EQUILIBRIUM OF CHARGED PLASMAS WITH WEAK AXISYMMETRIC MAGNETIC PERTURBATIONS

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The effect of weak axisymmetric magnetic and/or electrostatic perturbations on the equilibrium of a non-neutral plasma in a Malmberg-Penning trap is analyzed. A semi-analytic solution for the potential variations inside the trap is found in a paraxial limit of the perturbations for the case of global thermal equilibrium. The fraction of magnetically and electrostatically trapped particles is calculated for a bi-Maxwellian distribution function.

I. INTRODUCTION

A model of a long pure electron plasma column contained in a cylindrical conducting chamber of radius R and immersed in an axisymmetric magnetic field B is adopted, with z being the coordinate along the symmetry axis, as shown in Fig. 1. Column-end effects are neglected and the attention is focused on the central part of the confining chamber, with a grounded conducting wall. In the unperturbed state, characterized by a uniform magnetic field B_0 and a constant wall radius R_0, the plasma density is constant along field lines. The goal of this paper is to fully characterize the electric potential in the plasma in those regions of the device where the magnetic field $B = B_0 + B_1(z)$ and the wall radius $R = R_0 + R_1(z)$ are perturbed by small quantities $B_1(z) \ll B_0$ and $R_1(z) \ll R_0$, respectively (the latter can model a potential variation along the chamber wall).

II. PARAXIAL APPROXIMATION

In a long-thin (paraxial) approximation, i.e., when the variations of B and R are both: (i) axisymmetric; and (ii) smooth, so that their characteristic axial length, ℓ, substantially exceeds the wall radius, $\ell \gg R$, Poisson's equation takes the form

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial\phi}{\partial\rho} = -\frac{n/n_*}{\lambda_D^2}\,\frac{B_0}{B}, \qquad (1)$$

where $\phi = e\varphi/T$ is the dimensionless potential, $\rho \approx r\sqrt{B/B_0}$ is the flux radius labelling a magnetic field line starting at a radius r outside of the perturbation region, $\lambda_D = \sqrt{T/4\pi e^2 n_*}$,

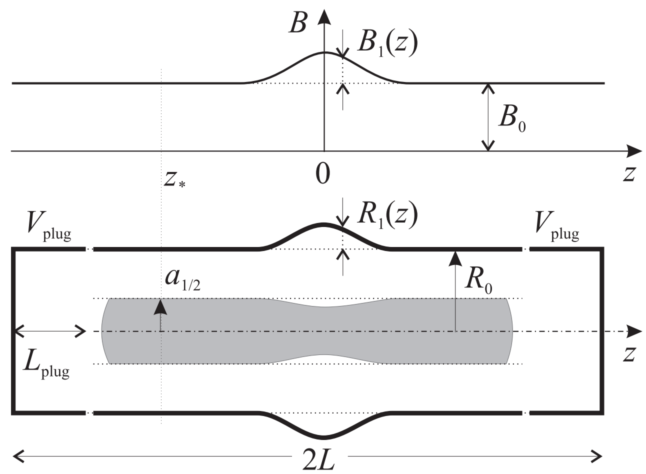

Figure 1: The variations of B and R are assumed to be: (i) axisymmetric; and (ii) smooth.

and $n/n_* = N(\rho)\exp(-\phi)$ is the plasma density normalized over its value n_* at a reference point $r = 0$, $z = z_*$; note that a Maxwell-Boltzmann distribution of plasma particles with a given temperature T is assumed here. Applying a standard perturbation analysis yields the equation

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial\phi_0}{\partial\rho} = -\frac{N_0(\rho)}{\lambda_D^2} \qquad (2)$$

for the unperturbed part of the potential ϕ_0, and

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial\phi_1}{\partial\rho} = \frac{N_0(\rho)}{\lambda_D^2}\,(\phi_1 + \delta B) \qquad (3)$$

for the perturbation ϕ_1 induced by the variation $\delta B \equiv B_1/B_0$ of the magnetic field. Here and below $N_0(\rho) = N(\rho)\times\exp[-\phi_0(\rho)]$, $n_* = N(0)\exp(-\phi_0(0))$. The unperturbed potential ϕ_0 satisfies $\phi_0(R_0) = 0$, while Eq. (3) is supplemented with the boundary condition $\phi_1(R_0) = -R_0\,\phi_0'(R_0)\,(\delta R + \delta B/2)$, which represents the linearized form of the boundary condition $\phi_0 + \phi_1 = 0$ for the total potential at the perturbed

