

Hadron physics in holographic QCD

A. B. Santra^a, U. Lombardo^b, A. Bonanno^c

^a Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai 400085, India.

^b INFN-LNS and University of Catania, 44 Via S.-Sofia, I-95125 Catania, Italy.

^c INAF- Catania Astrophysical Observatory, Via S.Sofia 78, 95123 Catania, Italy.

E-mail: santra@barc.gov.in

Abstract.

Hadron physics deals with the study of strongly interacting subatomic particles such as neutrons, protons, pions and others, collectively known as baryons and mesons. Physics of strong interaction is difficult. There are several approaches to understand it. However, in the recent years, an approach called, holographic QCD, based on string theory (or gauge-gravity duality) is becoming popular providing an alternative description of strong interaction physics. In this article, we aim to discuss development of strong interaction physics through QCD and string theory, leading to holographic QCD.

1. Introduction

The accepted theory of strong nuclear forces, or strong interactions, is Quantum Chromodynamics (QCD) [1], an Yang-Mills gauge theory with $SU(3)$ gauge group. It can be studied at high energies using perturbation theory with enormous success, but, it is very hard, known to be intractable for the analytical analysis at low energies because of the strong coupling problem. The low energy regime of QCD contains the most interesting phenomena related to hadron physics, thus it is of great theoretical interest. Due to unavailability of suitable analytical tools for understanding QCD in the non-perturbative regime, QCD inspired heuristic models and approximation schemes, ranging from the bag models [2, 3] to chiral perturbation theory [4, 5] have been used to get partial information about the dynamics of QCD in the strongly coupled regime. These models use simple frameworks for the analysis of some aspects of non-perturbative QCD, but give impressive results. Though the simplicity of the effective models make them very interesting and useful for the phenomenology of strong interactions, no rigorous relation has been established with these models to QCD in spite of much efforts. Another approach of studying non-perturbative QCD is by numerical simulation using lattice gauge theory or lattice QCD [6, 7]. This procedure, though computationally intensive, appears to be successful for the calculation of static quantities such as the vacuum structure and the spectrum.

Almost immediately after the discovery of asymptotic freedom in QCD, existence of a new phase of matter called Quark-Gluon-Plasma (QGP) was predicted [8, 9] at very high temperature and density. QGP was expected to be made of de-confined free quarks and gluons. The programme of heavy ion collision at relativistic energies started to realize QGP experimentally. The experimental observations, over the last decade, in relativistic heavy ion collisions have revealed that QGP is, contrary to expectations, also a strongly coupled system instead of a gas

of free quarks and gluons [10, 11]. Lattice QCD has been an effective tool for understanding the static properties related to equilibrium thermodynamics of QGP, but for the real time dynamic properties of strongly coupled plasma, the transport coefficients particularly, it is much more difficult [12]. This is mainly because lattice QCD methods are inherently Euclidean, and it is very difficult to extrapolate numerical results to Minkowski signature. Thus, unfortunately we do not have appropriate theoretical methods that would be applicable to the problems of QCD at strong coupling.

Another attempt of understanding strong interactions had been through S-matrix formalism [13, 14, 15]. It eventually led to the development of string theory [16, 17, 18]. The string description reproduces the Regge trajectories of hadrons very well, however this model was unable to reproduce the experimental results for hard and deep inelastic scatterings. Such a process described by string theory in flat space-time is soft, that is the amplitudes decay exponentially with energy, while both experimental data and QCD theoretical predictions [19, 20] indicate a hard behavior in which the amplitudes decay with a power of energy. So, the interest in string theory shifted towards making it a description of quantum gravity [21, 22]. This happened primarily due to the huge success of QCD at high energies, and consistency problem of string theory as well. Though terms like string tension, flux tubes are frequently used in hadron physics, still it was generally believed that string theory had very little to do as description of hadron physics.

A new way of thinking about strongly coupled gauge theories has emerged now, due to progress in string theory over the past decade, in the form of gauge-gravity duality [23, 24, 25] which connects a gauge theory in d -dimensional space-time to a gravity theory in $(d + 1)$ -dimensional spacetime. Thus, the duality is holographic. The most widely studied example of the duality is known as AdS/CFT correspondence. AdS stands for anti de Sitter space and CFT for conformal field theory. The AdS/CFT correspondence connects a maximally supersymmetric Yang-Mills theory in the gauge theory side to a string theory in a particular ten-dimensional space-time, $AdS_5 \times S^5$, in the gravity side. This duality, between strongly-coupled Yang-Mills theories and weakly-coupled gravity is very attractive for its potential application in understanding QCD in strongly coupled regime.

Direct application of the AdS/CFT correspondence to QCD is not possible, since QCD is neither supersymmetric nor it is fully conformal, though there are some energy regime in which it is ‘quasi conformal.’ So, gravity dual of QCD is not known yet. However, inspired by AdS/CFT correspondence, and assuming that dual gravity theory to QCD exists, Witten [26] first proposed to deform the AdS/CFT correspondence to describe strong interactions. A large amount of activity started since then in building holographic models of QCD or AdS/QCD models.

The idea behind AdS/QCD is to introduce an additional spatial direction to the four dimensional space-time, which roughly corresponds to the energy scale of the field theory, and try to construct a model that captures important non-perturbative aspects of the original 4-dimensional field theory, such as confinement and chiral symmetry breaking. These models have been proved to be successful in understanding these non-perturbative effects in simple terms.

The gravity dual of gauge theory at finite temperature has also been constructed introducing black hole horizon in AdS_5 [26]. Using the dual theory it becomes relatively easy to compute [27] transport coefficients, such as viscosity and diffusion constants, for strongly coupled plasma. The analysis of scattering amplitudes in the AdS black hole background led to the universal viscosity bound [28], which plays an important role in understanding the physics of the elliptic flow of QGP observed in relativistic heavy ion collisions.

In this article we aim to describe pedagogically the evolution of the understanding of strong interaction from QCD to holographic QCD. In section (2), we give an introduction to strong interaction in terms of QCD and S-matrix formalism. In section (3), we give very elementary

description of string theory as least as required to appreciate the terms used in AdS/CFT correspondence. In section (4), we introduce gauge gravity duality and holographic principle. In section (5), we discuss AdS/CFT correspondence through Maldacena conjecture introducing anti de Sitter space and conformal field theory. In section (6), we discuss different approaches of AdS/QCD models. In section (7), we discuss application of AdS/CFT correspondence to gauge theories at finite temperature. Finally, section (8) contains the summary and conclusions.

2. Strong interactions

Microscopic understanding of strong interaction (nuclear force) started with Yukawa [29] postulating that protons and neutrons (nucleons) interact by exchanging pions, pseudo scalar particles (0^-) of mass about 140 MeV. This idea got experimental approval after pions were discovered [30] in cosmic rays. However, it was soon realized that, (a) nucleons and pions show excitations (resonances), *i.e* these are not point like elementary particles, and (b) exchange of pions alone cannot explain the nuclear force. Strong interaction appears to be far more complicated.

With the availability of higher and higher energy accelerators, more and more particles other than the protons, neutrons and pions were produced. Fermions among them were named baryons, and bosons were named mesons. Some of these have strange properties, namely copious production and slow decay. A new quantum number called strangeness (S) was assigned to these particles. Strangeness has been found to be conserved in strong interactions.

To face this situation of understanding a large number of particles and their interactions, two proposals came by almost concurrently, (1) approach based on symmetry, that gave rise to the quark model and QCD, (2) approach based on S-matrix, that led to hadronic strings and string theory.

2.1. Symmetry approach of strong interaction

Patterns were found, when mesons and baryons of same spin and parity were placed on a two dimensional plot with charge (Q) and strangeness (S) as axes [31]. These patterns were identified with higher dimensional representations of $SU(3)$ group. Strong interaction must have $SU(3)$ symmetry. As higher dimensional representations of $SU(3)$ can be constructed taking direct product of fundamental representations, it was thus suggested that the particles corresponding to higher dimensional representations of $SU(3)$ could be made up of the particles corresponding to its fundamental (3-dimensional) representation.

Fundamental representation of $SU(3)$ constitutes three spin- $\frac{1}{2}$ particle states with fractional charge, and baryon number ($B = \frac{1}{3}$), namely, up (u) with charge, $Q = \frac{2}{3}$, strangeness, $S = 0$, isospin, $I = \frac{1}{2}$, third component of isospin, $I_z = \frac{1}{2}$, down (d) with charge, $Q = \frac{-1}{3}$, strangeness, $S = 0$, isospin, $I = \frac{1}{2}$, third component of isospin, $I_z = \frac{-1}{2}$, and strange (s) with charge, $Q = \frac{-1}{3}$, strangeness, $S = -1$, isospin, $I = 0$. These are collectively called quarks [32, 33], and are not observed in free state. Quark Model was developed to understand baryons as bound states of three quarks (qqq), ($p \equiv (uud), n \equiv (udd)$ etc) and mesons as bound state of quark and antiquark ($q\bar{q}$), ($\pi^+ \equiv (u\bar{d}), \pi^- \equiv (d\bar{u})$ etc.)

However, immediately, quark model was found to suffer from spin-statistics problem. In ($J = \frac{3}{2}, J_z = \frac{3}{2}$) state of Δ^{++} there are three u quarks with spin aligned in the same direction. Identical quarks in the same quantum state, giving rise to symmetric wavefunction for a fermion, violates Pauli principle. The problem disappears if quarks are not identical. A new quantum number, called color, was introduced for quarks [34, 35]. Each quark must appear in three colours (say, red q_r , green q_g and blue q_b). Quark model became extremely successful book-keeping model for strongly interacting particles. However, quarks were considered at that time as just “mathematical entities” having little to do with physical reality.

It was known quite early that quantum field theory is the appropriate framework to understand the dynamics of a system obeying principles of quantum mechanics and relativity. It was very successful in describing electromagnetic interactions. However, for strong interactions, there were some basic difficulties in constructing a quantum field theory, namely, (1) there are too many hadrons and too many forces to construct a sensible interaction lagrangian to build a useful field theory with predictive power, (2) perturbative expansion of field theory becomes meaningless as the coupling constant in strong interaction is large. So, at the end of the 1960's, a fundamental theory of strong interactions did not exist.

Finally, conclusive evidences of point like constituents (quarks) of nucleons were found in deep inelastic scattering of electrons off nucleons [36]. However, isolated quarks with color charge were not "seen". Quarks are found to be, (1) almost non-interacting at high energy or short distances (asymptotic freedom), and, (2) very strongly interacting at low energy or large distances (infrared slavery).

Quarks, instead of baryons and mesons, were identified as the fundamental degrees of freedom of strong interaction. There are 6 flavors of quarks, as shown in table 1 and each flavor is a three component vector in color space.

Flavor	$Q(e)$	I	I_3	S	C	B	T	Mass
u	2/3	1/2	1/2	0	0	0	0	2 MeV
d	-1/3	1/2	-1/2	0	0	0	0	5 MeV
s	-1/3	0	0	-1	0	0	0	100 MeV
c	2/3	0	0	0	1	0	0	1.3 GeV
b	-1/3	0	0	0	0	1	0	4.2 GeV
t	2/3	0	0	0	0	0	1	175 GeV

Table 1. Different flavors of quarks with respective quantum numbers and mass.

After quarks were identified as the fundamental degrees of freedom of strong interaction, complete understanding of strong interaction reduced to, (1) knowing interaction of quarks that explains observations of deep inelastic scattering experiment, (2) constructing bound states of three quarks for baryons, and of quark and anti-quark for mesons with this interaction, and (3) deriving the interaction between baryons and mesons. The methodology to achieve the above objective is quantum field theory with perturbative techniques that worked extremely well for electromagnetic interactions.

Interaction of quarks is modeled in the same way as interaction of electrons in electromagnetism. Quarks interact through its color charge by exchanging gluons, G_μ^a , which are, like photon, mass-less spin 1 particle having two polarizations (μ), and unlike photon, carry color charge ($a = 1, \dots, 8$). Quantum field theoretic description of the dynamics of quarks and gluons is called Quantum Chromodynamics (QCD) [37] in which the form of the interaction lagrangian is determined by the principle of local gauge invariance; that is, by demanding the total lagrangian to be invariant under the transformations belonging to the local (color) $SU(3)$ group. Such theories are called gauge theories or Yang-Mills theories [38]. The QCD lagrangian is written as,

$$\mathcal{L}_{QCD} = \bar{q}^f i\gamma^\mu (D_\mu - m_f) q^f - \frac{1}{2} Tr [G_{\mu\nu} G^{\mu\nu}] , \quad (1)$$

where the covariant derivative, $D_\mu = \partial_\mu - igT^a G_\mu^a$, the gluon field tensor, $G_{\mu\nu} = \frac{i}{g} [D_\mu, D_\nu]$, m_f is the mass of quark of flavor f , T^a are $SU(3)$ generators and g is quark-gluon coupling constant.

Expanding \mathcal{L}_{QCD} of (1) we get,

$$\begin{aligned} \mathcal{L}_{QCD} = & \underbrace{\bar{q}^f (i\gamma_\mu \partial^\mu - m) q^f - \frac{1}{4} (\partial_\mu G_\nu^a - \partial_\nu G_\mu^a)^2}_{\mathcal{L}_0} \\ & + \underbrace{g \bar{q}^f \gamma_\mu T^a G^{a\mu} q^f}_{\mathcal{L}_I} \\ & - \underbrace{\left(g f^{abc} \partial_\mu G_\nu^a G^{b\mu} G^{c\nu} + \frac{1}{4} g^2 f^{eab} f^{ecd} G_\nu^a G_\mu^b G^{c\nu} G^{d\mu} \right)}_{\mathcal{L}_{II}}, \end{aligned} \quad (2)$$

where \mathcal{L}_0 represents the lagrangian for free quarks and gluons, \mathcal{L}_I represents quark-gluon interaction, and \mathcal{L}_{II} represents gluon self interaction of gluons. Self interactions of gluons, a feature of non-abelian gauge theory, occurs because gluons have color charges. Such interactions make QCD non-linear and thus complicated. The quark-gluon interaction as well as the gluon self interaction strength is governed by the same coupling constant, g .

The QCD lagrangian, \mathcal{L}_{QCD} , possess, beside Poincare and local SU(3) (color) gauge symmetry, chiral symmetry and scale symmetry. In the limit, $m_u, m_d \approx 0$, ($m_u \simeq 1.7 - 3.3 \text{ MeV}$, $m_d \simeq 4.1 - 5.8 \text{ MeV}$), \mathcal{L}_{QCD} in the (u, d) sector is invariant under $SU(2) \times SU(2)$ chiral symmetry. However, due to absense of parity doublets in hadronic states the chiral symmetry is spontaneously broken. The QCD action,

$$S_{QCD} = \int d^4x \mathcal{L}_{QCD}, \quad (3)$$

is invariant under the scale (conformal) transformations,

$$x^\mu \rightarrow \lambda x^\mu, \quad q \rightarrow \frac{1}{\lambda^{\frac{3}{2}}} q, \quad G_\mu^a \rightarrow \frac{1}{\lambda} G_\mu^a, \quad (4)$$

in absence of the quark mass term in QCD lagrangian. This is called scale symmetry (or conformal symmetry), however, this symmetry is also broken due to quantum effects.

After quantization and renormalization, the QCD lagrangian, \mathcal{L}_{QCD} , uniquely defines the algorithm (or Feynman rules) for calculating amplitudes for all physical processes of strong interaction in perturbative expansion. The quadratic terms give the propagators, whereas the cubic and quartic terms give the interaction vertices.

