Determination of an Optimum Set of Testable Components in the Fault Diagnosis of Analog Linear Circuits

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***Abstract*— A procedure for the determination of an optimum set of testable components in the fault diagnosis of analog linear circuits is presented. The proposed method has its theoretical foundation in the testability concept and in the canonical ambiguity group concept. New considerations relevant to the existence of unique solution in the k-fault diagnosis problem of analog linear circuits are presented, and examples of application of the developed procedure are considered by exploiting a software package based on symbolic analysis techniques.**

***Index Terms*— Analog system fault diagnosis, analog system testing, fault location.**

I. Introduction

FAULT diagnosis and fault location in analog circuits are of fundamental importance for design validation and prototype characterization in order to improve yield through design modification. However, at present, while for digital circuits well-consolidated fully automatized techniques for fault diagnosis are commonly used, for analog circuits the development level is less advanced.

In the analog fault diagnosis field, an essential point is constituted by the concept of testability which, independently of the method that will be effectively used in fault location, gives theoretical and rigorous upper limits to the degree of solvability of the problem, once the test point set has been chosen. A well-defined quantitative measure of testability can be deduced by referring to fault diagnosis techniques of the parametric kind. These techniques, starting from a series of measurements carried out on previously selected test points, are aimed at determining the effective values of the circuit parameters by solving a set of equations (the fault diagnosis equations) nonlinear with respect to the component values. The solvability degree of these nonlinear equations constitutes the most used definition of testability measure [1]–[3], which indicates the ambiguity resulting from an attempt to solve such equations in a neighborhood of almost any failure. In other words, the testability measure provides information about the number of testable components with the selected test point set. When the testability value is not maximum, that is when it is less than the total number of potentially faulty circuit components, the problem is not uniquely solvable and it is necessary to consider further measurements, i.e., other test points, or accept a reduced number of potentially faulty components in order to locate the elements which have caused the incorrect behavior of the circuit under consideration. Generally, the second approach is used for two reasons. First, not all the possible test points actually can be considered because of practical and economic measurement problems strictly tied with the used technology and with the application field of the circuit under consideration. Second, the number of faulty components is generally smaller than the total number of circuit components. The single fault case is the most frequent, double or triple cases are less frequent, and the case of all faulty components is almost impossible. Therefore, as the testability is normally not maximum, the fault diagnosis problem is dealt with by assuming the quite realistic hypothesis that the number of faulty components is bounded, that is, the k-fault hypothesis is made. Under this hypothesis, in order to locate the faulty elements with as low as possible ambiguity, it is of fundamental importance to determine a set of components that is representative of all the circuit elements.

In this paper a procedure for the determination of an optimum set of testable components in the k-fault diagnosis of analog linear circuits is presented, where for the optimum set we mean a set of components representing all the circuit elements and giving a unique solution. The procedure is based on the testability evaluation of the circuit and on the determination of the canonical ambiguity groups. Referring again, for the sake of simplicity, to parametric fault diagnosis techniques, an ambiguity group can be defined as a set of components that, if used as unknowns (i.e., if considered as potentially faulty), gives infinite solutions in a phase of fault location (in literature the ambiguity group concept has already been introduced in [4], but in this paper it is considered with a wider meaning). A canonical ambiguity group is simply an ambiguity group that does not contain other ambiguity groups. It is worth pointing out that the proposed procedure gives information independently of the method that will be effectively used in the fault location phase (both simulation after test and simulation before test methods), even if it has been developed by referring to parametric fault diagnosis techniques. Furthermore, in the automation of the procedure the use of symbolic techniques is

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of fundamental importance because symbolic analysis, due to the fact that it gives symbolic rather than numerical results, is particularly suitable for applications such as testability and canonical ambiguity group determination, as will be shown in the following.

The paper is organized as follows. In Section I the theoretical basis for the determination of both the testability value and the canonical ambiguity groups is considered. In Section II, new considerations relevant to the existence of unique solution for the k-fault diagnosis problem of analog linear circuits are presented and the procedure for the determination of an optimum set of testable components is described. Finally, in Section III a software package, based on symbolic analysis techniques, for the determination of both testability and canonical ambiguity groups is briefly described and examples of application of the developed procedure are presented.

II. Testability and Ambiguity Groups

As was previously mentioned, a well-defined quantitative measure of testability can be deduced by referring to fault diagnosis techniques of the parametric kind. In these techniques it is necessary to determine a set of equations describing the circuit under test and solve it with respect to the component values. In the case of analog linear time-invariant circuits, the fault diagnosis equations can be constituted by the network functions relevant to the selected test points [5], [6] which are nonlinear with respect to the potentially faulty circuit parameters. By assuming that the faults can be expressed as parameter variations without influencing the circuit topology (i.e., faults as short and open are not considered), the testability measure T is given by the maximum number of linearly independent columns of the Jacobian matrix associated with the fault diagnosis equations, and it represents a measure of the solvability degree of the nonlinear fault diagnosis equations [1]–[3]. The entries of the Jacobian matrix are rational functions depending on the complex frequency s and the potentially faulty parameters. Thus, in order to evaluate the testability it is necessary to fix the potentially faulty parameter values and the complex frequency s. In [2] it has been shown that, once the frequency values are fixed (generally a multifrequency approach is considered in order to use a reasonable number of test points), the rank of the obtained Jacobian matrix is constant almost everywhere, i.e., for all the potentially faulty parameter values except those lying in an algebraic variety. Using this approach, the testability value, although independent of component values, is very difficult to handle and subject to roundoff errors if a numerical approach is used in its automation. In order to overcome these problems it has been demonstrated that starting from the network functions expressed in the following way:

