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ASSESSMENT OF THERMAL DISCOMFORT IN NON-UNIFORMLY HEATED ENCLOSURES: TWO INDICES IN THE TIME–SPACE DOMAIN

G. CAMMARATA, A. FICHERA AND L. MARLETTA

Istituto di Macchine, Facoltà di Ingegneria, Università di Catania, Viale A. Doria 6, 95125 Catania, Italy

SUMMARY

In this paper two indices are proposed to assess the thermal discomfort in not-uniformly heated enclosures in the time–space domain. The discussion of some meaningful cases reveals that living spaces are subject to significant non-uniform radiant heat fields, resulting in thermal discomfort. In order to quantify such effects, the concept of 'uniform equivalent temperature' is invoked. This allows two indices i^+ and i^- to be developed for assessing the thermal discomfort in intensity and duration. Such indices have a clear physical meaning and therefore may provide substantial help in detecting causes and/or locations of thermal unpleasantness. On this basis a necessary but not sufficient condition for thermal comfort is stated in the form: $i^+ = i^- = 0$, to be sought everywhere within the room. Further examples illustrate the value of the procedure in thermal comfort conscious design.

KEY WORDS *Thermal comfort* *Radiative heat transfer* *Building*

INTRODUCTION

In order to quantify the heat transfer rate between the human body and its surroundings through a single number, many temperature indices have been introduced such as the mean radiant temperature (T_{mr}), the operative temperature (T_o) and the 'new effective temperature' (ET^*).

In this list there is an ascending trend toward generality. T_{mr} accounts only for radiant heat transfer; T_o includes both the radiative and convective; while ET^*/contains radiative, convective and evaporative contributions.

As the reader will realize, these indexes are totally or partially non-representative of wellbeing as a whole. Rather they are tricks aimed at simplifying the mathematical formalism of the comfort equation.

A significant further step was achieved with the equivalent uniform temperature (T_{eu}), introduced by W. O. Wray (1980). This is defined as the uniform temperature of an imaginary enclosure in which a person will experience the same degree of thermal comfort as in the actual non-uniform environment. As such, it implies the thermal comfort in its entirety, providing the highest level of generality.

As we will see later, T_{eu} can be put in very simple form as a function of T_a, T_{mr} and a newly introduced parameter 's', including the rest of terms which the thermal comfort deals with. Hence T_{eu} is a time-varying function, because of the link to the dynamic behaviour of the enclosure.

In the subsequent sections it will be shown that, on the basis of T_{eu}, it is possible to derive two of indices to assess thermal discomfort, i.e. the duration and intensity of coldness (i^-) and/or warmth (i^+), throughout the day, for any location of the enclosure. To this aim the main topics of the W. O. Wray paper will be shortly reviewed and the procedure for both the building thermal analysis and the assessment of T_{mr} will be summarized. Then the procedure for the operative evaluation of (i^-) and (i^+) will be described in detail. Finally a number of study cases will be discussed.

As a result one should note that, by means of the abovementioned indices, the thermal comfort appears in the design process as a true state variable.

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THE EQUIVALENT UNIFORM TEMPERATURE AND THE INDICES OF DISCOMFORT

As shown in W. O. Wray's original paper (1980), for

$$|T_{cl} - T_{mr}| \ll T_{cl}$$

the Fanger equation (Fanger, 1972) can be linearized and represented by a straight line in the cartesian plane (T_a, T_{mr}) with a slope:

$$s(M, I_{cl}, \phi) = \left.\frac{dT_{mr}}{dT_a}\right|_{L=0}$$

$L = 0$ being the condition for thermal equilibrium for the human body. It is possible to show that T_{eu} is linked to T_a and T_{mr} by the following relationship:

$$T_{eu} = \left(\frac{1}{1-s}\right)T_{mr} + \left(\frac{s}{s-1}\right)T_a \qquad (1)$$

Now let us consider the 'predicted mean vote' equation (Fanger, 1972):

$$PMV = 0{\cdot}86L\ (0{\cdot}352e^{-(0{\cdot}036M)} + 0{\cdot}032)$$

Under the hypothesis that $T_{mr} = T_a$, and $PMV = 0$ we obtain $T_u = T_{mr} = T_a$, the optimum uniform temperature which is the temperature of an imaginary, uniformly heated enclosure in which a person will feel the same thermal sensation as in the actual non-uniform environment.