The wisdom of mixing quantum mechanics and relativity is that the vacuum or the ground state of a relativistic quantum mechanical system is not trivial, *i.e.* it is not just the empty space. Actually it can be thought of as a medium of virtual particles. Because of this, a general feature of quantum field theory is that the coupling constant, g , depends on the energy scale, μ at which observations are made. This dependence is given by the renormalization group (RG) equation, relating the beta function,

$$\beta(\alpha) = \mu \frac{d\alpha}{d\mu} \quad (5)$$

with the coupling constant. In case of QCD the RG equation is given as,

$$\mu \frac{d\alpha}{d\mu} = -\beta_0 \frac{\alpha^2}{2\pi} - \beta_1 \frac{\alpha^3}{4\pi^2} - \dots, \quad (6)$$

where the strong coupling constant, $\alpha = \frac{g^2}{4\pi}$. In the lowest order, solution of (6) is,

$$\alpha(\mu) = \frac{2\pi}{\beta_0 \ln\left(\frac{\mu}{\Lambda}\right)}, \quad (7)$$

with $\beta_0 = 11 - \frac{2}{3}n_f$. The integration constant, Λ , is an intrinsic energy scale of QCD, which is introduced dynamically. Several measurements of the strong coupling constant at the Z -boson mass scale fix [41] the QCD scale parameter, Λ , as,

$$\alpha(M_Z) = 0.117, \quad M_Z = 91 \text{ GeV}, \quad \Rightarrow \quad \Lambda = 217 \text{ MeV}. \quad (8)$$

It is seen from (7) that the coupling constant decreases with increasing energy, thus explaining asymptotic freedom, and the growth of the coupling at small energy is consistent with the non-observation of isolated quarks [39, 40]. The quarks are always confined to form hadrons. It is also seen from (7) that $\alpha(\mu)$ diverges at Λ . This is known as strong coupling problem of QCD. Moreover, the appearance of an intrinsic energy scale (Λ) in QCD breaks the scale symmetry (conformal symmetry).

Observationally to test the correctness of QCD, we require to compute the correlation functions, $\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle$, of gauge invariant operators, $\mathcal{O}(x_1)$. At high energy when the strong coupling constant, α , is small ($\alpha < 1$, weak coupling domain) the correlation functions are expressed as a perturbative expansion in α , and a few terms in the expansion give correct result. However, at low energies, below Λ ($\approx 217 \text{ MeV}$), the strong coupling constant, α , is large ($\alpha > 1$, strong coupling domain), the perturbative expansion is invalid, consequently interesting domain of strong interaction remains intractable.

At low energy QCD possesses no expansion parameter. 't Hooft proposed to consider a generalization of QCD obtained by replacing the gauge group $SU(3)$ by $SU(N_c)$ and to perform an expansion in $\frac{1}{N_c}$ [42, 43] in the limit $N_c \rightarrow \infty$. It was found that $SU(N_c)$ Yang Mills gauge theory has a well defined perturbative expansion in the parameter, $\frac{1}{N_c}$, only if the number of colors, N_c , is large, while keeping the 't Hooft coupling, $\lambda = g^2 N_c$ fixed. This is what is called large N_c limit of QCD. Representing gluon lines in double line notation of color and anti-color, it can be shown that a certain class of Feynman diagrams called planer diagrams survive in this limit. The diagrams look like two-dimensional surfaces. The topological classification of the Feynman diagrams can be made precise by associating a Riemann surface to each Feynman diagram. It turns out that contribution of a given Feynman diagram to the amplitude is governed by the factor N_c^χ , where χ is the Euler number of the Riemann surface corresponding to that Feynman diagram. Thus, the expansion of any amplitude, \mathcal{A} , of QCD in terms of Feynman diagrams can be expressed as,

$$\mathcal{A} = \sum_{\chi} N_c^\chi A^{(\chi)}(\lambda) \quad (9)$$

where the coefficients $A^{(\chi)}(\lambda)$ are to be computed separately for the diagrams. The large N_c QCD is clearly not the same as QCD, however, in many cases the results for large N_c are almost the same as those for $N_c = 3$ theories. There are two regimes in large N_c QCD when parameterized by the coupling λ . For $\lambda \ll 1$ the theory is perturbative in λ and Feynman diagram summation can be used to calculate amplitudes, however, for $\lambda \gg 1$ the theory is in the non-perturbative domain, the diagrams become discretized two dimensional surfaces which are too difficult to compute.

In this situation, when QCD is known to be highly difficult for the analytical analysis at low energies because of the strong coupling problem, two paths were followed, namely, (1) QCD inspired models and effective theories, (2) numerical calculations using lattice QCD.

QCD inspired models aim to understand structure of baryons and mesons and their interactions incorporating some features of QCD, mainly, asymptotic freedom, confinement and chiral symmetry. These models include several versions of phenomenological bag models [2]. In the (u, d) sector, QCD lagrangian (1) has chiral symmetry which is spontaneously broken as parity doublets are not found in the hadronic ground states. The chiral symmetry and its spontaneous breaking was identified to be the key feature of low energy QCD. Taking pseudoscalar mesons, the Goldstone bosons of spontaneous chiral symmetry breaking, as the fundamental degrees of freedom of strong interaction at low energy, an effective theory called chiral perturbation theory was developed to replace QCD for low energy hadron physics [44]. Though the QCD inspired models and effective theories describe low energy hadron physics phenomena quite well, but these are not rigorously derivable from QCD.

Lattice QCD appears to be the only known way to study QCD without approximations [6, 7, 45]. It aims to solve QCD numerically by discretizing space-time on a four dimensional lattice, so that it can be simulated on a computer using methods analogous to those used for systems of statistical mechanics. Fields representing quarks are defined at lattice sites while the gluon fields are defined on the links connecting neighboring sites. The input parameters in these simulations are the strong coupling constant, α_s , and the quark masses, m_q . The discrete space-time lattice can be thought of as a nonperturbative regularization scheme. At finite values of the lattice spacing a , (providing an ultraviolet cutoff of the order of $\frac{1}{a}$) there are no infinities, and in the limit, $a \rightarrow 0$, finite renormalized physical quantities are recovered. These simulations allow to calculate correlation functions of hadronic operators and matrix elements of any operator between hadronic states in terms of the fundamental quark and gluon degrees of freedom.

These calculations being computationally demanding require the use of largest available supercomputers. It seems to be the only hope for understanding non-perturbative QCD. In the past several years, it has produced quantitatively impressive results, however, there exists conceptual and technical problems to apply it in all situations of strongly coupled QCD.

One of the major successes of $SU(3)$ lattice QCD calculation is to demonstrate that a confining potential naturally emerges between a static, infinitely heavy quark (q) and antiquark (\bar{q}) by considering the Wilson loop [46, 47]. The $(q\bar{q})$ potential can be well reproduced by a sum of the Coulomb term due to the perturbative one-gluon-exchange process, a linear confinement term and a constant,

$$V_{q\bar{q}}(r) = -\frac{A}{r} + \sigma r + V_0, \quad (10)$$

where $\sigma \simeq 0.89 \text{ GeV/fm}$ of the confining term is like string tension. The linear potential at the long distance can be physically interpreted with the flux-tube picture or the string picture for hadrons, in which the quark and the antiquark are linked with a one-dimensional flux-tube with the string tension σ . This flux-tube picture (or the string picture) in the infrared region is supported by the Regge trajectory of hadrons.

Therefore, the starting problem of hadron physics, that is, how to construct the spectrum of mesons and baryons from the fundamental constituents, quarks and gluons, remains one of the most challenging unsolved problems of strong interaction dynamics. In addition, data from RHIC experiments also have convincingly suggested, contrary to expectations, that the quark gluon plasma (QGP) is strongly coupled [10]. Actually, every experimental datum of hadron physics contains at least a part that depends on strong coupling physics. It is ironical that though QCD is a beautiful physical theory, but in the low energy domain, that contains interesting phenomena of hadron physics, QCD remains inaccessible. No escape from strongly coupled regime !

2.2. S-matrix approach to strong interaction

In the 1960's, when list of strongly interacting particles, mesons and baryons, was ever increasing, it was clear that none of these particles were elementary. The idea of "nuclear democracy" was invoked, in which the notion that some particles are more elementary than others was rejected. All hadrons have similar status and the concept of elementary constituent was thought to be unnecessary. The form of a quantum theory to describe the strongly interacting particles can be determined by using very general consistency criteria, and features of the spectrum of particles.

It was observed that the resonances (very short lived strongly interacting particles) can be arranged in straight lines in two dimensional plots with spin (J) and square of mass (m) as axes. These are called Chew-Frautschi plots [48], and the straight lines are called Regge trajectories which follow the relation,

$$J(= \alpha(s)) = \alpha s + \alpha(0), \quad (11)$$

with $s = m^2$. The Regge trajectories can be derived from a simple assumption that the hadrons are described by rotating relativistic strings. Energy (E) and the angular momentum (J) of a rigidly rotating string of length L with tension T , the endpoints of which move at near the speed of light c , can be written as,

$$E = T \int_{-L/2}^{L/2} \frac{dr}{\sqrt{1-v(r)^2}} = \frac{\pi}{2} TL, \quad J = T \int_{-L/2}^{L/2} \frac{v(r) r dr}{\sqrt{1-v(r)^2}} = \frac{\pi}{8} TL^2. \quad (12)$$

that reproduces the Regge trajectory as,

$$J = \frac{1}{2\pi T} E^2 = \alpha s, \quad (13)$$

taking $E = m$. Understanding Regge trajectories in terms of rotating string marks the beginning of string description of hadrons.

A self consistent theory of strong interaction in which there are no elementary constituents, all particles are composites (lying on Regge trajectories), and they scatter by exchanging composite particles was formulated using S-matrix (scattering matrix) that describe what happens when particles collide. The S-matrix transforms the initial state, $|i\rangle$, at $t \rightarrow -\infty$ into the evolved state, $|f\rangle$, at $t \rightarrow +\infty$,

$$|f\rangle = S |i\rangle, \quad (14)$$

satisfying general principles of unitarity, analyticity, crossing symmetry, and asymptotic behavior. The transition matrix T is defined as,

$$S = I + iT, \quad (15)$$

and the scattering amplitude, M_{fi} , for a scattering process $|i\rangle \rightarrow |f\rangle$, is defines as,

$$\langle f|T|i\rangle = (2\pi)^4 \delta(p_f - p_i) M_{fi}. \quad (16)$$

The scattering amplitude, M_{fi} , can be expressed as product of two factors,

$$M_{fi} = K_{fi} A(s, t, u). \quad (17)$$

where K_{fi} is a covariant factor depending on spin, isotopic spin, and momentum components of the external particles, and the invariant amplitudes $A(s, t, u)$ is the dynamical part depending only on the Mandelstam variables, s, t and u of which only two are independent. The scattering

in the “t-channel” and “s-channel” must produce the same amplitudes, because same set of poles (resonances) participate in scattering in both channels. Veneziano [15] constructed the amplitude of a scattering process as,

$$A(s, t, u) = A(s, t) + A(s, u) + A(t, u), \quad (18)$$

with

$$\begin{aligned} A(x, y) &= \frac{\Gamma(-\alpha(x))\Gamma(-\alpha(y))}{\Gamma(-\alpha(x) - \alpha(y))} \\ &= \int_0^1 dz z^{-\alpha(x)-1} (1-z)^{-\alpha(y)-1}, \end{aligned} \quad (19)$$

where $\alpha(x)$ is a linear function of x , satisfying Regge trajectory. Veneziano formula was extremely successful in understanding low energy data of meson scattering.

The 4-particle Veneziano formula was generalized to an N-particle amplitudes which were factorized using operators consisting of an infinite number of harmonic oscillators. The complete spectrum of mesons was determined. It turned out that the degeneracy of states grew up exponentially with the mass that agrees with Hagedorn observation [49]. This suggested that Veneziano formula might be indicating something more than just approximate phenomenological descriptions of hadron scattering. These could be regarded as the lowest order approximation of amplitudes from some new quantum theory.

The Veneziano amplitude grew out of S-matrix theory where the scattering amplitude is the only observable object, the action or the lagrangian does not play any role. Nambu [16], Nielsen [17] and Susskind [18] found that the infinite number of oscillators, that one gets through the factorization of N-point Veneziano amplitude, naturally comes out from quantization of the lagrangian for a one-dimensional relativistic string, and Veneziano formula can be obtained as the tree-level string interaction amplitude. This is the beginning of string theory to understand hadronic physics. In the following section we give a very brief introduction to string theory.

3. Elements of string theory

We discuss some basic concepts of string theory mostly following the text books [50, 51, 52, 53, 54, 55, 56, 57]. Some of these concepts will be required to appreciate the connection between gauge theories and string theories through Ads/CFT correspondence.

3.1. Bosnic strings

String are objects extended in one spatial dimension, having no thickness but only length, typically of the order of Planck length $\left(l_P = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-33} \text{ cm}\right)$, thus practically look like point particles in the hadronic length scale. The strings could be open ended or closed. As the string moves, it sweeps out a two-dimensional surface (sheet), called the worldsheet, similar to the world-line generated by the motion of a point particle. We need two coordinates, (σ, τ) , to describe the worldsheet, where σ is a coordinate along the string, and τ a timelike coordinate, with the limits, $0 \leq \sigma \leq l_s$ and $-\infty \leq \tau \leq \infty$, where l_s is the length of the string. The world sheet is embedded in a d -dimensional space-time, \mathcal{M} . Thus, any point P having worldsheet coordinates (σ, τ) can also be specified by d -coordinates, $X^\mu(\sigma, \tau)$ ($\mu = 0, 1, \dots, d-1$), of \mathcal{M} .

The action to describe the dynamics of string can be taken as proportional to the invariant area of the worldsheet,

$$S \propto \int dA. \quad (20)$$

The infinitesimal area, dA , can be written as,

$$dA = d\sigma d\tau \sqrt{-\det(h_{ab})}, \quad (21)$$

where

$$h_{ab} = G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu, \quad (a, b) \in (\sigma, \tau), \quad (22)$$

is the induced metric on the worldsheet. $G_{\mu\nu}$ is the metric of \mathcal{M} , it governs all the geometric properties of \mathcal{M} . The Nambu-Goto action is written as,

$$S_{NG} = -T \int d\sigma d\tau \sqrt{-\det(h_{ab})} = -T \int d\sigma d\tau \sqrt{-\det(\partial_a X^\mu \partial_b X_\mu)}, \quad (23)$$

where T , called ‘string tension’, has dimension of force, and can be related to the Regge slope, α , as,

$$T = \frac{1}{2\pi\alpha}, \quad \alpha \equiv l_s^2, \quad (24)$$

with fundamental length scale, l_s , (string length). This action is difficult to quantize, because of the square root. An alternate action, equivalent to the Nambu-Goto action, S_{NG} , the so called ‘Polyakov action’ is constructed by introducing an auxiliary field, γ_{ab} , as,

$$S_P = -\frac{T}{2} \int d\sigma d\tau \sqrt{-\det(\gamma_{ab})} \gamma^{ab} G_{\mu\nu} \partial_a X^\mu \partial_b X_\mu. \quad (25)$$

The polyakov action is similar to the action of d bosonic fields, $X^\mu(\sigma, \tau)$ in 2-dimensions (one space and one time). It is invariant under Lorentz transformations and translations of $X^\mu(\sigma, \tau)$, in d -dimensions, thus exhibits Poincare symmetry. In addition, S_P is invariant under 2-dimensional diffeomorphism or reparametrization, $\tau \rightarrow \tau'(\sigma, \tau)$, $\sigma \rightarrow \sigma'(\sigma, \tau)$, and the Weyl rescaling of the auxiliary field, $\gamma_{ab} \rightarrow \gamma_{ab} e^{\phi(\tau, \sigma)}$.

The equations of motion of $X^\mu(\sigma, \tau)$ can be obtained by varying S_P with respect to $X^\mu(\sigma, \tau)$ as,

$$\frac{\delta S_P}{\delta X^\mu} = 0 \quad \implies \quad \partial_a \left(\sqrt{-\gamma} \gamma^{ab} \partial_b X_\mu \right) = 0, \quad (26)$$

which are constrained by the equation of motion for γ^{ab} , obtained by varying S_P with respect to γ^{ab} as,

$$\frac{\delta S_P}{\delta \gamma^{ab}} = 0 \quad \implies \quad \partial_a X^\mu \partial_b X_\mu - \frac{\gamma^{ab}}{2} \gamma^{cd} \partial_c X^\mu \partial_d X_\mu = 0. \quad (27)$$

These constraints are called the Virasoro constraints. The equation of motion for X^μ are supplemented with boundary conditions that make the boundary terms vanish in variation. The closed string satisfies the periodic boundary conditions, $X^\mu(\sigma, \tau) = X^\mu(\sigma + l_s, \tau)$, and the open string satisfies either Neumann boundary conditions, $\partial_\sigma X^\mu(0, \tau) = \partial_\sigma X^\mu(l_s, \tau) = 0$, or Dirichlet boundary conditions, $\delta X^\mu(0, \tau) = \delta X^\mu(l_s, \tau) = 0$, at the end points of the string. For an open string there could be Neumann boundary conditions in p directions, and Dirichlet boundary conditions in $d - p$ directions.

The re-parametrization and Weyl symmetry of S_P are local gauge symmetry on the worldsheet. These can be utilized to fix the auxiliary field, γ^{ab} . However, even after gauge fixing, S_P is left with a residual 2-dimensional conformal symmetry.