$$h_l(\mathbf{p}, s) = \frac{N_l(\mathbf{p}, s)}{D(\mathbf{p}, s)} = \frac{\displaystyle\sum_{i=0}^{n_l} \frac{a_i^{(l)}(\mathbf{p})}{b_m(\mathbf{p})} \cdot s^i}{s^m + \displaystyle\sum_{j=0}^{m-1} \frac{b_j(\mathbf{p})}{b_m(\mathbf{p})} \cdot s^j}, \quad l = 1, \cdots K \tag{1}$$

where $\mathbf{p} = [p_1, p_2, \cdots, p_R]^t$ is the vector of the potentially faulty parameters and K is the total number of equations, the testability is equal to the rank of a matrix $\mathbf{B}$ independent of the complex frequency s whose entries are constituted by the derivatives of the coefficients of the fault diagnosis equations with respect to the potentially faulty circuit parameters [6]. Therefore, the testability matrix can be considered in the following form:

$$\mathbf{B} = \begin{bmatrix} \dfrac{\partial \frac{a_0^{(1)}}{b_m}}{\partial p_1} & \dfrac{\partial \frac{a_0^{(1)}}{b_m}}{\partial p_2} & \cdots & \dfrac{\partial \frac{a_0^{(1)}}{b_m}}{\partial p_R} \\ \cdots & \cdots & & \cdots \\ \dfrac{\partial \frac{a_{n_1}^{(1)}}{b_m}}{\partial p_1} & \dfrac{\partial \frac{a_{n_1}^{(1)}}{b_m}}{\partial p_2} & \cdots & \dfrac{\partial \frac{a_{n_1}^{(1)}}{b_m}}{\partial p_R} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial \frac{a_0^{(K)}}{b_m}}{\partial p_1} & \dfrac{\partial \frac{a_0^{(K)}}{b_m}}{\partial p_2} & & \dfrac{\partial \frac{a_0^{(K)}}{b_m}}{\partial p_R} \\ \cdots & \cdots & & \cdots \\ \dfrac{\partial \frac{a_{n_K}^{(K)}}{b_m}}{\partial p_1} & \dfrac{\partial \frac{a_{n_K}^{(K)}}{b_m}}{\partial p_2} & \cdots & \dfrac{\partial \frac{a_{n_K}^{(K)}}{b_m}}{\partial p_R} \\ \dfrac{\partial \frac{b_0}{b_m}}{\partial p_1} & \dfrac{\partial \frac{b_0}{b_m}}{\partial p_2} & \cdots & \dfrac{\partial \frac{b_0}{b_m}}{\partial p_R} \\ \cdots & \cdots & & \cdots \\ \dfrac{\partial \frac{b_{m-1}}{b_m}}{\partial p_1} & \dfrac{\partial \frac{b_{m-1}}{b_m}}{\partial p_2} & \cdots & \dfrac{\partial \frac{b_{m-1}}{b_m}}{\partial p_R} \end{bmatrix}. \tag{2}$$

If the fault diagnosis equations are generated in a completely symbolic form, the testability evaluation becomes easy to perform. In this case, the entries of the matrix $\mathbf{B}$ can be simply led back to derivatives of sums of products and the computational errors are drastically reduced in the automation phase. Once the matrix $\mathbf{B}$ has been determined, testability evaluation can be performed by triangularizing $\mathbf{B}$ and assigning arbitrary values to the components (as was previously mentioned, testability does not depend on component values). The disadvantage of considering as the testability matrix the matrix $\mathbf{B}$ instead of the Jacobian matrix consists in the fact that the testability meaning of the solvability measure of the fault diagnosis equations is less immediate. However, this limitation can be overcome by splitting the fault diagnosis equation solution into two phases. In the first phase, starting from the measurements carried out on the selected test points at different frequencies, the coefficients of the fault diagnosis equations are evaluated, eventually exploiting a least-squares procedure in order to minimize the error due to measurement inaccuracy [7]. In the second phase, the component values are obtained by solving the nonlinear system constituted by the equations expressing the previously determined coefficients as functions of the circuit parameters. In this way the following nonlinear system has to be solved:

$$\begin{cases} \dfrac{a_0^{(1)}(\mathbf{p})}{b_m(\mathbf{p})} = A_0^{(1)} & \cdots \quad \dfrac{a_{n_1}^{(1)}(\mathbf{p})}{b_m(\mathbf{p})} = A_{n_1}^{(1)} \\ \quad\vdots & \\ \dfrac{a_0^{(K)}(\mathbf{p})}{b_m(\mathbf{p})} = A_0^{(K)} & \cdots \quad \dfrac{a_{n_K}^{(K)}(\mathbf{p})}{b_m(\mathbf{p})} = A_{n_K}^{(K)} \\ \dfrac{b_0(\mathbf{p})}{b_m(\mathbf{p})} = B_0 & \cdots \quad \dfrac{b_{m-1}(\mathbf{p})}{b_m(\mathbf{p})} = B_{m-1} \end{cases} \tag{3}$$

where $A_i^{(l)}$ and $B_j (i = 0, \cdots, n_l, j = 0, \cdots, m-1)$ are the coefficients of the fault diagnosis equations in (1) which have been calculated in the previous phase. The Jacobian matrix of this system coincides with the matrix $\mathbf{B}$ in (2), hence, all the information provided by a Jacobian matrix with respect to its corresponding nonlinear system can be obtained from the matrix $\mathbf{B}$. In particular, if rank$\mathbf{B}$ is equal to the number of unknown parameters, the component values can be uniquely determined by solving the equations in (3) through the consideration of a set of measurements carried out on the test points. If the testability T ($T = \text{rank}\mathbf{B}$) is less than the number of unknown parameters R, a locally unique solution can be determined only if $R - T$ components are considered not faulty.

The matrix $\mathbf{B}$ does not give only information about the global solvability degree of the fault diagnosis problem. In fact, by noting that each column is relevant to a specific component of the circuit and by considering the linearly dependent columns of $\mathbf{B}$, other information can be obtained. For example, if a column is linearly dependent with respect to another one, this means that a variation of the corresponding component provides a variation on the fault-equation coefficients, indistinguishable with respect to that produced by the variation of the component corresponding to the other column. This means that the two components are not testable and they constitute an ambiguity group of the second order. By extending this reasoning to groups of linearly dependent columns of $\mathbf{B}$, ambiguity groups of a higher order can be found. Then, summarizing, the following definition can be formulated.

Definition 1: A set of j components constitutes an ambiguity group of order j if the corresponding j columns of the testability matrix $\mathbf{B}$ are linearly dependent.

In other words, the ambiguity groups of a circuit in which a certain test point set has been chosen can be determined by locating the linearly dependent columns of the testability matrix $\mathbf{B}$. Furthermore, as was mentioned, an ambiguity group that does not contain other ambiguity groups is called canonical. Therefore, a canonical ambiguity group can be defined as follows.

Definition 2: A set of k components constitutes a canonical ambiguity group of order k if the corresponding k columns of the testability matrix $\mathbf{B}$ are linearly dependent and every subset of this group of columns is constituted by linearly independent columns.

It is important to notice that with this definition, the order of the canonical ambiguity groups cannot be greater than the testability value plus one. Moreover, it is worth pointing out that, while the ambiguity group definition in [4] is strictly tied with small deviations of the component values with respect to the nominal values, in our definition there is not this limitation because both testability and linearly dependent columns of the matrix $\mathbf{B}$ are independent of the component values.

In most cases the canonical ambiguity groups have some components in common. By unifying these types of groups, another ambiguity group, corresponding again to linearly dependent columns of the matrix $\mathbf{B}$ is obtained. We define as global an ambiguity group of the following type.

Definition 3: A set of m components constitutes a global ambiguity group of order m if it is obtained by unifying canonical ambiguity groups having at least one element in common.

Obviously, a canonical ambiguity group which does not have components in common with any other canonical ambiguity group can be considered as a global ambiguity group. Finally, the columns of the matrix $\mathbf{B}$ that do not belong to any ambiguity group are linearly independent. We define as surely testable a group of components of the following kind.

Definition 4: A set of n components whose corresponding columns of the testability matrix $\mathbf{B}$ do not belong to any ambiguity group constitutes a surely testable group of order n.

Obviously, the number of surely testable components cannot be greater than the testability value, that is, the rank of the matrix $\mathbf{B}$.

III. Determination of an Optimum Set of Testable Components

As affirmed in the previous section, if the testability value is equal to the number of unknown parameters, the fault diagnosis problem can be uniquely solved. This means that, independent of the used fault location method, it is theoretically possible to determine the faulty components, starting from the measurements carried out on the selected test points. It has been previously affirmed also that, if the testability is less than the number of unknown parameters R, a locally unique solution can be determined only if $R - T$ components are considered not faulty, that is, with a value equal to the nominal one. In this case (very frequent in the practical applications because the testability is usually not maximum), the k-fault hypothesis is made and at most a number of faults equal to the testability value can be considered. However, under this hypothesis whatever fault location method is used, it is necessary to be able to select as potentially faulty components a set of elements that represents as well as possible all the circuit components. To this end, the determination of both the canonical ambiguity groups and surely testable group is of fundamental importance. In order to better understand the role of these groups, we refer again to the parametric fault diagnosis techniques.