Further, under the constraints $PMV = +1$ and $PMV = -1$, the previous equation provides respectively T_1 and T_2; hence $\Delta T = T_1 - T_2$. The ΔT is referred to as the 'comfort range', defined as the temperature range about the optimum uniform temperature T_u, within which a subject will experience only slight discomfort, by virtue of being either too warm ($PMV = +1$) or too cool ($PMV = -1$).

It follows that as long as the T_{eu} lies within the comfort range ΔT, the enclosure can be considered reasonably comfortable, while thermal discomfort occurs when T_{eu} lies outside ΔT.

Consequently it is suitable to define two indices of thermal discomfort as follows:

$$i^- = [T_{eu}(\tau) - (T_u + \Delta T)]d\tau \quad \text{if } T_{eu} < T_u - \Delta T$$
$$i^+ = [T_{eu}(\tau) - (T_u + \Delta T)]d\tau \quad \text{if } T_{eu} > T_u + \Delta T \qquad (2)$$

represented by the shaded area in Figure 1.

These indices are meaningful i.e. representative of the comfort as a whole since, through T_{eu}, they include all the variables involved in the comfort equation. Further, since they also have a physical meaning, they are helpful and easy to use in optimizing the design from the wellbeing point of view.

Now other topics are to be covered, such as the mathematical model for the building thermal analysis and the evaluation of the T_{mr} in the time–space domain.

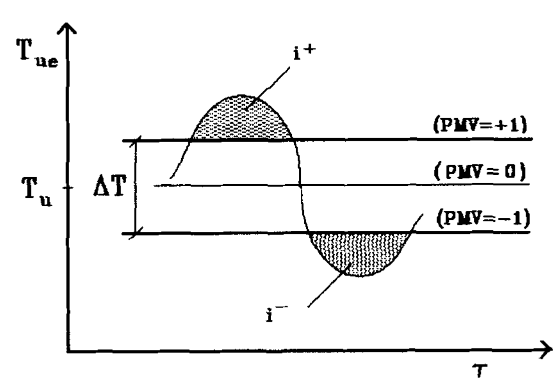

Figure 1.

THE MODEL FOR THE BUILDING THERMAL ANALYSIS AND THE MEAN RADIANT TEMPERATURE

In order to assess the building thermal response, a dynamic model was used, previously developed by the authors and experimentally validated (Cammarata *et al.*, 1987; Aleo *et al.*, 1989). Although a full description of the model was given by Cammarata (Cammarata *et al.*, 1987), its main features are summarized here.

It is a 'distributed parameter model' (DPM) based on linear equations. Walls can be discretized in a number of layers. An energy balance equation can be stated for every single layer as follows:

$$mc_j \frac{dT_j}{d\tau} = \frac{T_{j-1} - T_j}{R_{j,j-1}} - \frac{T_{j+1} - T_j}{R_{j,j+1}} + q_s$$

The left-hand side represents the heat stored in the layer, and the right-hand side includes all the involved heat fluxes. That is to say, the first two terms are the conductive and/or convective contributions, while q_s accounts for heat treansfer with the environment. More precisely, q_s for internal layers is zero, for the outermost layer accounts for the solar radiation directly impinging on the outer surface of the wall, whereas for the internal surface it is the flux due to solar radiation admitted through windows and shared among the walls.

This latest term in the model was assessed by assuming that the enclosure behaves as an Ulbricht sphere, i.e. any wall receives an amount of energy proportional to its thermal capacity and absorptance.

The set of equations for the walls can be put in the (canonical) form:

$$[\dot{X}] = [A][X] + [B][U]$$

Here $[X]$ is the state vector and $[\dot{X}]$ its time derivative; they both include the state variables, i.e. the temperature of every single layer T_j; $[U]$ is the input vector containing the boundary conditions, such as outdoor temperature and solar radiation; $[A]$ and $[B]$ respectively referred to as state array and input array gather the system structural data. The canonical equation can be solved according to the typical techniques of the 'linear system theory' (Cadzow and Martens, 1970) and the output is the temperature behaviour of any single layer (T_j) and hence the surface temperature T_s of any wall.

