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WHAT CAN THE KINEMATIC COMPLEXITY OF ASTROPHYSICAL SHEAR FLOWS LEAD TO?

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ABSTRACT

We develop a method for investigating waves and instabilities in astrophysical shear flows with complex kinematics. Using new tools, we find an unexpected richness in the spectrum of compressible fluctuations sustained by such a flow. The principal characteristic of the revealed exotic phenomena is their *asymptotic persistence* in the absence of viscosity. "Echoing" as well as strongly unstable (including parametrically driven) solutions are identified. Examples of astrophysical shear flows with nontrivial velocity structure where these method and results can be applied are also discussed.

Subject headings: hydrodynamics — instabilities — waves

1. INTRODUCTION

Astrophysical shear flows are attractive objects for scientific investigation both for their physical content and for their astrophysical significance. A shear flow is an interesting example of a system in which the linear dynamics is governed by a "nonnormal" (non–self-adjoint) set of equations (Trefethen et al. 1993). This nonnormality has motivated several recent studies of the initial-value problem in different kinds of parallel shear flows. A host of phenomena, overlooked (in fact, inaccessible) in the standard normal mode approach, have been identified, including (1) the ability of the shear-modified waves to extract energy from the mean shear flow (Chagelishvili, Rogava, & Segal 1994); (2) the existence of shear-induced coupling and mutual transformations of waves in flows sustaining $n > 1$ modes of wave motion (Chagelishvili, Rogava, & Tsiklauri 1996); (3) the excitation of shear-driven beat waves (Rogava & Mahajan 1997); (4) the appearance of nonperiodic, algebraically unstable modes of collective behavior—"shear vortices" (Rogava, Chagelishvili, & Mahajan 1998); and (5) the occurrence of abrupt vortex-wave conversions in flows with moderate and high shear rates (Chagelishvili et al. 1997).

In astrophysics, the importance of local, shear-induced phenomena has been recognized for a long time. Some obvious examples are the investigation of galactic density waves (Toomre 1969; Goldreich & Tremaine 1978; Fan & Lou 1997) using the Goldreich & Lynden-Bell (1965) model of a shearing gas disk, and the study of nonaxisymmetric, transiently growing, incompressible modes in nonmagnetized accretion disks (Lominadze, Chagelishvili, & Chanishvili 1988) and the strongly unstable nonaxisymmetric modes in weakly magnetized (high-β) accretion disks (Balbus & Hawley 1992), and of similar unstable spiral modes in strongly magnetized low-β disks (Tagger, Pellat, & Coroniti 1992). It has been argued that the local shear instability (Velikhov 1959; Chandrasekhar 1960) identified recently for weakly magnetized disks (Balbus & Hawley 1991; Hawley & Balbus 1991) is one of the most powerful processes feeding off the differential shear. More recently, it has been further argued that shear-induced wave transformations may play an important role in the generation of pulsar radio emission (Mahajan, Machabeli, & Rogava 1997), the generation of solar magnetohydrodynamic (MHD) waves, and the acceleration of the solar wind (Poedts, Rogava, & Mahajan 1998).

One common feature of the existing astrophysical applications is that they assume rather simple, locally plane-parallel (or reducible to plane-parallel) forms of the background shear flow with *linear* velocity profiles. Fortunately, this approximation works for plane-parallel flows with *nonlinear* velocity profiles as well, because locally, on length scales that are small compared to the outward dimensions of the flow, any piecewise linear or smoothly curvilinear profile can be considered as approximately linear, i.e., with a constant shear rate. The latter assumption is crucial for the validity of the often-used Kelvin method of changing variables (Kelvin 1887).

However, a majority of the astrophysical shear flows are non–plane-parallel, and some are not parallel at all. The flows in the solar atmosphere (Parker 1979), for instance, fall into this category. In the photosphere, complicated three-dimensional convective motions are superimposed on the background effect of the solar differential rotation. The slender magnetic filaments rooted in the convective zone, where their footpoints are continually massaged and squeezed by the convective granules (Parker 1974), exhibit a surging of the fluid up and down along their length. There is ample observational evidence for complicated oscillatory, transient, or sustained motions in the corona (especially within plasma loops), and in the chromosphere and the transition region, where the loops are rooted (Pneuman & Orrall 1986). Spicules, the notable patterns of chromospheric fine structure, beginning in the chromosphere and threading through the transition region into the low corona, exhibit an upward mass flux, measured by Doppler shifts of spectral lines. These motions are matched in a geometrically nontrivial fashion with the motions within horizontally lying fibrils and with the network downflow in the transition region (Athay 1986). The presence of these complicated patterns of plasma motion, coupled with magnetic fields, should heavily influence the generation and propagation of MHD waves in the solar atmosphere, processes that could contribute to the coronal heating and the acceleration of the solar wind.

Another class of astrophysical shear flows with complex kinematics are the galactic and extragalactic jets (Lovelace, Wang, & Sulkanen 1987; Hughes 1991; Burgarella, Livio, & O'Dea 1993; Shibata 1996). The complex three-dimensional topology of ordered magnetic fields, responsible for the collimation of the jets and/or the acceleration of matter up to relativistic velocities (Lovelace, Berk, & Contopoulos 1991), ensures the complex kinematic inhomogeneity of the background mean flow within the jets. This complexity, again, should strongly affect the observational appearance of these objects.