In [8], a theorem has been demonstrated showing that a circuit is k-fault diagnosable in a neighborhood of the nominal value of the components $\boldsymbol{p_o}$ if every combination of $k + 1$ columns of the Jacobian matrix $\mathbf{J}$ associated with the nonlinear fault diagnosis equations evaluated in a previously selected set of frequencies and in the nominal value of the components, is independent. In [8], k-fault diagnosable at $\boldsymbol{p_o}$ means that all the possible combinations of k elements γ_i among the total number R of circuit components are distinguishable from each other (the total number of combinations γ_i is equal to $R!/((R-k)!k!)$, that is, $i = 1, 2 \cdots R!/((R-k)!k!)$) where two combinations γ_i and γ_j are distinguishable if they do not have totally coincident measurements in any neighborhood of $\boldsymbol{p_o}$. Furthermore, in [8] there is a lemma affirming that, under the k-fault hypothesis, two different combinations of k elements γ_i and γ_j are distinguishable if the rank of the matrix constituted by the columns of the Jacobian matrix $\mathbf{J}$, relevant to each element belonging to γ_i and to each element belonging to γ_j, is greater or equal to $k+1$. The previously mentioned theorem implicitly involves as well the solvability with respect to a combination of k elements γ_i, that is guaranteed if the matrix obtained from the Jacobian matrix $\mathbf{J}$ by selecting all the columns relevant to the elements of γ_i has a rank equal to k. Finally, in [8] the theorem and the lemma, enunciated with reference to the nominal value of the components $\boldsymbol{p_o}$, are also extended to almost all $p \in \Re^R$. In our case the word diagnosable has been replaced with the word testable and the Jacobian matrix $\mathbf{J}$ corresponds to the matrix $\mathbf{B}$ [5], [6]. Now we apply the lemma and the theorem in [8] to explain the role of the canonical ambiguity groups and the testable group in the selection of an optimum set of testable components. Furthermore, it is worth pointing out that, as the linear independence of the columns of $\mathbf{B}$ is not influenced by the component values, the use of the matrix $\mathbf{B}$ instead of the matrix $\mathbf{J}$ automatically extends the theorem and the lemma in [8] to almost all $p \in \Re^R$.

On the basis of the previous considerations the following useful lemmas can be introduced.

Lemma 1: Two combinations of k elements γ_i and γ_j belonging to the surely testable group (of order greater than k) are distinguishable.

This result directly derives from the lemma in [8]. In fact, the columns of $\mathbf{B}$ relevant to the parameters of γ_i and γ_j form a matrix with at least $k+1$ linearly independent columns because the two combinations γ_i and γ_j differ by at least one element.

On the basis of this lemma it is possible to deduce that if k elements belonging to the surely testable group are considered as unknowns, they give a unique solution in the phase of fault location.

Lemma 2: Two combinations of k elements γ_i and γ_j, belonging to a canonical ambiguity group of order $k+1$, are not distinguishable.

In fact, by applying the lemma in [8] the columns of $\mathbf{B}$ relevant to the elements of γ_i and γ_j form a matrix with $k+1$ different columns (γ_i and γ_j can differ by only one element) that, by the definition of the canonical ambiguity group, are not linearly independent.

On the basis of this lemma, it is possible to deduce that, even if k columns relevant to a canonical ambiguity group of order $k+1$ are linearly independent, the corresponding combination of k elements γ_i is not testable. Hence, the k elements considered as unknowns do not give a unique solution in the phase of fault location.

Lemma 3: Two combinations of k elements γ_i and γ_j belonging to a canonical ambiguity group of order greater or equal to $k+2$ are distinguishable.

In fact, by applying the lemma in [8] the columns of $\mathbf{B}$ relevant to the elements of γ_i and γ_j form a matrix with at least $k+1$ different columns (γ_i and γ_j can differ by at least one element) that are also linearly independent by the definition of canonical ambiguity group.

On the basis of this lemma it is possible to deduce that k elements of a canonical ambiguity group of order greater or equal to $k+2$ are testable, that is, the k elements considered as unknowns give a unique solution in phase of fault location.

In conclusion, in the case that all the canonical ambiguity groups of a circuit have no elements in common, it is sufficient to verify that all the possible combinations of k elements γ_i, belonging to every canonical ambiguity group, are distinguishable from each other because generic combinations of k elements belonging to different canonical ambiguity groups (and/or eventually belonging to the testable group) surely correspond to linearly independent columns and satisfy the lemma in [8]. By exploiting all these considerations we can reformulate the theorem in [8] in the following way.

Theorem: A circuit, characterized by canonical ambiguity groups without any elements in common, is k-fault testable if the smallest canonical ambiguity group has order greater or equal to $k+2$.

At this point we define an optimum set of testable components as follows.

Definition 5: A group of components constitutes an optimum set of testable components if it represents all the circuit components and if it gives a unique solution for the fault diagnosis equations under the k-fault hypothesis.

As a consequence of the previous theorem, a procedure for the selection of such a set of components can be summarized as follows:

1) evaluation of the circuit testability T;
2) determination of all the canonical ambiguity groups;
3) subsequent determination of the surely testable group;
4) supposition of k-fault hypothesis, with $k \le k_a - 2$ and k_a order of the smallest canonical ambiguity group;
5) selection of the components belonging to the surely testable group;
6) for each canonical ambiguity group, selection of at most $k_i - 2$ components as representatives of the corresponding canonical ambiguity group, with k_i order of the ith canonical ambiguity group.

