Received: 10 December 2019 | Revised: 31 March 2020 | Accepted: 19 April 2020
DOI: 10.1002/2050-7038.12469

RESEARCH ARTICLE

WILEY

Measurement devices allocation in distribution system using state estimation: A multi-objective approach

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Peer Review
The peer review history for this article is available at https://publons.com/publon/10.1002/2050-7038.12469.

Summary
Optimal allocation of measurement devices is a necessity in order to carry out state estimation of a distribution system. In this paper, the placement problem of power measurement devices is modeled using a multi-objective method. The objectives of the problem are to minimize measurement devices' costs while increasing the accuracy of state estimation and improving the state estimation quality. Also, operational priorities are considered as another objective, which are based on power losses, lines' capacities, the number of lines connected to a specific line, and the change in lines' flows direction. A multi-objective evolutionary algorithm based on decomposition (MOEA/D) is used to optimize the allocation of measurement devices within the problem of distribution system state estimation. The state estimation problem is optimized by particle swarm optimization (PSO) algorithm and the Monte Carlo simulation is used to develop some conditions within the network to guarantee the robustness of the proposed method. The method is tested by simulation results on an IEEE 33-bus and IEEE 123-bus radial network.

KEYWORDS
allocation, allocation quality, distribution system state estimation, measurement device, multi-objective optimization

List of Symbols and Abbreviations: $C_{pf,\,j}$, Cost of j^{th} measurement device; ERR_V, Relative error of voltage magnitude; ERR_δ, Relative error of voltage phase; $F_{N^{direction}}$, The normalizer function for $N^{direction}$ in the range [0,1]; $F_{N^{connection}}$, The normalizer function for $N^{connection}$ matrix in the range [0,1]; $F_{P^{loss}}$, The normalizer function for P^{loss} matrix in the range [0,1]; $F_{S_{\max}^{flow}}$, The normalizer function for $S_{\max}^{flow}$ matrix in the range [0,1]; $F_{pf,\,j}$, Total priority indices corresponding to line j; h_i, System state function (nonlinear function of the measurements data); J, State estimation problem cost function (Jacobian matrix); m, Number of measurements; n_l and l and n_b, Number of network's branches; $N^{connection}$, Number of lines connected to a line for all network lines matrix; $N^{direction}$, The number of network's lines power flow direction changes matrix; N, Normal distribution function; σ, Normal distribution variance; P_x, Error covariance matrix of estimated states; P_i^{inj}, Injected active power from bus i; P_j^{inj}, Injected active power from bus j; $P_{pf,\,j}$, Binary variable which shows a measurement device located on line j ($P_{pf,\,j}=1$) or not ($P_{pf,\,j}=0$); P_i^D, Active power of bus i; $P_{i,j}^{loss}$, Power losses of line i and line j; P^{loss}, Matrix of network's lines power loss; Q_i^D, Reactive power of bus i; R_Z, Measurements data error covariance matrix; $S_{\max}^{flow}$, Lines' maximum power capacities matrix of distribution network; V_k^t, Real values of voltage magnitude at node k; δ_k^t, Real values of voltage phase at node k; V_k^{es}, Estimated values of voltage magnitude at node k; δ_k^{es}, Estimated values of voltage phase at node k; w_i, Weighting factor for i^{th} measurement data; $\omega_{F_{S_{\max}^{flow}}}$, The weight (worthiness) of $S_{\max}^{flow}$ normalizer function; $\omega_{F_{P^{loss}}}$, The weight (worthiness) of $F_{P^{loss}}$ normalizer function; $\omega_{F_{N^{direction}}}$, The weight (worthiness) of $F_{N^{direction}}$ normalizer function; $\omega_{F_{N^{connection}}}$, The weight (worthiness) of $F_{N^{connection}}$ normalizer function; x, System state variables; z_i, The value that measured by meters; $Z1$, Cost function of measurement devices; $Z2$, Cost function of operational priority; $Z3$, Cost function of voltage magnitude relative error; $Z4$, Cost function of voltage phase relative error; $Z5$, Cost function of quality of measurement; Z, Multi-objective cost function.

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1 | INTRODUCTION

Today, accurate supervision on distribution systems requires high accuracy in estimating the distribution network parameters and acceptable accuracy for distribution system supervision can be achieved by installing measurement devices. In a distribution network, measurement devices can be positioned at every node to estimate the states of the system; however, it is not economically acceptable. On the other hand, these devices may also provide some inaccurate data, which are called measurement error. To solve this issue, a state estimation approach can be used. State estimation can accurately determine the states of the system from the noisy data by taking into account available measured data and the network topology. However, the number of real-time measurements in distribution systems is limited. In this case, a large number of historical data, which are retrieved from a priori knowledge, should be available to maintain observability of the network, as well as for the convergence of the state estimation algorithm. Historical data are called pseudo measurements and their accuracy is also comparatively low. So, the estimation accuracy is not as expected. Consequently, some additional measurement devices require to be located at proper locations in distribution networks to achieve a better estimation accuracy. Some essential tools of state estimation are bad data detection and identification.

In Ref.,[1] the placement of phasor measurement units (PMUs) aiming at establishing a desired level of robustness has been studied. This strategic placement method has also considered passive buses at which zero injections are taken into account as virtual measurements with zero cost and they lead to reduce the number of required PMUs. In Ref.,[2] two graph-theoretic algorithms for locating phasor measurement units in a multi-area power system network have been developed. The method tried to identify its dynamic equivalent model by dividing the transmission network into clusters of synchronous generators and loads. In Ref.,[3] a bad data filter for measurement data has been introduced based on the weighted least square method. In Ref.,[4] a branch current-based state estimation algorithm has been presented. The method picked the magnitude and phase angle of the branch current within a distribution network as the state variables. Also, the effect of type and the location of the measurement on distribution system state estimation have been studied. An analytical technique for meter placement aiming at improving the quality of voltage and angle estimations has been presented in Ref.[5] The meter placement problem has been simplified by transforming it into a probability bound reduction problem. The accuracy of the estimates has been reflected by the area of the error ellipse. Also, the location with the largest area of the covariance error ellipse has been searched as a potential location for meter placement. The procedure sequentially continues until the desired level of accuracy in estimates is reached. To improve this procedure, an ordinal optimization for the meter placement problem has been proposed. The method aims at seeking a set of meter locations minimizing the probability that the peak value of the relative errors in voltage magnitudes and angle estimates through the network exceeds a specified threshold. In Ref.,[6] a meter placement problem for distribution system state estimation using ordinal optimization has been presented. The method tried to find a set of meter locations aiming at minimizing the probability at which the relative errors in voltage magnitudes and angle go beyond a specified threshold. In Ref.,[7] a multi-objective optimization method aiming at seeking the number and location of the measurement devices for accurate distribution system state estimation has been proposed. The objectives, which should be minimized, are the total cost and the average relative percentage error of bus voltage magnitude and voltage angle. However, the cost function has been modeled as a simple and fixed term for each measurement device by assigning pre-determined weighting factors. In this paper, differently from what has been proposed in Ref.,[7] five objective functions are defined where only two objectives, including the minimization of the average relative percentage error of bus voltage magnitude and voltage angle, are similar to the ones proposed in Ref.[7] In Ref.,[8] a meter placement method considering state estimation for distribution networks has been presented. Also, the meter placement has been conducted simultaneously with the distribution network reconfiguration aiming at annual energy loss reduction. The problem has been solved by the multi-objective biased random-key genetic algorithm. In Ref.,[9] a multi-objective mixed-integer linear programming method has been presented for the optimal phasor measurement unit placement problem. The problem aimed at minimizing cost while guaranteeing system observability. In Ref.,[10] a linear formulation for power system state estimation has been proposed in which simultaneously conventional and synchrophasor measurements have been considered. Moreover, the state estimation problem minimized measurement errors. The authors in Ref.[11] evidenced that the judicious design of the neural network training cost function helps with improving the overall distribution network state estimation performance. The method used historical or simulation-derived data to train a shallow neural network. In Ref.,[12] an optimal phasor measurement units' placement method aiming at ensuring observability has been presented. Also, The PMU allocation problem has been optimized based on measurement observability criteria using the branch-and-bound algorithm and binary-coded genetic algorithm. In Ref.,[13] a robust measurement device placement model considering network reconfiguration and the uncertain nature of distributed generation units has been presented. Various weights in the robust measurement placement method have been computed by the Markov chain and analytic hierarchy process. Also, the Gaussian mixture model is implemented to approximate the intermittent power of distributed generations. In Ref.,[14] a circuit representation model to calculate state estimation errors has been presented. Also, the disjunctive model has been used to transform the optimal meter placement problem to a mixed-integer linear programming problem. In Ref.,[15] the performance of state estimation in the power system paradigm in the presence of false data injection attacks has been evaluated. Also, an algorithmic solution has been presented to address the problem of installing additional PMUs considering cyber security constraints. In Ref.,[16] a multi-objective optimization method has been used for pleacement of PMUs and intelligent electronic devices in distribution networks. For accurate state estimation, a hybrid estimation of distribution algorithm aiming at minimizing the total cost of measurement devices has been implemented to find the optimal number and location of them. In Ref.,[17] the optimal PMU placement problem has been assessed in which zonal voltage controllability and voltage magnitude estimation in cooperation with SCADA have been considered. Also, spectral clustering has been employed to efficiently seek the optimal clustering boundaries for placement layout. In Ref.,[18] the placement of phasor measurement units based on the genetic algorithm method has been presented. The method considered observability and security of the network by allocating the least number of PMUs as well as providing the most redundant set of measurements. In Ref.,[7] a multi-objective hybrid particle swarm optimization algorithm for optimal placement of the measurement devices has been presented. The objective functions were the total cost, the average relative percentage error of bus voltage magnitude, and angle. Also, the random variation in loads, the impact of distributed generations, and the metrological error of the measurement devices have been considered. In Ref.,[19] an algorithm for placement of PMUs aiming at making the system observable has been developed. The method focused on the limitation of the number of measurements provided by each PMU.