The ubiquity of astrophysical shear flows with complex (multidimensional) kinematics demands that appropriate nonasymptotic methods be devised to examine the responses of such flows to perturbations. Here we propose a simple method that reduces the initial-value problem to a set of manageable ordinary differential equations in time. A similar method, designed for the *incompressible* flows with spatially uniform shearing rates, has been used in hydrodynamics (Lagnado, Phan-Thien, & Leal 1984; Craik & Criminale 1986). We will see that even a *slightly* complex velocity inhomogeneity imparts an immense richness to the temporal behavior of the perturbations.

In the next section, we consider the evolution of fluctuations in a relatively simple, two-dimensional hydrodynamic system. The motivation is to expose the salient features of the new physics without being swamped by algebraic complications. In the concluding part of the article we will chart out the scope of this calculation, and discuss those astrophysical situations for which our results may be relevant. It is hoped that our methods and results will shed some light on the observational puzzles associated with such systems.

2. MAIN CONSIDERATION

Our analysis deals with only small-scale perturbations, those with characteristic length scales l_i ($i = x, y, z$) much smaller than the characteristic scales L_i of the basic shear flow. For $l_i \ll L_i$, the spatial variation of a general mean velocity field $\boldsymbol{U}(x, y, z)$ could be approximated, in the close neighborhood of a point $A(x_0, y_0, z_0)$ ($|x - x_0|/|x_0| \ll 1$, etc.), by the linear terms in its Taylor expansion. The set of nine constants $U_{i,k}(x_0, y_0, z_0)$ ($i, k = x, y, z$) forming the shear matrix $\mathscr{S}$,

$$\mathscr{S} \equiv \begin{pmatrix} U_{x,x} & U_{x,y} & U_{x,z} \\ U_{y,x} & U_{y,y} & U_{y,z} \\ U_{z,x} & U_{z,y} & U_{z,z} \end{pmatrix} \equiv \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \tag{1}$$

will then fully characterize the flow in the region. For flows with *homogeneous* equilibrium density, the velocity field is divergence-free—$\nabla \cdot \boldsymbol{U} = 0$—which translates into the constraint that the shear matrix is traceless. The components of the mean velocity can now be expressed as $U_i(x, y, z) \simeq U_{0i} + a_{ik} x_k$, where $U_{0i} = U_i(x_0, y_0, z_0)$ and $x_i \equiv x_i - x_{0i}$. This will convert the linearized convective derivative operating on the perturbations to $\mathscr{D} u_i + a_{ik} u_k$, where $\mathscr{D}$ stands for the spatially inhomogeneous operator $\mathscr{D} \equiv \partial_t + U_i(x, y, z)\partial_i$.

It should be remembered that not *all* $\boldsymbol{U}(x, y, z)$'s are possible; the relevant flow must satisfy the stationary state equations for a given physical system. In this paper we do not restrict the range of possible mean velocity fields, but simply concentrate on developing an approach for studying the initial value problem valid for arbitrary ambient shear flows.

Our first task is to find a transformation that will annihilate the spatial dependence in the operator $\mathscr{D}$. For any fluctuation $F(x, y, z; t)$, the *Ansatz*

$$F(x, y, z; t) \equiv \hat{F}[\boldsymbol{k}(t), t] \exp(i\varphi), \tag{2a}$$

$$\varphi(t) \equiv k_i(t)x_i - U_{0i} \int_0^t k_i(t')\, dt', \tag{2b}$$

seems to do the job, provided that the wavevector $\boldsymbol{k}(t)$ acquires the time dependence given by

$$\partial_t \boldsymbol{k} + \mathscr{S}^T \cdot \boldsymbol{k} = 0, \tag{3}$$

where $\mathscr{S}^T$ is the transposed shear matrix (Craik & Criminale 1986). These equations, even in their most general form, have analytic solutions that embrace all possible kinds of background flows. For example, the plane Couette flow, corresponding to $a_{12} = A$ and all other $a_{ik} = 0$, leads to the obvious solutions $k_x(t) = k_x(0)$, $k_z(t) = k_z(0)$, and $k_y(t) = k_y(0) - Atk_x(0)$. For shear flows with more sophisticated kinematics, the temporal dependence of $\boldsymbol{k}(t)$ would be more complex, encompassing both exponentially evolving and periodic behavior.

The convective derivative now becomes an ordinary derivative in time: $\mathscr{D}F = \exp(i\varphi) d_t \hat{F}$. The resulting nonautonomous ordinary differential equations (ODEs) can be analyzed to "smoke out" overlooked modes of collective behavior excited by the nontrivial velocity fields.

The scope of this method is best demonstrated by examining the *compressible* fluctuation dynamics of a two-dimensional hydrodynamic system with a homogeneous mean density. For the ambient flow velocity $\boldsymbol{U}(x, y) \equiv U_x(x, y)\boldsymbol{e}_x + U_y(x, y)\boldsymbol{e}_y$, with $a_{11} = -a_{22} \equiv \sigma$, $a_{12} \equiv a$, and $a_{21} \equiv b$, we can readily derive the set of linearized equations

$$\mathscr{D}\rho' + \rho_0(\partial_x u_x + \partial_y u_y) = 0, \tag{4a}$$

$$\mathscr{D}u_x + \sigma u_x + a u_y = -c_s^2\, \partial_x(\rho'/\rho_0), \tag{4b}$$