At this stage, one can state a further balance equation for the indoor air which provides either the indoor temperature T_a or, for rooms under thermostat constraints, the thermal load in the time domain. The surface temperatures of walls (T_s) also allow the mean radiant temperature T_{mr} for the enclosure to be determined. Now the theoretical formula:

$$T_{mr}^4 = \sum_{s=1}^{n} T_s^4 F_{p-s}$$

implies some computational difficulties for the view factors F_{p-s}, because of the complex geometry of the human body. To overcome the problem Fanger provides a number of diagrams, derived from an experimental approach (Fanger, 1972). However, as usual, graphical tools are not suitable for automatic procedures.

An alternative may be the model provided by Bonavita *et al.* (1989). Under the hypothesis that the human body is much smaller than the surfaces in question, they give a simplified procedure for the evaluation of the view factor as a function of the geometrical co-ordinates of the subject and those of two opposite corners of the surface, such as a wall, window, heater and/or any other source of radiant heat.

The computational procedure is then as follows. Perform first the thermal transient analysis of the whole system, in order to get the time sequence of the infoor air temperature and that of any radiant surface. Then evaluate the mean radiant temperature for any point of a mesh, previously drawn within the room. Then assess the T_{eu}, according to equation (1) and finally the indices i^+ and i^-, following equation (2). It is understood that this procedure can be implemented in any computer code for the thermal analysis of buildings in the time domain.

In subsequent sections a number of study-cases will be discussed to illustrate the usefulness of this method for assessing the thermal comfort in non-uniformly heated enclosures.

RESULTS

The abovementioned procedure was applied to a room of regular shape ($5 \times 5 \times 3m^3$) in a Mediterranean climate, for winter performance.

The mesh has a pitch of 0·5 m and the subject is supposed to be at 1·2 m above the floor. Let us first observe the T_{mr} patterns, shown in Figures 2 to 5. Every picture can be considered a photogram of a temporal sequence, taken at regular time intervals. The plots allow the effect of the thermal radiation emitted by cold or hot surfaces, such as windows or heaters. The scale appearing at the leftmost edge of the room refers to T_{mr} (° C).

Figure 2 refers to a room of heavyweight construction, taken at 20 ° C, having solid walls, except for one which faces due south and is provided with a window of 2·5 m^2. This is supposed to have a single glass, 3 mm thick.

The cold effect reaches a distance of about 1 m from the surface in daytime and less than 2 m at night. Elsewhere, the T_{mr} seems to be constant throughout the day, owing to the stabilizing effect of other walls, supposed to be adjacent to rooms at the same temperature.

