosity segregation is not taken into account, all error bars lie well below SCDM estimates, even for the most favorable case of $\epsilon = 0.7$ (Fig. 3, *upper dashed line*), which means a linear increase of clustering. The disagreement is dramatic for data set 2, where estimates exceed data by more than 10 σ, but disagreements also exceed 3 σ for some points of the HDF-N. In an LCDM model, on the other hand, even when luminosity segregation is not considered, the disagreement between data and estimates is significantly reduced. Once again, the best estimates are obtained for $\epsilon = 0.7$, and the main disagreement concerns data set 2. Although statistical estimators can hardly be used in such cases, when error bars from different authors are compared, the feeling one has, looking at the upper curves of Figure 3*b*,

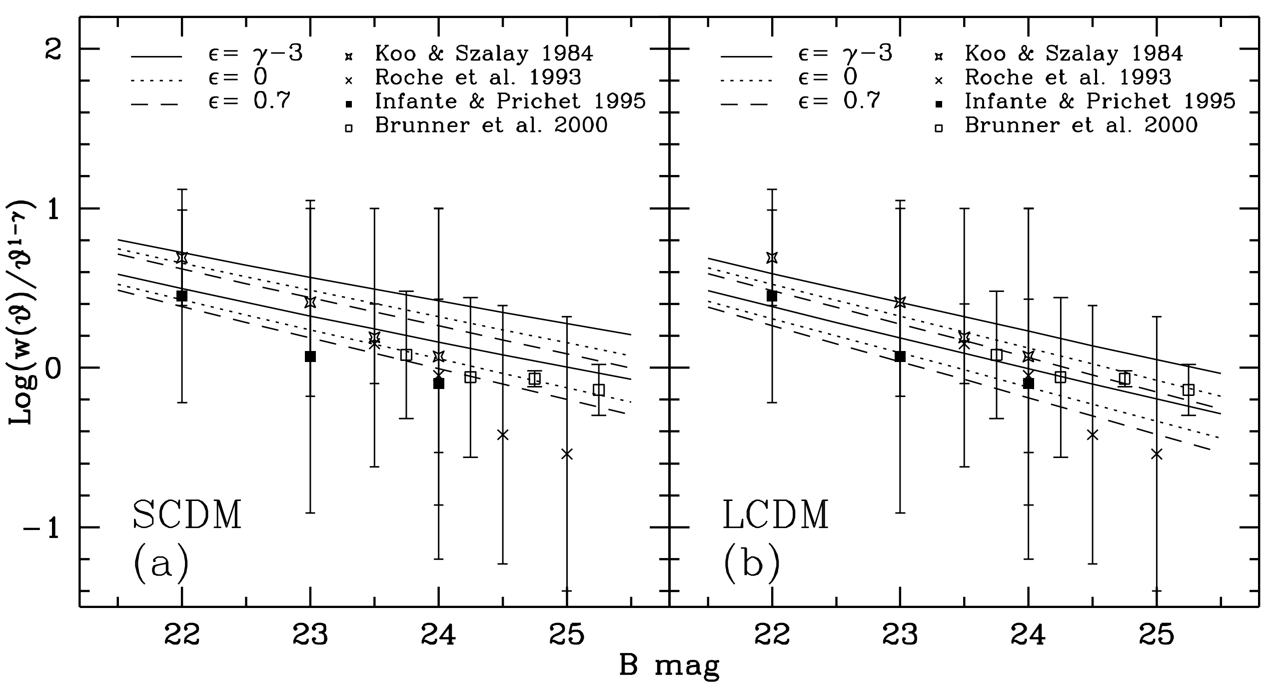

FIG. 2.—Logarithmic amplitude of the angular two-point function, $A_w \equiv w(\vartheta)/\vartheta^{1-\gamma}$, as a function of the limiting B magnitude. The superimposed curves show the predicted behavior for A_w for different values of the evolutionary parameter ϵ and different regimes of luminosity segregation, assuming either (*a*) a SCDM ($\Omega_m = 1.0$, $\Omega_\Lambda = 0$) or (*b*) LCDM ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$) cosmological model. In particular, for a given ϵ value, the upper curve corresponds to L-independent correlations, while the lower curve assumes the LP expression $r_0 = 0.4D_L + 2.2$.