The existence of an optimum set of testable components is guaranteed only if the order of the smallest canonical ambiguity group is greater or equal to $k+2$. For example, if there is at least a canonical ambiguity group of the second order, the optimum set does not exist. However, when it does exist, it is not unique (it is unique only in the case of maximum testability). By following the previous procedure, each element belonging to the surely testable group is representative of itself, while the elements selected for each canonical ambiguity group are representative of all the elements of the corresponding canonical ambiguity group. When the number

k of possible simultaneous faults is chosen *a priori* and an optimum set of testable components does not exist (or when for whatever value of k the optimum set does not exist, as in the case of presence of canonical ambiguity groups of the second order), only one component has to be selected for the canonical ambiguity groups with an order less than or equal to $k+1$, while for the surely testable group and for the canonical ambiguity groups with an order greater or equal to $k+2$, steps 5) and 6) of the procedure have to be applied. Even if a unique solution does not exist, by proceeding in this way we are able to choose a set of components which represents as well as possible all the circuit elements and, as shown in the next section, in the phase of fault location it will be eventually possible to confine the presence of faults to well-defined groups of components belonging to canonical ambiguity groups.

All the considerations described until now refer to the case in which all the canonical ambiguity groups are distinct. However, in most cases there are two or more canonical ambiguity groups that have some components in common, so the eventual presence of a unique solution is strongly influenced by this not-null intersection among the canonical ambiguity groups, and it is necessary to consider the global ambiguity groups for the determination of the optimum set of testable components. For example, when there are several distinct canonical ambiguity groups of the second order, even if a unique solution does not exist we can, however, characterize them by selecting one element for every one because the chosen components are distinguishable on the base of the lemma in [8]. However, if there are canonical ambiguity groups of the second order with a not-null intersection, we cannot select one element for every one of them because, on the basis of the lemma in [8], these elements are not distinguishable. Thus, only one element has to be chosen as representative of all these groups. In other words, the global ambiguity group derived from the union of all the canonical ambiguity groups with a not-null intersection has to be considered, and only one element of this new group has to be selected as representative of all the other elements belonging to it. In general, when canonical ambiguity groups also of a different order have elements in common, it is not sufficient to verify that all the combinations of k components belonging to the same canonical ambiguity group are distinguishable in order to have a k-fault testable circuit, but it is necessary to verify this for every global ambiguity group. Having all the global ambiguity groups of a circuit null intersection, each column corresponding to an element of a global ambiguity group is linearly independent with respect to columns corresponding to another global ambiguity group. This means that, in order to verify if a circuit is k-fault testable, it is sufficient to verify that all the possible combinations of k elements belonging to each global ambiguity group are distinguishable. To this aim the following lemma can be introduced.

Lemma 4: Two combinations of k elements γ_i and γ_j, belonging to a global ambiguity group obtained by unifying canonical ambiguity groups of order at least equal to $k+2$, are distinguishable.

This result directly derives from Lemma 3 and it does not need further explanations.

At this point, by recalling that a canonical ambiguity group that has a null intersection with respect to all the other canonical ambiguity groups can be considered as a global ambiguity group, we can reformulate the previous theorem in the following form.

Theorem: A circuit is k-fault testable if all the global ambiguity groups have been obtained by unifying canonical ambiguity groups of an order at least equal to $k+2$.

When the canonical ambiguity groups do not have a null intersection, the procedure of selection of the optimum set of testable components consists of the following steps:

1) evaluation of the circuit testability T;
2) determination of all the canonical ambiguity groups;
3) subsequent determination of all the global ambiguity groups;
4) subsequent determination of the surely testable group;
5) supposition of k-fault hypothesis, with $k \leq k_a - 2$ and k_a order of the smallest canonical ambiguity group;
6) selection of components belonging to the surely testable group;
7) for each global ambiguity group, selection of at most $k_i - 2$ components as representatives of the corresponding global ambiguity group with k_i minimum order of the canonical ambiguity groups, constituting the ith global ambiguity group.

In this case also an optimum set of testable components could not exist but, if it does exist, it is not unique. With this kind of selection each element belonging to the surely testable group is representative of itself, while the elements selected for each global ambiguity group are representative of all the elements of the corresponding global ambiguity group. When the number k of possible simultaneous faults is chosen *a priori* and an optimum set of testable components does not exist (or when for whatever value of k the optimum set does not exist, as in the case of presence of canonical ambiguity groups of the second order), only one component has to be selected as representative for the global ambiguity groups obtained by unifying canonical ambiguity groups of an order less than or equal to $k+1$, while for the surely testable group and for the other global ambiguity groups, the steps 6) and 7) of the procedure have to be applied. As in the case of distinct canonical ambiguity groups, if a unique solution does not exist, by proceeding in this way we are able to choose a set of components which represents as well as possible all the circuit elements and, as will be shown in the next section, in the phase of fault location it will be eventually possible to confine the presence of faults to well-defined groups of components belonging to global ambiguity groups.