Moreover, in this paper, the cost objective function is modeled in a different way containing different details and parameters. On the other hand, two new goals are added in the objective functions named operational priorities and quality of placement. Table 1 indicates a comparison between the strategies and objectives presented in the literature as well as the ones proposed in this paper.

To the best of our knowledge, the quality of error in the distribution system state estimation problem has not been considered as an objective. In this paper, the quality of state estimation error as an objective is taken into account within the measurement devices placement problem. This objective may be in contrast with the other objectives

TABLE 1 A comparison between objective functions and strategies

	Objective				
Refs.	Decrease in number of measurement devices	Reliability	Increase in accuracy of estimated values	Cost	Strategies
6			✓		Probabilistic model based on error of state estimation values
13	✓		✓	✓	Robust and fast algorithm for measurement devices placement
14	✓		✓	✓	To model measurement devices using circuit elements
15		✓	✓		Considering measurement data security
16	✓		✓	✓	Multi-objective optimization
17		✓	✓		Probabilistic multi-objective optimization
18	✓	✓		✓	Using genetic algorithm
7	✓		✓	✓	Probabilistic model based on error of state estimation values
19	✓				A measurement devices placement algorithm
This Paper	✓	✓	✓	✓	A measurement devices placement algorithm

because it is achieved while the number of measurement devices increases. Also, the operational priorities indices including power losses monitoring, high capacity lines, the number of connected lines to a main line, and the variation in power flow direction should be maximized by installing measurement devices. Moreover, relative errors of voltage magnitude and angle as an objective are minimized. In addition to the two objectives, the quality of state estimation variables as a new objective is maximized. In summary, the main target of this paper is to optimally allocate measurement devices of active and reactive power within the network aiming at improving the observability index. Accordingly, the five objective functions of the proposed method can be summarized as (a) minimizing the allocation costs of measurement devices, (b) maximizing the operational priorities indices, (c) minimizing the relative error of the voltage magnitude for each network nodes, (d) minimizing the relative error of the voltage angle for each network nodes, and (e) maximizing the quality of measurement devices allocation. The contributions of this paper are summarized as follows:

- To implement a multi-objective evolutionary algorithm based on decomposition to model the placement problem of measurement devices.
- To improve the measurement devices placement by introducing the new objective called the quality of the placement.
- To consider distribution system operators' attitudes and requirements in location of measurement devices.

The rest of the paper is organized as follows: The problem of distribution system state estimation is described in Section 2. The formulation of the problem is given in Section 3. In Section 4, the proposed meter placement flow is presented. The simulation and results are discussed in Section 5, and finally, the conclusion of this paper is presented in Section 6.

2 | DISTRIBUTION SYSTEM STATE ESTIMATION

Generally, the weighted least square (WLS) method is used to solve the state estimation problem. In this paper, however, the particle swarm optimization is implemented to solve the state estimation problem because this method can be more accurate than the WLS method.[20,21] Nevertheless, the objective function of the distribution system state estimation problem can be defined as follows:

$$\min J(x) = \sum_{i=1}^{m} w_i (z_i - h_i(x))^2. \tag{1}$$

In this problem, the magnitude and phase of lines' currents indicate the system state variables [x in Equation (1)] and the active and reactive power flows measured by devices are considered as the real measurement data. In Figure 1, pseudo measurements data modeling by artificial neural network (ANN) is illustrated. By using historical data of load consumptions, a load model is generated using multiplayer perceptron neural network, which is considered as pseudo

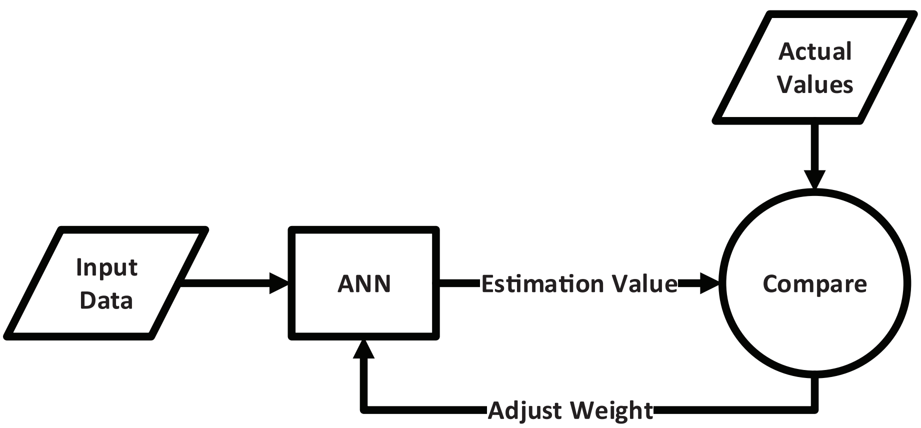


FIGURE 1 Pseudo measurements data modeling by ANN

measurement data in the state estimation. The input data and actual values are active and reactive power injection and load demand in each node, respectively.[22] Historical data of load demands are modeled using white noise with 50% and 20% variances as given in Equations (2) and (3).[5]

$$P_{i,new}^{D} = P_i^D + N(0,\tilde{\sigma}^2) \tag{2}$$

$$Q_{i,new}^{D} = Q_i^D + N(0,\tilde{\sigma}^2), \tag{3}$$

where $N(0, \sigma^2)$ represents normal distribution function with mean zero and varianceσ^2.

3 | PROBLEM DEFINITION

The main objective of this paper is to optimally allocate active and reactive power flow measurement devices within the network aiming at improving the observability index. Moreover, the measurement devices cost and the state estimation accuracy are considered. On the other hand, the quality of the state estimation output should be maximized, which is mainly conflicting with minimizing measurement devices cost function. The five objectives of the problem are described as follows.

3.1 | Cost of measurement devices

The cost is always an important aspect of placement problems. Hence, in this paper, one of the objective functions is the measurement devices placement cost which should be minimized. It is assumed that all devices have the same capital cost.[7]

$$Z1 = \min \sum_{j=1}^{n_l} C_{pf,j} P_{pf,j}. \tag{4}$$

If a device is allocated on a network branch, the values of $P_{pf,\ j}$ are 1, otherwise is 0. Placement costs of measurement devices do only include capital costs of devices.

3.2 | Operational priorities indices

The available method for measurement devices placement presented in the literature may not cover the requirements and priorities of the experts who work in operation, planning, and decision-making sections of utilities. In addition to cost and observability objectives, there are some important aspects for distribution operators according to which they prefer to have the measurements in specific nodes in their networks. The aspects have been called operational priorities and described as follows:

1 Usually, lines with high power capacity are important to the operators because a considerable share of current flows through these lines. As a result, measurements in lines with higher capacity can give a better monitoring view to the operators;
2 One of the critical issues in distribution networks is power losses, which operators try to continuously monitor and reduce. So, a measurement in lines with higher power losses may help operators to reach the goal;
3 Regarding protection issues, measurements in lines in which the power flow direction frequently changes are also important for the operator;
4 From the operator point of view, the main lines, which directly feed many connected lines, are important. So, they prefer to have a measurement in these lines.

In this paper, according to the distribution operators' attitudes and viewpoints, a term called operational priorities has been added in the objective function. As the importance of each item among the priorities may be different for operators, weighting factors are assigned to them.

3.2.1 | Line power capacity

To monitor the power flow in distribution networks, the measurement data of lines with high capacity may be helpful. So, the line with high power flow has a higher priority for installing measurement devices.[23] This term is imported as an option in total operational priorities' objective function of the placement problem. Let assume the maximum power capacities of lines ($S_{\max}^{flow}$) are represented by the following matrix.

$$S_{\max}^{flow} = \left[s_{1,\max}^{flow} \; s_{2,\max}^{flow} \; \cdots\right]_{1\times l}. \tag{5}$$

To model the maximum power capacity of lines as a term in operational priorities' objective function, Equation (6) is defined to normalized this term between [0,1].

$$F_{S_{\max}^{flow}} = \frac{\max\left(S_{\max}^{flow}\right) - S_{\max}^{flow}}{\max\left(S_{\max}^{flow}\right) - \min\left(S_{\max}^{flow}\right)}. \tag{6}$$

3.2.2 | Power losses

Power loss reduction is an important mission of electric utilities. Accurate as well as appropriate measurement data through the network provide more accurate power loss calculation, which helps the operator with deploying loss reduction plans like distributed generation sitting and sizing. So, the line with high power losses needs more attention and accurate measurement.[24] The power losses of lines are calculated according to Equation (7).

$$P_{i,j}^{loss} = \left|P_i^{inj} - P_j^{inj}\right|, \tag{7}$$

where i and j are the number of buses. All of lines power losses are collected in a matrix as given by Equation (8).

$$P^{loss} = \left[p_1^{loss} \;\; p_2^{loss} \;\; \cdots\right]_{1\times l}. \tag{8}$$

Then, the power loss is modeled as a normalized term in the operational priorities' function by Equation (9).

$$F_{P^{loss}} = \frac{\max\left(P^{loss}\right) - P^{loss}}{\max\left(P^{loss}\right) - \min\left(P^{loss}\right)}. \tag{9}$$

3.2.3 | Change in power flow direction

Due to the presence of protection devices in a network, the change in power flow direction through a line is important. Let assume a line in which the current direction changes due to small variations in loads. Protection devices may measure this variation as a fault and send a trip command. On the other hand, the high number of changes in power flow direction in a line shows the strategic position of the line within a network. So, the measurement of this type of lines is useful and has a higher priority in installing measurement devices.[25] So, the pseudo code in Algorithm 1 is used to calculate the number of changes in power flow direction through a line.