and

$$\mathscr{D}u_y + b u_x - \sigma u_y = -c_s^2\, \partial_y(\rho'/\rho_0), \tag{4c}$$

governing the evolution of small-scale perturbations in this flow. Applying the *Ansatz* (eq. [2]), we convert the system to the set of first-order ODEs:

$$d_\tau \varrho = \mathscr{K}_x v_x + \mathscr{K}_y v_y, \tag{5a}$$

$$d_\tau v_x + \varepsilon v_x + R_1 v_y = -\mathscr{K}_x \varrho, \tag{5b}$$

$$d_\tau v_y + R_2 v_x - \varepsilon v_y = -\mathscr{K}_y \varrho, \tag{5c}$$

with the components of the variable wave vector obeying

$$d_\tau \mathscr{K}_x + \varepsilon \mathscr{K}_x + R_2 \mathscr{K}_y = 0, \tag{6a}$$

$$d_\tau \mathscr{K}_y + R_1 \mathscr{K}_x - \varepsilon \mathscr{K}_y = 0, \tag{6b}$$

where we have used the dimensionless notation $\tau \equiv c_s k_x(0)t$, $\varepsilon \equiv \sigma/c_s k_x(0)$, $R_1 \equiv a/c_s k_x(0)$, $R_2 \equiv b/c_s k_x(0)$, $\mathscr{K}_{x,y} \equiv k_{x,y}/k_x(0)$, $\varrho \equiv i(\rho'/\rho_0)$, and $v_{x,y} \equiv \hat{u}_{x,y}/c_s$.

Equations (5) and (6) sustain *two* conserved quantities: $\Delta \equiv \mathscr{K}_y d_\tau \mathscr{K}_x - \mathscr{K}_x d_\tau \mathscr{K}_y$, relating the components of $\mathscr{K}$, and $\mathscr{C} \equiv \mathscr{K}_y v_x - \mathscr{K}_x v_y + (R_1 - R_2)\varrho$, which links the physical variables of the system with one another.

Defining $\mathscr{K}^2 \equiv \mathscr{K}_x^2 + \mathscr{K}_y^2$, $\varrho \equiv \mathscr{K}\Psi$, and realizing that Δ and $\mathscr{C}$ are constants, we can derive from equations (5) and (6) the following explicit second-order ODE (where $\Lambda^2 \equiv R_1 R_2 + \varepsilon^2$):

$$d_\tau^2 \Psi + \left(\mathscr{K}^2 - \Lambda^2 + \frac{3\Delta^2}{\mathscr{K}^4}\right)\Psi = \frac{2\mathscr{C}\Delta}{\mathscr{K}^3}\,, \tag{7}$$

which determines the time behavior of the perturbations.

The constant of integration, $\mathscr{C}$, appearing in the inhomogeneous term, can be interpreted as the source of vortical fluctuations (Chagelishvili, Rogava, & Segal 1994). The entire time dependence of both coefficients in equation (7) comes through the time dependence of $\mathscr{K}$. There are three distinct classes of solutions:

1. $\Lambda^2 = 0$: In this simple case, $\mathscr{K}^2 = \mathscr{K}^2(0) + d_\tau \mathscr{K}^2(0)\tau + (R_1 - R_2)\Delta\tau^2$. This form of $\mathscr{K}$ pertains to the extensively studied case of the plane Couette flow.

2. $\Lambda^2 > 0$: Here $\mathscr{K}^2 = \delta + \mathscr{A}\cosh(2\Lambda\tau + \psi_0)$, where $\delta \equiv -(R_1 - R_2)\Delta/2\Lambda^2$, and $\mathscr{A}$ and ψ_0 are determined by initial values of the wavenumbers. This case allows for a simple asymptotic analysis. For $\Lambda\tau \to \infty$, $\mathscr{K}^2 \approx ae^{2\Lambda\tau}$, and equation (7) can be approximated by a Bessel equation,

$$d_\tau^2 \Psi + (ae^{2\Lambda\tau} - \Lambda^2)\Psi = 0\,, \tag{8}$$

with the solution

$$\Psi = J_1\left(\frac{a^{1/2}}{\Lambda}\, e^{\Lambda\tau}\right) \to \left(\frac{2\Lambda}{\pi a^{1/2}}\right)^{1/2} e^{-\Lambda\tau/2} \cos\left(\frac{a^{1/2}}{\Lambda}\, e^{\Lambda\tau} - \frac{3\pi}{4}\right),$$

leading to an exponential growth for the physical density perturbation:

$$\varrho = \mathscr{K}\Psi \approx e^{\Lambda\tau/2} \cos\left(\frac{a^{1/2}}{\Lambda}\, e^{\Lambda\tau} - \frac{3\pi}{4}\right). \tag{9}$$

The numerical solution displayed in Figure 1, in which we have plotted ϱ as a function of time, clearly confirms the predictions of equation (9). It must be stressed, however, that for this class of flows ($\mathscr{K}^2 \to e^{2\lambda\tau}$), the viscous damping will tend to kick in in due time and will eventually damp the mode. This is indeed found to be the case when viscosity is incorporated into the original setup.