The solution of (27) can be written as an expansion in terms of different oscillation modes of string. The string action, S_P , can be quantized using canonical quantization procedure, by making the dynamical variable, $X^\mu(\sigma, \tau)$, and its conjugate momenta as operators and employing commutation relations between them. The expansion coefficients of oscillation modes become creation and annihilation operators. A Fock space of quantum states corresponding to different oscillation modes are generated by applying these creation operators on the vacuum. The residual symmetry is implemented on quantum states using Gupta-Bleuler like methodology of quantum electrodynamics. The particles corresponding to the excitations of the open string propagate in the world-volume on which the end points of the string is fixed, while particles corresponding to the excitations of the closed string propagate in the full d -dimensional space-time. The main results of bosonic string theory can be listed as,

- (i) The bosonic string can be quantized consistently only if the dimension of \mathcal{M} , $d = 26$,
- (ii) Each vibrational mode of the string corresponds to a particle with distinct spin (J) and mass (M). The spectrum contains a finite number of massless and infinitely many massive excitations with, $M^2 = \frac{n}{2\alpha}$ with $n \in N$.
- (iii) The lowest energy state (ground state) is tachyonic, $M^2 < 0$,
- (iv) The spectrum of the closed string contains a massless spin-2 (graviton) particle.

The above results were not encouraging enough to sustain interest in string theory as a candidate to describe strong interaction physics. The graviton-like particle was a great embarrassment as the string was first developed as a model of hadrons. However, progress in string theory continued to make it free from above anomalies, particularly the tachyonic states. The major development in string theory at this stage involved inclusion of fermions in string action as any fundamental theory of nature must contain both of these types of particles. In doing this, a new kind of symmetry called ‘supersymmetry’ was developed.

3.2. Supersymmetry

The supersymmetry is a symmetry between bosons and fermions [58, 59]. It came about in trying to unify gravity with other interactions. According to Einstein’s general theory of relativity, gravity is related to the space-time symmetries of Poincare group, a non-compact Lie group, whereas other interactions, namely strong, weak and electromagnetic, are based on local gauge invariance of internal symmetries (or gauge groups) of compact Lie groups namely, $SU(3)$, $SU(2)$ and $U(1)$ respectively. Therefore, any attempt to unify gravity with other interactions requires, the Poincare group and the internal symmetry group to be part of same algebra. However, the answer to such attempts turned out to be no, in the form of the Coleman-Mandula ‘no-go theorem’, which says that if the Poincare symmetry and internal symmetries were to combine in a Lie algebra, the S-matrices for all processes would be identically zero [60]. However, like all theorems, this theorem also was only as strong as its assumptions, and one of the assumption was that the final algebra is a Lie algebra. It was realized that the notion of Lie algebra can be generalized to a graded Lie algebra (or super algebra) to circumvent the no-go theorem.

A super algebra is an algebra that contains some generators, Q^i , which satisfy anticommutation relations among themselves, and commutation relations with other generators which satisfy commutation relations among themselves. An anticommuting operator Q^i is fermionic, so, these operators generate transformations shown schematically as,

$$Q^i|\text{Boson}\rangle = |\text{Fermion}\rangle, \quad Q^i|\text{Fermion}\rangle = |\text{Boson}\rangle.$$

This is called supersymmetry (SUSY). Fermions and bosons are grouped together into supermultiplets which are related under this symmetry. Thus, according to supersymmetry,

corresponding to every boson there must be a fermion with the same mass and internal quantum numbers, and vice-versa.

Supersymmetry generators, Q^i , are intrinsically complex objects, so $Q^{i\dagger}$ are also distinct symmetry generators. The index $i = 1, \dots, \mathcal{N}$ represents different supersymmetric generators in case there are more than one pair.

Supersymmetric partners of currently known particles have not yet been observed experimentally. It is believed that the reason for non observation of these particles is because supersymmetry is a broken symmetry, and as a result the superpartners are much heavier than the known particles. Particle accelerators (LHC) may be on the verge of finding evidence for high energy supersymmetric partners in the near future!

3.3. Superstring theory

Superstring theory attempts to describe both bosons and fermions in terms of the vibrational modes of the fundamental string. In doing that, the field content of the world-sheet is enlarged to include fermionic (anticommuting) variables, Ψ_α^μ , in addition to the bosonic variables, X^μ . For every μ , the two component Ψ_α^μ is a fermionic variable on the world-sheet, quantization of which would give rise to particle states that behave as space-time fermions. The Polyakov string action with world-sheet fermions can be written in conformal gauge as,

$$S_P = -\frac{T}{2} \int d\sigma d\tau (\partial_\alpha X^\mu \partial^\alpha X_\mu - i \bar{\Psi}^\mu \rho^\alpha \partial_\alpha \Psi_\mu), \quad (28)$$

where ρ^α are 2-dimensional Dirac matrices,

$$\{\rho^\alpha, \rho^\beta\} = -2\eta^{\alpha\beta}, \quad \rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad (29)$$

and the world-sheet fermions, $\Psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix}$, with $\bar{\Psi}^\mu = \Psi^\dagger \rho^0$.

The original action, from which (28) is obtained after gauge fixing, is invariant under: (i) Poincare transformations, (ii) Worldsheet reparametrizations, (iii) Weyl transformations, (iv) super-Weyl transformations (for fermionic variables), and (v) worldsheet supersymmetry. The worldsheet supersymmetry transforms fermions into bosons and vice versa as,

$$\delta X^\mu = \epsilon \Psi^\mu, \quad \delta \Psi^\mu = -i \rho^\alpha \partial_\alpha X^\mu \epsilon, \quad (30)$$

where ϵ is a constant anti-commuting spinor.

The equation of motion for the X^μ fields is the same to that in the bosonic case (Laplace equation), and the equation of motion for the fermionic variables Ψ^μ is the Dirac equation in two dimensions. These are supplemented with boundary conditions. For the closed superstring there are two possibilities for the boundary conditions of fermions, (i) periodic boundary conditions (Ramond (R) sector) $\psi_{+/-}^\mu(\sigma, \tau) = \psi_{+/-}^\mu(\sigma + l, \tau)$ and (ii) anti-periodic boundary conditions (Neveu-Schwarz (NS) sector) $\psi_{+/-}^\mu(\sigma, \tau) = -\psi_{+/-}^\mu(\sigma + l, \tau)$.

In addition there are constraints, coming from the equation of motion of the auxiliary field, to be satisfied by the fermionic variables. These are more involved and called the super-Virasoro constraints.

The quantization of the superstring is achieved, in canonical quantization procedure, by promoting the bosonic variable, X^μ , and the fermionic variable, Ψ^μ , to operators, and imposing commutation and anticommutation relations respectively between them and their canonical conjugates. In doing that the coefficients corresponding to vibrational modes become creation and annihilation operators.

A Fock space of quantum states corresponding to different oscillation modes of superstring are generated by applying these creation operators on the vacuum. The residual symmetry is implemented on quantum states. These vibrational modes of the superstring correspond to an infinite tower of particle states with definite spin (J) and mass (M), satisfying a linear Regge trajectory relation. It also turns out that the spectrum of the superstring can be organized into spacetime supersymmetry multiplets. The spectrum of the fermionic string is truncated by an additional projection, the Gliozzi-Scherk-Olive (GSO) projection [61]. This projection can be chosen in such a way that the tachyon is projected out, and the remaining spectrum is space-time supersymmetric. The main results of superstring theory can be listed as,

- (i) The quantized theory of superstring shows that vibrational mode corresponds bosonic and fermionic particles with distinct spin (J) and mass (M),
- (ii) The superstring can be quantized consistently only if the dimension of the space-time is 10.
- (iii) The ground state is not tachyonic.
- (iv) The spectrum of the closed superstring contains a massless spin-2 excitation ($J = 2, M = 0$), whereas the spectrum of the open string contains a massless spin-1 excitation ($J = 1, M = 0$), along with their space-time supersymmetric partners.

The massless spin-1 excitations of open string were identified with gluons and the massless spin-2 excitation of closed string was identified with graviton. A bold assertion was made that string theory could be a framework for understanding quantum gravity [21, 22].

There are five types of perturbatively consistent superstring theories in 10 space-time dimensions. These are called, Type-I, Type-IIA, Type-IIB, Heterotic $SO(32)$ and Heterotic $E8 \times E8$. All of these are free from tachyon problem, and contain closed strings implying that string theory is inconsistent without closed strings (or gravity).

3.4. String interaction

The simplest string interaction is mediated by joining two open strings at their end points, or splitting an open string into two open strings. For example, the Feynman diagram corresponding to annihilation of two particles is obtained by merging of two open strings into a single open string.

The strength of the interaction is controlled by the value of the dimensionless string coupling constant g_s , which is dynamically determined through the vacuum expectation value, Φ_0 , of the dilaton, Φ , as $g_s = e^{\Phi_0}$. The dilaton is a massless excitation present in every string theory.

In quantum field theory, computation of scattering amplitudes for a given process involves summation over all possible Feynman diagrams corresponding to that process. However, in string theory, it is replaced by the summation over world-sheets of different topologies, depending on the choice of the type of the string theory. For example, the worldsheet corresponding to scattering of two closed strings, in the tree level, is topologically equivalent to a sphere with four discs cut out, and in the one-loop it is a torus. These are two dimensional surfaces with holes. Thus, interactions are introduced in string theory through the inclusion of topologically non-trivial surfaces.

Scattering amplitudes, \mathcal{A} , of a given process can be computed in perturbation theory, where \mathcal{A} is expanded in a power series in the string coupling g_s as,

$$\mathcal{A} = \sum_{\chi} g_s^{-\chi} A^{(\chi)}, \quad (31)$$

where each term $A^{(\chi)}$ is computed separately. The validity of perturbation theory requires $g_s \ll 1$. The strength with which a given world-sheet contributes to the scattering amplitude is governed by χ , the Euler number of the worldsheet, determined by its topology. Though

the scattering amplitude is formally given in (31), however, extracting answers from diagrams with more than two loops is very difficult due to the complexity of the mathematics involved in dealing with these surfaces.

The scattering amplitudes in string theory at each order in perturbation theory consist of only one diagram which is not plagued with infinity, in contrast to that in quantum field theory where there are many Feynman diagrams at each order and diagrams containing loops suffer from ultra-violet divergences. Heuristically this can be understood as if the point-like interaction vertices of quantum field theory are now smeared due to finite extent of the string.

The scattering amplitude (31) of string perturbation theory is similar to the scattering amplitude (9) in 't Hooft large N expansion of $SU(N)$ gauge theory with the identification, $g_s \sim \frac{1}{N}$. Feynman diagrams of the gauge theory become surfaces that represent interacting strings. This gives the strongest hint that there could be some connections between string theory and gauge theory.

3.5. Compactification

Superstrings are consistently quantized in 10-dimensional space-time, but observationally space-time of our universe is 4-dimensional. To make the superstrings to describe the universe, the extra 6 dimensions have to be curled up into a small compact space of dimension of the order of the string scale (10^{-33} cm), so that the presence of these extra dimensions will not be detected. Thus, a compact 6-dimensional space needs to be hidden at every point in the 4-dimensional space-time.

It is possible to formulate string theory in non-trivial ten dimensional space-times where only four dimensions are infinitely extended and the remaining six are curled up and compact. One possible realization of compactification can be achieved by making a direct product or factorization ansatz, $\mathcal{M}^{10} = M^4 \times K^6$ for the 10-dimensional space-time, where M^4 is 4-dimensional Minkowski space, and K^6 a 6-dimensional compact manifold. The resulting theory in M^4 depends on the geometry and topology of the compact manifold, K^6 . A particular scheme of compactification is known as Kaluza-Klein compactification, in which the extra 6 spatial dimensions may be put on 6 circles, forming a 6-dimensional torus.

The compactification leads to a tower of states, called Kaluza-Klein states, because if one of the spatial dimensions is circular of radius R (periodic), we know from quantum mechanics that the momentum in that dimension is quantized as, $p = \frac{n}{R}$ ($n = 0, 1, 2, 3, \dots$). These states may show up in the mass spectrum, however, choosing the spatial extension, R , of the compactified dimension appropriately the Kaluza-Klein states may be made unobservable in the uncompactified 4-dimensional world.

Another interesting implication of compactification is that strings can wind around a compact dimension. This leads to winding modes in the mass spectrum. A closed string can wind around a periodic (circular) dimension an integral number of times. The energy of the winding mode is equal to the string tension, T , times the total length of the wrapped string, $E_w = w2\pi RT = \frac{wR}{\alpha}$, with w ($= 0, 1, 2, \dots$) being winding the number.

Another compactification scheme is by taking the compact manifold K^6 to be Calabi-Yau space. In fact, the string dynamics, in particular the assumption of the factorization ansatz, $\mathcal{M}^{10} = M^4 \times K^6$, and supersymmetry in uncompactified M^4 , restricts the compactified space, K^6 , to be Calabi-Yau space. It is possible to cleverly compactify [62] the superstring to get realistic gauge groups, $SU(3) \times SU(2) \times U(1)$, of standard model in M^4 . The topology of Calabi-Yau space controls the number of families of low mass fermions, quarks and leptons.

3.6. Non-perturbative string theory

The formulation of string theory is of perturbative nature. The Feynman diagrams correspond to surfaces. There is no a priori reason, however, to assume that the string coupling constant,

g_s , which controls the expansion in terms of surfaces, is small. In addition, there are five independent perturbative superstring theories with different basic properties. Thus, for a complete understanding of the string theory we need to include non-perturbative effects. Understanding non-perturbative effects in string theory was possible by the discovery of duality symmetries [63, 64]. Dualities actually are equivalences between two apparently different theories. The idea of duality is very powerful, since mapping to a dual set of variables we can transform a difficult question (strong-coupling behavior) of one theory into a easy one (weak-coupling behavior) of the other. There are two types of duality symmetry in string theory, namely S-duality and T-duality.

The S-duality, which has a correspondence in quantum field theory, relates strong coupling regime of one theory to the weak coupling regime of the other. The T-duality, on the other hand, has no analogue in quantum field theory, it belongs solely to string theory, and originates due to compactification of extra dimensions in string theory. It relates a theory in which extra dimension is compactified on a circle of radius R , to another theory in which extra dimension is compactified on a circle with radius $1/R$.

These dualities play a very important role in unifying five different perturbative superstring theories. Using a combination of S and T dualities it was realized that the five perturbatively defined string theories are actually different phases of a unified theory called M-theory which is defined in 11-dimensions [65, 66]. In this picture, different ground states of the M-theory correspond to various string theories. The perturbation theory only probes the vicinity of each ground state, whereas duality makes non-perturbative correlations across different vacua.

Using T-duality Polchinski [67, 68] showed that string theory necessarily includes, in addition to strings, various higher dimensional objects known as branes [69]. One important class of branes are called Dp-branes (or Dirichlet-p-branes, with $0 \leq p \leq 9$) which are $(p+1)$ -dimensional hyper-surfaces in the $(9+1)$ -dimensional target space, \mathcal{M} . D0-branes are particle-like, D1-branes are string-like, and D2-branes are membrane-like objects, and so on.

There are two ways to think about Dp-branes. On the one hand, in closed string theory these are solitonic solutions of the equations of motion, characterized by a charge and mass per unit volume, called brane tension, which is inversely proportional to the string coupling constant g_s . This dependence on the coupling constant is typical of the solitons in field theory. Because of their masses, Dp-branes can deform the space-time metric around them as manifestation of the gravitational effects.

On the other hand, in open string theory these are $(p + 1)$ -dimensional hyper-surfaces. The end points of open strings, satisfying Neumann boundary conditions in p longitudinal directions and Dirichlet boundary conditions in $9 - p$ transverse directions, lie on these surfaces, and move freely along the p -directions of the D-brane, but cannot leave it by moving in the transverse directions. The end points of an open string while moving on a Dp-brane may come in contact and become a closed string which can leave the brane, and closed strings can break touching the Dp-branes and become open strings. The massless excitations of a open string whose end points lie on a Dp-brane are gauge fields and their super-partners constitute a maximally supersymmetric $U(1)$ gauge theory in $p+1$ dimensions. This gauge theory is localized on the world-volume of the Dp-branes. The gauge coupling constant, g , is related to the string coupling constant g_s as, $g^2 = 4\pi g_s$, for $p = 3$.