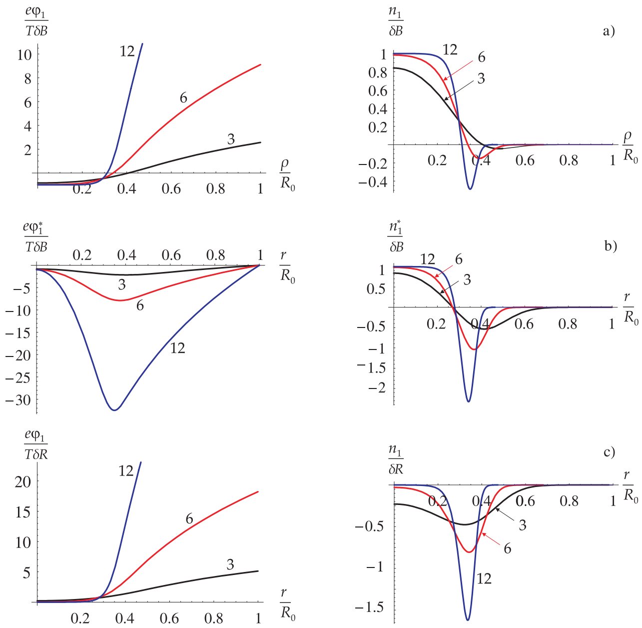

Figure 2: Perturbed potential and density for different ratios $a_{1/2}/\lambda_D$ (indicated directly on the plots): a) magnetically induced perturbations as a function of ρ, b) the same as a function of r, c) electrostatically induced perturbations (in this case $\rho = r$).

flux radius of the wall, $\rho_W = R_0\,(1 + \delta R + \delta B/2)$, where $\delta R \equiv R_1/R_0$.

A straightforward analysis of Eq. (3) reveals that the potential perturbation induced by magnetic field ripples qualitatively differs from that induced by chamber wall ripples. The former has opposite signs in the inner and outer parts of the plasma column, while the latter has always the same sign at all radii as shown in Fig. 2 relevant to a plasma in a state of global thermal equilibrium[1,2] with fixed plasma radius $a_{1/2}$ (computed at the level of $1/2$ of the maximum density) and three different values of λ_D such that $a_{1/2}/\lambda_D = 3, 6, 12$. For magnetically induced perturbations, both the perturbation ϕ_1 in flux coordinates and the variation of electric potential in ordinary coordinates

$$\phi_1^*(r,z) = \phi_0'(r)\, r\, \delta B/2 + \phi_1(r,z) \tag{4}$$

tend to $-\delta B$ at $r = 0$, if $a_{1/2}/\lambda_D \to \infty$ (Figs. 2a and 2b). On the contrary, electrostatically induced perturbations, characterized by the relative amplitude δR of the variation of the conducting wall radius, are shielded by the perturbed electric charge at the column edge, so that $\phi_1 \to 0$ at $r = 0$ as $a_{1/2}/\lambda_D \to \infty$ (Fig. 2c).

Since $\phi_1/\delta B < 0$ in the bulk of the plasma, the global thermal equilibrium state of a non-neutral plasma confined in a magnetic mirror field exhibits a curious feature, qualitatively discussed previously.[3] If $a_{1/2}/\lambda_D \gtrsim 3$, the plasma density increases linearly with the mirror ratio, so that the plasma is denser in the high magnetic field region since the magnetic squeeze forms a potential trap for low energy particles.