Algorithm 1 Pseudo code for calculating the number of power flow direction changes

```
SET initial iterator to 1.
SET an empty array of changes in the power flow direction counter.
WHILE iterator is lesser than 10 000:
    Generate white noise with zero mean and 50% SD.
    Add the generated white noise to each network's node including an active load.
    Run the distribution power flow with these loads.
    Extract the sign of line power flows.
    SET our initial InternalIterator to 1.
    WHILE InternalIterator is lesser than the number of network lines:
      IF the line power flow is positive then:
          increase our counter by 1.
      IF NOT:
          decrease our counter by 1.
      END IF
      increase InternalIterator by 1.
    END WHILE
    increase iterator by 1.
END WHILE
```

The output is a matrix with integer values corresponding to each line as given in Equation (10).

$$N^{direction} = \left[n_1^{direction} \quad n_2^{direction} \quad \dots \right]_{1 \times l}. \tag{10}$$

Then, the changes in power flow direction are modeled as a normalized term in the operational priorities' objective function by Equation (11).

$$F_{N^{direction}} = \frac{\max\left(N^{direction}\right) - N^{direction}}{\max\left(N^{direction}\right) - \min\left(N^{direction}\right)}. \tag{11}$$

3.2.4 | The number of lines connected to a line

In a radial distribution network, a line connected to more lines has a larger capacity to flow power and is considered as the main line of the network. So, the accurate measurement of these lines is important and has a priority to install measurement devices.[13] To calculate the number of lines connected to a line, the pseudo code shown in Algorithm 2 is used.

Algorithm 2 Pseudo code for calculation of the number of lines connected to a line

```
extract FromBusMatrix which is a matrix array of sending buses from the network.
extract ToBusMatrix which is a matrix array of receiving busses from the network.
SET iterator to 1.
initialize the LinesConnectionMatrix which represents the number of lines connected to a line.
initialize FIT, FIF, TIT, and TIF matrixes which have the same size of network lines.
WHILE iterator is lesser than the number of FromBusMatrix elements:
    FIT(iterator) ← find how many items of ToBusMatrix is the same as FromBusMatrix(iterator).
    FIF(iterator) ← find how many items of FromBusMatrix is the same as FromBusMatrix(iterator).
    TIT(iterator) ← find how many items of ToBusMatrix is the same as ToBusMatrix (iterator).
    TIF(iterator) ← find how many items of FromBusMatrix is the same as ToBusMatrix (iterator).
    LinesConnectionMatrix(iterator) ← FIT(iterator)+ FIF(iterator)+ TIT(iterator)+ TIF(iterator)
    increase iterator by 1.
END WHILE
```

The Algorithm 2 results in a matrix with real values shown in Equation (12).

$$N^{connection} = \left[n_1^{connection} \quad n_2^{connection} \quad \dots \right]_{1 \times l}. \tag{12}$$

Then, the normalized value for this parameter can be calculated as given in Equation (13).

$$F_{N^{connection}} = \frac{\max\left(N^{connection}\right) - N^{connection}}{\max\left(N^{connection}\right) - \min\left(N^{connection}\right)}. \tag{13}$$

Now, the four parameters which are normalized in the range of [0, 1] can be gathered in a coefficient named operational consideration index which is given in Equation (14). Also, from the operators' point of view, some of these parameters may have more importance. So, a weighting coefficient (ω) is defined corresponding to each priority.

$$F_{pf} = \omega_{F_{S_{\max}^{flow}}} F_{S_{\max}^{flow}} + \omega_{F_{ploss}} F_{P^{loss}} + \omega_{F_{N^{direction}}} F_{N^{direction}} + \omega_{F_{N^{connection}}} F_{N^{connection}}. \tag{14}$$

The objective corresponding to operational priority ($Z2$) which should be maximized is given as follows

$$Z2 = \max \sum_{j=1}^{n_l} F_{pf\,j} P_{pf\,j}. \tag{15}$$

3.3 | Relative errors of voltage magnitude and phase

The relative error for the voltages' magnitude and phase are defined as Equations (16) and (17), respectively.

$$ERR_V = \frac{100}{n_b} \sum_{k=1}^{n_b} \left| \frac{V_k^t - V_k^{es}}{V_k^t} \right| \tag{16}$$

$$ERR_\delta = \frac{100}{n_b} \sum_{k=1}^{n_b} \left| \frac{\delta_k^t - \delta_k^{es}}{\delta_k^t} \right|. \tag{17}$$

The relative error is calculated using scenarios corresponding to different states and conditions of the network and created by a random Monte Carlo simulation. If in 95% of scenarios, the relative error is lower than a threshold value, the placement is acceptable. In this paper, the relative error threshold values of the voltage magnitude and voltage angle are 5% and 1%, respectively.[6] Also, the minimization of voltage magnitude and voltage phase relative error represents the two other objectives, which are formulated as Equations (18) and (19).

$$Z3 = \min ERR_V \tag{18}$$

$$Z4 = \min ERR_\delta. \tag{19}$$

3.4 | Quality of placement

The quality of measurement devices allocation is defined as follows.[5]

$$Z5 = \max Q = \max \frac{1}{P_x}, \tag{20}$$

where P_x is obtained from Equation (21); Q is calculated by reversing each element of the matrix P_x.

$$P_x = \left(H^T(x)R_Z^{-1}H(x)\right)^{-1}, \tag{21}$$

where R_Z is the inverse of the w_i in Equation (1) and H is a Jacobian matrix of measurement function that is defined as Equation (22).

$$H(x) = \frac{\partial h(x)}{\partial x}. \tag{22}$$

To improve the quality of estimated values, the number of measurement devices should be increased. Therefore, the quality objective function ought to be maximized. Actually, the problem is a trade-off between the cost and the quality. According to the above-mentioned objective functions, the multi-objective problem is formulated as a vector with five dimensions, which is given in Equation (23).

$$Z = \begin{bmatrix} \min \sum_{j=1}^{n_l} C_{pf\,j} P_{pf\,j} \\ \max \sum_{j=1}^{n_l} F_{pf\,j} P_{pf\,j} \\ \min \frac{100}{n} \sum_{k=1}^{n_b} \left| \frac{V_k^t - V_k^{es}}{V_k^t} \right| \\ \min \frac{100}{n} \sum_{k=1}^{n_b} \left| \frac{\delta_k^t - \delta_k^{es}}{\delta_k^t} \right| \\ \max \frac{1}{\left(H^T(x)R_Z^{-1}H(x)\right)^{-1}} \end{bmatrix}. \tag{23}$$

Subject to

$$\left| \frac{V_k^t - V_k^{es}}{\delta_k^t} \right| \le 0.01, \left| \frac{\delta_k^t - \delta_k^{es}}{\delta_k^t} \right| \le 0.05, \tag{24}$$

where Z is the multi-objective function of the placement problem which is optimized by the evolutionary algorithm based on decomposition. The maximum relative percentage deviation in voltage angle and voltage magnitude is lower equal than 5% and 1%, respectively.[5]

4 | MULTI-OBJECTIVE OPTIMIZATION

In this paper, the state estimation problem has been formulated by PSO. To model the measurement device placement problem, the multi-objective evolutionary algorithm based on decomposition (MOEA/D) has been implemented. The procedure of the proposed placement method is illustrated in Figure 2.

According to Figure 2, in the first step of layer one, the parameters of MOEA/D are set to use in the algorithm. In step 2 of layer one, the equations related to vector Z are defied. In step 3 of layer one, the initial population for MOEA/D is produced and after evaluating them, the primary Pareto front is generated. In the main loop of the algorithm, a crossover is carried out between populations in order to generate new children. Then, the generated children called the new generation are analyzed and compared with the previous generation; after that, the Pareto front is updated. This procedure is repeated until the termination target is achieved. In layer 3, which corresponds to the multi-objective Z vector definition, the measurement data and variables produced by MOEA/D are received in steps 1 and 2, respectively. Then, in step 3, K number of scenarios related to measurement data errors are created. In layer 4 and