3. $-\omega^2 \equiv \Lambda^2 < 0$: The solution for $\mathscr{K}^2$ is now periodic: $\mathscr{K}^2 = \delta + \mathscr{B}\cos(2\omega\tau + \phi_0)$, with $\mathscr{B}$ and ϕ_0 depending, again, on the initial values of the wavenumbers. Equation (7) acquires the from of an inhomogeneous Hill equation,

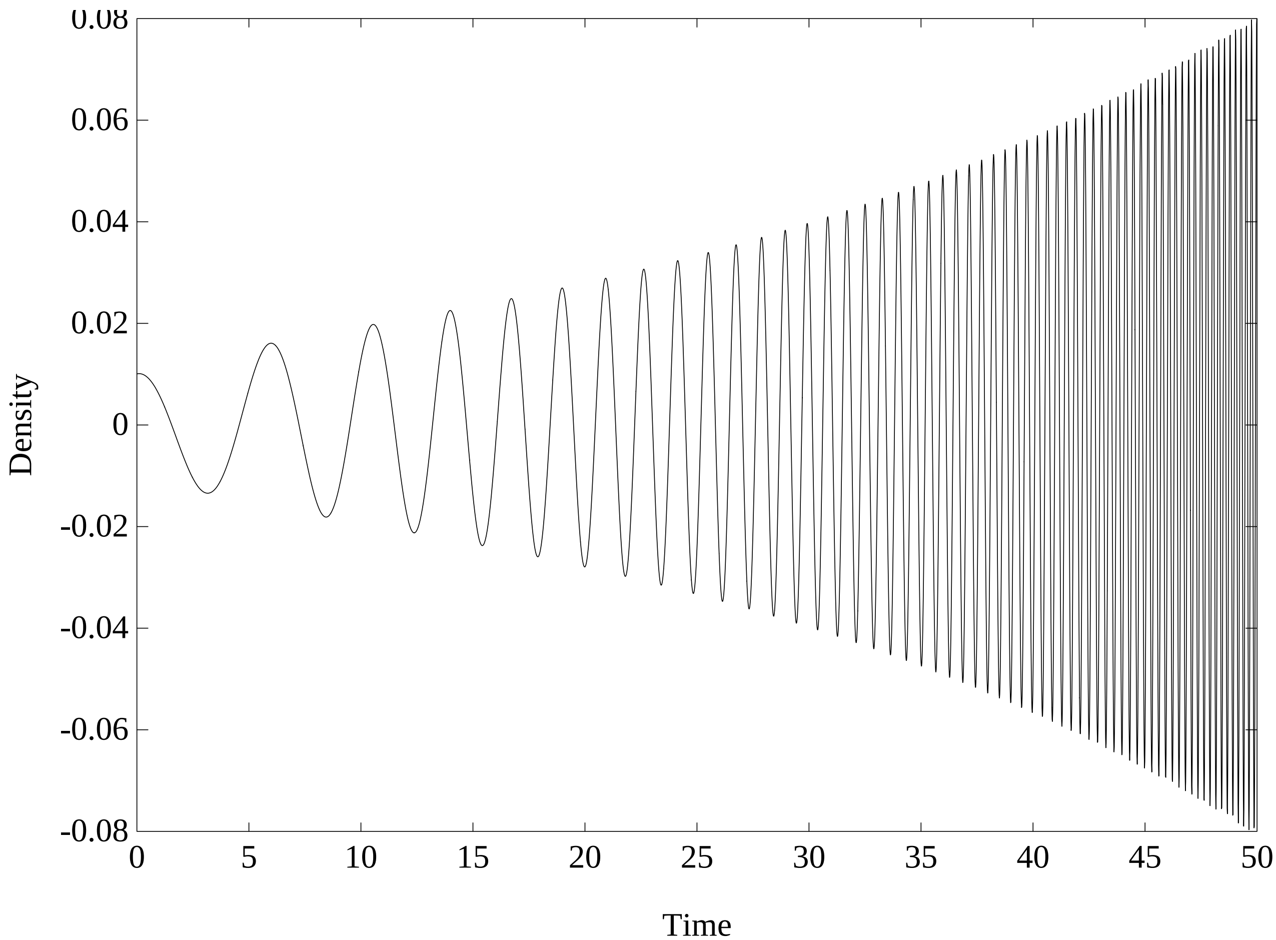

FIG. 1.—Density perturbation $\varrho(\tau) = \mathscr{K}(\tau)\Psi(\tau)$, for $R_1 = 0.1$, $R_2 = 0.05$, $\varepsilon = 0$, $\mathscr{K}_y(0) = 0.1$, $\mathscr{C} = 0$, $\Psi(0) = 10^{-2}$, and $d_\tau \Psi(0) = 0$

and its numerical analysis reveals the following three subclasses of solutions:

A. *Vortical solutions* interacting with the acoustic wave. These solutions are expected whenever $\mathscr{C}$ is large and the frequency $\omega \equiv [-(R_1 R_2 + \varepsilon^2)]^{1/2}$ is small. These are new, very peculiar versions of the Kelvin modes (Kelvin 1887), with the difference that the new vortex (Fig. 2) is not a transient; it does have a transient growth, but it repeats with the periodicity 2ω. The vortex forms at a given time, gives up its energy back to the flow and the acoustic oscillations, and echoes back after a time $T = \pi/\omega$. We call this picturesque phenomenon *asymptotic persistence*, and we believe it is a hallmark of shear flows with complex kinematics.