The comparison of Fig. 2a and Fig. 2b shows that the perturbation of the electric potential in ordinary cylindrical coordinates is much greater than that expressed in flux coordinates; however, the dominant first term in Eq. (4) does not affect the particle motion along a magnetic field line. The function ϕ_1 nowhere exceeds the value of δB within the plasma column whereas ϕ_1^* reaches a much greater value near the column edge, where $\phi_1^* \sim (a_{1/2}/\lambda_D)^2\delta B$. As a consequence, within the range of validity of the 1D approximation the evaluation of the small quantity $\phi_1(\rho,z)$ from realistic 2D simulations of the potential $\phi(r,z)$ requires a very high accuracy of computation. 2D plasma equilibrium simulations are addressed elsewhere[4] to evaluate the accuracy of the above described 1D paraxial theory. These 2D simulations show reasonable agreement with 1D theory for plasma parameters relevant to the CamV experiment.[5]

III. ANISOTROPIC PLASMA

Experimentally, a non-neutral plasma may remain anisotropic for a relatively long time, with the longitudinal temperature typically strongly exceeding the perpendicular temperature, $T_\parallel \gg T_\perp$.[6] The opposite case, $T_\parallel \ll T_\perp$, may also have its own peculiarities. The previous discussion can readily be extended to the case of a bi-Maxwellian distribution function

$$f(\varepsilon,\mu,\rho) = \frac{m^{3/2} n_* N(\rho)}{(2\pi)^{3/2} T_\parallel^{1/2} T_\perp} \exp\!\left[-\frac{\varepsilon - \mu B_0}{T_\parallel} - \frac{\mu B_0}{T_\perp}\right], \tag{5}$$

where ε and μ denote energy and magnetic moment, respectively. In this case

$$n/n_* = N(\rho)\, \exp(-e\varphi/T_\parallel) \left[\frac{T_\parallel B}{(T_\parallel - T_\perp)B_0 + T_\perp B}\right]. \tag{6}$$

Poisson's equation (2) for unperturbed electric potential remains formally valid for a redefined function $\phi_0 = e\varphi_0/T_\parallel$ and a Debye length $\lambda_D = \sqrt{T_\parallel/4\pi e^2 n_*}$. Eq. (3) for the perturbed potential $\phi_1 = e\varphi_1/T_\parallel$ is only slightly modified,

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial\phi_1}{\partial\rho} = \frac{N_0(\rho)}{\lambda_D^2}\left(\phi_1 + \frac{T_\perp}{T_\parallel}\delta B\right), \tag{3'}$$

while the boundary condition at the conducting wall remains unchanged. Thus, the magnetic perturbation δB enters the boundary-value problem multiplied by the factor $T_\perp/T_\parallel$, but through the boundary conditions at the wall without this factor. The effect of these boundary conditions is effectively shielded at the plasma edge, if $\lambda_D \ll a_{1/2}$. One can therefore expect that $\phi_1 \approx -(T_\perp/T_\parallel)\,\delta B$ in the bulk of the plasma, and a magnetic perturbation induces a potential perturbation $\varphi_1 \approx (T_\perp/e)\delta B$ near the column axis, which is proportional to $T_\perp$.

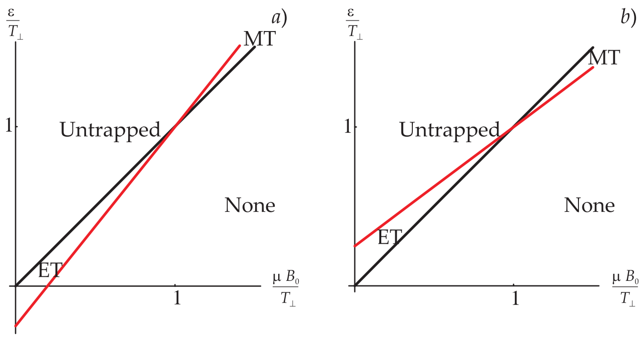

Figure 3: Phase space for anisotropic plasma. a) $\delta B > 0$ (magnetic squeeze); ET—electrostatically trapped particles, located within a magnetic squeeze; MT—magnetically trapped particles, reflected from a magnetic squeeze. b) $\delta B < 0$ (magnetic trap); ET—electrostatically trapped particles, reflected from the potential squeeze at a magnetic well; MT—magnetically trapped particles, localized within a magnetic well.

IV. TRAPPED PARTICLE FRACTIONS

Two distinct groups of trapped particles exist:[3] particles with low parallel energy are trapped in the low magnetic field region (magnetic trap), while particles with low total energy are trapped in the high field region (magnetic squeeze). The fraction of trapped particles is computed for the bi-Maxwellian distribution (5). The computation is performed for $a_{1/2} \gg \lambda_D$ (i.e., at low $T_\parallel$), and assuming that $\phi_1 = -(T_\perp/T_\parallel)\,\delta B$ is uniform over the radius r except for a narrow region close to the plasma column edge with a width of the order of λ_D.