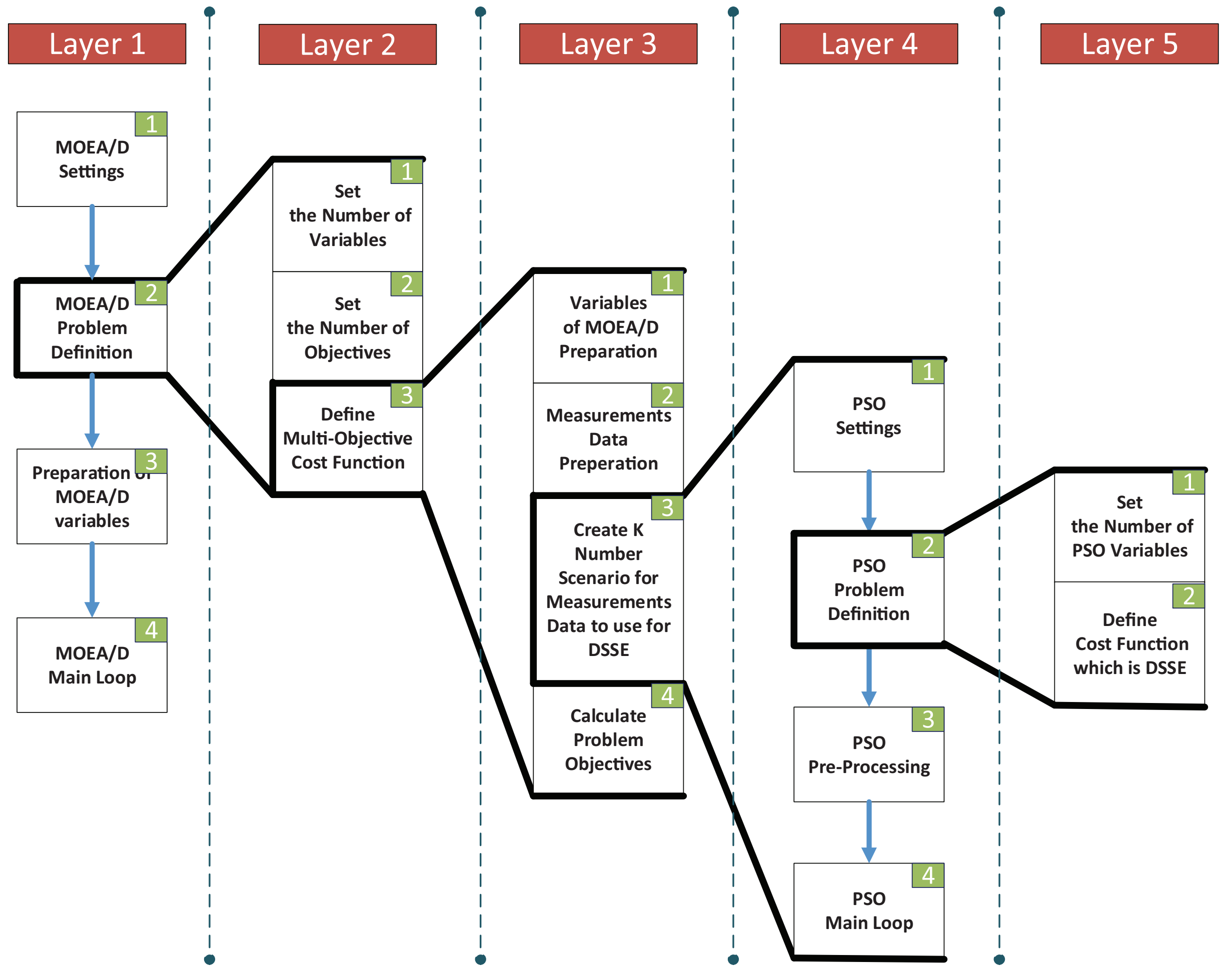


FIGURE 2 Diagram of the proposed measurement device placement method based on state estimation

layer 5, the state estimation problem is optimized by PSO. In multi-objective problems, the final step is to select a solution among several Pareto fronts. This selection is desirably carried out by each user. However, there are some selection methods called multiple attribute decision-making (MADM) that help users. The MADM includes several methods to make a preference decision over the available solutions. A decision-making process can be categorized into three general parts including evaluation, prioritization, and selection. Three scoring methods used in many kinds of literature are the simple additive weighting method (SAW),[26] the weighted product method (WPM),[27,28] and the technique for order preference by similarity to ideal solution (TOPSIS).[29]

It is worth to be noted that the configuration distribution network may be changed to improve the system performances in terms of voltage regulation, system reliability, and power losses. The network reconfiguration has not been directly considered within the proposed method. However, the proposed method can be conducted for different configurations of the distribution network. Then, the distribution system operator, as a decision-maker, can compare and analyze the optimum place of measurement devices corresponding to each configuration in terms of probability of occurrence. As future work, distribution network reconfiguration can be considered within the proposed method. The idea would be to assign the weight or probability of occurrence to each configuration.

In this paper, generating the Pareto front is the main goal of the proposed measurement device placement method and the selection of one of the Pareto solutions is done only for analyzing the results. It means that the final decision is made by distribution system operators as decision-makers based on their priorities and preferences, not by the analyst. However, the evaluation part of this method is based on TOPSIS method.

It should be noted that the measurement device placement problem has been carried out in offline mode. So, the run time, as well as network scalability, is not as a challenge. Also, the number and the location of sensors are defined as variables and determined based on the optimization algorithm. In order to show the scalability of the problem, the

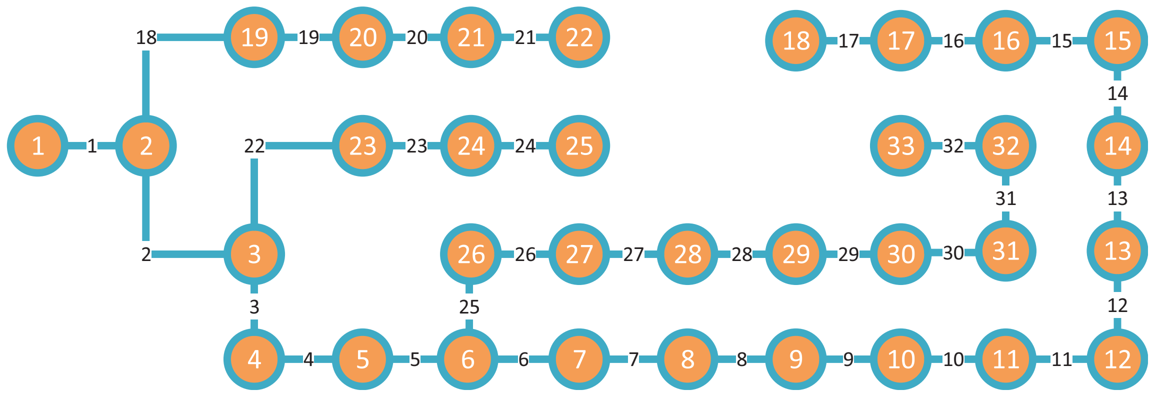


FIGURE 3 IEEE 33-bus test network

proposed method has been also tested on a large distribution network and the results have been presented in the simulation section.

5 | CASE STUDY

In this paper, the IEEE 33-bus radial distribution network[30] illustrated in Figure 3 is used to evaluate and analyze the proposed method. Nine best solutions among 50 solutions in Pareto front in different scenarios depending on measurements' error are presented in Table 2.

5.1 | Analysis of results

As shown in Table 2, lines 2, 21, 29, and 32 are important in placing the measurement devices. Line 21 and line 32 experience maximum number of changes in power flow direction; so, they also have the priority in installing measurement devices. Line 3 is also important because the bus 3 supplies more than half of the network. The line number 29 is also important due to the high capacity of active and reactive power as well as high power losses. In Figure 4, the operational priorities corresponding to the cost of measurement devices placement in all scenarios are shown.

According to Figure 4A, the operational priorities have a direct relation with the measurement devices allocation cost. This direct relation is also approximated by a linear and 4th-degree curve. On the other hand, the norm of the real and estimated values of priorities and the costs corresponding to the number of measurement devices are illustrated in Figure 3B. As shown, the 4th-degree approximation curve with a lower norm than linear approximation provides an accurate assessment of the relation between priorities and the cost of measurement devices.

5.2 | Placement quality analysis

As shown in Table 2, the Pareto number 17 and 5 in scenario 1 have the same values in the measurement devices place and priorities. However, the Pareto 17 has achieved a better value in relative error percentage of voltage magnitude and phase if compared with the one in Pareto 5. According to the results given in Table 2, the placement quality corresponding to estimated values of voltage magnitude and phase in scenario 3 is better than the one in the other scenarios.

The relation between the quality of state estimation and cost of measurement devices has been illustrated in Figure 5.

As shown in Figure 5A, some approximations for the relation between the state estimation quality and the number of devices in scenario 1 have been drawn. The result showed that the quality index has been significantly raised due to the increase in the number of measurement devices. According to Figure 5B, the 3rd-degree (Cubic) approximation has the lower norm than the one in 4th-degree, 2nd-degree (Quadratic), and linear approximations.