4. Gauge-Gravity duality

Strong interaction appears to be too difficult to understand in both the approaches described above, because of QCD becoming strongly coupled at low energies, and string theory becoming more of a theory of quantum gravity. In this situation, it is an interesting idea to look for a duality, that is, to find a theory which is equivalent to QCD, but weakly coupled when QCD is strongly coupled.

There are several examples of duality in physics. A famous example of duality in $(1 + 1)$ -dimensions is between Sine-Gordon model describing massless bosons and Thirring model describing massive fermions. The Sine-Gordon model [70] is a quantum field theory in $(1+1)$ -dimensional space-time of a single scalar field $\phi(x)$ which is self-coupled as described by the lagrangian,

$$\mathcal{L}_{SG} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{\alpha}{\beta^2} (\cos(\beta\phi) - 1), \quad (32)$$

where α is a parameter and β is the coupling constant. The classical equations of motion corresponding to the lagrangian (32) give rise to stable solutions known as solitons and antisolitons. Interestingly, many-soliton solutions obey Pauli's exclusion principle, which can be interpreted as a fermion-like behavior [70]. In turn, the Thirring model [71] is a theory of a self-coupled Dirac field $\psi(x)$ in $(1+1)$ -dimensional space-time described by the lagrangian,

$$\mathcal{L}_T = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m_f\bar{\psi}\psi - \frac{\kappa}{2} (\bar{\psi}\gamma^\mu\psi)^2, \quad (33)$$

where m_f is the mass of the fermion field and κ is a dimensionless coupling constant. Coleman [72] compared the n -point Green functions of the Sine-Gordon and Thirring model computed using a perturbative approach in coordinate representation and concluded that they should be equivalent if the coupling constants β and κ are related by,

$$\frac{4\pi}{\beta^2} = 1 + \frac{\kappa}{\pi}. \quad (34)$$

Weak Coupling in the Sine-Gordon model corresponds to strong coupling in the Thirring model. The Thirring model is S-dual to the sine-Gordon model. The fundamental fermions of the Thirring model correspond to the solitons of the Sine-Gordon model.

Given the above example of duality between two completely different theories we may think that a dual to strongly coupled gauge theory like QCD may be found in string theory, because there are several connections between QCD and string theory, namely,

- (i) Understanding Regge trajectories (11) in terms of semi-classical spinning relativistic string,
- (ii) The confining term in quark-antiquark potential (10) appears to be string-like,
- (iii) Feynman diagrams of the $SU(N)$ gauge theory in the large N limit can be identified as surfaces that represent interacting strings.

However, in course of time, strong interaction and string theory got disconnected mainly because of overwhelming success of QCD at high energy, and string theory becoming more of a theory of quantum gravity than strong interaction.

A very important development namely holographic principle, has played the crucial role in finding a dual theory of QCD that brings strong interaction and string theory together again.

4.1. Holographic principle

The holographic principle originated in trying to understand thermodynamics of black holes. A black hole is a region of space from which nothing, not even light, can escape. Thus, a region of space of radius r containing mass M becomes a black-hole, in newtonian description of gravity, if escape speed from that region is greater than the speed of light,

$$\sqrt{\frac{2GM}{r}} > c. \quad (35)$$

Dense enough matter collapses into a black hole [73]. In Einstein's general theory of relativity, which is relativistic description of gravity, black holes appear as the singularities of space-time, where the curvature diverges. The singularities are conjectured [74] to be hidden behind the event horizon, the surface, $r = r_H$,

$$r_H = \frac{2GM}{c^2}, \quad (36)$$

through which things can go in but can not come out. Thus, black holes are an essential feature of gravity. Physically black holes may be created in astrophysical processes when stars run out of nuclear fuel and implode under gravitational force. If the mass of the leftover matter after supernova explosion exceeds about 3 solar masses, it will collapse into a black hole.

Using general relativity Hawking [75] proved an useful theorem called the horizon area theorem, according to which the total horizon area, A , in a closed system containing black holes never decreases. It can only increase or stay the same,

$$\Delta A \geq 0. \quad (37)$$

Increase in total horizon area comes from the growth of black holes by accretion of "normal" matter or by the coalescence of black holes.

Stationary black holes are unique, in the sense that any arbitrary system having mass M , angular momentum J and charge, Q , will collapse to form a unique black hole. This is known as black hole uniqueness theorem or 'no hair theorem' [76]. Thus, black holes are actually extremely simple macroscopic objects, which can be parametrized by only a few numbers, similar to elementary particles which are characterized by a few quantum numbers.

The black hole uniqueness theorem apparently leads to a paradox of violating second law of thermodynamics, since in the initial state the matter that forms or falls into a black hole has arbitrarily large phase space or entropy, and in the final state the black hole occupies a point in phase space having zero entropy. Resolution of the paradox was given by Bekenstein [77]. Noting striking similarity between second law of thermodynamics, $\Delta S \geq 0$ and Hawking area theorem: $\Delta A \geq 0$, he proposed that black holes must have entropy, S_{bh} , which is proportional to the area of their event horizons,

$$S_{\text{bh}} \propto A. \quad (38)$$

Black holes having entropy gives rise to paradox again, since objects having entropy are hot, so they must radiate, but nothing can come out of black holes. Resolution of this paradox was given by Hawking using quantum mechanics [78]. Virtual pairs of particles and antiparticles are produced and annihilated in vacuum all the time violating energy conservation for short durations allowed by Heisenberg's uncertainty principle. If such an event occurs near the event horizon of a black hole, the virtual pair can be split up by the strong gravity so that it is possible for one with negative energy to fall in the black hole and the other with positive energy to escape. The energy conservation "debt" involved in the vacuum fluctuation is paid by the black hole itself, the negative energy particle makes a negative contribution to the mass of the black hole when it goes in. The black hole's mass decreases by the mass of the escaping particle. The escaping particle is seen by a distant observer as emission by the black hole.

The spectrum of the emitted particles is similar to black body radiation with a temperature. Using semi-classical method Hawking [78] estimated the temperature to be,

$$T_H = \frac{\hbar c^3}{16\pi^2 k_B GM}. \quad (39)$$

Using, $dE = c^2 dM$, and first law of thermodynamics, $dE = T_H dS_{\text{bh}}$, we get the black hole entropy as

$$S_{BH} = \int \frac{c^2}{T_H} dM = \frac{k_B A}{4l_P^2}, \quad (40)$$

where $A = 4\pi r_H^2$ is the area of the event horizon. In contrast to statistical physics or local field theory where entropy is a measure of the phase space volume and proportional to volume of the system, black hole entropy, as proposed by Bekenstein, is proportional to the area of the horizon. Proportionality of black hole entropy to the area rather than volume indicates that the degrees of freedom of (quantum) gravity may not be similar to that of local field theories. In the context of string theory using D-branes, the Bekenstein-Hawking formula (40) of black hole entropy has been derived [79] for some special class of black holes.

Application of second law of thermodynamics shows that the entropy of any region is bounded and the upper bound is equal to the entropy of the largest black hole that can be accommodated in the region. Let the entropy, S , of a region of volume V with area A be greater than the entropy, S_{BH} , of the largest black hole which can be accommodated in the region ($S > S_{BH}$). Then, putting in some matter in that region a black hole can be formed, the entropy of the region becomes S_{BH} which is less than original entropy S . It violates second law of thermodynamics. The conundrum of violating second law of thermodynamics can be resolved if the original assumption is wrong. Therefore, in a system with gravity, maximum entropy of any region is proportional to the boundary area of the region.

The perplexing result of quantum gravity that maximum entropy of a given region is proportional to its boundary area, led 't Hooft [80] and Susskind [81] to formulate holographic principle. It states that any theory of gravity in d dimensional space-time should have a description in terms of a quantum field theory defined in a flat (*i.e* without gravitational interactions) $(d - 1)$ -dimensional space-time. Both the theories have an entropy proportional to the area in d -dimensions, because surface in d -dimension is the same as volume in $(d - 1)$ -dimensions. In other words, quantum gravity in any space of d -dimension can be formulated in terms of degrees of freedom of a local field theory defined in the boundary space of $(d - 1)$ dimension. This situation is analogous to an optical hologram, which stores all information of a 3-dimensional object in a 2-dimensional surface.

The holographic principle suggests that there should exist a relation (equivalence) between gauge theory in $(3 + 1)$ -dimensions with a gravity theory in $(4 + 1)$ -dimensions. This equivalence of a gravity theory in bulk to a gauge theory on surface is called gauge-gravity duality. The remarkable utility of this duality is that, in particular limiting situations when quantum field theory is strongly coupled, the corresponding gravity theory becomes classical. Thus, strongly coupled regime of quantum field theories can be approached doing calculations in classical gravity theory.

An extensively studied particular case of the gauge-gravity duality in the context of string theory is the AdS/CFT correspondence proposed by Maldacena [23]. In the recent time, with the conjecture of Maldacena, string theory has become a powerful analytic tool for studying strongly coupled gauge theories. The most interesting strongly coupled gauge theory is QCD at low energies. Though gravity dual of QCD has not been found yet, but theories close (supersymmetric generalizations) to QCD have been studied using the so-called 'gauge gravity duality.

5. AdS/CFT correspondence

The AdS/CFT correspondence is a special case of gauge-gravity duality, where the gravity theory is defined on a 5-dimensional Anti de-Sitter (AdS₅) space, and the dual gauge theory

is a conformal field theory (CFT) defined in the boundary of AdS_5 , which is 4-dimensional Minkowski space. Before giving a heuristic derivation of AdS/CFT correspondence, we discuss briefly about the Anti de-Sitter space and conformal field theory.

5.1. Anti de-Sitter space

The anti-de Sitter space is a maximally symmetric, homogeneous and isotropic space-time with constant negative curvature (hyperboloid) [82, 83, 84]. Roughly, a negative curvature can be described by the fact that geodesics (shortest line between two points) which are parallel to begin with start moving away from each other while in a spherical or elliptical surface (with positive curvature) they come closer to each other.

Five dimensional anti-de Sitter space, AdS_5 , with constant negative curvature, R , can be conveniently defined by the equation,

$$\left[(X^0)^2 - (X^1)^2 - (X^2)^2 - (X^3)^2 \right] - (X^4)^2 + (X^5)^2 = R^2 \quad (41)$$

as an embedding in a 6-dimensional flat space-time with co-ordinates, (X^0, \dots, X^5) , and having (2, 4) signature. The defining equation (41) is symmetric under the transformation,

$$X^\mu \rightarrow X'^\mu = \Lambda^\mu_\nu X^\nu, \quad (42)$$

where Λ^μ_ν belongs to $SO(2,4)$, the conformal group in 4-dimension. Thus, $SO(2,4)$ is the isometry group of AdS_5 . The line element can be written as,

$$ds^2 = - (dX^0)^2 + (dX^1)^2 + (dX^2)^2 + (dX^3)^2 + (dX^4)^2 - (dX^5)^2, \quad (43)$$

with the constraint (41). Introducing a set of coordinates, called Poincare co-ordinates, given by a four dimensional Lorentz vector, $x^\mu \equiv (x^0, \vec{x})$, and a fifth coordinate, z , as,

$$\begin{aligned} X^i &= \frac{R x^i}{z}, \quad i = 1, \dots, 3; & X^5 &= \frac{R x^0}{z}, \\ X^0 + X^4 &= z - \frac{\eta_{\mu\nu} x^\mu x^\nu}{z}, & X^0 - X^4 &= \frac{R^2}{z}, \end{aligned} \quad (44)$$

with $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, the AdS_5 metric in (x^μ, z) co-ordinates can be written as,

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2) = g_{MN} dx^M dx^N, \quad (45)$$

where

$$g_{MN} = \frac{R^2}{z^2} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} = \frac{R^2}{z^2} \eta_{MN}, \quad (46)$$

with η_{MN} as 5-dimensional Minkowski metric and $\det(g) = \frac{L^{10}}{z^{10}}$. The AdS_5 has slices isomorphic to four-dimensional Minkowski space-time put on z -axis. The Minkowski metric is multiplied by a warp factor $\frac{R^2}{z^2}$, so, an observer living on a Minkowski slice sees all lengths rescaled by a factor of z depending on its position in the fifth dimension.

The Ricci tensor, R_{MN} , calculated from the AdS_5 metric g_{MN} is,

$$R_{MN} = -\frac{4}{R^2}g_{MN}, \quad (47)$$

which gives the Ricci scalar, \mathcal{R} , of AdS_5 metric as, $\mathcal{R} = -\frac{20}{L^2}$. We see that AdS_5 metric is solution of source free Einstein's equation of general relativity,

$$R_{MN} - \frac{1}{2}\mathcal{R}g_{MN} + \Lambda g_{MN} = 0, \quad (48)$$

in five dimension having coordinates (t, x^1, x^2, x^3, z) with cosmological constant, $\Lambda = -\frac{12}{R^2}$.

To understand physically the significance of the fifth coordinate, let us consider a 4-dimensional space with coordinates, $[x^\mu \equiv (t, \vec{x})]$, with a variable, l , that corresponds to energy scale,

$$l \equiv E = ' \partial_t '.$$

Under a scale transformation the coordinates transform as,

$$x^\mu \rightarrow x'^\mu = \lambda x^\mu, \quad l \rightarrow l' = \frac{l}{\lambda}. \quad (49)$$

We can construct a 5-dimensional space with co-ordinates, (x^μ, l) , that encodes the requirement of scale invariance should have the line element as,

$$ds^2 = R^2 \left(l^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{dl^2}{l^2} \right), \quad (50)$$

where R is the radius of curvature of the AdS_5 space. By changing the coordinate, $l \rightarrow \frac{1}{z}$, in (50), we get the line element (45) of AdS_5 . Thus, the fifth co-ordinate, z , of AdS_5 is inversely related to the energy scale. Large energy scale, or the ultra violet region, corresponds to small z , and small energy scale, infra red region, corresponds to large z . Thus, physics in the Minkowski space at different energy scale can be explored from AdS_5 -space at different values of z .

5.2. Conformal field theories and $\mathcal{N} = 4$ Supersymmetric Yang Mills theory

The conformal transformations include Poincare transformations and scale transformations that preserve the metric up to a scale factor, $g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x)$. These transformations form a group called conformal group. The generators are the usual Lorentz generators, $M_{\mu\nu}$, the Poincare translation operators P_μ , and in addition two generators D and K_μ corresponding to scale transformations. The conformal group in 4-dimensions is isomorphic to $SO(2,4)$. A field theory which is invariant under conformal group is called a conformal field theory. If we want to combine the conformal algebra with the supersymmetry algebra, then additional fermionic generators are required to be included in the algebra. The scaling dimension Δ of a field $\phi(x)$ is governed by the transformation rule under scaling of coordinates,

$$D : \quad x^\mu \rightarrow \lambda x^\mu, \quad \Rightarrow \quad \phi(x) \rightarrow \phi'(x) = \lambda^\Delta \phi(\lambda x). \quad (51)$$

Representations of the conformal group are labeled by the scaling dimension Δ and Casimir invariants of Poincare group. The structure of the correlation functions of primary fields, that is n-point functions, in a conformal field theory are completely determined by conformal symmetry.

$\mathcal{N} = 4$ $SU(N_c)$ super Yang-Mills theory in 3 +1 dimensions is an example of superconformal field theory [61, 85, 86], which is invariant under supersymmetry as well as conformal

transformations. Because of exact conformal invariance even after quantization, unlike non-supersymmetric 3+1 dimensional pure Yang-Mills theory, it is scale invariant, that is beta function is zero in all orders of perturbation theory and the same is supposed to be true at the non-perturbative level. It means that the couplings do not depend on the energy scale. It contains two types of fermionic operators (generators of supersymmetry), Q_α^A and \bar{Q}_β^A ($A = 1, 2, 3, 4$) which transform under lorentz transformations as four spinors. The theory has the maximal amount of supersymmetry. The supersymmetric algebra is invariant under a $SU(4)$ rotation of the fermionic operators, consequently the lagrangian of $\mathcal{N} = 4$ $SU(N_c)$ SYM is invariant under $SU(4) \sim SO(6)$ global transformations. This is called R-symmetry. The degrees of freedom of this theory are as follows:

- (i) A vector field A_μ in the adjoint representation of $SU(N_c)$, which is a singlet under $SO(6)$.
- (ii) Six real scalars χ^a in the 6 dimensional vector representation of $SO(6)$, which also transform in the adjoint representation of $SU(N_c)$.
- (iii) Four Weyl fermions λ_α^A also transforming in the adjoint representation of $SU(N_c)$ and 4 spinor representation of $SO(6)$, corresponding to the fundamental representation of $SU(4)$.