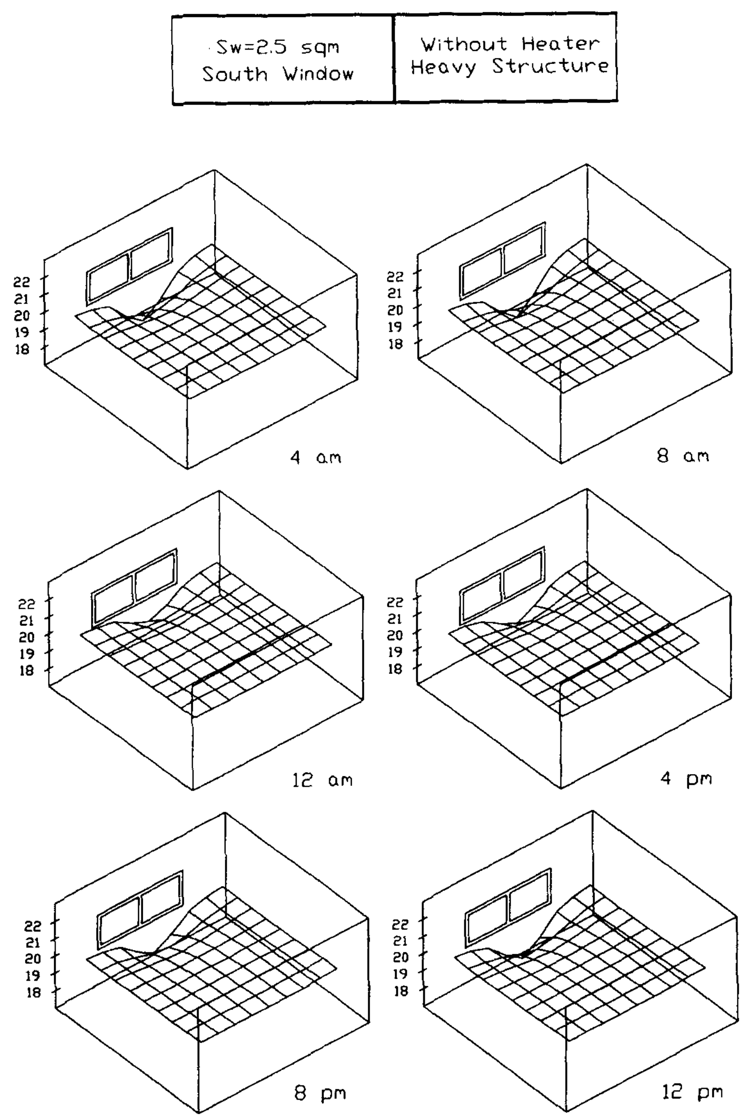

Figure 2.

Figure 3 shows the same enclosure in the same mode of operation, but provided with a 15-m^2 single glazed surface. The cold effect now is dominant during the night, whereas in the daylight hours, owing to the large amount of solar radiation, the wall temperature rises, making T_{mr} rise in turn. As a result the gridlines translate almost rigidly.

In Figure 4 the room has two glazed surfaces, 2·5 m^2 each, and a heater fed with hot water from 8 till 10 a.m. and from 8 till 10 p.m. The dramatic effect of the heater (surface temperature 70 °C, surface area 1 m^2) on the spatial distribution of T_{mr} is self-evident. We shall see later the consequences of such a performance on thermal comfort.

The first remark, then, can be as follows: during heater operation, although the room is under thermostat control (i.e. dry bulb temperature constant), T_{mr} is heavily affected by the sources of radiant heat, and this in turn will affect the thermal comfort. How much it happens can be assessed by using the indexes i^+ and i^- as shown below.

Figure 5, for instance, refers to a room with 2·5 m^2 single glazed surface above a heater, which