TABLE 2 Simulation results for top 9 Pareto

Scenario	Pseudo measurement data error (%)	Actual measurement data error (%)	Pareto number	Line number of measurement device location	Objective function's values: Operational priorities	Allocation Cost	Voltage magnitude relative error (%)	Voltage angle relative error (%)	Quality
1	50	3	17	2,18,21,29,32	0.786	5	0.062	0.18	6 193 631
			6	2,8,9,17,19,21,23,24,29,32	1.249	10	0.066	0.172	6 251 634
			25	2,3,5,10,12,18,21,24,25,29,32	1.705	11	0.059	0.167	6 158 703
			18	16,17,18,21,29,32	0.535	6	0.073	5.604	6 197 340
			5	2,18,21,29,32	0.786	5	0.064	2.412	6 147 578
			3	2,12,21,29,32	0.778	5	0.072	2.79	6 205 497
			7	2,4,18,21,29,32	0.986	6	0.057	0.809	6 168 935
			35	2,4,6,13,21,29,30,32	1.192	8	0.052	0.438	6 206 521
			4	2,3,12,17,21,29,32	1.084	7	0.058	0.646	6 170 107
2	50	1	1	6,9,11,21,22,28	0.624	6	0.06	1.733	5 502 006
			40	11,21,22,28	0.422	4	0.065	6.57	5 528 269
			21	6,9,11,21,22,28	0.624	6	0.061	5.359	5 540 717
			3	3,4,5,15,16,19,21,30,31	1.277	9	0.057	2.747	5 526 346
			6	6,8,14,19,21,23,25,26,28	0.938	9	0.065	3.531	5 546 269
			13	3,4,5,10,12,14,15,16,19,21,30,31	1.5	12	0.06	2.765	5 521 234
			31	3,4,5,10,12,15,16,19,21,28,30,31	1.566	12	0.071	2.339	5 521 076
			5	6,8,9,13,14,21,22,26,28,32	0.978	10	0.066	4.081	5 543 178
			4	1,4,7,8,11,12,16,17,20,21,24,27,29,31	1.547	14	0.069	5.899	5 531 704
3	20	3	28	2,3,5,12,14,16,20,21,32	1.461	9	0.052	0	6 136 349
			7	3,4,5,21,32	0.916	5	0.059	0	6 189 908
			20	3,4,5,6,16,21,23,25,28,29,31,32	1.723	12	0.042	0	6 226 931
			18	2,3,20,21,25,32	1.007	6	0.04	0.085	6 176 973
			5	3,5,20,21,23,32	0.931	6	0.05	0.198	6 192 494
			8	3,4,5,20,21,23,32	1.13	7	0.052	0.111	6 237 443
			6	3,4,5,21,22,23,32	1.179	7	0.046	0.125	6 227 611
			1	3,4,19,21,23,29,31	0.93	7	0.048	0.896	6 203 677
			25	3,4,8,20,21,22,25,32	1.006	8	0.057	1.023	6 225 639
4	20	1	15	2,4,7,9,13,21,26	1.059	7	0.051	0.494	5 270 989

TABLE 2 (Continued)

Scenario	Pseudo measurement data error (%)	Actual measurement data error (%)	Pareto number	Line number of measurement device location	Objective function's values				
					Operational priorities	Allocation Cost	Voltage magnitude relative error (%)	Voltage angle relative error (%)	Quality
			20	2,4,7,15,21	0.869	5	0.055	1.57	
			9	1,2,4,7,15,21,30	1.191	7	0.05	0.553	5 317 455
			29	2,4,7,13,16,21	0.955	6	0.047	1.373	5 282 134
			1	1,2,3,4,7,9,14,17,21,27	1.631	10	0.056	0.315	5 253 647
			13	2,3,7,13,16,20,21,24,25	1.276	9	0.057	0.4	5 324 565
			22	2,7,9,13,21,24	0.844	6	0.054	2.023	5 346 074
			14	2,7,8,13,20,21,30,32	1.018	8	0.053	0.666	5 307 198
			5	2,3,7,13,20,21,24	1.072	7	0.052	1.103	5 333 563

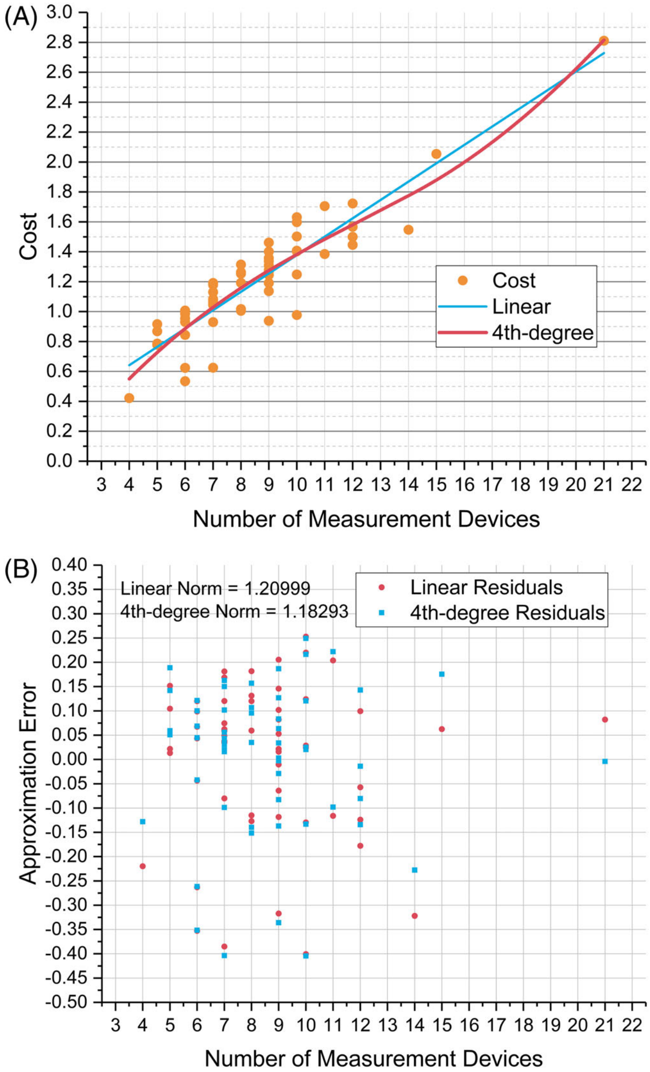


FIGURE 4 Tracking behavior of measurement devices cost with their priorities

5.3 | Analysis of voltage magnitude and phase

In this part, the state estimation is carried out based on the result of the measurement device placement, which has been provided by MOEA/D. The PSO method is used to optimize the state estimation problem, which their parameters are summarized in Table 3. Hence, an overall analysis of this estimator has been presented in Table 4. In cases of 20% error in pseudo measurements, the relative error of voltage magnitude and voltage phase is lower than the case with 50% error in pseudo measurements. However, the main focus of the measurement devices allocation was to minimize the total cost. According to Table 4, the relative error of the voltage phase is zero; also, the maximum value of the relative error is almost 1%, which is lower than the one in other scenarios. To accurate analyze of the state estimation, the actual and estimated values of voltage magnitude and voltage phases have been presented in Figure 6. As shown in Figure 6, the estimator can accurately estimate the system states with the minimum number of measurement devices. In Figure 7A, it is clear that the relative error of voltage magnitude has experienced a declining trend with increasing the number (cost) of measurement devices. However, utilization of a large number of measurement devices has not a good economic result. The trade-off between the relative error value of voltage and number of measurement devices is determined by system operators.

The proposed method provides an accurate estimation of system states even with a low cost of measurement devices. The robustness of this estimator can be verified in Figure 7. The curve of voltage magnitude relative error vs the cost of measurement devices is approximated with 3rd-degree (cubic) and linear polynomial curves that are illustrated in Figure 7A. The 3rd-degree approximation has a lower norm value than the one in the linear approximation. Because of the similarity of these norms, they both can follow the behavior of the original curve. However, according to

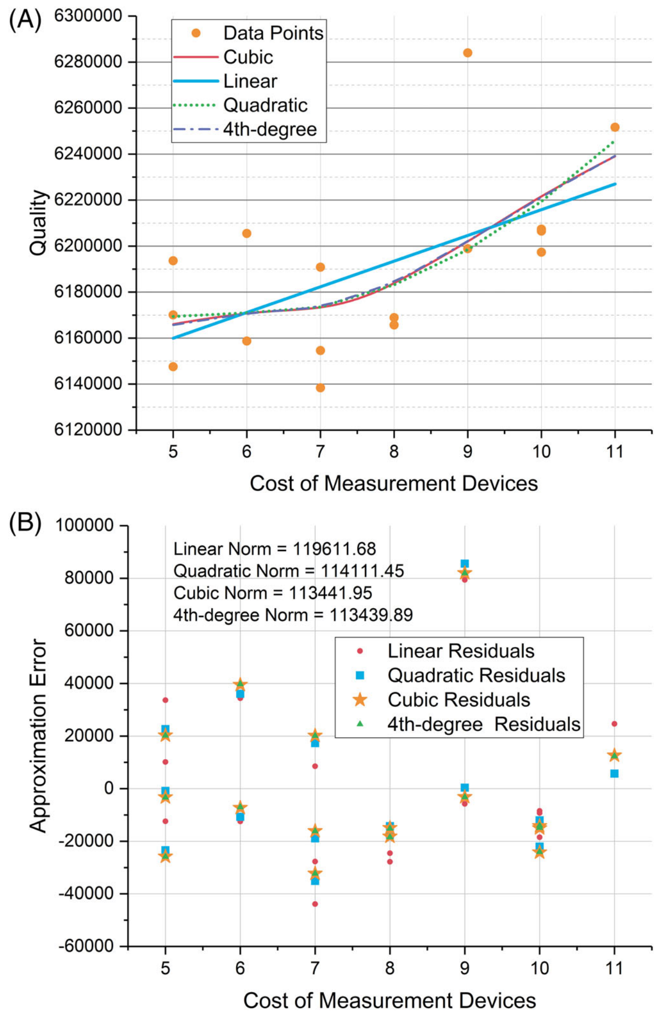


FIGURE 5 Tracking behavior of measurement devices cost with allocation quality

TABLE 3 PSO parameters setting

Nomenclature	Description	Value
n_{pop}	Population size (Swarm size)	50
n_{iter}	Maximum number of generation (Termination condition)	10
φ_1	Personal acceleration coefficient	2.05
φ_2	Social acceleration coefficient	2.05
κ	Constriction coefficient	1
$\chi = \frac{2\kappa}{\left\|2-(\varphi_1+\varphi_2)-\sqrt{(\varphi_1+\varphi_2)^2-4(\varphi_1+\varphi_2)}\right\|}$	Inertia coefficient	0.73
ω_{damp}	Damping ratio of inertia	0.99

Figure 7C, the 4th-degree approximation of voltage phase relative error vs the number of measurement devices curve has better norm value if compared with the one in the linear approximation.