5.3. The Maldacena Conjecture

The Maldacena conjecture about the duality between gauge theory and gravity, or the AdS/CFT correspondence, arises from two different descriptions of a system of N_c coincident D3 branes.

Let us consider type IIB superstring theory in the presence of N_c D3 branes. On this background, string theory contains perturbative excitations corresponding to both closed string and open strings. Closed strings propagate on the bulk, whereas open strings are attached to the D3 branes and go from one D3 brane to another. The stack of N_c D3 branes are connected by N_c^2 different type of open strings because the strings can begin or end on any of the D3 branes. N_c^2 is the dimension of the adjoint representation of $U(N_c)$. At energies lower than energy of string scale ($1/l_s$), that is in the limit, $l_s \rightarrow 0$, only the massless string states can be excited and the interaction between closed string modes in the bulk and the open string modes in the the stack of D3 branes vanishes. The closed string modes describe free gravity in the bulk, and the N_c^2 massless open string modes and their superpartners constitute maximally supersymmetric $\mathcal{N} = 4$ $U(N_c)$ super Yang-Mills gauge theory on 4 dimensional worldvolume of the stack of branes. However, the rigid motion of the entire system of branes as a whole can be described by the excitations of the $U(1)$ subgroup of $U(N_c)$. Because of the overall translation invariance, this mode decouples from the remaining $SU(N_c)$ modes that describe motion of the branes relative to one another. Thus, there are two decoupled systems, one of free gravity in the bulk and the other a 4-dimensional $\mathcal{N} = 4$ $SU(N_c)$ super Yang-Mills gauge theory on the brane.

The same system can considered from a different point of view. D branes are massive, so a stack of N_c D3 branes is heavy and naturally deform the 10 dimensional space-time. The solution of the equation of motion of low energy string theory effective action for a stack of N_c D3 branes can be written as,

$$ds^2 = \frac{1}{\sqrt{f(r)}} \left(-dt^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right) + \sqrt{f(r)} \left(dr^2 + r^2 d\Omega_5 \right). \quad (52)$$

The metric inside the parentheses in the first term is the 4-dimensional Minkowski metric of the D3 brane and that in the second term is the flat metric of the 6 transverse coordinates written in spherical coordinates. The function $f(r)$ is given by,

$$f(r) = 1 + \frac{R^4}{r^4}. \quad (53)$$

In the limit, $r \gg R$, $f \simeq 1$, the metric (52) reduces to that of flat space with a small correction corresponding to the gravitational potential due to the massive stack of N_c D3 branes.

On the other hand, for $r \ll R$, $f(r) = \frac{R^4}{r^4}$, in which case the metric (52) reduces to,

$$ds^2 = ds_{AdS_5}^2 + R^2 d\Omega_5^2, \quad (54)$$

where

$$\begin{aligned} ds_{AdS_5}^2 &= \frac{r^2}{R^2} \left(-dt^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right) + \frac{R^2}{r^2} dr^2, \\ &= \frac{R^2}{z^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right), \quad \left(\text{with, } r = \frac{R^2}{z} \right), \end{aligned} \quad (55)$$

is the metric (45) of five-dimensional anti-de Sitter space-time. Thus, the ten-dimensional metric factorizes into $AdS_5 \times S^5$ in the strong gravity region. The geometry of 10-dimensional space-time due to the stack of N_c D3-branes is flat at regions far away from the branes, whereas close to the branes a ‘throat’ geometry of the form $AdS_5 \times S^5$ develops. The size of the throat is set by the curvature, R of AdS_5 . There are two distinct sets of degrees of freedom corresponding to gravity, one propagating in the flat region and the other of closed string excitations propagating near the throat. At low energies these two modes decouple from each other.

Thus, in the low-energy limit of both descriptions of a stack of N_c D3-branes there are two decoupled theories, one of which is supergravity in flat space. So, the other theories that appear in both descriptions can be identified. This leads to the Maldacena conjecture: $\mathcal{N} = 4$ $SU(N_c)$ super Yang-Mills theory in 3+1 dimensions is equivalent (or dual) to type II B string theory on $AdS_5 \times S^5$.

There are two parameters in the gauge theory side, namely, (1) the gauge coupling constant, g , and (2) number of colors, N_c . The value of ‘t Hooft coupling constant, $\lambda = g^2 N_c$ decides whether gauge theory is in weakly coupled or strongly coupled regimes. When $\lambda \gg 1$, gauge theory is strongly coupled, and $\lambda \ll 1$ corresponds to weakly coupled regime. On the other hand, string theory on $AdS_5 \times S^5$ has three parameters, namely, (1) string length l_s , (2) string coupling g_s , and (3) radius of curvature of AdS_5 space, R . When $R \gg l_s$, closed string theory reduces to Einstein’s classical gravity. The parameters of two sides are related as,

$$g^2 = 4\pi g_s, \quad R^4 = 4\pi g_s l_s^4 N_c. \quad (56)$$

The weak coupling regime of gauge theory, $\lambda \ll 1$, corresponds to $g_s N_c \ll 1$, we see from (56) that $R \ll l_s$. It means the radius characterizing the gravitational effect of the D-branes becomes small in string units. Closed strings feel a flat space-time everywhere except very close to the locations of D-branes. In the regime of weak coupling, thus the closed string description is not useful since it would involve complicated sub-string-scale geometry. On the other hand, in the strong coupling regime of gauge theory, $\lambda \gg 1$ corresponds to $g_s N_c \gg 1$, we see from (56) that $R \gg l_s$. It means the radius characterizing the gravitational effect of the D-branes becomes large in string units, so the geometry becomes weakly curved. In this limit the closed string description simplifies and essentially reduces to classical gravity. Thus, the difficult strongly coupled regime of gauge theory is mapped to classical gravity through this remarkable correspondence.

We see that when the gravity side is strongly coupled the gauge theory side is weakly coupled and vice versa. It is a strong-weak coupling duality. This is the reason why this duality is so exciting to study strongly coupled gauge theories. However, finding the dual theory, in general cases is difficult. It has been possible only for those cases which have a D-brane interpretation, and whose low-energy theory is understood.

Though the conjecture has not been proved rigorously, there are many tests that indicate that the conjecture is correct. One of them is symmetry consideration. Symmetries on both sides of the duality are identical. On the gravity side, $AdS_5 \times S^5$ has two symmetry groups, the $SO(6)$ of the 5-sphere, and $SO(4, 2)$ of the of the AdS_5 space-time. On the other side, $\mathcal{N} = 4$ super Yang-Mills has $SU(4)_R \sim SO(6)$ group as the R-symmetry and $SO(4, 2)$ as the conformal symmetry in 4-dimensions.

5.4. Correlation functions

We now wish to understand how the AdS/CFT correspondence can be used to extract information about the strongly coupled gauge theory in 4-dimensions. Let us consider a gauge theory defined by the action, say $S_{4D}[q]$. The correlation functions of suitable gauge invariant operators, $\mathcal{O}(x^\mu) = f(q)$, which are functions of the field variable q encode all information about the gauge theory. The general idea in calculating the correlation functions using AdS/CFT correspondence is that the operators in gauge theory side are related to a field in the AdS_5 space. Corresponding to every operator $\mathcal{O}(x^\mu)$ of the conformal field theory in 4-dimensions, there exists a unique $\phi(x^\mu, z)$ field living in the 5-dimensional AdS_5 space. These fields are called bulk fields, which are related to the boundary field, $\phi_0(x^\mu)$, on the boundary of AdS_5 by the relation,

$$\phi(x^\mu, z) = z^{4-\Delta} \phi_0(x^\mu), \quad (57)$$

where Δ is the conformal dimension of the operator $\mathcal{O}(x^\mu)$. The connected correlation functions can be calculated from the generating functional,

$$Z_{4D}[\phi_0] \equiv e^{W_{4D}[\phi_0]} = \int \mathcal{D}[q] \exp\left(iS_{4D}[q] + i \int d^4x \mathcal{O}(x)\phi_0(x)\right), \quad (58)$$

by taking the successive functional derivative of the generating functional as,

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2) \cdots \mathcal{O}(x_n) \rangle_c = (-i)^n \frac{\delta^{(n)} W_{4D}[\phi_0]}{\delta \phi_0(x_1) \delta \phi_0(x_2) \cdots \delta \phi_0(x_n)} \Big|_{\phi_0=0}. \quad (59)$$

These are very difficult to calculate in the strongly coupled regime of gauge theory. The AdS/CFT correspondence provides a procedure to calculate these correlation functions using the gravity theory in AdS_5 .

Given the nature of the field, $\phi(x^\mu, z)$, depending on the operator $\mathcal{O}(x^\mu)$, we construct the lagrangian density, $\mathcal{L}(\phi, \partial_M \phi)$, and construct action, $S_{5D}[\phi(x^\mu, z)]$ in 5-dimensional space AdS_5 as,

$$S_{5D}[\phi] = \int d^5x \sqrt{\det(g_{MN})} \mathcal{L}(\phi, \partial_M \phi), \quad (60)$$

where g_{MN} is the metric of AdS_5 .

The AdS/CFT correspondence [24, 25] identifies the generating functional, $W_{4D}[\phi_0]$, of connected correlation functions of the gauge-invariant operator, $\mathcal{O}(x^\mu)$, in 4-dimensional gauge theory with the 5-dimensional action, $S_{5D}[\phi]$, as,

$$W_{4D}[\phi_0] = i \lim_{\epsilon \rightarrow 0} S_{5D}[\phi(x^\mu, \epsilon)], \quad (61)$$

so that the correlation functions can be calculated using $S_{5D}[\phi]$,

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2) \cdots \mathcal{O}(x_n) \rangle_c = (-i)^{n-1} \lim_{\epsilon \rightarrow 0} \frac{\delta^{(n)} S_{5D}[\phi(x, \epsilon)]}{\delta \phi(x_1, \epsilon) \delta \phi(x_2, \epsilon) \cdots \delta \phi(x_n, \epsilon)} \Big|_{\phi_0=0}. \quad (62)$$

This can be viewed as the definition of gauge theory correlation functions in terms of the action, S_{5D} , in a higher dimensional space. This is an useful formulation of the AdS/CFT correspondence for computing correlation functions of the strongly coupled gauge theory.

6. Holographic QCD

Holographic QCD aims to study strongly coupled regime of QCD using AdS/CFT to get analytical model for hadron structure and their interactions [87, 88]. The AdS/CFT correspondence gives an equivalence between classical gravity theory in 5-dimensions and $\mathcal{N} = 4$ $SU(N_c)$ supersymmetric Yang-Mills theories (SYM) in the large N_c limit. QCD is fundamentally different from SYM theories, it is non-supersymmetric and conformal symmetry of classical QCD is broken by quantum effects. Exact string theory (or gravity) dual of QCD is not known yet. In this situation, to apply the remarkable AdS/CFT correspondence to QCD, we need to modify the AdS geometry to build a realistic gravity dual of QCD. Two Approaches are usually followed to construct holographic models of QCD.

- (i) Top down approach: In this approach, one starts from string theory and chooses an appropriate D-brane configuration to get as close as possible to the realistic QCD [89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110]. The most closely related to QCD is of Sakai and Sugimoto [93, 94], where the 'color' Yang-Mills fields are introduced by the massless open string fluctuations of a stack of N_c D4-branes wrapped on a circle, and flavor degrees of freedom through fermionic open string fluctuations between D4-branes and a pair of N_f D8-branes and $\overline{D8}$ -branes. The model is nonsupersymmetric and describes $SU(N_c)$ Yang-Mills theory, giving a geometrical picture of chiral symmetry breaking. In the strong coupling limit (large N_c), the stack of N_c color branes has a dual description in terms of a classical gravity theory. we shall not discuss this approach in this article.
- (ii) Bottom up approach: In this approach, one looks at the hadron phenomenology of QCD first and then attempts to build its 5D-gravity equivalent model in AdS_5 space-time by introducing appropriate fields and modifying the geometry of the extra dimension to reproduce the phenomenology [111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127]. The Kaluza-Klein modes of five dimensional fields with quantum numbers of QCD states are identified with the hadronic resonances. The most important aspect of QCD at low energy is confinement. This means that there exists a maximum separation for quarks in the hadrons. The five dimensional model of QCD in AdS_5 must take this into account. Because of confinement, there must exist limiting separation of quarks and therefore a minimum value of energy scale r , or a maximum value of z . Therefore, AdS_5 space should end at a finite value in the fifth axis, $z_0 = \frac{1}{r_0} \equiv \frac{1}{\Lambda R^2}$. Cutoff at z_0 breaks conformal invariance and allows the introduction of the QCD scale. Introduction of cut-off in z -axis (to implement confinement) can be done in two ways,
 - (a) Hard wall model: The lagrangian is multiplied by a step function, so that it vanishes beyond $z = z_0$. This is similar to the MIT bag model, but with the boundary condition on the fifth dimension.
 - (b) Soft wall model: Introduce an additional field called dilaton field that breaks the scale symmetry. The lagrangian is multiplied by $e^{-D(z)}$, so that it vanishes slowly as z increases.

We shall discuss in the following some elements of how the mass spectrum of scalar and vector particles are generated in the gravity dual models models. We shall also discuss how chiral symmetry appears in holographic QCD.

6.1. Scalar mesons

We start with the action for a scalar (or pseudoscalar) field $\Phi(x^\mu, z)$ in five dimensional AdS_5 , in which x^μ represent the four dimensional Minkowski space-time coordinates and z represents

the fifth coordinate. The action is written as,

$$S = \int d^4x dz \sqrt{|g|} e^{-D(z)} \frac{1}{2} \left(g^{MN} \partial_M \Phi \partial_N \Phi - \mu^2 \Phi^2 \right), \quad (63)$$

where the term $\mu^2 \Phi^2$, though looks like a mass term in the five dimensional space, but, as we shall see, the parameter μ is not related to the masses in Minkowski space. A background dilaton field, $D(z)$, has been introduced in the action, which can be taken of the form as,

$$D(z) = \begin{cases} 0, & \text{Hard wall,} \\ \lambda^2 z^2, & \text{Soft wall.} \end{cases}$$

for the hard wall and soft wall scenarios. The metric of AdS_5 as given in (46) can be written, introducing, $A(z) = -\log z + \log R$, for convenience, as,

$$g_{AB} = e^{2A(z)} \eta_{AB}, \quad g^{AB} = e^{-2A(z)} \eta^{AB}, \quad \sqrt{|g|} = e^{5A(z)} \quad (64)$$

with η_{AB} as the five dimensional Minkowski metric. We can read off from (63) the lagrangian density as,

$$\mathcal{L} = e^{B(z)} \frac{1}{2} \left(\eta^{MN} \partial_M \Phi \partial_N \Phi - e^{2A(z)} \mu^2 \Phi^2 \right), \quad (65)$$

where $B(z) = 3A(z) - D(z)$. The equation of motion of $\Phi(x^\mu, z)$ can be written as the usual Euler-Lagrange equation as,

$$\partial_A \left(\frac{\partial \mathcal{L}}{\partial \partial_A \Phi} \right) - \frac{\partial \mathcal{L}}{\partial \Phi} = 0, \quad \Rightarrow \quad \eta^{\alpha\beta} \partial_\alpha \partial_\beta \Phi - \partial_z^2 \Phi - \partial_z B(z) \partial_z \Phi + e^{2A(z)} \mu^2 \Phi = 0. \quad (66)$$

We decompose, $\Phi(x, z)$, as,

$$\Phi(x, z) = e^{ip_\mu x^\mu} \phi(z), \quad (67)$$

a product of a plane wave part in four dimensional Minkowski space-time and a function $\phi(z)$ depending only on the fifth coordinate z . Substituting $\Phi(x, z)$ in (66) we get the equation for $\phi(z)$ as,

$$-\partial_z^2 \phi(z) - \partial_z B(z) \partial_z \phi(z) + e^{2A(z)} \mu^2 \phi(z) = m^2 \phi(z), \quad (68)$$

where, $m^2 = p^\mu p_\mu$, corresponds to the mass of the scalar(pseudoscalar) particle in four dimensions. Putting, $\phi(z) = e^{-\frac{B(z)}{2}} \psi(z)$ we can eliminate the term containing first derivative in (68) and transform it to a Schrodinger like equation in $\psi(z)$ as,

$$-\partial_z^2 \psi(z) + V(z) \psi(z) = m^2 \psi(z), \quad (69)$$

with

$$V(z) = \frac{1}{2} \partial_z^2 B(z) + \left(\frac{1}{2} \partial_z B(z) \right)^2 + e^{2A(z)} \mu^2. \quad (70)$$

We now solve the equation for $\psi(z)$, (69), in hard wall and soft wall scenarios.