It is important to confirm that the proposed procedure of component selection, even if developed by referring to parametric fault diagnosis techniques, is independent of the method used in the phase of fault location. For example, if a technique based on neural networks is used [9], the presented procedure can be very useful for sizing (that is for the choice of the neuron number) and for training the neural network because, for example, it is useless to train the network with data relevant to indistinguishable components. Furthermore, once the elements representative of all the circuit components have been chosen on the basis of the previous procedure, the research of the faulty components is up to the chosen fault location method. If the selected component set is optimum, the result given by the used fault location method can be theoretically unique. Otherwise, always on the basis of the selected components, it is possible to interpret in the best way the obtained results, as it will be shown in the next section. Finally, as the set of components to be selected is not unique, the eligibility of the most suitable one could be given by practical considerations as, for example, the set containing the highest number of components with less reliability or by the features of the subsequent chosen fault location method (algorithms using a symbolic approach, neural networks, fuzzy analyzer, etc.).

IV. Examples

Fundamental steps of the presented procedure are the testability evaluation and the canonical ambiguity group determination. In order to automatize these operations, a program has been developed by the authors [10]. It is based on symbolic analysis techniques which, as was previously mentioned, are particularly suitable for this kind of application because they not only simplify the determination of both testability and canonical ambiguity groups, but also strongly reduce the unavoidable roundoff errors introduced by the use of numerical techniques applied to problems relevant to sensitivity evaluation. The program has been included in the software package SAPWIN [11], [12], which permits, in a completely symbolic form, the evaluation of the network functions relevant to the selected test points. It requires a schematic entry of the circuit and all the selected test points. Once the symbolic network functions have been determined, a procedure for the testability evaluation starts and, subsequently, an algorithm of canonical ambiguity group determination is activated. The program yields as output the circuit testability and all the canonical ambiguity groups. In the following we present examples of the application of the developed procedure by exploiting the realized program for the testability evaluation and canonical ambiguity group determination.

As a first example, let us consider the Sallen–Key bandpass filter, whose schematic entry for the program is shown in Fig. 1 (V_o is the chosen test point). The program results for this circuit are shown in Fig. 2. As can be seen, there are two canonical ambiguity groups without any elements in common that can be considered also as global ambiguity groups. The first group is of the second order and it is not possible to select a set of components giving a unique solution. The surely testable group is the following:

G1 C1.

As the testability is equal to three, we can take into account at most a three-fault hypothesis in order to obtain a possible solution. On the basis of our procedure, the elements to select as representative of the circuit components are the surely testable group components and only one component for each canonical ambiguity group. Let us suppose, for example, a situation of single fault. Independent of the used fault location method, if the obtained solution gives as faulty element C1 or G1, we can localize the fault with certainty because both C1 and G1 belong to the surely testable group. If we locate as a potentially faulty element a component belonging to the second-order canonical ambiguity group, we can only know that there is a fault in this ambiguity group, but we cannot locate it exactly because there is not a unique solution. Instead, if we obtain as a faulty element a component belonging to the third-order ambiguity group, we have unique solution and then we can localize the fault with certainty. In other words, a fault in a component of this group can be counterbalanced only by simultaneous faults on all the other components of the same group. However, by the hypothesis of single fault, this situation can not occur.

As second example let us consider the Tow–Thomas filter shown in Fig. 3. In this circuit we have chosen as test points the voltages V_a and V_b and the program results are reported in Fig. 4. In this case the global ambiguity groups are the following:

1) G2 G3 G4 C2
2) G1 G5 G6 C1.

It is worth pointing out that the second group is also a canonical ambiguity group. Moreover, there is not the surely testable group and, as the testability is equal to four, a solution can be obtained by supposing, at most, a four-fault hypothesis.

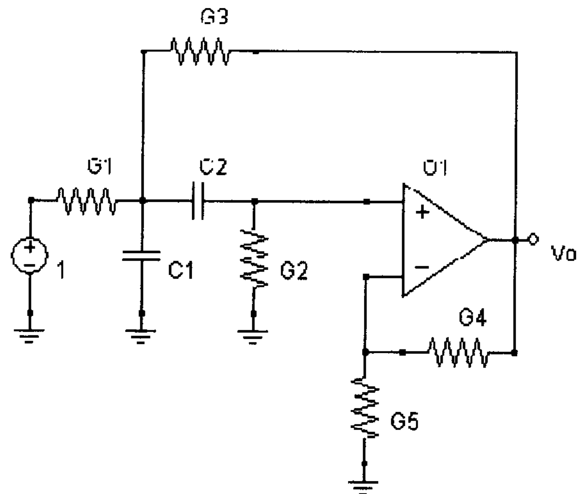

Fig. 1. Program schematic entry for the Sallen–Key bandpass filter.

```
Testability Value: 3

Total Number of Components: 7

Canonical Ambiguity Groups:
G5 G4
C2 G2 G3
```

Fig. 2. Program results for the circuit in Fig. 1.