B. *Unstable Acoustic Waves*: It is well known that equations with periodic coefficients allow unstable solutions in certain ranges of parameters. Note that for $\Delta = 0$, equation (7) becomes a Mathieu equation, the solutions of which are very well known. It is reasonable to expect that even for $\Delta \neq 0$, the solutions will retain the peculiarities of the Mathieu solution, e.g., the regions of instability. Numerical solutions show that equation (7) has a number of unstable regions in the parameter space defined by $[\omega, R_2, \mathscr{K}_y(0)]$. One such parametrically unstable solution is displayed in Figure 3, where the initial perturbation with $\Psi(0) = 10^{-4}$ $[\mathscr{K}_y(0) = 8]$ exhibits powerful exponential growth by several orders of magnitude.

C. *Stable Acoustic Waves*: the basic solutions of equation (7) are some combination of the vortical and acoustic types of perturbation. For a variety of initial conditions, acoustic type will dominate the vortical type. In this case, the frequency as well as the amplitude of the mode changes periodically. A pair of typical plots is shown in Figures 4*a* and 4*b*. These figures are drawn for the same set of parameters as Figure 3, but with the difference that $\mathscr{K}_y(0) = 10$ for Figure 4*a*, and $\mathscr{K}_y(0) = 6$ for Figure 4*b*. Comparison of these solutions with the unstable one shows that the acoustic waves become unstable for relatively narrow and restricted ranges of the system parameters. This feature clearly reflects the parametric nature of the solution displayed in Figure 3.

In all these cases, the fluctuations do not go away; they are not *transient*, but amplify and/or persist. This is of course true for the linear stage of their evolution in an inviscid (zero viscosity) fluid. The effects of viscosity can be readily incorporated into this model problem, and its details will be reported later. Note that the strongest effect of viscosity is on perturbations whose $\mathscr{K}$ increases with time. These short-scale perturbations will readily damp due to viscosity, converting perturbation energy into heat. It is quite remarkable that, through the agency of the multidimensional shear, the linear theory is able to mimic the most essential element of the fully developed fluid turbulence; the