- Hard wall model: In this model there is no dilation field, so,

$$D(z) = 0, \quad \Rightarrow \quad B(z) = 3A(z) = 3(\ln L - \ln z), \quad (71)$$

consequently the potential $V(z)$ from (70) becomes,

$$V(z) = \frac{1}{4z^2} (4\nu^2 - 1), \quad (72)$$

with, $\nu^2 = \mu^2 R^2 + 4$. In addition, the space of AdS_5 is restricted in the fifth direction, $0 \leq z \leq z_0$, with the boundary condition that $\phi(z)$ vanishes at z_0 ,

$$\phi(z_0) = 0. \quad (73)$$

Changing the variable, $y = mz$, and $\psi(y) = \sqrt{y} f(y)$ the equation of motion can be transformed to the Bessel equation,

$$\left(-\partial_y^2 - \frac{1}{y} \partial_y + \frac{\nu^2}{y^2} \right) f(y) = f(y) \quad (74)$$

with solutions $f(y) = J_\nu(y)$. The solution of the field equation can be written as,

$$\phi(z) = \left(\frac{z}{R} \right)^{\frac{3}{2}} (mz)^{\frac{1}{2}} J_\nu(mz) \sim z^2 J_\nu(mz). \quad (75)$$

The solution is valid for any value of m , thus it does not yield a discrete hadron mass spectrum. To get the discrete mass spectrum, we impose the boundary condition, (73), on the solution, which gives,

$$J_\nu(mz_0) = 0. \quad (76)$$

A discrete spectrum of masses of scalar (pseudoscalar) meson is generated through zeros of Bessel function. We get not only one, but a whole series with same index ν . In addition, there are resonances corresponding to the different values of ν . A general feature of string picture is realized. We would get similar pattern of the Regge trajectory, however these excitations are not linear in m^2 , but, from the properties of zeros of Bessel function it is seen that these are approximately linear in m .

- Soft wall model: In this model there is a dilation field, $D(z)$, which is taken as,

$$D(z) = \lambda^2 z^2, \quad \Rightarrow \quad B(z) = 3A(z) - D(z) = 3(\ln L - k \ln z) - \lambda^2 z^2, \quad (77)$$

and the five dimensional space-time of AdS_5 is not restricted. The equation for ψ , (69), becomes,

$$-\partial_u^2 \psi(u) + \left(\frac{4\nu^2 - 1}{4u^2} + u^2 \right) \psi(u) = \left(\frac{m^2}{\lambda^2} - 2 \right) \psi(u). \quad (78)$$

with $u = \lambda z$. The solution of this equation are the eigenfunctions,

$$\psi_n(u) = e^{-\frac{u^2}{2}} u^{\nu+\frac{1}{2}} L_n^\nu(u^2) \quad \Rightarrow \quad \phi_n(z) \sim e^{-\frac{\lambda^2 z^2}{2}} z^{2+\nu} L_n^\nu(\lambda^2 z^2), \quad (79)$$

and eigenvalues,

$$m^2 = 2\lambda^2 (2(n+1) + \nu), \quad (80)$$

where L_n^ν is the associated Laguerre polynomials. We see that the soft wall model has the desired feature that the excitations are proportional to the squared mass, m^2 . The agreement for the experimental values and the theoretical results from the soft wall model is satisfactory.

6.2. Vector particles

The action for the vector field, $A_M(x^\mu, z)$, in five dimensional AdS_5 is written as,

$$S = \int d^4x dz \sqrt{|g|} e^{-D(z)} \left(g^{MM'} g^{NN'} F_{MN} F_{M'N'} \right), \quad (81)$$

where, F_{MN} is the field tensor given as,

$$F_{MN} = \partial_M A_N - \partial_N A_M, \quad (82)$$

and $D(z)$ is the dilaton field. The lagrangian can be read off from the action as,

$$\mathcal{L} = \frac{1}{4} e^{B(z)} \left(\eta^{MM'} \eta^{NN'} F_{MN} F_{M'N'} \right), \quad (83)$$

with, $B(z) = A(z) - D(z)$. In Minkowski space there are three spin components of a vector particle. We impose two gauge conditions to reduce 5 components of A_M to three components. The gauge conditions are,

$$A_5 = 0, \quad \eta^{MK} \partial_K A_M = 0. \quad (84)$$

The equation of motion are the usual Euler-Lagrange equations which can be written as,

$$\frac{\partial \mathcal{L}}{\partial A_B} - \partial_A \left(\frac{\partial \mathcal{L}}{\partial A_{AB}} \right) = 0, \quad \Rightarrow \quad -\partial_z B(z) \partial_z A_\alpha - \partial_z^2 A_\alpha + \eta^{\mu\nu} \partial_\mu \partial_\nu A_\alpha = 0. \quad (85)$$

We decompose A_α into a plane wave part in the Minkowski space and another part depending only on the fifth coordinate z as,

$$A_\alpha(x^\mu, z) = e^{-ip_\mu x^\mu} \epsilon_\alpha(p) \phi(z) \quad (86)$$

where $\epsilon_\alpha(p)$ is the polarization vector of a transverse vector field, i.e. $\epsilon \cdot p = 0$. The equation for $\phi(z)$ becomes,

$$\left(-\partial_z^2 - \partial_z B(z) \partial_z \right) \phi = m^2 \phi, \quad (87)$$

where, $m^2 = p^\mu p_\mu$ corresponds to the mass of vector particles. To eliminate the term containing first derivative we substitute, $\phi(z) = e^{-\frac{B(z)}{2}} \psi(z)$, so that the equation for $\psi(z)$ becomes a Schrodinger like equation,

$$-\partial_z^2 \psi(z) + V(z) \psi(z) = m^2 \psi(z), \quad (88)$$

with the potential, $V(z)$, as,

$$V(z) = \frac{1}{4} (\partial_z B(z))^2 + \frac{1}{2} \partial_z^2 B(z). \quad (89)$$

We solve (88) in hard wall and soft wall scenarios,

- Hard Wall Model: In this model there is no dilation field, so,

$$D(z) = 0, \quad \Rightarrow \quad B(z) = A(z) = (\ln L - \ln z), \quad (90)$$

consequently the potential $V(z)$ from (89) becomes, $V(z) = \frac{3}{4z^2}$, and the space of AdS₅ is restricted in the fifth direction, $0 \leq z \leq z_0$, with the boundary condition that $\phi(z)$ vanishes at z_0 , $\phi(z_0) = 0$. The equation for $\psi(z)$ becomes,

$$\partial_z^2 \psi(z) + \frac{3}{4z^2} \psi(z) = m^2 \psi(z), \quad (91)$$

The solution of this equation can be written in terms of the Bessel function as,

$$\psi(z) = \sqrt{mz} J_1(mz), \quad \Rightarrow \quad \phi(z) = e^{-\frac{B(z)}{2}} \psi(z) = \sqrt{\frac{m}{R}} z J_1(mz) \quad (92)$$

The five dimensional field vanishes at the hard wall, $\phi(z_0) = 0$. The masses, m_n , of the vector meson resonances are then determined by the zeros of first order Bessel function. The mass spectrum, as found in the scalar particle case, is not linear in m^2 .

- Soft Wall Model: In this model there is a dilaton field, $D(z) = \lambda^2 z^2$, and the AdS₅ is not restricted. The potential, $V(z)$ can be obtained from (89) as,

$$V(z) = \frac{1}{4} (\partial_z B(z))^2 + \frac{1}{2} \partial_z^2 B(z) = \frac{3}{4z^2} + \lambda^4 z^2 \quad (93)$$

The equation for ψ becomes,

$$\partial_z^2 \psi(z) + \left(\frac{3}{4z^2} + \lambda^4 z^2 \right) \psi(z) = m^2 \psi(z), \quad (94)$$

the solution of which can be written as,

$$\psi_n(z) = e^{-\frac{\lambda^2 z^2}{2}} (\lambda z)^{\frac{3}{2}} L_n^1(\lambda^2 z^2), \quad \Rightarrow \quad \phi_n(z) = e^{-\frac{B(z)}{2}} \psi(z) = \sqrt{\frac{\lambda^3}{L}} z^2 L_n^1(\lambda^2 z^2), \quad (95)$$

where L_n^1 are the Laguerre polynomials. The mass spectrum is obtained from the eigenvalues,

$$m^2 = 4\lambda^2 (n + 1). \quad (96)$$

The mass spectrum is linear in m^2 in the soft wall scenario for the vector particles also.

6.3. Holographic model of QCD

In the holographic QCD model we consider a 5-dimensional dual of the two flavor, (u, d) sector of QCD. In this case, QCD has a global $SU(2)_L \times SU(2)_R$ chiral symmetry. The conserved currents corresponding to this symmetry are the vector and axial-vector currents which can be expressed in terms of left handed and right handed currents, $j_L^{\mu a} = \bar{q}_L \gamma^\mu \tau^a q_L$ and $j_R^{\mu a} = \bar{q}_R \gamma^\mu \tau^a q_R$, where τ^a 's are $SU(2)$ flavor generators. To describe the chiral dynamics, we shall also need an operator called chiral condensate, $\bar{q}_R^\alpha q_L^\beta$, where α and β are $SU(2)$ flavor indices in the fundamental representation. The holographic QCD does not refer explicitly to quark and gluon degrees of freedom, only deals with the bound states of QCD. According to the AdS/CFT correspondence, there is a one-to-one map between 4-dimensional operators in field theory living on the boundary and 5-dimensional bulk fields in gravity side. The global chiral symmetry $SU(2)_L \times SU(2)_R$ of QCD is made local symmetry in the gravity side. Therefore, we need to introduce local gauge (vector) fields L_M^a and R_M^a of which the values at $z = 0$ play a role of external sources for $SU(2)_L$ and $SU(2)_R$ currents, $j_L^{\mu a}$ and $j_R^{\mu a}$ respectively. The vector and axial-vector gauge

	4D: $\mathcal{O}(x)$	5D: $\phi(x^\mu, z)$	p	Δ	m_5^2
Current Operator :	$j_{L\mu}^a = \bar{q}_L \gamma_\mu t^a q_L$	$L_M^a(x^\mu, z)$	1	3	0
	$j_{R\mu}^a = \bar{q}_R \gamma_\mu t^a q_R$	$R_M^a(x^\mu, z)$	1	3	0
Quark bilinear :	$\bar{q}_L^\alpha q_R^\beta$	$\frac{2}{z} X^{\alpha\beta}(x^\mu, z)$	0	3	-3

Table 2. Correspondence of operators in gauge theory side to fields in gravity side.

fields are defined as $V_M^a = (L_M^a + R_M^a)/\sqrt{2}$ and $A_M^a = (L_M^a - R_M^a)/\sqrt{2}$. Since chiral symmetry is known to be broken to $SU(2)_V$ spontaneously, we introduce a field $X^{\alpha\beta}$ corresponding to $\bar{q}_R^\alpha q_L^\beta$ in order to realize the spontaneous chiral symmetry in the AdS_5 side. In table (2) we give the correspondence of operators and fields in QCD side and gravity side respectively. The 5-dimensional mass of the bulk gauge field L, R and X is determined by the relation,

$$m_5^2 R^2 = (\Delta - p)(\Delta + p - 4), \quad (97)$$

[24, 25] where Δ denotes the conformal dimension of the corresponding operator with spin p and R is the AdS_5 curvature. The mass of the gauge field in the bulk of five dimensional space-time turns out to be, $m_5^2 = 0$, which is expected because of gauge symmetry, and the mass of the scalar X is, $m_5^2 = -\frac{3}{R^2}$.

The action in five dimensional space-time for the meson sector in holographic QCD is constructed taking the fields of interest with the appropriate quantum numbers and Lorentz structures as [116, 117],

$$S_{5D} = \int d^4x \int dz \sqrt{|g|} e^{-D(z)} \text{Tr} \left[|DX|^2 + \frac{3}{R^2} X^2 - \frac{1}{4g_5^2} \left(|F^L|^2 + |F^R|^2 \right) \right], \quad (98)$$

where, $|DX|^2 = (D_M X)^\dagger (D^M X)$ with the covariant derivative, $D_M X = \partial_M X - iL_M X + iX R_M$, $|F^L|^2 = F_{MN}^L F^{LMN}$ with the field tensors, $F_{MN}^L = \partial_M L_N - \partial_N L_M - i[L_M, L_N]$ (and analogous expression for $|F^R|^2$) and $L_M = \tau^a L_M^a, R_M = \tau^a R_M^a$. The gauge coupling, g_5 , in five dimensional space-time is fixed by matching the vector correlation function in five dimension to that from the operator product expansion, $g_5 = 12\pi^2/N_c$ [116]. The action (98) contains the dilaton field $D(z)$ for soft wall model, however, by making, $D(z) = 0$, we can consider this lagrangian for hard wall model.

The scalar field, $X(x^\mu, z)$, in five dimensional space is taken as a product,

$$X^{\alpha\beta}(x^\mu, z) = X_0^{\alpha\beta}(z) e^{i\tau^a \pi^a(x^\mu, z)}, \quad (99)$$

in which the vacuum expectation value, $X_0^{\alpha\beta}(z) = \langle X^{\alpha\beta}(x^\mu, z) \rangle$ depends on z coordinate alone. The pion field $\pi^a(x^\mu, z)$ appearing in (99) is dimensionless and related to the canonically-normalized pion field $\tilde{\pi}^a(x^\mu, z)$ of chiral lagrangians via $\pi^a = \frac{\tilde{\pi}^a}{f_\pi}$ with $f_\pi = 93 \text{ MeV}$. The vacuum expectation value $X_0^{\alpha\beta}(z)$ spontaneously breaks the (approximate) chiral symmetry by forming the quark condensate. The field $X_0^{\alpha\beta}(z)$ is determined solving the equation of motion of this field obtained from the action (98) by turning off all fields except $X_0^{\alpha\beta}(z)$. The equation of motion for $X_0^{\alpha\beta}(z)$ can be derived using usual variational procedure as,

$$\partial_z \left(\frac{1}{z^3} \partial_z X_0^{\alpha\beta}(z) \right) + \frac{3}{z^5} X_0^{\alpha\beta}(z) = 0. \quad (100)$$

The general solution to this homogeneous linear second order ordinary differential equation is given by the polynomial,

$$2X_0^{\alpha\beta}(z) = M^{\alpha\beta}z + \Sigma^{\alpha\beta}z^3, \quad (101)$$

where two integration constants, $M^{\alpha\beta}$ and $\Sigma^{\alpha\beta}$ are determined by the boundary conditions at $z = 0$ and $z = z_0$ in the hard wall model as,

$$M^{\alpha\beta} = \lim_{z \rightarrow 0} \frac{2X_0^{\alpha\beta}(z)}{z}, \quad \Sigma^{\alpha\beta} = \frac{2X_0^{\alpha\beta}(z_0) - M^{\alpha\beta}z_0}{z_0^3}.$$

The physical meanings of $M^{\alpha\beta}$ and $\Sigma^{\alpha\beta}$ are respectively the bare quark mass responsible for explicit breaking of chiral symmetry and the chiral condensate responsible for spontaneous chiral symmetry breaking.