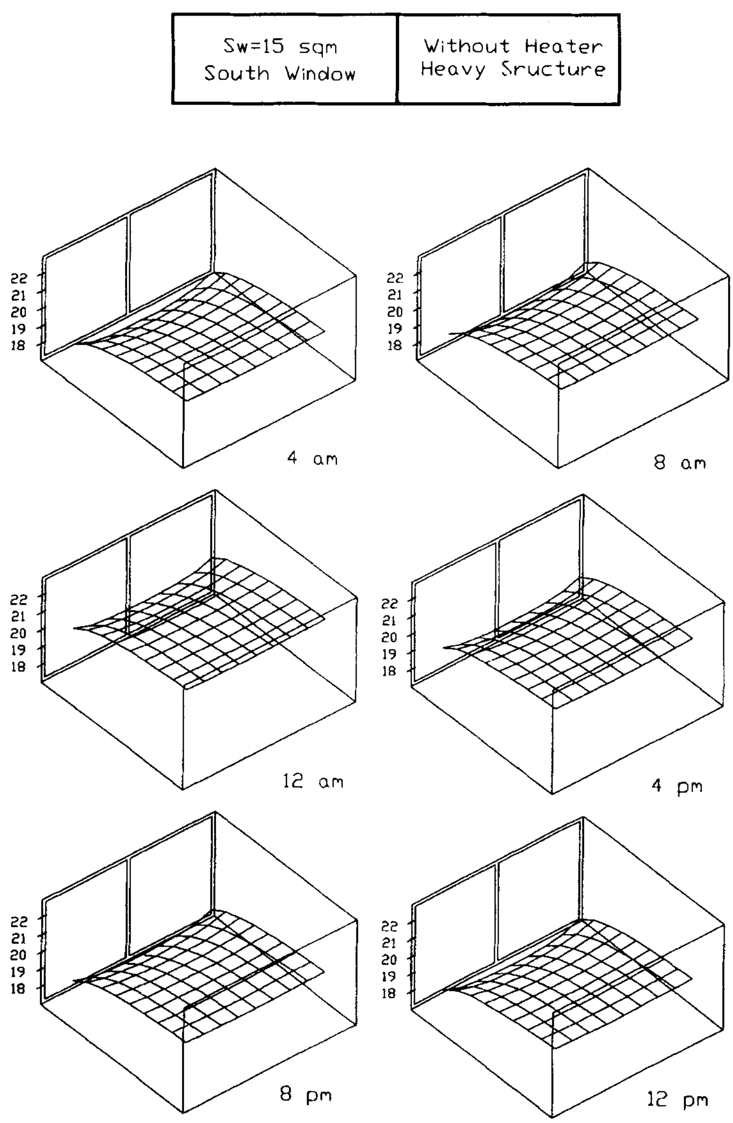

Figure 3.

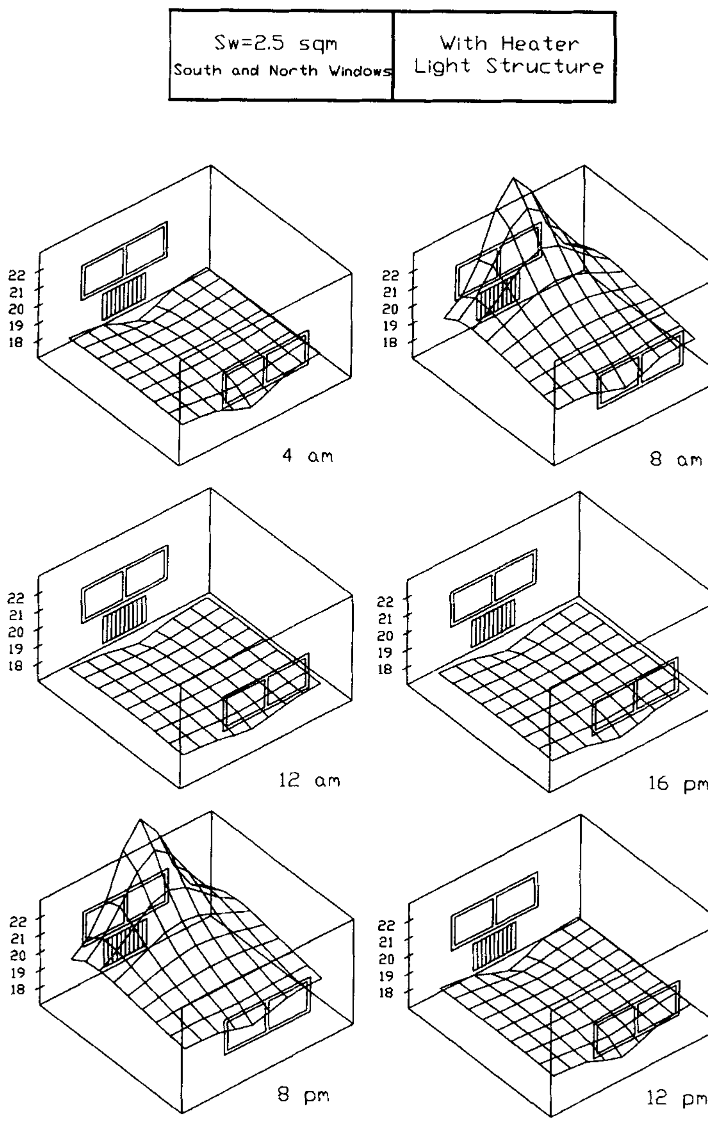

Figure 4.

maintains the indoor air at 20 °C and irradiates at 70 °C. Only warmth generates discomfort (the i^- being zero anywhere) and just close to the heater.

A more unpleasant situation occurs in the case of a room not thermostated (i.e. without heaters) with two windows in opposite walls (north and south) (Figure 6). The generally negative values of the index of discomfort indicate that coldness is felt almost everywhere, but especially close to the glazed surfaces.

Figures 7(a) and 7(b) respectively show i^+ and i^- for a room of ($5 \times 5 \times 3$ m^3) in a severe climate (minimum outdoor temperature −7 °C) in winter mode. Apart one internal partition, the room has three external walls, each one with windows of 15 m^2 and three heaters, heating the room all day long.

As expected, warmth is mostly felt in front of the heaters, whereas close to the corners coldness is dominant. This happens because the view factor in the proximity of the corners is small with respect to the heater surface area and large with respect to the cold glazed surface.