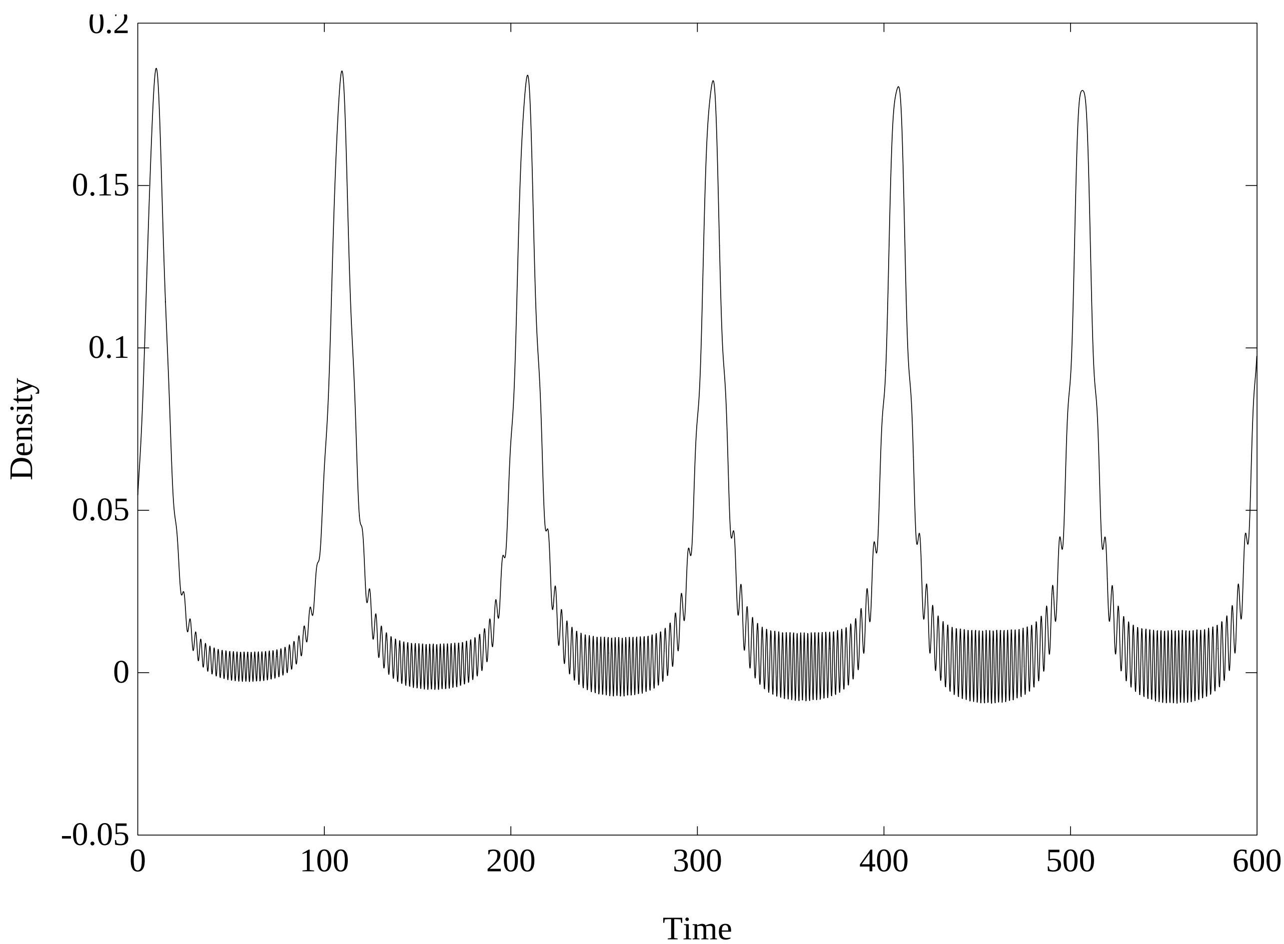

FIG. 2.—Asymptotic persistence of "echoing" Kelvin vortices for density perturbation $\varrho(\tau)$ when $R_1 = 0.1$, $R_2 = -0.01$, $\varepsilon = 0$, $\mathscr{K}_y(0) = 1$, $\mathscr{C} = 1$, $\Psi(0) = 3.87 \times 10^{-2}$, and $d_\tau \Psi(0) = 8.66 \times 10^{-3}$.

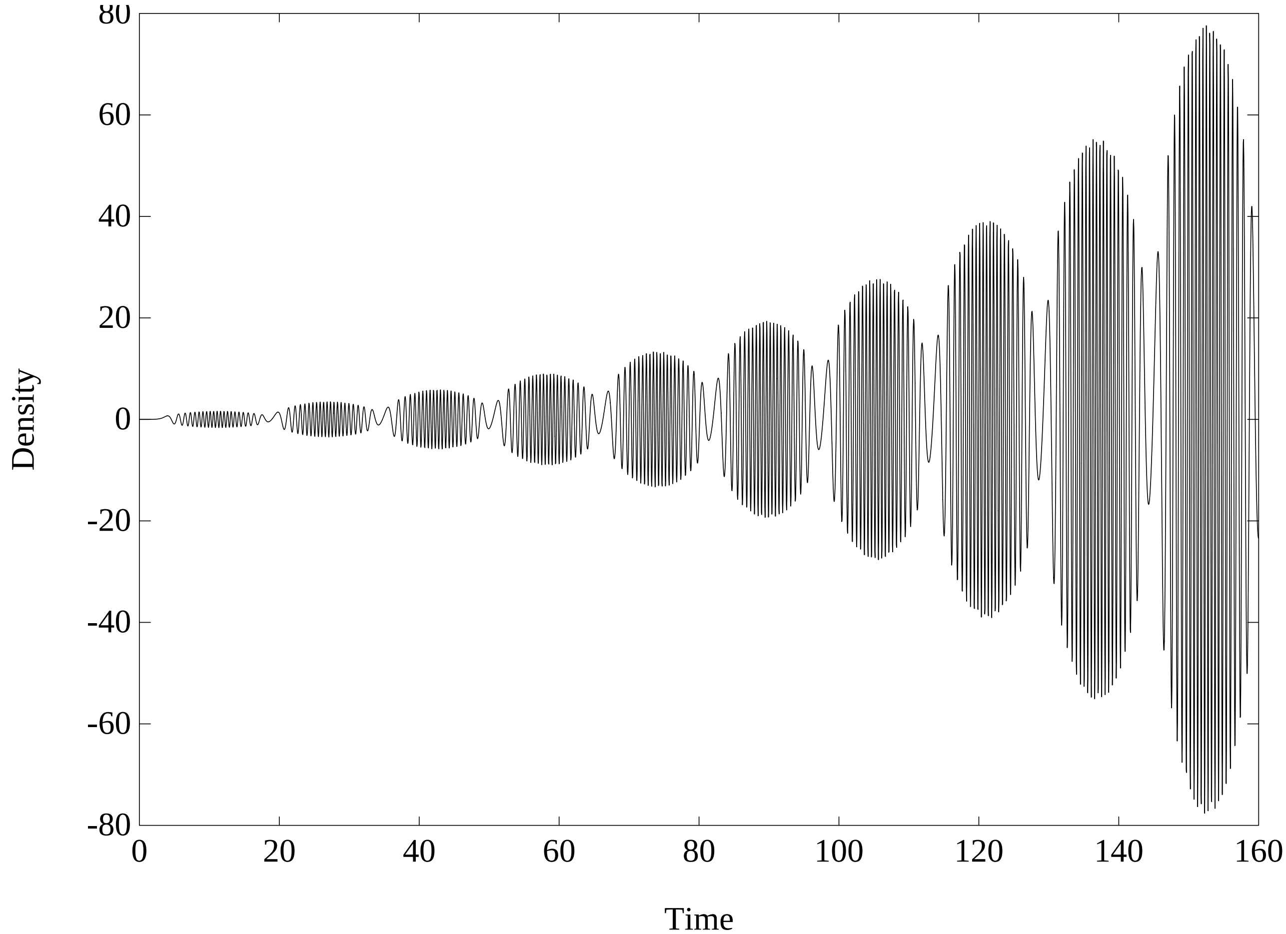

FIG. 3.—Parametrically unstable acoustic wave solution plotted for $\varrho(\tau)$ when $R_1 = 2.0$, $R_2 = -2 \times 10^{-2}$, $\varepsilon = 0$, $\mathscr{K}_y(0) = 8$, $\mathscr{C} = 1$, $\Psi(0) = 10^{-4}$, and $d_\tau \Psi(0) = 0$.

transfer of energy from long- to short-wavelength perturbations that find themselves, eventually, in the dissipation range.

