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Reconstruction of Galileo Galilei's experiment: the inclined plane

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Abstract

In the 'Third Day' of the *Discourses and Mathematical Demonstrations Concerning Two New Sciences* Galileo Galilei describes the famous experiment of the inclined plane and uses it to bring an experimental confirmation to the laws of uniformly accelerated motion. We describe a reconstruction of the experiment and how the results can be used for students when studying the laws of motion.

Introduction

In the 'Third Day' of his *Two New Sciences* [1] Galileo talks about the *local motion*, the principles of uniform and uniformly accelerated motion are presented and their consequences are discussed[1]. The treatise begins with a theoretical analysis of the former (as a motion in which equal times—no matter how they are chosen—correspond to equal travelled distances) and continues with a list of corollaries resulting from the definition. He then discusses accelerated motion and gives it a formal definition (as a gain of equal 'speed levels' in equal times) and, through mathematical steps, characteristics of the uniformly accelerated motion are deduced, most importantly that travelled distances and squared times are proportional. The following discussion moves to the physical aspects of the problem of whether this type of motion can exactly reproduce the natural fall of the bodies. The experiment Galileo carried out with the inclined plane confirms this.

According to Galileo, free-fall is a natural motion towards 'the common centre of heavy bodies' (i.e. the centre of the Earth). He assumes the inclined plane is a device which can 'slow-down' this kind of motion, without changing the mathematical ratios of the quantities involved.

Several works have been dedicated to reproducing Galilean experiments (an interesting discussion on Galileo's reasoning appeared recently in *Phys. Educ.* [4]). In the 1960s, Settle [5] built a device very similar to Galileo's and reproduced his experiment. Settle wanted to know if Galileo, using the technology available in the 17th century, could actually verify the law of accelerated motion. Some authors (i.e. Koyré [6]) believed that this experiment and other similar ones, described by Galileo in his books, were executed only in the scientist's mind. Settle's experiment gave an undoubted answer to this question and, according to the author, the inclined plane and the water clock, built following Galileo's specifications, are precise enough to prove the direct proportionality between distances and squared times.

We carried out the same experiment with the aim of reproducing the results obtained by

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[1] This work was published in 1638 in Leida, but almost all the science historians agree that the content had been written several years before (1602–1609), while Galileo lived in Padua (see the work by Geymonat [2] or the more recent work by Drake [3], which both provide a detailed analysis of his life and work). The *Two New Sciences* is a dialogue, divided into four days, in which three characters defend different points of view, but all converge to the author's ideas at the end.

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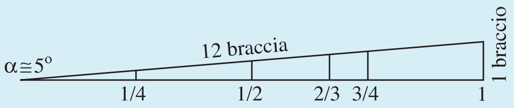

Figure 1. Sketch of Galileo's inclined plane (the inclination is about 5°). The starting points of the ball are also indicated. Lengths are measured in *braccia* (cubits).

Galileo, but using a procedure that can be used in secondary-school classrooms.

Experiment in Galileo's book

We quote the text from the *Two New Sciences*, below, which describes the experiment and which we have divided into four parts for ease of reference. In figure 1 the inclined plane is drawn to scale, according to the instructions contained in the passage.

'(i) *A piece of wooden moulding or scantling, about 12 cubits*[2] *long, half a cubit wide, and three finger-breadths thick, was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball.* (ii) *Having placed this board in a sloping position, by lifting one end some one or two cubits above the other, we rolled the ball, as I was just saying, along the channel, noting, in a manner presently to be described, the time required to make the descent. We repeated this experiment more than once in order to measure the time with an accuracy such that the deviation between two observations never exceeded one-tenth of a pulse-beat.* (iii) *Having performed this operation and having assured ourselves of its reliability, we now rolled the ball only one-quarter the length of the channel; and having measured the time of its descent, we found it precisely one-half of the former. Next we tried other distances, comparing the time for the whole length with that for the half, or with that for two-thirds, or three-fourths, or indeed for any fraction; in such experiments, repeated a full hundred times, we always found that the spaces traversed were to each other as the squares of the times, and this was true for all inclinations of the plane, i.e., of the channel, along which we rolled the ball. We also observed that the times of descent, for various inclinations of the plane, bore to one another precisely that ratio which, as we shall see later, the Author had predicted and demonstrated for them.* (iv) *For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small glass during the time of each descent, whether for the whole length of the channel or for a part of its length; the water thus collected was weighed, after each descent, on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the times, and this with such accuracy that although the operation was repeated many, many times, there was no appreciable discrepancy in the results*.'