5.4 | Confidence region

In statistics, a confidence region is defined as an n-dimensional generalization of a confidence interval. It is a set of points in an n-dimensional space, often illustrated as an ellipsoid around a point that is an estimated solution to a

TABLE 4 Distribution system state estimation evaluation

Scenario	Pseudo measurement data error (%)	Actual measurement data error (%)	Maximum voltage magnitude relative error (%)	Minimum voltage magnitude relative error (%)	Maximum voltage angle relative error (%)	Minimum voltage angle relative error (%)
1	50	3	0.073	0.052	5.604	0.167
2	50	1	0.071	0.057	6.57	1.733
3	20	3	0.059	0.04	1.023	0
4	20	1	0.061	0.047	5.863	0.306

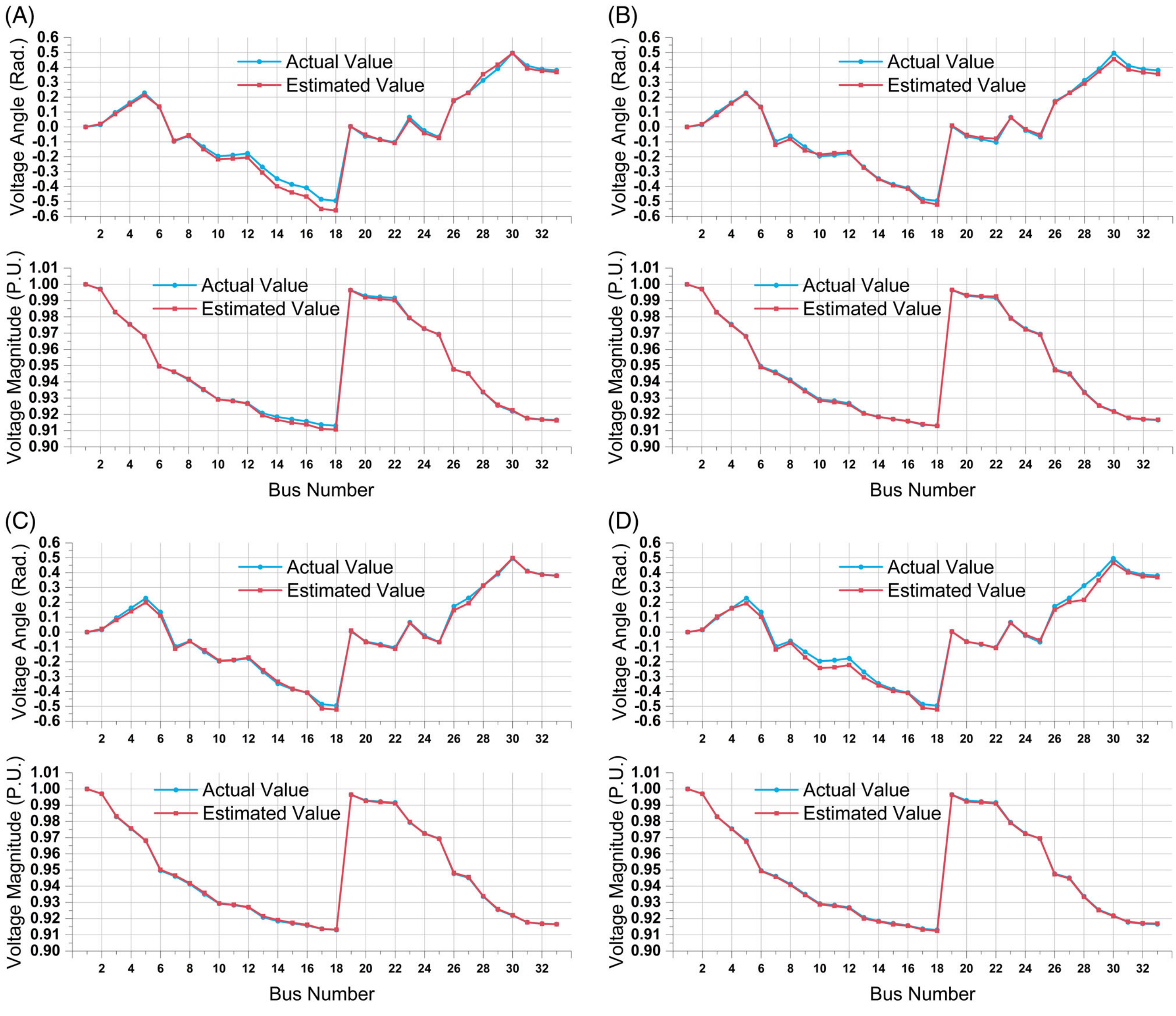


FIGURE 6 Estimation and actual values for voltage magnitude and voltage angle of best Pareto

problem. Moreover, a confidence interval is computed from the statistics of the observed data. It may contain the true value of an unknown population parameter, which has an associated confidence level. The confidence level measures the level of confidence that the parameter lies in the interval.[31,32] In a mono-dimensional problem, the confidence interval is represented as two parallel lines around an estimated solution. On the other hand, in a multi-dimensional problem, a region is defined for confidence instead of a line. Afterward, the confidence region is a set of interconnected

FIGURE 7 The curve of relative error for voltage magnitude and voltage angle with cost of measurement devices

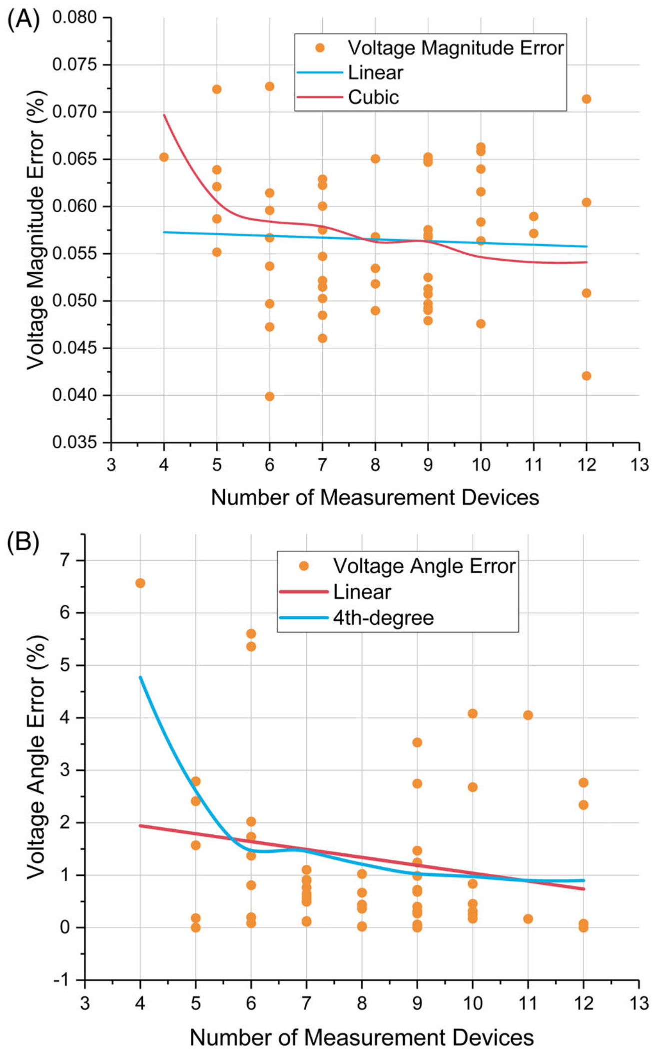
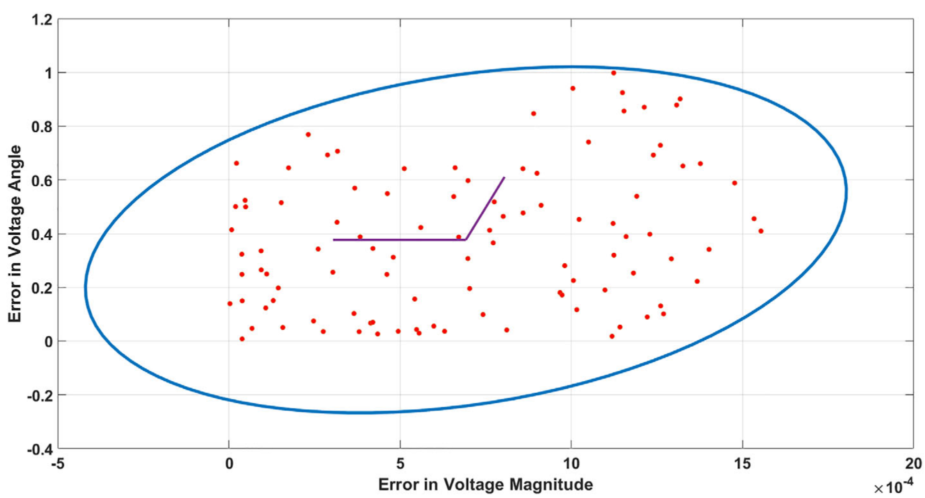