One must also couple the current theory with nonlinear processes that lead to a fragmentation of the perturbation scale and their angular redistribution in ***k***-space. The combination of viscous and nonlinear effects will decide the eventual fate of the perturbations. There exists the possibility that the picture developed in this paper could substantially add to our understanding of the transition to turbulence in these kinds of flows. This fascinating problem will be the next element in the further development of this approach.

3. DISCUSSION

It seems reasonable to extrapolate that the phenomenon of asymptotic persistence of fluctuations is quite general; it should manifest itself in most astrophysical systems (neutral fluids as well as plasmas) with kinematically nontrivial sheared background mean flows. A comprehensive review of all plausible astrophysical applications is beyond the scope of this paper. Instead, we concentrate on a subclass of the flows in which the asymptotically persistent shear-driven phenomena are likely to occur.

1. The very first example comes from the solar atmosphere. As noted earlier, there is plenty of direct and indirect observational evidence for complicated oscillatory, transient, or sustained motions in the solar photosphere, chromosphere, transition region, and corona (Pneuman & Orrall 1986; Athay 1986; Thomas 1996). These observations suggest that in solar MHD flows (1) the magnetic fields are widespread, playing a crucial role in the physics of the solar atmosphere; and (2) the plasma motions are of large amplitudes; they are coupled with the magnetic field, and they also tend to influence the overall solar activity.

However, the exact sequence of physical processes that leads to the ultimate coronal heating and acceleration of the solar wind is still unclear. Propagating MHD waves, network magnetic fields, and direct plasma outflows are likely physical factors that should contribute to the transmission of the mechanical energy to the chromosphere and corona. Clearly, different patterns of the solar plasma MHD flows (e.g., photospheric siphon flows [Thomas 1996], chromospheric spicules and fibrils [Athay 1986], coronal loops, etc.) comprise good examples of astrophysical shear flows with immanently complex kinematics.

In our attempts to make simple, solvable models, we can sometimes oversimplify the problem. For example, the siphon flows in the solar magnetic flux tubes and sunspots, which offer the most likely explanation of the Evershed effect in sunspots, are usually considered to be one-dimensional, locally parallel, and uniform across the cross section of the tube. However, we do know that even in the simplest prototype of the siphon flow (the hydrodynamical Hagen-Poiseuille flow) the velocity is *not* uniform across the cross section of the pipe. Moreover, realizing that the foot-