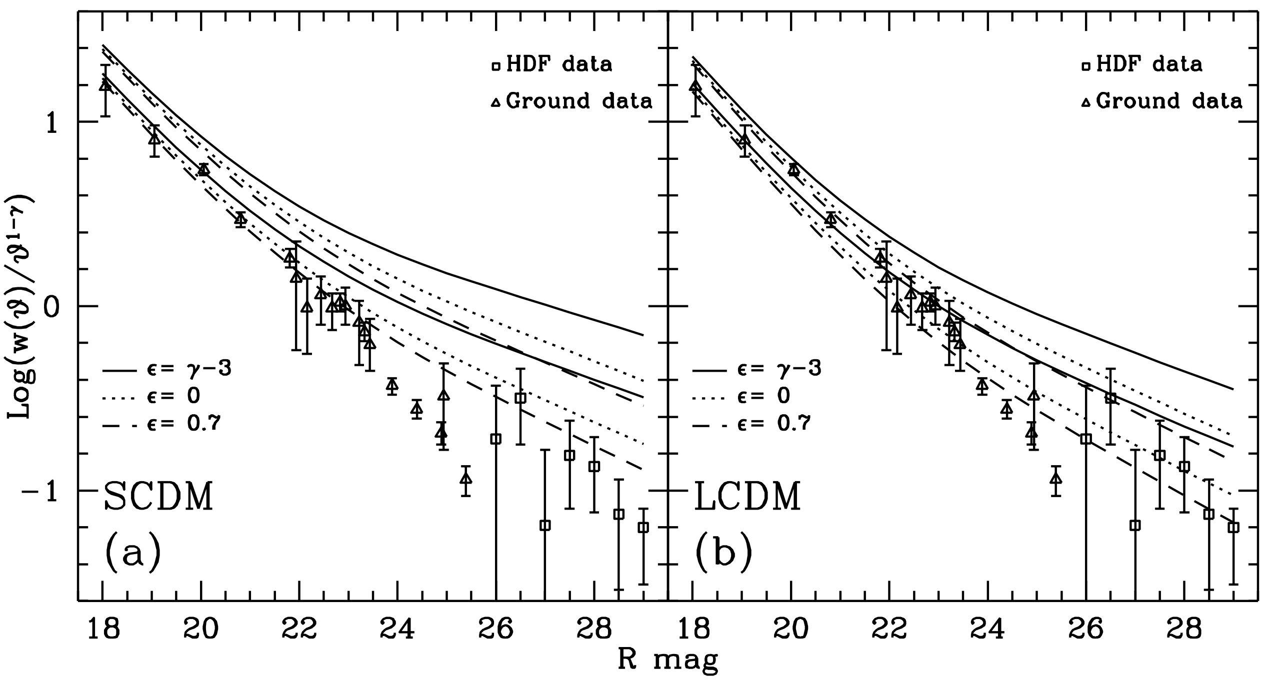

FIG. 3.—Same as Fig. 2, but for R-band data. See Villumsen et al. (1997) and references therein for details on observations and data reduction. The disagreement between predictions and data is mainly due the uncertainties on k-corrections, which become significative for deep samples, such as the one considered here. However, we applied the same k-correction model to both luminosity-independent and luminosity-dependent correlation estimates; therefore, the relative separation between the two sets of curves is largely unaffected and shows the effects of luminosity segregation on a galaxy scale.

is that the agreement is good only with the data set 1; not only the data from set 2, but also HDF-N data systematically lie below estimates, even for $\epsilon = 0.7$.

The situation is different when luminosity segregation is taken into account. For SCDM, the behavior of estimates is just slightly worse than what we have for LCDM, if luminosity segregation is neglected. The best theoretical estimates are instead obtained for LCDM, taking into account luminosity segregation. Once again, $\epsilon = 0.7$ seems favored. For the data set 1, estimates miss two points with very narrow error bars. The four points of data set 2, on the other hand, are approached at the 2–3 σ level (except one), and the HDF-N error bars are satisfactorily met. Altogether, it may be legitimate to conclude that, taking into account luminosity segregation, LCDM meets data for $0 \lesssim \epsilon \lesssim 0.7$.

