

## Erratum: constraining flavour symmetries at the EW scale I: the $A_4$ Higgs potential

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In this Erratum we clarify that in contrast to the statements made in the article the CP symmetry is not broken in the context of a  $A_4$  symmetric Higgs potential. We acknowledge that these conclusions match the results appeared in ref. [1].

In general, CP violation can be either explicit if it appears directly at the level of the Higgs potential or implicit if it occurs due to the vacuum expectation values (vevs) of scalar fields. The solutions that minimize the scalar potential studied in the section 5 have an explicit complex phase  $\omega_1$  in some of the vevs. One might thus wonder whether the Higgs sector in  $A_4$  models gives rise to extra sources of CP violation.

We first investigate whether the potential in eq. (2.2) exhibits explicit CP violation. We find that the potential is not invariant under a ‘naive’ CP transformation

$$\Phi_i \xrightarrow{CP} \Phi_i^* . \quad (1)$$

Under this transformation,  $\epsilon$  and  $-\epsilon$  get interchanged in the potential in eq. (2.2). The expression in (1) does not describe the most general CP transformation however. A more

general CP transformation follows when the ‘pure’ CP transformation in (1) is combined with a Higgs basis transformation

$$\Phi_i \xrightarrow{CP} \mathcal{U}_{ij} \Phi_j^*. \quad (2)$$

Here  $\mathcal{U}$  is a unitary matrix in the space of the three Higgs fields. It was shown in ref. [2] that the Higgs potential conserves CP explicitly if a matrix  $\mathcal{U}$  exists such that the ‘new’ CP transformation in (2) leaves the potential invariant. For the potential in eq. (2.2) it is not hard to find such a matrix. An example is the matrix that parameterizes the interchange of the first and second Higgs fields

$$\mathcal{U} = e^{i\alpha} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

In this case, the CP transformation is defined according to

$$\Phi_1 \xrightarrow{CP} e^{i\alpha} \Phi_2^*, \quad \Phi_2 \xrightarrow{CP} e^{i\alpha} \Phi_1^*, \quad \Phi_3 \xrightarrow{CP} e^{i\alpha} \Phi_3^*. \quad (4)$$

We conclude that the  $A_4$  invariant Higgs potential does not violate CP explicitly.

There is still the possibility of spontaneous CP violation through the complex vacua discussed in the previous section. In refs. [2, 3], it is shown that a vacuum does not give rise to spontaneous CP violation if there is a matrix  $\mathcal{U}$  such that the CP transformation in (2) also leaves the vacuum invariant. In that case, the vacuum thus satisfies

$$\langle \Phi \rangle = \mathcal{U} \langle \Phi \rangle^*. \quad (5)$$

In other words, each component  $v_i e^{i\omega_i}$  of the vector of vevs should be written as a linear combination of the complex conjugates of the vevs  $v_j e^{i\omega_j}$  with the coefficients given by  $\mathcal{U}_{ij}$

$$v_i e^{i\omega_i} = \mathcal{U}_{ij} v_j e^{i\omega_j}. \quad (6)$$

In the specific case under investigation, where  $\mathcal{U}$  has the form in (3), this is represented by

$$v_1 e^{i\omega_1} = v_2 e^{i(\alpha-\omega_2)}, \quad v_2 e^{i\omega_2} = v_1 e^{i(\alpha-\omega_1)}, \quad v_3 e^{i\omega_3} = v_3 e^{i(\alpha-\omega_3)}. \quad (7)$$

The first two equations are dependent: they require  $v_1$  and  $v_2$  to be each others complex conjugate. The third equation requires the third vev to be real. The two vacua that could lead to spontaneous CP violation,  $(v e^{i\omega_1}, v, 0)$  and  $(v e^{i\omega_1}, v e^{-i\omega_1}, r v)$ , both satisfy the conditions in (7), for  $\alpha = \omega_1$  and  $\alpha = 0$ , respectively. As a result, they do not break CP spontaneously, notwithstanding the fact that they are inherently complex.

The criterium of conserving or violating CP depending on whether the transformation matrix  $\mathcal{U}$  exists, is not always a very practical one. Even if such a transformation exists, it may not be easy to find. An alternative test is in the straightforward calculation of CP-odd basis invariants that vanish if CP is conserved and that are non-zero if CP is violated (or, at least one of them is). Invariants for the potential in eq. (2.2) and the vacua of the previous subsection were calculated in ref. [4]. As expected, they are all zero.

Clarified that no CP violation arises in the context of  $A_4$  symmetric scalar potential as studied in this article, one could wonder on the correctness of the analysis presented in section 7.2. However, no specific investigation on CP violating processes have been presented and neither the conclusions nor the plots are changed. In particular, in section 7.2.1, we concluded that no lower bounds on  $m_1$  and  $m_2$  can be recovered in figure 4. This followed from assuming the existence of a limit situation in which the two lightest states have almost the same CP parity and therefore the Z boson does not decay into them. Since CP is preserved, the two lightest states can have the same CP parity without invoking any particular limit and therefore no lower bound on  $m_1$  and  $m_2$  can indeed be recovered in figure 4.

## References

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