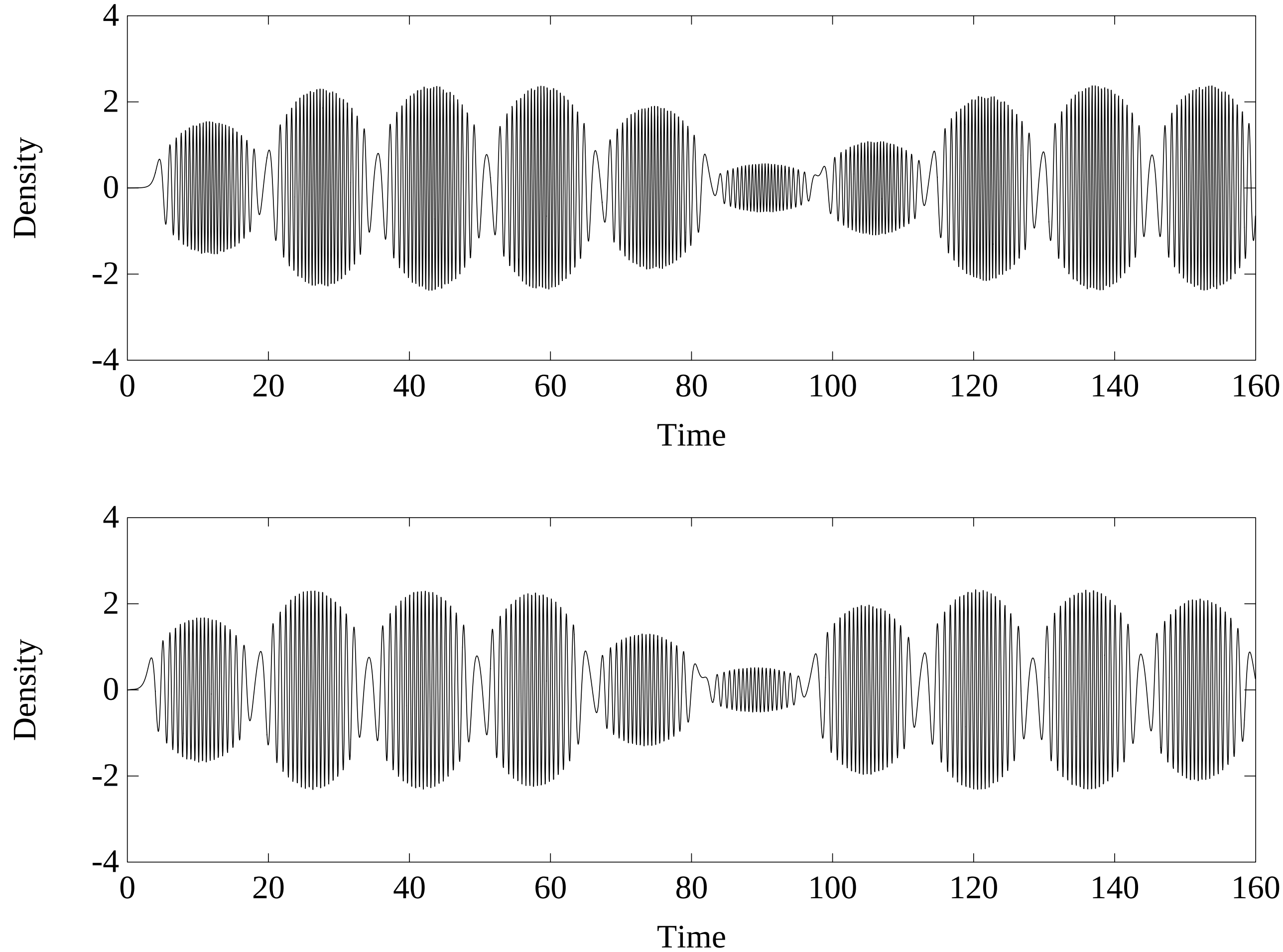

FIG. 4.—Stable acoustic wave solutions presented by $\varrho(\tau)$ graphs, for the same sample of system parameters as in Fig. 3, except that here (*a*) $\mathscr{K}_y(0) = 10$ and (*b*) $\mathscr{K}_y(0) = 6$.

points are most probably twisted and squeezed by the convective granules (Parker 1974), it is reasonable to admit that the solar siphon flows (as well as real flows within spicules and fibrils) would be kinematically complex.

We must remark that studies of even the simplest plane-parallel shear flow nonmodal effects in solar plasma flows are in their infancy (Zaqarashvili 1997; Poedts, Rogava, & Mahajan 1998). Going a step further, we can admit that magnetic fields, coupled with kinematically complex fluid motions within different layers of the solar atmosphere and within different elements of the solar atmospheric fine structure, may naturally exhibit asymptotically persistent shear-induced phenomena, similar to the ones that are identified in the present paper. These processes may strongly influence the propagation and the morphology of MHD waves traveling throughout the solar atmosphere. This important study needs to be undertaken.

2. Accretion-ejection flows are yet another large class of astrophysical flows with nontrivially sheared velocity fields. It is meaningful to consider accretion disks and jets in the same framework, since existing observations suggest that the formation of galactic and extragalactic jets is intimately related to the existence of gaseous accretion disks around central objects (Camenzind 1996). A delineation of the disk-outflow transition is a problem whose solution will necessarily involve flows with complicated kinematics.

Jets, or narrow high-velocity plasma streams, are a common occurrence in our own Galaxy. Just in the regions near the Earth, there are several young stellar objects (YSOs) accreting nearby material and emanating jets of neutral or ionized atomic matter. Distant galaxies exhibit extragalactic jets consisting of magnetized, relativistic flows of electron-proton and electron-positron plasmas. Recent extensive observations by the *Hubble Space Telescope* (*HST*) helped to give us a close look at the region near a jet's origin, where the accreting mode of motion changes into the outflow mode of motion. For a better interpretation of the observations, a study of the properties of jet-forming flows, characterized by rather nontrivial geometry and fluid (plasma) dynamics, is likely to be essential.

Kinematic complexity seems to be a general characteristic of astrophysical jet flows. It is true for YSOs, the newly formed stars still embedded in their parent molecular cloud. These objects are a common site for jet formation, and the accretion of the molecular material is a probable energy source for this process (Ray 1996). The *HST* observations provide detailed diagnostics of the physical conditions within the jets and in their surroundings. In Herbig-Haro 30 (HH 30), for instance (Burrows et al. 1996), *HST* images show an accretion disk surrounding the newly forming star and a narrow jet. This confirms the basic picture of an accretion-driven jet. For molecular outflows from protostars, observations imply (Henriksen 1996) that the outflows extend down to the stellar scale, and the transition between the outflow region and the star is expected to play a crucial role in the collimation of these outflows. The same is true for jets associated with different types of symbiotic stars (with R Aquarii as a prominent example). They involve magnetized accretion disks, which produce highly collimated bipolar winds and jets. There is some observational evidence (Kafatos 1996) that the jet in R Aqr is expanding in a helical-like structure. The jets in all these Galactic objects comprise rotating, velocity-sorted sheared outflows with presumably highly nontrivial flow geometry.

Extragalactic jets are in many ways similar to the stellar jets, but have speeds approaching the speed of light, are much larger in terms of their length scales, and are greatly scaled up in terms of energy (Biretta 1996; Ferrari et al. 1996). Interesting morphological features of the extragalactic radio jets, showing a remarkable periodicity in some examples, are likely to be the signatures of waves and instabilities within them and should be closely related to the kinematic portrait of the involved flows. The role of velocity shear–induced effects should be significant in this context. The discovery of a gas disk orbiting the nucleus in M87 tends to confirm the role of disk-jet transition region (another example of a kinematically complex shear flow) for the production and effective collimation of the jet.

Thus, we can conclude that there is an abundance of astrophysical shear flows that may sustain (and be affected by) the modes of collective behavior identified in the present study. These linear modes, driven by a multidimensional velocity shear, are new and quite exotic in that they are endowed with many fascinating features of nonlinear physics (frequency and wavenumber changing with time and amplitude). Renewed investigations of the astrophysical flows in terms of these modes could, indeed, provide some of the missing elements in our understanding of the observed phenomena.

In addition, since the shear flows are being recognized as major determinants of the fate of the fusion plasmas, we believe that the shear-generated and maintained fluctuation spectrum could also become a crucial new element in understanding the anomalous transport in magnetic-confinement experiments.

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