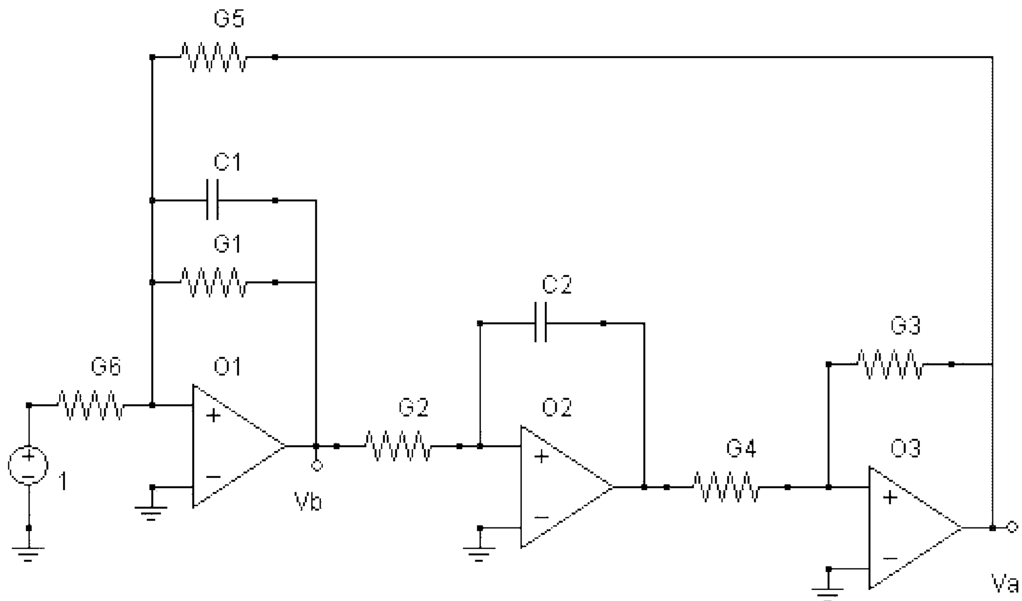

Fig. 3. Program schematic entry for the Tow–Thomas filter.

```
Testability Value: 4

Total Number of Components: 8

Canonical Ambiguity Groups:
G2  G3
G2  G4
G2  C2
G3  G4
C3  C2
G4  C2
G1  G5  G6  C1
```

Fig. 4. Program results for the circuit in Fig. 3.

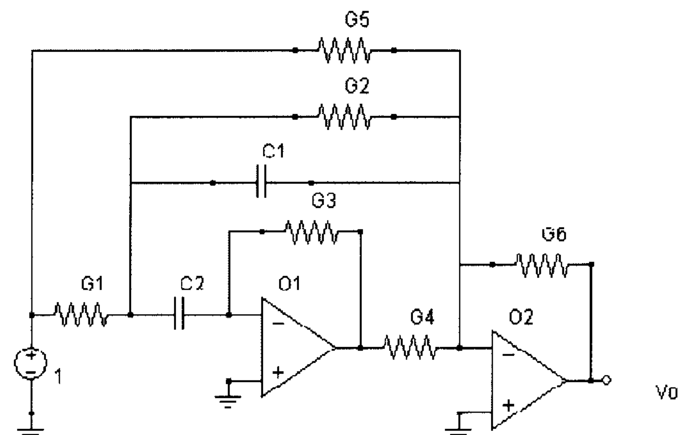

Fig. 5. Program schematic entry for the notch filter.

```
Testability Value: 5

Total Number of Components: 8

Canonical Ambiguity Groups:

G3  C1  C2  G4
G1  G2  G3  C1  C2
G1  G2  G3  C1  G4
G1  G2  G3  C2  G4
G1  G2  C1  C2  G4
G1  G2  G4  G5  G6
```

Fig. 6. Program results for the circuit in Fig. 5.

There is not an optimum set of testable components, that is, there is not a unique solution because the smallest order of the canonical ambiguity groups is two. On the basis of our procedure, the elements to be selected as representative of the circuit components are only one element for the first global ambiguity group and two elements of the second one. Let us suppose, for example, a situation of double fault. Independently of the used fault location method, if the obtained solution gives as faulty elements two components belonging to the second global ambiguity group, we have a unique solution and then we can localize the fault with certainty. If we locate as potentially faulty elements one component belonging to the first group and the other belonging to the second one, we can affirm that the element belonging to the second group is surely faulty. Instead, we can only know that there is a fault in the first ambiguity group but we cannot locate it exactly because there is not a unique solution. On the other hand, if we want to determine a possible solution for the fault diagnosis equations associated to the determined testability value ($T = 4$), we must choose only one element of the first global ambiguity group and three components of the second one. It is necessary to consider only one element of the first group because this global ambiguity group is constituted by the union of second-order canonical ambiguity groups. This means that, if we choose two elements of this group, we consider a submatrix of the testability matrix $\mathbf{B}$ with determinant equal to zero and then it

