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Periodic waves travelling along an unsmooth boundary via the fractal variational theory

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ABSTRACT

The solitary waves of the fractal Korteweg-de Vries (KdV) equation travelling along an unsmooth boundary is studied by Ji-Huan He, et al (Results in Physics, 2021, 104104 [1]). In this letter, we obtain its periodic waves travelling along an unsmooth boundary via the fractal variational theory, which is expected to open the new perspectives on the study of the fractal travelling wave theory.

Introduction

Ji-Huan He, et al. obtain the solitary waves of the fractal Korteweg-de Vries (KdV) equation travelling along an unsmooth boundary in [1], which makes an unprecedented contribution on the study of the fractal travelling wave theory in physics. Inspired by this work, we aim to seek the periodic wave travelling along an unsmooth boundary in this short paper. The fractal KdV equation that can work under the unsmooth boundary (such as the fractal boundary in Fig. 1) is expressed as [1–4]:

$$\frac{\partial\varphi}{\partial t^{\gamma}}+\varphi\frac{\partial\varphi}{\partial x^{\gamma}}+\frac{\partial^{3}\varphi}{\partial x^{3\gamma}}=0. \tag{1.1}$$

where γ represents the two-scale fractal dimension. The definitions of the fractal derivatives $\partial/\partial t^{\gamma}$ and $\partial/\partial x^{\gamma}$ are detailedly given in [1–4] as:

$$\frac{\partial}{dt^{\alpha}}\varphi(x,t_0)=\Gamma(1+\gamma)\lim_{\substack{t-t_0=\Delta t\\ \Delta t\neq 0}}\frac{u(x,t)-u(x,t_0)}{(t-t_0)^{\gamma}}, \tag{1.2}$$

$$\frac{\partial}{dx^{\gamma}}\varphi(x_0,t)=\Gamma(1+\gamma)\lim_{\substack{x-x_0=\Delta x\\ \Delta x\neq 0}}\frac{u(x,t)-u(x_0,t)}{(x-x_0)^{\gamma}}, \tag{1.3}$$

where Δt is the period required for the motion through a fractal space Δx. For $\gamma=1$, Eq. (1.1) becomes the classic KdV equation.

Periodic wave solution

For obtaining the periodic wave solution, the following transform is introduced:

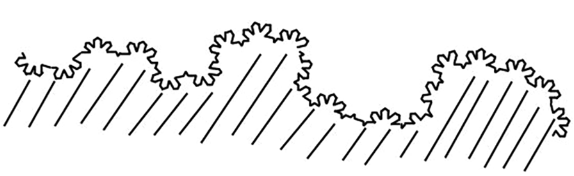

Fig. 1. The unsmooth boundary (fractal boundary).

$$\varepsilon^{\gamma}=x^{\gamma}-mt^{\gamma}. \tag{2.1}$$

Eq. (1.1) can be converted as:

$$-m\frac{\partial\varphi}{\partial\varepsilon^{\gamma}}+\varphi\frac{\partial\varphi}{\partial\varepsilon^{\gamma}}+\frac{\partial^{3}\varphi}{\partial\varepsilon^{3\gamma}}=0. \tag{2.2}$$

Integrating Eq. (2.2) with respect to $\varepsilon^{\gamma}$and ignoring the integral constant, yields:

$$-m\varphi+\frac{1}{2}\varphi^{2}+\frac{\partial^{2}\varphi}{\partial\varepsilon^{2\gamma}}=0. \tag{2.3}$$

We can establish its fractal variational principle as [5,6]:

$$J(\varphi)=\int\left\{-\frac{1}{2}m\varphi^{2}+\frac{1}{6}\varphi^{3}-\frac{1}{2}\left(\frac{\partial\varphi}{\partial\varepsilon^{\gamma}}\right)^{2}\right\}d\varepsilon^{\gamma}. \tag{2.4}$$

The periodic wave solution of Eq. (2.3) is assumed as:

$$\varphi(\varepsilon^{\gamma})=\psi cos(\omega\varepsilon^{\gamma}),\omega>0. \tag{2.5}$$

Taking Eq. (2.5) into Eq. (2.4) and applying He's frequency formulation [7,8], we have:

$$\omega=\sqrt{\frac{-(4\psi+3m\pi)}{3\pi}}>0. \tag{2.6}$$

So the periodic wave solution can be obtained as:

$$\varphi(x,t)=\psi cos\left[\sqrt{\frac{-(4\psi+3m\pi)}{3\pi}}(x^{\gamma}-mt^{\gamma})\right] \tag{2.7}$$

By selecting $\psi=-1$, $m=-1$, we draw the behaviors of Eq. (2.7) with different fractal orders γ in Fig. 2. Obviously, it is found that the value of γ has a great influence on the periodic characteristics of the periodic wave, that is the smaller the value of γ, the greater the period is, but the amplitude remains unchanged.

Conclusion

In this letter, we presented the periodic wave solution of the fractal KdV equation travelling along an unsmooth boundary via the fractal variational theory, which offers a promising and simple approach to

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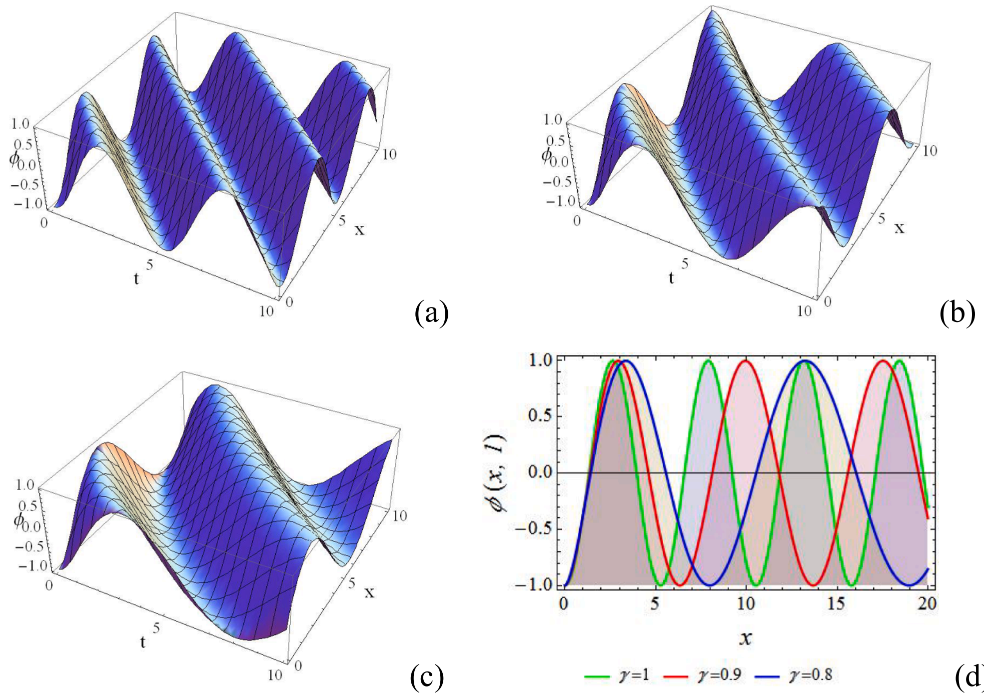

Fig. 2. The behaviors of Eq. (2.7), (a) $\gamma = 1$. (b) $\gamma = 0.9$. (c) $\gamma = 0.8$. (d) $t = 1$.

construct the periodic solution. The results show that the smaller the value of the fractal order γ is, the greater the period is, but the amplitude remains unchanged. The finding of paper is expected to shed a new light on the fractal and fractional travelling wave theory [9,10].

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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