A magnetic squeeze, $\delta B = B_1/B_0 > 0$, creates a potential well for the particles with small magnetic moment since $\phi_1 = -(T_\perp/T_\parallel)\,\delta B < 0$. Electrostatically trapped particles are located within the region ET in Fig. 3a. Particles with a greater magnetic moment are reflected from the magnetic squeeze; they are magnetically trapped outside of the magnetic squeeze region that corresponds to the region MT in Fig. 3a. Both kind of trapped particles are located between the lines $\varepsilon = \mu B_0$ and $\varepsilon = \mu(B_0 + B_1) + \phi_1$, which intersect in the point $\varepsilon = \mu B_0 = T_\perp$. Consequently, electrostatically trapped particles have an energy below the plasma perpendicular temperature, $\varepsilon < T_\perp$, while magnetically trapped particles are characterized by a higher energy, $\varepsilon > T_\perp$.

A local depression of the magnetic field, $\delta B = B_1/B_0 < 0$, yields a potential squeeze $\phi_1 = -B_1/B_0 > 0$. Consequently, the particles with low energies, $\varepsilon < T_\perp$, are electrostatically trapped outside of the perturbation region (Fig. 3b).

Independently of whether the trapping is caused by a magnetic squeeze or a well, it turns out that the density fractions are given by the following universal formulas

$$\frac{n_{\mathrm{ET}}}{n} = 0.52\sqrt{\frac{T_\perp|B_1|}{T_\parallel B_0}}, \qquad \frac{n_{\mathrm{MT}}}{n} = 0.37\sqrt{\frac{T_\perp|B_1|}{T_\parallel B_0}}. \qquad (7)$$

V. DISCUSSION

A paraxial theory of the equilibrium of a non-neutral plasma for weak axial perturbations of magnetic and electric fields has been developed and the fractions of magnetically and electrostatically trapped particles have been computed.

It has been shown that a magnetic barrier (trap) with $\delta B = B_1/B_0 > 0$ ($\delta B < 0$) creates a potential trap (barrier) with a depth (height) $\varphi_1 \approx -(T_\perp/e)\,\delta B$ in the plasma bulk. The perturbation of the electric potential induced by a variation of the conducting wall radius (or by a variation of φ over the wall of the confining chamber) is effectively shielded inside the plasma column. In contrast to electrostatically induced perturbations, a magnetically induced φ_1 usually changes its sign at a certain radius within the column edge.

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REFERENCES

[1] T.M. O'NEIL and C.F. DRISCOLL, "Transport to Thermal Equilibrium of a Pure Electron Plasma", *Phys. Fluids*, **22**, 266 (1979).

[2] I.A. KOTELNIKOV, R. POZZOLI and M. ROMÉ, "Thermodynamic Equilibrium of Hollow Nonneutral Plasmas", *Phys. Plasmas*, **11**, 4396 (2000).

[3] J. FAJANS, "Non-Neutral Plasma Equilibria, Trapping, Separatrices, and Separatrix Crossing in Magnetic Mirrors", *Phys. Plasmas*, **10**, 1209 (2003).

[4] M. ROMÉ, I. KOTELNIKOV and A. KABANTSEV, "Equilibrium of Non-Neutral Plasmas with Weak Axisymmetric Magnetic Perturbations", Proceedings of the Workshop on Non-Neutral Plasmas, Aarhus, Denmark, 26–30 June (2006), to be published.

[5] A.A. KABANTSEV, J.H. YU, R.B. LYNCH and C.F. DRISCOLL, "Trapped Particles and Asymmetry-Induced Transport", *Phys. Plasmas*, **10**, 1628 (2003).

[6] D.L. EGGLESTON, K.J. MCMURTRY, A.A. KABANTSEV and C.F. DRISCOLL, "Effect of Axial Magnetic Field Variations on Asymmetry-Induced Transport in a Non-Neutral Plasma Trap", *Phys. Plasmas*, **13**, 032303 (2006).