The equations of motion of other fields of interest are obtained varying the action with respect to the corresponding fields. The axial-like gauge, $V_5(x^\mu, z) = 0$, $A_5(x^\mu, z) = 0$ has been found to be convenient to work with. The axial vector field A_μ is written as a sum of a transverse (divergence less) part, $A_{\mu\perp}$, and a longitudinal part ϕ , $A_\mu = A_{\mu\perp} + \phi$ with $\partial_\mu A_{\mu\perp} = 0$. The equations of motion for V_μ^a (vector particles), $A_{\mu\perp}^a$ (transverse component of vector particles), ϕ (longitudinal component of vector particles) and π^a (pseudoscalar particles) become, after taking the Fourier transform with respect to the four dimensional coordinates x^μ , one dimensional equations as,

$$\partial_z \left(\frac{e^{-D(z)}}{2} \partial_z V_\mu^a(q^\mu, z) \right) + \frac{q^2}{2} e^{-D(z)} V_\mu^a(q^\mu, z) = 0, \quad (102)$$

$$\left[\partial_z \left(\frac{e^{-D(z)}}{z} \partial_z A_\mu^a(q^\mu, z) \right) + \frac{q^2}{2} e^{-D(z)} A_\mu^a(q^\mu, z) - \frac{g_5^2}{z^3} e^{-D(z)} v(z)^2 A_\mu^a(q^\mu, z) \right]_\perp = 0, \quad (103)$$

$$\partial_z \left(\frac{e^{-D(z)}}{z} \partial_z \phi^a(q^\mu, z) \right) + \frac{g_5^2}{z^3} e^{-D(z)} v(z)^2 (\pi^a(q^\mu, z) - \phi^a(q^\mu, z)) = 0, \quad (104)$$

$$-q^2 \partial_z \phi^a(q^\mu, z) + \frac{g_5^2}{z^2} v(z)^2 \partial_z \pi^a(q^\mu, z) = 0, \quad (105)$$

with q^μ as the momentum in four dimensions. These equations are solved as eigenvalue equations subject to appropriate boundary conditions of hard wall and soft wall models. In the hard wall model, the masses of the mesons appear from the zeros of the solutions at the infra red cut off z_0 . As discussed earlier, the hard wall model does not reproduce the linear relation in square of mass. In soft wall model the mass spectrum appears from the eigenvalues and does reproduce the linear relation in square of mass. The mass spectrum of soft wall model matches well with the experimental values. The solutions can be used to calculate form factors of the mesons and other correlators.

The electromagnetic form factor, for example of a scalar particle, with momentum transfer $q = p - p'$ in holographic QCD can be defined in the same way as the form factor in four dimension,

$$F(Q^2) (2\pi)^4 \delta^4(p - p' - q) \epsilon \cdot q = \int d^4x dz \sqrt{|g|} g^{LL'} \Phi_p^*(x^\mu, z) i \partial_L \Phi_p(x^\mu, z) A_{L'}(x^\mu, z), \quad (106)$$

where $\Phi_p(x^\mu, z)$ are the solutions of equations of motion for scalar particles and $A_{L'}(x^\mu, z)$ is an external electromagnetic field in 5 dimensions with the polarization vector ϵ . The electromagnetic field $A_{L'}$ satisfies Maxwell's equations in 5 dimensions. Using solutions of the

equations of motion for appropriate particles we can find the electromagnetic form factors of mesons in hard wall and soft wall scenarios. Pion electromagnetic form factors have been calculated using holographic QCD. The form factor reproduce the experimental data quite well.

The holographic QCD can be extended to baryons [128]. A spin $\frac{1}{2}$ baryon operator in the Minkowski space (boundary of AdS_5) will correspond to a Dirac field in the bulk of AdS_5 . The action in five dimensions for the Dirac field can be written with constraints of Lorentz invariance. By solving equation of motion with appropriate boundary conditions tower of hadronic resonance states can be generated. Similar procedure can be adapted as well for higher spin particles.

7. Thermal Gauge Gravity Duality

We want to construct gravity dual of gauge theory at finite temperature to describe, systems like quark gluon plasma. The tool of AdS/CFT correspondence can be applied in these systems to learn about strongly coupled gauge theories at finite temperature by doing calculations in classical gravity [26]. In order to introduce a temperature into the gauge theory there are two approaches which are valid in two different regimes,

- (i) Thermal AdS: Temperature can be introduced in the gravity theory in AdS_5 space-time by Wick rotating the Minkowski time of AdS to euclidean time ($t \rightarrow i\tau$) in AdS_5 metric,

$$ds_{AdS_5}^2 = \frac{R^2}{z^2} \left(d\tau^2 + dx^i dx_i + dz^2 \right), \quad (107)$$

and making τ periodic (anti-periodic) for bosons (fermions), by identifying, $\tau \sim \tau + \beta$, as done in equilibrium thermal field theory. The period (circumference of the ‘thermal circle’) is related to the temperature by, $\beta = \frac{1}{T}$, so that at high temperature the euclidean time is on a small circle, and at low temperature the circle unwinds and the original theory on Minkowski space is recovered. The temperature T is identified with the temperature of the field theory on the boundary, namely SYM theory since, τ also corresponds to the euclidean time coordinate of the boundary theory. In AdS/CFT correspondence, this description corresponds to the low temperature regime of gauge theory.

- (ii) Black hole AdS: In this case, temperature is introduced in the gravity theory in AdS_5 space-time through a black hole metric,

$$ds_{AdS_5}^2 = \frac{R^2}{z^2} \left(f(z) d\tau^2 + dx^i dx_i \right) + \frac{R^2}{z^2 f(z)} dz^2, \quad f(z) = 1 - \frac{z^4}{z_0^4}, \quad (108)$$

since black holes have a temperature due to Hawking process [78]. The event horizon is located at $z = z_0$, which has three flat directions \vec{x} . The metric (108) is thus called a black three-brane metric. Hawking temperature, T_H , for the metric (108) is, $T_H = \frac{1}{\pi z_0}$. This description corresponds to high temperature regime of gauge theory.

There should be a temperature where the description of finite temperature SYM by thermal AdS and the black hole AdS coincide. It was shown by Hawking and Page [129] that at some critical temperature, T_{HP} , there is a first order phase transition where thermal AdS metric tunnels to black hole AdS metric. Through gauge gravity correspondence this transition can be interpreted in the gauge theory as a phase transition between confined to de-confined phase. The de-confinement temperature, T_c , obtained using holgraphic QCD in the hard and soft wall models is 122 MeV and 191 MeV respectively [130].

The quark-gluon plasma was expected to be a free gas of quarks and gluons, however, at the experimentally accessible energies the observations seem to indicate that it behaves like a almost perfect-fluid with small shear viscosity [10]. This observation leads to the conclusion that quark-gluon plasma is a strongly coupled system, within the non-perturbative regime of QCD.

The AdS/CFT correspondence can be applied to such situations. It identifies strongly coupled plasma at finite temperature in the gauge theory side with a black brane in gravity side. There exists a one-to-one correspondence between the thermodynamic properties of the black hole and those of the plasma, namely its temperature and entropy. The plasma properties thus should be calculable using black branes. The AdS/CFT correspondence relates the correlators of the energy-momentum tensor, $T_{xy}(t, \vec{x})$, in two space-time points at zero frequency, $\omega = 0$, and the absorption cross section, $\sigma(\omega)$, of a graviton by the static black brane in the bulk,

$$\sigma(\omega) = \frac{8\pi G}{\hbar\omega} \int d^4x e^{i\omega t} \langle [T_{xy}(t, \vec{x}), T_{xy}(0, 0)] \rangle . \quad (109)$$

The shear viscosity, η , of plasma is given by a Kubo formula,

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{2\hbar\omega} \int d^4x e^{i\omega t} [T_{xy}(t, \vec{x}), T_{xy}(0, 0)] , \quad (110)$$

from which we can relate the shear viscosity and graviton absorption cross section by black brane as,

$$\eta = \frac{\lim_{\omega \rightarrow 0} \sigma(\omega)}{16\pi G} . \quad (111)$$

According to a general theorem [132], the graviton absorption cross-section of black hole in the low energy limit ($\omega \rightarrow 0$) is, $\sigma(\omega) = A$, where A is the horizon area. The entropy density, s , of the black brane is, $s = \frac{A}{4G}$. We get the viscosity to entropy density ratio of strongly coupled plasma of $\mathcal{N} = 4$ SYM theory as,

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} . \quad (112)$$

The shear viscosity was first computed [27] for the D3 brane corresponding to the $\mathcal{N} = 4$ SYM which is a conformal field theory. This result is extremely robust because it depends only on the universal properties of black hole horizons. All gauge theory dual to gravity should fulfill this relation in spite of their different field contents and type of supersymmetry [28]. This is a kind of universality relation. However, in the absence of gravity dual of QCD it is not clear yet if the result (112) holds for QGP.

8. Summary

In this article we have tried to present in a simplistic way an overview of the developments that has taken place over many years in understanding strong interaction.

In the 1970's QCD was established as the quantum field theoretical description of strong interaction, which by now is extremely well tested experimentally. The degrees of freedom of QCD are quarks and gluons that transform respectively in the fundamental and adjoint representation of the non-abelian local (color) gauge group $SU(3)$. The interaction between quarks is mediated by gluons. QCD has the desired feature of asymptotic freedom. However, QCD remained intractable for the problems of hadron physics at low energies. The mechanism of confinement of quarks into hadron is still not clear.

In the late 1960's phenomenological understanding of strong interaction in terms of S-matrix and Regge trajectories led to the formulation of string theory. It was soon realized that string theory including both bosons and fermions need 10-dimensional space-time to be consistent. It also had inescapably a graviton like massless spin-2 excitation mode. It became a promising candidate for quantum gravity. Energy regime of string theory is in the Planck scale ($\sim 10^{19}$

GeV), making the predictions of string theory almost impossible to verify experimentally. So, string theory was given up as a theory of the strong interactions.

QCD and string theory became two independent successful theory. However, it was always hoped from both sides to find a link between them, from string theory side the link would provide some way for its experimental verification, and from QCD side it may give some new path to understand the low energy phenomena of confinement, chiral symmetry breaking and hadronic interactions. There had been many hints to expect that string theory may be a potential candidate to provide new insights in QCD.

The holographic principle developed in the context of black hole thermodynamics proposed that there must be a duality between a QCD like gauge theory and a gravity theory in higher dimensional space-time. However, it was not possible to find an useful connection between QCD and string theory until the discovery of D-branes in superstring theory.

D branes are special non-perturbative objects in string theory. On the one hand, from the closed string point of view, D branes are soliton like solutions of low energy effective theory of superstring theories. The excitations here are the gravitational modes of closed string vibrations in the space-time curved by the mass of D branes. On the other hand, from open string point of view, these are hypersurfaces in 10-dimensional space-time, on which open strings end. The excitations in this case are gauge bosons produced due to the motion of charged ends of open strings on the D branes.

The dream of finding an useful connection between QCD and string theory was partially fulfilled with the AdS/CFT correspondence conjectured by Maldacena in 1997, noting two different descriptions of the same object D branes. In the original form of the correspondence, the gauge theory is, $\mathcal{N} = 4 SU(N_c)$ supersymmetric Yang-Mills theory in 4-dimensional Minkowski space-time, and in the gravity theory is type IIB string theory in $AdS_5 \times S^5$ space-time. In the large N_c limit the gravity theory becomes classical.

QCD is very different from $\mathcal{N} = 4 SU(N_c)$ supersymmetric Yang-Mills theory. So it is necessary to break supersymmetry and conformal invariance by modifying AdS_5 space to apply AdS/CFT correspondence to QCD. Two approaches, namely top-down and bottom-up, have been developed to achieve the above goal. There are considerable success in both the approaches in understanding low energy features of QCD.

Strongly coupled gauge theories at finite temperature have also been studied with the help of AdS/CFT correspondence by introducing black hole horizon in AdS_5 space. These studies give an universal lower limit for the viscosity to entropy ratio for strongly coupled plasma.

Criticisms have been raised against the holographic QCD approach because strictly AdS/CFT works for SYM theories. However, as long as gravity dual to QCD not discovered, we may consider holographic QCD as a effective low-energy model independent of the AdS/CFT correspondence.

We may conclude this article by saying that strong interaction is indeed very difficult to understand, and we have a long way ahead to go to fully understand it.

9. Acknowledgements

ABS is grateful to INFN-LNS, Catania, Italy for hospitality, where part of the work was done.

References

- [1] S. Narison, "QCD as a Theory of Hadrons: From Partons to Confinement", Cambridge Univ. Press (2004).
- [2] C. E. DeTar, J. F. Donoghue, "Bag Models of Hadrons", Ann. Rev. Nucl. Part. Sc., 33 (1983) 235.
- [3] A. W. Thomas, W. Weise, "The structure of the nucleon", Wiley (2001).
- [4] A Pich, "Chiral perturbation theory" Rep. Prog. Phys. 58 (1995) 563.
- [5] B. Kubis, "An introduction to chiral perturbation theory" in "Physics and Astrophysics of Hadrons and Hadronic Matter", Ed. A. B. Santra, Narosa, New Delhi (2008), arXiv:hep-ph/0703274.
- [6] H. J. Rothe, "Lattice Gauge Theories: An Introduction", World Scientific (2005).

- [7] C. Gattringer, C. B. Lang, “Quantum chromodynamics on the lattice”, Springer (2010).
- [8] J. C. Collins, M. J. Perry, “Superdense Matter: Neutrons Or Asymptotically Free Quarks? ,” Phys. Rev. Lett. 34 (1975) 1353
- [9] N. Cabibbo, G. Parisi, “Exponential Hadronic Spectrum and Quark Liberation,” Phys.Lett. B59 (1975) 67.
- [10] E. V. Shuryak, “What RHIC experiments and theory tell us about properties of quark-gluon plasma?,” Nucl. Phys. A 750 (2005) 64, arXiv:hep-ph/0405066.
- [11] J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal, U. A. Wiedemann, “Gauge/String Duality, Hot QCD and Heavy Ion Collisions,” arXiv:hep-ph/1101.0618.
- [12] D. Mateos, “Gauge/string duality applied to heavy-ion collisions: limitations, insights and prospects”, J. Phys. G: Nucl. Part. Phys. 38 (2011)124030, arXiv:hep-th/1106.3295.
- [13] R. Dolen, D. Horn, C. Schmid, “Finite-Energy Sum Rules and Their Application to N Charge Exchange”, Phys. Rev. 166 (1968) 1768.
- [14] C. Schmid, “Direct-Channel Resonances from Regge-Pole Exchange”, Phys. Rev. Letters 20, 689 (1968).
- [15] G. Veneziano, “Construction of a crossing-symmetric, Regge-behaved amplitude for linearly rising trajectories”, Nuovo Cimento A 57 (1968) 190.
- [16] Y. Nambu, “Quark model and the factorization of the Veneziano amplitude,” in Symmetries and Quark Models, ed. R. Chand, Gordon and Breach (1970).
- [17] H. B. Nielsen, “An almost physical interpretation of the integrand of the n-point Veneziano amplitude,” submitted to the 15th International Conference on High Energy Physics, Kiev (1970).
- [18] L. Susskind, “Dual-symmetric theory of hadrons,” Nuovo Cim. 69A (1970) 457.
- [19] V. A. Matveev, R.M. Muradian, A. hep-th/ 9805156.N. Tavkhelidze, “Automodellism in the large - angle elastic scattering and structure of hadrons,” Lett. Nuovo Cim. 7 (1973) 719.
- [20] S. J. Brodsky, G. R. Farrar, “Scaling Laws at Large Transverse Momentum.,” Phys. Rev. Lett 31 (1973) 1153.
- [21] J. Scherk and J. Schwarz, Dual models for non-hadrons, Nucl. Phys. B81 (1974) 118.
- [22] T. Yoneya, “Connection of Dual Models to Electrodynamics and Gravidynamics,” Prog. Theor. Phys. 51 (1974) 1907.
- [23] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2 (1998) 231, arXiv:hep-th/9711200.
- [24] S. S. Gubser, I. R. Klebanov, “Gauge theory correlators from non-critical string theory,” A. M. Polyakov, Phys. Lett. B428 (1998) 105, arXiv:hep-th/9802109.
- [25] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2 (1998) 253, arXiv:hep-th/9802150.
- [26] E. Witten, “Anti-de Sitter space, thermal phase transition, and confinement in gauge theories” Adv. Theor. Math. Phys. 2 (1998) 505, arXiv:hep-th/9803131.
- [27] G. Policastro, D. T. Son and A. O. Starinets, “The shear viscosity of strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma, Phys. Rev. Lett. 87 (2001) 081601, arXiv:hep-th/0104066.
- [28] P. Kovtun, D. T. Son and A. O. Starinets, “Holography and hydrodynamics: Diffusion on stretched horizons,” JHEP 0310 (2003) 064, arXiv:hep-th/0309213.
- [29] H. Yukawa, “On the Interaction of Elementary Particles,” Proc. Phys. Math. Soc. Jap. 17 (1935) 48.
- [30] G. P. S. Occhialini, C. F. Powell, “Processes involving charged mesons,” Nature 159 (1947) 186.
- [31] M. Gellmann, Y. Neeman, “The Eightfold Way,” W. A. Benjamin, 1964, New York.
- [32] M. Gell-Mann, “A Schematic Model of Baryons and Mesons,” Phys.Lett. 8 (1964) 214.
- [33] G. Zweig, “An SU(3) model for strong interaction symmetry and its breaking,” in Developments In The Quark Theory Of Hadrons, Vol. 1, CERN (1964)
- [34] O. W. Greenberg, “Spin and Unitary Spin Independence in a Paraquark Model of Baryons and Mesons,” Phys. Rev. Lett. 13 (1964) 598.
- [35] M. Y. Han and Y. Nambu, “Three-Triplet Model with Double SU(3) Symmetry,” Phys. Rev. 139 B1006.
- [36] J. I Friedman and H. W. Kendall, “Deep Inelastic Electron Scattering,” Ann. Rev. Nucl. Sci., 22 (1972) 203.
- [37] H. Fritzsche, M. Gell-Mann and H. Leutwyler, “Advantages of the Color Octet Gluon Picture,” Phys. Lett. B 47 (1973) 365.
- [38] C. N. Yang, R. L. Mills, “Conservation of Isotopic Spin and Isotopic Gauge Invariance,” Phys. Rev. 96 (1954) 191.
- [39] D. J. Gross and F. Wilczek, “Ultraviolet Behavior of Non-Abelian Gauge Theories,” Phys. Rev. Lett. 30 (1973) 1343.
- [40] H. D. Politzer, “Reliable Perturbative Results for Strong Interactions?,” Phys. Rev. Lett. 30 (1973) 1346.
- [41] Particle Data Group, <http://pdg.lbl.gov>.
- [42] G. 't Hooft, “A Planar Diagram Theory for Strong Interactions,” Nucl. Phys. B72 (1974) 461.