As a result, as the comfort patterns are not evenly distributed, the room can be considered thermally

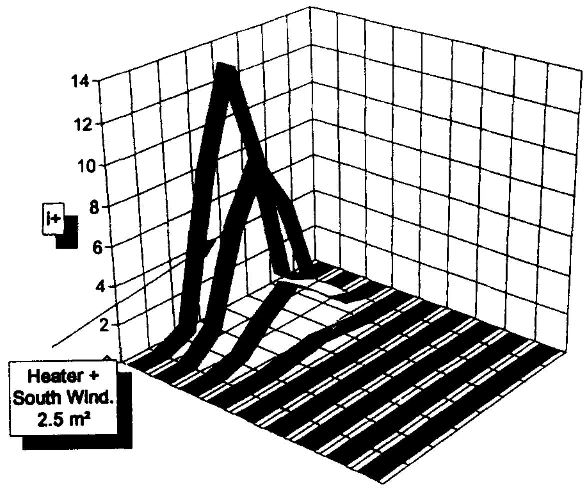

Figure 5.

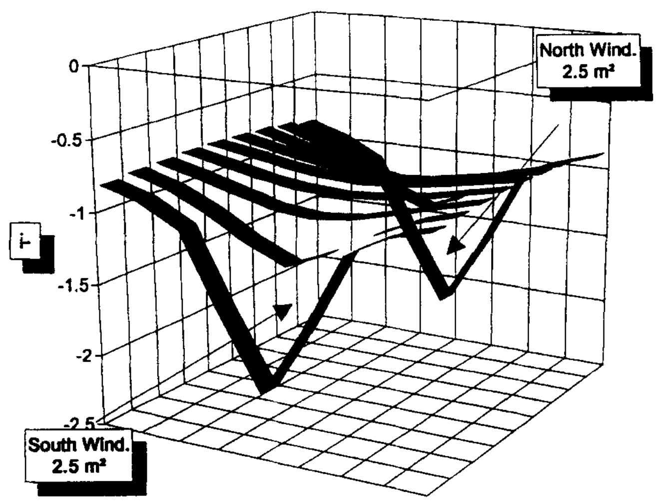

Figure 6.

unpleasant, although the heating system is correctly sized. Of course, if suitable corrections are made, a satisfactory design could be attained. In this case:

$$i^+ = i^- = 0$$

everywhere within the room.

It is understood that such a statement can be considered a necessary but not sufficient condition for thermal comfort in buildings.

DISCUSSION

It is appropriate to make some remarks about the range of reliability of the present method. Firstly, the simplified model for the view factors treats the human body as a sphere with a projected area much