FIGURE 8 Confidence region for voltage magnitude and voltage angle relative error



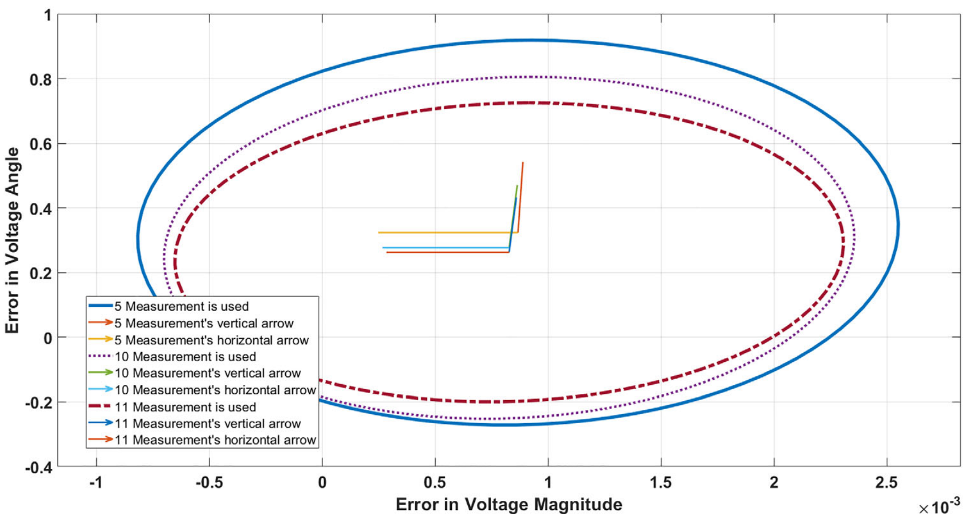


FIGURE 9 Changes in error ellipsoid area by changes in number of measurement devices

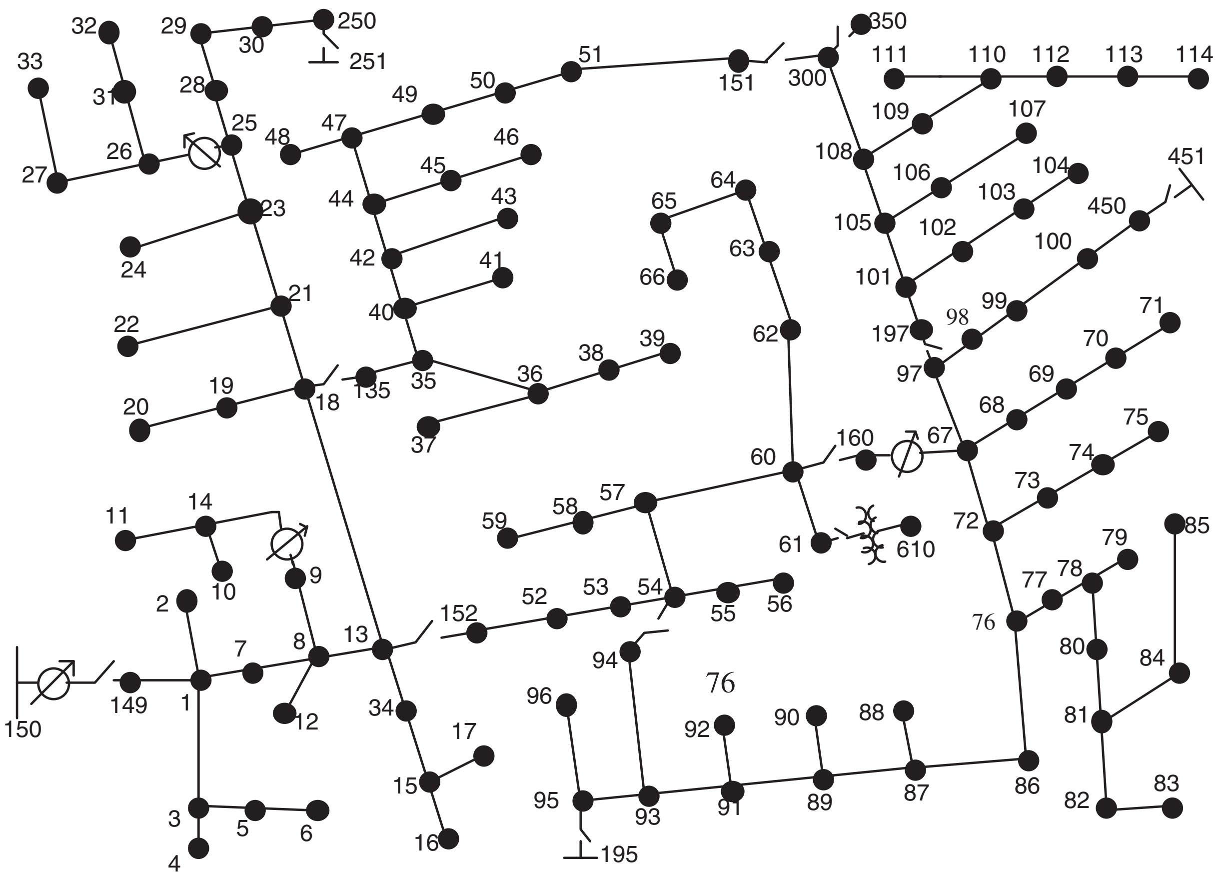


FIGURE 10 IEEE 123-node test feeder

points often represented by an ellipsoid shape for an estimated value of a problem's solution. The ellipsoid area is drawn using covariance between the dimensions. So, if the covariance between observed data was zero, the ellipsoid area would be zero and when the covariance increases, the area grows too. Hence, about the relative error in voltage as mentioned in Section 3.3, K = 100 is the number of scenarios created by Monte Carlo simulation based on different network

TABLE 5 Line segment data

From bus	To bus	Line number	From bus	To bus	Line number
1	2	1	63	64	63
1	3	2	64	65	64
1	7	3	65	66	65
3	4	4	67	68	66
3	5	5	67	72	67
5	6	6	67	97	68
7	8	7	68	69	69
8	12	8	69	70	70
8	9	9	70	71	71
8	13	10	72	73	72
9	14	11	72	76	73
13	34	12	73	74	74
13	18	13	74	75	75
14	11	14	76	77	76
14	10	15	76	86	77
15	16	16	77	78	78
15	17	17	78	79	79
18	19	18	78	80	80
18	21	19	80	81	81
19	20	20	81	82	82
21	22	21	81	84	83
21	23	22	82	83	84
23	24	23	84	85	85
23	25	24	86	87	86
25	26	25	87	88	87
25	28	26	87	89	88
26	27	27	89	90	89
26	31	28	89	91	90
27	33	29	91	92	91
28	29	30	91	93	92
29	30	31	93	94	93
30	250	32	93	95	94
31	32	33	95	96	95
34	15	34	97	98	96
35	36	35	98	99	97
35	40	36	99	100	98
36	37	37	100	450	99
36	38	38	101	102	100
38	39	39	101	105	101
40	41	40	102	103	102
40	42	41	103	104	103
42	43	42	105	106	104
42	44	43	105	108	105

(Continues)

TABLE 5 (Continued)

From bus	To bus	Line number	From bus	To bus	Line number
44	45	44	106	107	106
44	47	45	108	109	107
45	46	46	108	300	108
47	48	47	109	110	109
47	49	48	110	111	110
49	50	49	110	112	111
50	51	50	112	113	112
51	151	51	113	114	113
52	53	52	135	35	114
53	54	53	149	1	115
54	55	54	152	52	116
54	57	55	160	67	117
55	56	56	197	101	118
57	58	57	13	152	119
57	60	58	18	135	120
58	59	59	60	160	121
60	61	60	61	610	122
60	62	61	97	197	123
62	63	62	150	149	124
			610	1005	125

conditions and the relative error has been calculated for the scenarios.[5] If 95% of the relative errors were lower than a threshold, the estimator and the solution are acceptable. The concept behind this theory means if the probability of relative errors of voltage magnitude and voltage phase decreases to 95%, then the optimal solution can be found. Actually, an interval with 95% confidence is considered. But, due to the relative error have two dimensions, a region should be used instead of an interval. This problem has two viewpoints. The first one is the number of relative errors calculated for the probabilistic scenarios, which should be less than 95%. The formulation related to the confidence region is represented in the Appendix A. Figure 8 depicts the confidence region for the relative error of voltage magnitude and voltage phase at bus 20 in scenario 4. As shown, all samples are in the confidence zone, which demonstrates the robustness of the proposed method in allocating measurement devices. On the other hand, another sample of a confidence region for bus 28 in scenario 1 is illustrated in Figure 9. It is clear that if the number of measurement devices increases, the ellipsoid area would be smaller. It means that the boundary of this area gets narrower. This event is the result of low covariance between voltage magnitude and voltage phases' relative errors and can be proofed by Figure 6. According to Figure 9, when the number of measurement devices increases from 5 to 10, the area of the ellipsoid is more affected if compared with the case in which the number of measurement devices increases from 10 to 11.