- [43] E. Witten, "Baryons in the $1/N$ Expansion," Nucl. Phys. B160 (1979) 57.
- [44] J. Gasser and H. Leutwyler, "Chiral Perturbation Theory to One Loop," Annals Phys. 158 (1984) 142.
- [45] G. S. Bali, "QCD forces and heavy quark bound states," Phys. Rept. 343, 1 (2001), arXiv:hep-ph/0001312.
- [46] T.T. Takahashi, H. Matsufuru, Y. Nemoto and H. Suganuma, "Three-Quark Potential in SU(3) Lattice QCD," Phys. Rev. Lett. 86, 18 (2001), arXiv:hep-lat/0006005.
- [47] T. Kawanai, S. Sasaki, "Interquark potential with finite quark mass from lattice QCD," Phys. Rev. Lett. 107 (2011) 091601, arXiv:hep-lat/1102.3246.
- [48] G.F. Chew, S. C. Frautschi, "Regge trajectories and the principle of maximum strength for strong interactions," Phys. Rev. Lett. 8 (1962) 41.
- [49] R. Hagedorn, "Statistical thermodynamics of strong interactions at high-energies," Nuovo Cim. Suppl. 3 (1965) 147.
- [50] M.B. Green, J.H. Schwarz, E. Witten, "Superstring theory, Vols. I & II," Cambridge University Press (1987).
- [51] D. Lust, S. Theisen, "Lectures on string theory," Lect.Notes Phys. 346 (1989).
- [52] J. Polchinski, "String theory, Vols. I & II," Cambridge Univ. Press (1998)
- [53] C. V. Johnson, "D-branes," Cambridge Univ. Press (2003).
- [54] R. J. Szabo, "An Introduction to String Theory and D-Brane Dynamics," Imperial College Press, (2004).
- [55] B. Zwiebach, "A First Course in String Theory," Cambridge University Press (2004).
- [56] K. Becker, M. Becker, and J. Schwarz, "String Theory and M-Theory: A Modern Introduction," Cambridge Univ. Press (2007).
- [57] E. Kiritsis, "String theory in a nutshell," Princeton Univ. Press (2007).
- [58] J. Wess, J. Bagger, "Supersymmetry and supergravity," Princeton Univ. Press (1992).
- [59] S. Weinberg, "The Quantum Theory of Fields, Volume III: Supersymmetry," Cambridge Univ. Press (2000).
- [60] S. Coleman and J. Mandula, "All Possible Symmetries of the S Matrix," Phys. Rev. 159 (1967) 1251.
- [61] F. Gliozzi, D.I. Olive and J. Scherk, "Supersymmetry, Supergravity Theories and the Dual Spinor Model," Nucl. Phys. B122 (1977) 253.
- [62] P. Candelas, G. T. Horowitz, A. Strominger, E. Witten, "Vacuum Configurations for Superstrings," Nucl. Phys. B258 (1985) 46.
- [63] S. E. Hjeltnel and U. Lindstrom, "Duality for the non-specialist," arXiv:hep-th/9705122.
- [64] A. Sen, "An introduction to non-perturbative string theory," in "Cambridge 1997, Duality and supersymmetric theories", arXiv:hep-th/9802051.
- [65] C. Hull and P. K. Townsend, "Unity of superstring dualities," Nucl. Phys. B438 (1995) 109, arXiv:hep-th/9410167.
- [66] E. Witten, "String theory dynamics in various dimensions," Nucl. Phys. B443 (1995) 85, arXiv:hep-th/9503124.
- [67] J. Polchinski, "Dirichlet Branes and Ramond-Ramond charges," Phys. Rev. Lett. 75 (1995) 4724, arXiv:hep-th/9510017.
- [68] J. Polchinski, "TASI lectures on D-branes," arXiv:hep-th/9611050.
- [69] A. Giveon and D. Kutasov, "Brane Dynamics and Gauge Theory," Rev. Mod. Phys. 71 (1999) 983, arXiv:hep-th/9802067.
- [70] T. H. R. Skyrme, "A Nonlinear field theory," Proc. Roy. Soc. A 260 (1961) 127.
- [71] W. Thirring, "A Soluble relativistic field theory?," Ann. Phys. (N.Y.) 3 (1958) 91.
- [72] S. Coleman, "Quantum sine-Gordon equation as the massive Thirring model," Phys. Rev. D 11, (1975) 2088.
- [73] J. R. Oppenheimer, H. Snyder, "On continued gravitational contraction," Phys. Rev. 56 (1939) 455.
- [74] R. Penrose, "Gravitational collapse: the role of general relativity," Riv. del Nuovo Cimento 1, (1969) 252.
- [75] S. W. Hawking, "Black holes in general relativity," Commun. Math. Phys. 25 (1972) 152.
- [76] W. Israel, "Event Horizons in Static Vacuum Space-Times," Phys. Rev., 164 (1967) 1776.
- [77] J. D. Bekenstein, "Black holes and entropy," Phys. Rev. D7 (1973) 2333.
- [78] S. W. Hawking, "Particle Creation by Black Holes," Commun. Math. Phys. 43 (1975) 199.
- [79] A. Strominger, C. Vafa, "Microscopic origin of the Bekenstein-Hawking entropy," Phys. Lett. B379 (1996) 99, hep-th/9601029.
- [80] G. 't Hooft, "Dimensional Reduction in Quantum Gravity," in Salam Festschrift, eds. A. Aly, J. Ellis and S. Randjbar-Daemi, World Scientific, Singapore, 1993, arXiv:gr-qc/9310026.
- [81] L. Susskind, "The World as a Hologram," J. Math. Phys. 36 (1995) 6377, arXiv:hep-th/940989.
- [82] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, "Large N field theories, string theory and gravity," Phys. Rept. 323 (2000) 183, arXiv:hep-th/9905111.
- [83] J. L. Petersen, "Introduction to the Maldacena conjecture on AdS/CFT," Int. J. Mod. Phys. A14 (1999) 3597, arXiv:hep-th/9902131.

- [84] I. R. Klebanov, "TASI Lectures: Introduction to the AdS/CFT Correspondence," arXiv:hep-th/0009139.
- [85] E. D'Hoker, D. Z. Freedman, "Supersymmetric gauge theories and the AdS / CFT correspondence," arXiv:hep-th/0201253.
- [86] L. Brink, J. Scherk and J.H. Schwarz, "Supersymmetric Yang-Mills Theories,," Nucl. Phys. B121 (1977) 77.
- [87] I. R. Klebanov, "QCD and String Theory," Int. J. Mod. Phys. A21 (2006) 1831, arXiv:hep-ph/0509087.
- [88] S. S. Gubser, A. Karch, "From gauge-string duality to strong interactions: a Pedestrian's Guide," arXiv:hep-th/0901.0935.
- [89] I. R. Klebanov, M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and chi SB resolution of naked singularities," JHEP 0008 (2000) 052, arXiv:hep-th/0007191.
- [90] Karch, A., Katz, E. "Adding avor to AdS/CFT," JHEP 0206 (2002) 043, arXiv:hep-th/0205236.
- [91] M. Kruczenski, D. Mateos, R. C. Myers, D. J. Winters, "Towards a holographic dual of large N(c) QCD," JHEP 0405 (2004) 041, arXiv:hep-th/0311270.
- [92] J. Babington, J. Erdmenger, N. J. Evans, Z. Guralnik, I. Kirsch, "Chiral symmetry breaking and pions in nonsupersymmetric gauge / gravity duals,," Phys. Rev. D 69 (2004) 066007, arXiv:hep-th/0306018.
- [93] T. Sakai and S. Sugimoto, "Low energy hadron physics in holographic QCD," Prog. Theor. Phys. 113 (2005) 843, arXiv:hep-th/0412141.
- [94] T. Sakai and S. Sugimoto, "More on a holographic dual of QCD," Prog. Theor. Phys. 114 (2006) 1083, arXiv:hep-th/0507073.
- [95] J. Hirn, N. Rius and V. Sanz, "Geometric approach to condensates in holographic QCD," Phys. Rev. D 73 (2006) 085005, arXiv:hep-ph/0512240.
- [96] K. Ghoroku, N. Maru, M. Tachibana and M. Yahiro, "Holographic model for hadrons in deformed AdS(5) background," Phys. Lett. B 633 (2006) 602, arXiv: hep-ph/0510334.
- [97] E. Antonyan, J. A. Harvey, D. Kutasov, "Chiral symmetry breaking from intersecting D-branes," Nucl.Phys. B784 (2007) 1, arXiv:hep-th/0608177.
- [98] N. Evans, A. Tedder and T. Waterson, "Improving the infra-red of holographic descriptions of QCD," JHEP 0701 (2007) 058, arXiv:hep-ph/0603249.
- [99] C. Csaki and M. Reece, "Toward a systematic holographic QCD: A Braneless approach," JHEP 0705 (2007) 062, arXiv:hep-ph/0608266.
- [100] R. Casero, E. Kiritsis and A. Paredes, "Chiral symmetry breaking as open string tachyon condensation," Nucl.Phys. B787 (2007) 98, arXiv:hep-th/0702155.
- [101] J. Erdmenger, K. Ghoroku and I. Kirsch, "Holographic heavy-light mesons from non-Abelian DBI," JHEP 0709 (2007) 111, arXiv:hep-th/0706.3978.
- [102] O. Bergman, S. Seki and J. Sonnenschein, "Quark mass and condensate in HQCD," JHEP 0712 (2007) 037, arXiv: hep-th/0708.2839.
- [103] O. Bergman, G. Lifschytz, M. Lippert, "Holographic Nuclear Physics," JHEP 0711 (2007) 056, arXiv: hep-th/0708.0326.
- [104] A. Dhar and P. Nag, "Sakai-Sugimoto model, Tachyon Condensation and Chiral symmetry Breaking," JHEP 0801 (2008) 055, arXiv:hep-th/0708.3233.
- [105] U. Gursoy and E. Kiritsis, "Exploring improved holographic theories for QCD: Part I," JHEP 0802 (2008) 032, arXiv:hep-th/0707.1324.
- [106] U. Gursoy, E. Kiritsis and F. Nitti, "Exploring improved holographic theories for QCD: Part II," JHEP 0802 (2008) 019, arXiv:hep-th/0707.1349.
- [107] J. Erdmenger, N. Evans, I. Kirsch, "Mesons in Gauge/Gravity Duals - A Review," Eur. Phys. J. A35 (2008) 81, arXiv:hep-th/0711.4467.
- [108] B. A. Burrington, J. Sonnenschein, "Holographic Dual of QCD from Black D5 Branes," JHEP 0908 (2009) 057 arXiv:hep-th/0903.0628.
- [109] D. K. Hong, H. Yee, "Holographic aspects of three dimensional QCD from string theory," JHEP 1005 (2010) 036 arXiv:hep-th/1003.1306.
- [110] T. Imoto, T. Sakai, S. Sugimoto, "Mesons as Open Strings in a Holographic Dual of QCD," Prog. Theor. Phys. 124 (2010) 263, arXiv:hep-th/1005.0655.
- [111] J. Polchinski and M. J. Strassler, "Hard scattering and gauge/string duality," Phys. Rev. Lett. 88 (2002) 031601, arXiv:hep-th/0109174.
- [112] J. Polchinski, M. J. Strassler, "Deep inelastic scattering and gauge/string duality," JHEP 0305 (2003) 012, arXiv:hep-th/0209211.
- [113] H. Boschi-Filho, N. R.F. Braga, "Gauge/string duality and scalar glueball mass ratios," JHEP 0305 (2003)009, arXiv:hep-th/0212207.
- [114] H. Boschi-Filho, N. R. F. Braga, "QCD/string holographic mapping and glueball mass spectrum," Eur.Phys.J.C32 (2004)529, arXiv:hep-th/0209080.
- [115] S. J. Brodsky, G. F. de Teramond, "Light front hadron dynamics and AdS/CFT correspondence," Phys.

- Lett. B582 (2004) 211-221, arXiv:hep-th/0310227.
- [116] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, "QCD and a holographic model of hadrons," Phys. Rev. Lett. 95 (2005) 261602, arXiv:hep-ph/0501128.
- [117] L. Da Rold and A. Pomarol, "Chiral symmetry breaking from five dimensional spaces," Nucl. Phys. B721 (2005) 79, arXiv:hep-ph/0501218.
- [118] G. F. de Teramond, S. J. Brodsky, "Hadronic spectrum of a holographic dual of QCD," Phys. Rev. Lett. 94 (2005) 201601, arXiv:hep-th/0501022.
- [119] S. J. Brodsky and G. F. de Teramond, "Hadronic spectra and light-front wavefunctions in holographic QCD," Phys. Rev. Lett. 96 (2006) 201601, arXiv: hep-ph/0602252.
- [120] J. Erlich, G. D. Kribs and I. Low, "Emerging holography," Phys. Rev. D 73 (2006) 096001, arXiv:hep-ph/0509002.
- [121] L. Da Rold and A. Pomarol, "The Scalar and pseudoscalar sector in a five-dimensional approach to chiral symmetry breaking," JHEP 0601 (2006) 157, arXiv:hep-ph/0510268.
- [122] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, "Linear confinement and AdS/QCD," Phys. Rev. D 74 (2006) 015005, arXiv:hep-ph/0602229.
- [123] A. Pomarol, A. Wulzer, "Baryon Physics in Holographic QCD," Nucl.Phys. B809 (2009) 347, arXiv:hep-ph/0807.0316.
- [124] H. Kim, Y. Kim, U. Yakhshiev, "Mesons and nucleons from holographic QCD in a unified approach," JHEP 0911 (2009) 034, arXiv:hep-ph/0908.3406.
- [125] W. de Paula, T. Frederico, "Scalar mesons within a dynamical holographic QCD model.," Phys.Lett. B693 (2010) 287, arXiv:hep-ph/0908.4282.
- [126] K. Jo, B. Lee, C. Park, S. Sin, "Holographic QCD in medium: A Bottom up approach," JHEP 1006 (2010) 022 , arXiv:hep-ph/0909.3914.
- [127] S.S. Afonin, "Low-energy holographic models for QCD," Phys. Rev. C83 (2011) 048202, arXiv:hep-ph/1102.0156.
- [128] D. K. Hong, T. Inami, H. Yee, "Baryons in AdS/QCD," Phys. Lett. B646 (2007) 165, arXiv:hep-ph/0609270.
- [129] S. W. Hawking and D. N. Page, "Thermodynamics of Black Holes in anti-De Sitter Space', Commun. Math. Phys. 87 (1983) 577.
- [130] C. P. Herzog, "A Holographic Prediction of the Deconfinement Temperature," Phys. Rev. Lett. 98 (2007) 091601, arXiv:hep-th/0608151.
- [131] D. T. Son, A. O. Starinets, "Viscosity, Black Holes, and Quantum Field Theory," Ann. Rev. Nucl. Part. Sci. 57 (2007) 95, arXiv:hep-th/0704.0240.
- [132] S. R. Das, G. W. Gibbons and S. D. Mathur, "Universality of low energy absorption cross sections for black holes," Phys. Rev. Lett. 78 (1997) 417, arXiv:hep-th/9609052.