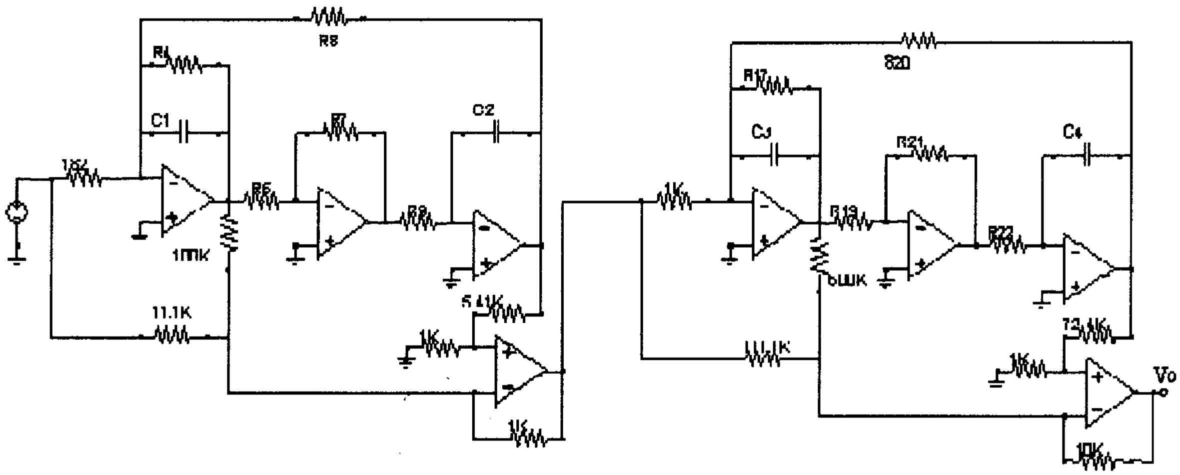

Fig. 7. A fourth-order elliptic low-pass filter.

is not possible to find a solution. Obviously, by choosing the four components as previously described, the obtained solution is not unique because all the possible combinations of the four elements are possible solutions.

As a third example let us consider the notch filter shown in Fig. 5. In this circuit we have chosen as a test point the output voltage V_o and the program results are reported in Fig. 6.

In this case, all the circuit components belong to a unique global ambiguity group and, consequently, there is no surely testable group. However, it is possible to notice that the global ambiguity group is formed by canonical ambiguity groups of at least an order of four. Hence, all the possible selections of groups constituted by the two components can be considered as optimum sets of testable components and we have the existence of a unique solution in any case where we take into account a group of two components (unique solution until two-fault hypothesis). This means that if, for example, a situation of double fault is considered, independent of the used fault location method, the obtained solution locates with certainty the faulty components.

As a last example let us apply the procedure to the circuit in Fig. 7 that is a slight modification of the circuit reported in [9]. In this case, only 13 components are considered potentially faulty and are indicated by their symbolic name, while the others are considered healthy and are indicated by their numerical value. The output voltage V_o has been chosen as the test point and the program results are reported in Fig. 8. In this case the surely testable group is the following:

C1 C3 R4 R17 R8.

For the remaining eight components belonging to canonical ambiguity groups of order two, it is easy to see that they can be grouped in the two following global ambiguity groups:

1) R6 R7 R9 C2

2) R19 R21 R22 C4.

As the testability is equal to seven, a solution can be obtained by supposing at most a seven-fault hypothesis. There is not an optimum set of testable components because the canonical ambiguity groups have order two. On the basis of our procedure, the elements to select as representative of the circuit

```
Testability Value: 7

Total Number of Components: 13

Canonical Ambiguity Groups:
R6 R7
R6 R9
R6 C2
R7 R9
R7 C2
R9 C2
R19 R21
R19 R22
R19 C4
R21 R22
R21 C4
R22 C4
```

Fig. 8. Program results for the circuit in Fig. 7.

components are those belonging to the surely testable group and only one component for each global ambiguity group. For whatever fault hypothesis from one to seven and, independent of the used fault location method, a component given as faulty by a solution is effectively faulty if it belongs to the surely testable group. Otherwise, it indicates only that there is a fault in the corresponding global ambiguity group without allowing the determination of the effectively faulty component.

V. Conclusion

By using the fundamental information given by the testability evaluation and the canonical ambiguity group determination, a new procedure for the selection of an optimum set of testable components in the k-fault diagnosis of analog linear circuits has been proposed, where optimum set means a set of components representing all the circuit elements and giving a unique solution. Furthermore, when an optimum set of components is not determinable, the developed procedure allows us to select the elements that represent all the circuit components. New considerations relevant to the existence of a unique solution in the k-fault diagnosis problem of analog linear circuits have been presented and examples of applications of the developed procedure have been considered by exploiting a software package, based on symbolic analysis techniques, for the determination of both testability and canonical ambiguity groups in analog linear circuits. Finally, the obtained results can constitute the first step in the development of whatever procedure for the fault location of analog linear circuits because they represent theoretical and rigorous upper limits to the degree of solvability of the problem of faulty component location.

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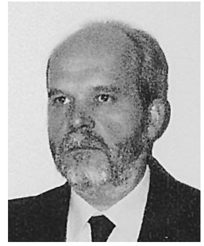

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