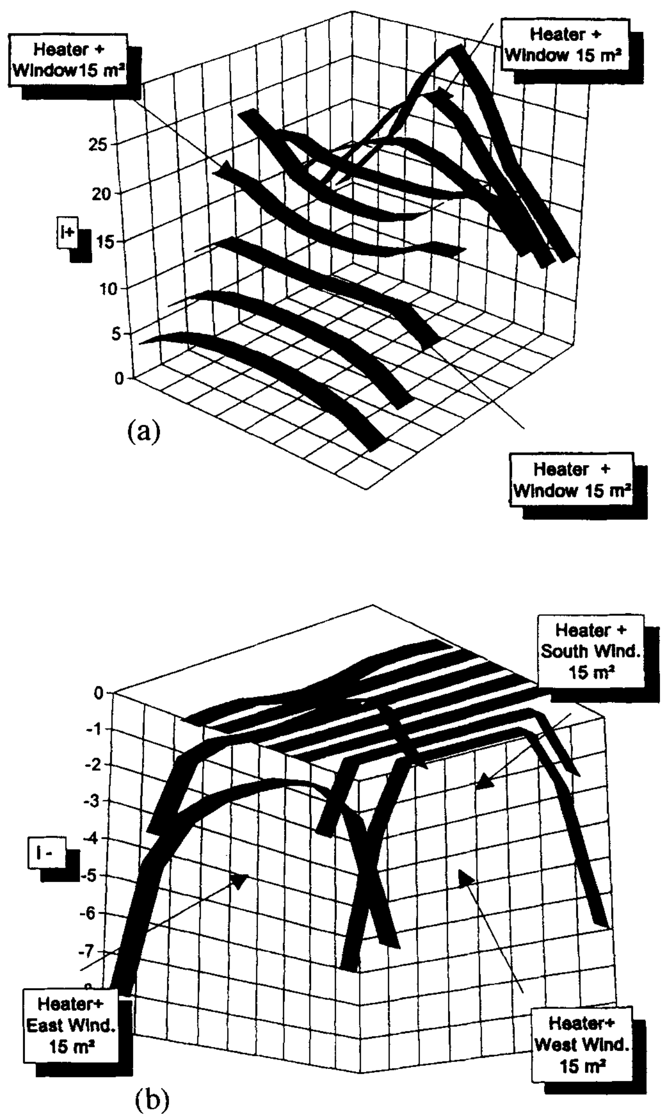

Figure 7.

smaller than the walls to which irradiates. Clearly this hypothesis does not hold thoroughly as the subject approaches the radiant surface in question. Secondly, in the Fanger equation the thermal comfort condition is expressed by $L = 0$, L being the thermal load. In general that implies the steady state, whereas T_{eu} is evaluated as a time-varying function. Hence it is to be assumed that the heat trnasfer between the subject and the environment is a quasi-steady-state process. Again, this is acceptable when heat transfer is dominated by the high thermal inertia of the building mass, but may be questionable during more or less rapid (unsteady-state) processes such as in fast turning on or off of the heaters etc.

Finally the hypothesis $|T_{cl} - T_{mr}| \ll T_{cd}$, which the linearization of the comfort equation is based on, fails close to the radiant surface. In this case the original Fanger equation must be resumed as well as the related *PMV*.

In conclusion, only an experimental validation can provide the ultimate assessment of this model. This is being carried out at present, and will be reported in a future paper.

CONCLUSIONS

This study has drawn attention to an aspect which is often disregarded, i.e. the integration of wellbeing into the design process or the use of the thermal comfort itself as a design tool. This paper has tried to give an answer by means of the indexes i^+ and i^-. In order to state these, in the paper a detailed procedure is provided. This is based on the linearized Fanger equation and the concept of the uniform equivalent temperature (T_{eu}). The use of a dynamic model is also required for determining the instantaneous values of T_a, the surface temperature of walls, and the view factors between the human body and its surroundings, from which the T_{mr} is to be derived. In this work the computer code for the energy analysis of buildings, which supports the thermal comfort model, is based on the state-space method, as available to the authors. But clearly any other code for time-varying conditions may be used, such as NBSLD, DOE-2, ESP etc., provided it has suitable routines for the view factor treatment. Finally, by combining T_{mr} and T_a in a simple formula, it is possible to derive T_{eu} and hence i^+ and i^-.

In conclusion, the proposed method may be reproduced in an automatic procedure for obtaining quick and immediately understandable results, since i^+ and i^- have indeed a clear physical meaning.

In the author's opinion, another valuable outcome of this paper is that a necessary although not sufficient condition for thermal comfort can be stated as follows:

$$i^+ = i^- = 0$$

to be sought everywhere within the enclosure and which can be assumed as a rule of thumb for thermal comfort conscious design.

NOMENCLATURE

F_{p-s}	= view factor between a person (p) and a radiant surface (s)
i^+	= index of thermal discomfort due to warmth
i^-	= index of thermal discomfort due to coldness
L	= thermal load per unit body surface area (W/m^2)
M	= activity level (W/m^2)
$(mc)_j$	= thermal capacity of the jth layer in a composite wall
PMV	= predicted mean vote
q_s	= heat transfer between the surface layer of wall and its environment
$R_{j,j-1}$	= thermal resistance between the consecutive layers jth and (j − 1)th
$R_{j,j+1}$	= thermal resistance between the consecutive layers jth and (j + 1)th
T_a	= indoor air dry bulb temperature (°C)
T_{mr}	= mean radiant temperature (°C)
T_{cl}	= outer surface clothing temperature (°C)
t_{eu}	= equivalent uniform temperature (°C)
T_u	= univorm temperature (°C)
T_j	= temperature of jth layer in a composite wall
T_s	= surface temperature of the wall (°C)
ϕ	= relative humidity
τ	= current time

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