5.5 | Simulation results on IEEE 123-bus test network

To evaluate the scalability and robustness of the proposed method, the simulation has been also conducted on the IEEE 123-bus test system as a large distribution network.[33] The single diagram of the network has been shown in Figure 10 where the line number has been given in Table 5. The simulation results for scenario 4 and the five best Pareto solutions have been given in Table 6. As shown, the lines 6, 9, 15, 19, 28, 40, 41, 57, 58, 69, 71, 74, 86, 87, 92, 107, 110, 113, 113, and 123 are common in all five Pareto solutions. For example, the lines 113, 110, 87, and 71 have high power losses and the lines 114 and 87 have large capacities. So, the measurement data of these lines are necessary for achieving the pre-defined

TABLE 6 Simulation results for top 5 Pareto

Pareto number	Line number of measurement device location	Number of measurement devices	Objective function's values			
			Allocation cost	Voltage magnitude relative error (%)	Voltage angle relative error (%)	Quality
13	1,2,5,6,8,9,12,15,16,17,18,19,24,26,27,28,29,30,31,32,3 3,34,36,37,39,40,41,43,44,47,52,53,57,58,62,65,68,69, 71,74,75,77,79,82,86,87,88,89,90,92,94,96,97,99,103,10 5,107,110,111,112,113,114,119,120,121,122,123	67	1,079	0,129 536	0	1 589 848
9	1,6,8,9,11,15,16,18,19,20,21,22,28,35,36,37,39,40,41,4 4,45,50,53,54,57,58,64,66,68,69,70,71,72,74,75,77,82,8 3,86,87,88,91,92,93,95,98,100,101,107,110,113,114,121,122,123,125	56	0,921	0,129 322	0	1 261 926
10	1,2,5,6,8,9,12,14,15,16,17,18,19,24,26,27,28,29,30,31,3 2,33,34,36,37,39,40,41,43,44,47,48,53,57,58,62,65,66,6 8,69,71,74,75,77,79,80,84,86,87,88,89,90,92,94,96,97,9 9,103,105,107,110,111,112,113,114,119,120,121,123,125	70	1,114	0,128 294	0	1 658 682
1	5,6,9,14,15,19,20,21,25,27,28,29,30,34,40,41,42,45,47, 49,50,51,54,55,57,58,69,70,71,74,81,82,84,85,86,87,91, 92,94,97,98,99,101,104,107,108,109,110,112,113,114,123	52	0,782	0,129 834	0	1 083 050
17	1,4,5,6,8,9,12,14,15,16,17,18,19,23,24,26,27,28,30,31,3 2,33,34,36,37,39,40,41,43,44,47,48,52,53,57,58,62,64,6 5,66,68,69,71,74,75,77,79,80,82,83,84,86,87,88,89,90,9 2,94,96,97,99,103,105,107,110,111,112,113,114,119,120,121,123	73	1,161	0,12 921	0	1 715 544

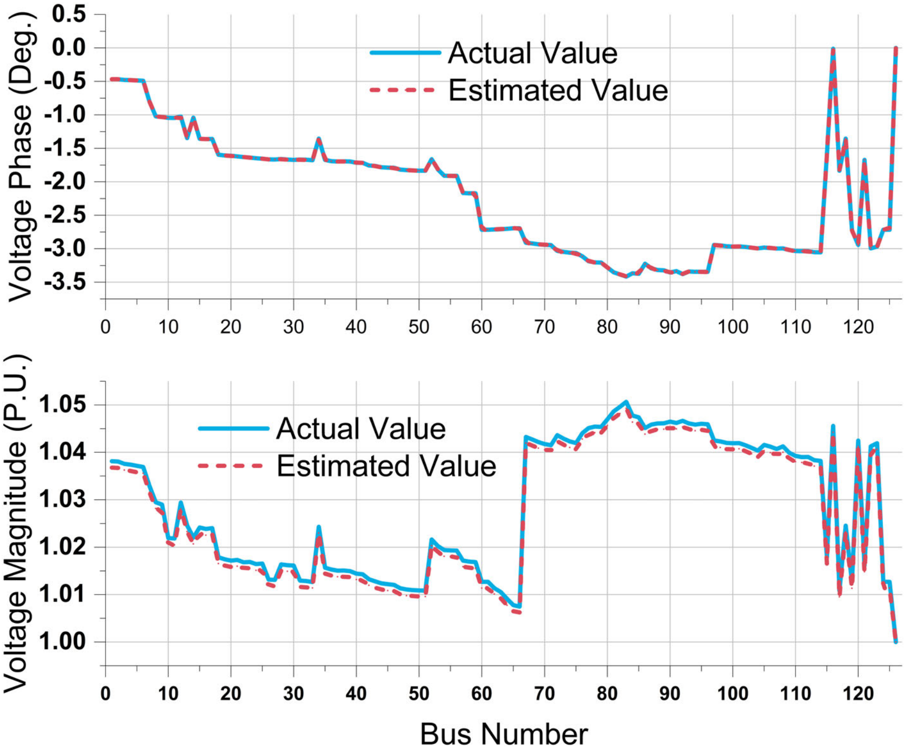


FIGURE 11 Estimation and actual values for voltage magnitude and voltage angle of scenario 4

objectives. The voltage magnitude and angles corresponding to scenario 4 have been illustrated in Figure 11. As shown, the estimations have an acceptable accuracy where in case of voltage angle, the estimation errors are near to zero.

6 | CONCLUSION

In this paper, a new approach for the placement problem of measurement devices using the state estimation algorithm has been proposed. To model the problem, four scenarios for data errors from measurement devices as well as errors in pseudo measurement data has been produced. Then, the meter placement has been modeled as a multi-objective problem and optimized using the MOEA/D algorithm. The proposed procedure for selecting the best Pareto front can help system operators with flexibly preferring their requirements and goals in the meter placement problem. The results showed that the parameters such as power losses and the capacity of a line as well as the number of changes in flow direction and the number of lines connected to a specific line can effect on locations of the measurement devices. Moreover, the robustness of the state estimation method in approximating the voltage magnitude and phase using the PSO algorithm has been shown. Also, the proposed method was able to guarantee that errors were lower than 0.1% for magnitude and 6% for the phase in all scenarios.

Future work will include modeling the measurement device placement problem using a distributed algorithm like ADMM. As the topology of distribution systems may vary many times during a year due to some operational actions such as reenergizing a disconnected load point or carrying out repairing and maintenance, the measurement device placement can be studied under different network configurations as the other future work.

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How to cite this article: Hassannejad Marzouni A, Zakariazadeh A, Siano P. Measurement devices allocation in distribution system using state estimation: A multi-objective approach. Int Trans Electr Energ Syst. 2020; e12469. https://doi.org/10.1002/2050-7038.12469

APPENDIX A GEOMETRIC INTERPRETATION OF ERROR CONFIDENCE REGION

Consider e_1 as the vector of the voltage magnitude relative errors and e_2 as the vector of the voltage angle relative errors. The covariance matrix between these two errors can be calculated as:

$$\sum = \begin{bmatrix} E[(e_1-\mu_1)(e_1-\mu_1)] & E[(e_1-\mu_1)(e_2-\mu_2)] \\ E[(e_2-\mu_2)(e_1-\mu_1)] & E[(e_2-\mu_2)(e_1-\mu_1)] \end{bmatrix}, \tag{A1}$$

where E is the expected value and μ_i is $E(e_i)$ and $\sum^{-1}$ is the inverse of the covariance matrix which is also known as the precision matrix. The precision matrix is a real symmetric matrix, there is an orthogonal matrix F such that

$$F^{-1} \times P \times F = F^T \times P \times F = \Lambda, \tag{A2}$$

where P is the precision matrix and Λ is the eigenvalues. Now, we need eigenvalues of the covariance matrix and it can be explained as follows

$$\sum \times \nu = \lambda \times \nu, \tag{A3}$$

where λ is eigenvalues of $\sum$ matrix and ν is a non-zero n-by-1 vector. The eigenvalues $\sum$ can be calculated as

$$\sum \times \nu = \lambda \times \nu \tag{A4}$$

$$\sum \times \nu - \lambda \times \nu = 0 \tag{A5}$$

$$\sum \times \nu - \lambda \times I \times \nu = 0 \tag{A6}$$

$$\left(\sum - \lambda \times I\right) \times \nu = 0 \tag{A7}$$

$$\left(\sum - \lambda \times I\right) = 0. \tag{A8}$$

This equation is called the characteristic equation of $\sum$ and is an n-th order polynomial in λ with n roots. These roots are called the eigenvalues of $\sum$. These roots can be shown as

$$\Lambda = (\lambda_1, \lambda_2). \tag{A9}$$

Now, the equation of the confidence region ellipsoid area can be explained as

$$e_1^T \times \sum \times e_1 = c, \tag{A10}$$

where c is a constant n-by-1 vector. Let G be the diagonal matrix as follows

$$G = \begin{bmatrix} \sqrt{\frac{c}{\lambda_1}} & 0 \\ 0 & \sqrt{\frac{c}{\lambda_2}} \end{bmatrix}. \tag{A11}$$

Now, consider $\hat{e}_1$ as the transformation of the below equation

$$e_1 = F \times G \times \hat{e}_1. \tag{A12}$$

As defined before, F is an orthogonal matrix and therefore has a unit determinant, $F \times G$ determinant can be calculated as

$$F \times G = \sqrt{c} \times \sqrt{\det\left(\sum\right)}. \tag{A13}$$

It is easy to see that, in the form of $\hat{e}_1$, the ellipsoid's equation is simply $\hat{e}^2{}_1 + \hat{e}^2{}_2 = 1$ and the unit two-dimensional sphere volume that scaled by determination of $F \times G$ can be expressed as $c \times \pi \times \sqrt{\det(\sum)}$, which is the area of the ellipsoid.