Values $\epsilon > 0.7$, bearing no physical interpretation, were not addressed in the above analysis. This restriction has no consequence in the case of luminosity segregation, where a behavior approaching data is obtainable with $\epsilon < 0.7$. With no luminosity segregation, instead, such a behavior would be obtainable only pushing ϵ to really unphysical values $\sim$2–3.

Recall, however, that all the above analyses are still partially preliminary. What they surely show is the impact of luminosity segregation on A_w estimates. However, various points should still be deepened to gain full reliability. For instance, a more precise evaluation of the effects of our approximations on k-correction for high-z objects should be performed. A further improvement can be obtained if photometric redshifts are available. They would provide us with data on the dependence of the galaxy number density on z, which could be compared with the Press & Schechter estimates used here.

In a previous analysis of the evolution of the angular function, performed also using photometric redshifts, Connolly, Szalay, & Brunner (1998) found that in order to keep r_0 and γ at around their canonical values, for a SCDM model, $\epsilon \simeq 2.1 \pm 0.5$ is needed. They also found that, in order to avoid such unphysical ϵ, r_0 had to be substantially lowered. Their best-fit value is $r_0 \simeq 2.4$. A direct comparison between such results and our own is perhaps premature. Taking luminosity segregation into account has different effects than drastically lowering r_0. It seems, however, that their results should be reconsidered, taking luminosity segregation into account. This might allow for a reconciliation between our predictions and the observations, at least within the frame of an LCDM cosmology.

6. CONCLUSIONS

In this work we have shown the effects of luminosity segregation on the scaling properties of the two-point angular function. In the Euclidean limit and in the absence of source evolution, fairly unexpectedly, no effect arises. On the contrary, the scaling expected for the angular functions with or without luminosity segregation differ significantly when samples extend to magnitudes $\gtrsim$24–25. In such cases, samples contain galaxies with $z \gtrsim 0.5$, where geometrical effects, and possibly evolution, are not negligible. Accordingly, quantitative conclusions also depend on the cosmological model.

However, quite independently of the model, it is clear that taking luminosity segregation into account causes significant shifts in the expected angular amplitude; these are comparable to the shifts due to variations of the parameter ϵ, which sets the rate of clustering evolution. Therefore, neglecting luminosity segregation surely yields misleading estimates of ϵ.

In this paper we have also tentatively analyzed recent clustering data in the B and R bands. Within the limits allowed by public materials, we can safely make a few points.

In the B band, for both SCDM and LCDM, recent data seem incompatible with stable clustering (in comoving coordinates), if an L-independent r_0 is assumed. For SCDM, even more stringent conclusions can be drawn, and an L-independent r_0 seems incompatible with data for any reasonable clustering evolution law.

In the R band, further difficulties arise from the scarcity of data on k-corrections for galaxies beyond $z \sim 1$–1.5. This problem arises, however, only because deeper data are available in the R band, where, because of the ease of obtaining deep R observations, there are considerably more clustering data. Hence, far from worsening, clustering data here are richer. Our analysis of Villumsen et al. (1997) results, in particular, strengthens the above conclusions from the B band. That standard values of r_0 and γ were hardly compatible with HDF-N data had already been outlined by Connolly et al. (1998). After exploring various hypotheses, they essentially concluded that the very parametrization of clustering was unsuitable and had to be modified. Although this may be true, here we note that, once luminosity segregation is taken into account, values of $\epsilon \sim 0.7$ (linear increase of the clustering rate) may become consistent with data for LCDM. On the other hand, SCDM is only in moderate disagreement with HDF-N data, but disagrees substantially with previous results of ground-based observations (Brainerd et al. 1995).

Let us finally remark that all our fits tend to show a better agreement between theory and data for models with a cosmological constant. This adds to other, more stringent, evidences in favor of this class of cosmological models.

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