This description is very clear and is almost a prototype for modern scientific language. In part (i) detailed information about the device is proposed (as if to enable the reader to repeat the experiment in the same conditions as Galileo's). In part (ii) the method for taking measurements is described and in part (iii) the results are enumerated. Finally in part (iv) there is a digression on the additional but necessary device, the water clock.

Galileo states that his results have been obtained from experiments *repeated a full hundred times*. He probably realized that any measurement has to be reproduced without significant changes in the results but also suggested the use of an *a posteriori* data analysis to *filter* raw data and obtain a unique, reliable value of the descent time, as we will do in our experiment.

Our reconstruction of the device

To reproduce the experiment we built an inclined plane from a pine plank, 3.20 m long (figure 2) with a channel cut into it and with a small piece of wood at the end. The ball was rolled down the edges of this channel rather than down the channel itself as this provided a smoother surface (figure 3). The inclination (that is, the height-to-length ratio) is 1/12, but 1/8 and 1/6 can also be used with an adjustable support. The dimensions of our inclined plane are not the same as Galileo's,

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[2] The original word is *braccio* (that is arm in English). The translation of the Two New Sciences uses 'cubit', and we follow this assumption.

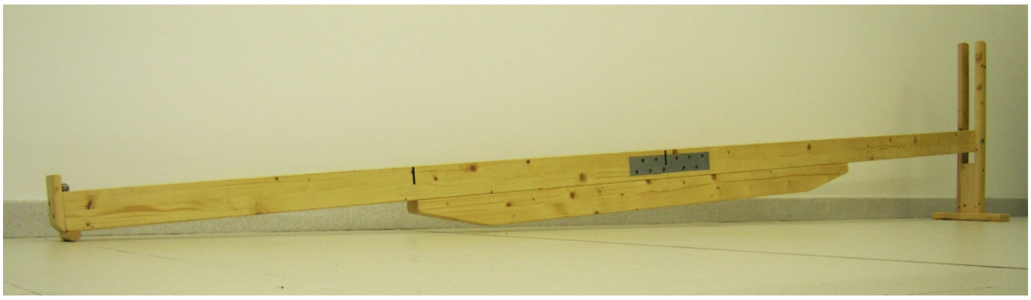

Figure 2. Photograph of our inclined plane.

Figure 3. The ball rolling down the edges of the groove in the wooden plank.

as they have been reduced to about 1:2 to make the experiment suitable for a school laboratory. The Florentine *braccio*, used by Galileo, was about 60 cm long, as can be seen in the reference bars fixed on the external walls of the town hall in Pistoia (figure 4). However, the different dimensions used in our experiment reproduce the original inclination.

Figure 4. The lengths of the metre and of the double cubit (*doppio braccio*) on the walls of the town hall in Pistoia. The inscription indicates that the double cubit (120 cm long, so giving a 60 cm long cubit) is an *ancient Tuscan measure*.

The water clock has been reproduced using a burette (figure 5), with a capacity of 25 ml and sensitivity 0.05 ml which gives very accurate time measurements. The measurements are made by opening the tap with one hand while using the other hand to roll the ball down the inclined plane. The tap is closed when the ball hits the small piece of wood at the end of the channel. It is important that the same person carries out the whole operation to reduce the time lapse between starting the ball rolling and opening the tap.

We noticed that after ‘training’ and repetition of this procedure better measurements were obtained as the time values had larger fluctuations when the experiment was first carried out. Probably Galileo had experienced this problem, when he said *the operation was repeated many, many times*.

We noticed that in our clock the water flux was not exactly constant in time as it depends on the pressure of the water column above the exit hole. However we verified that in our experiment this effect gives rise to a systematic shift of the

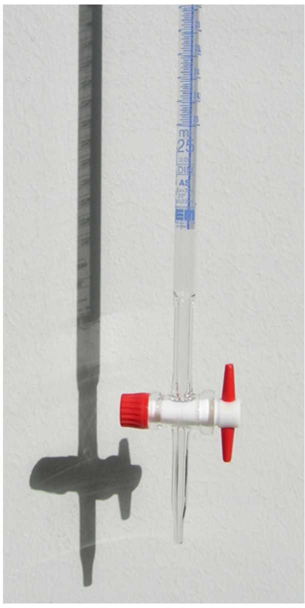

Figure 5. The burette.

water volumes that is negligible with respect to the statistical uncertainties of the measurement.

Results

In our measurements the time is given in ml of fallen water but the laws of motion can be verified without converting the time into more familiar units such as seconds. In figure 6 five histograms are depicted: each one corresponds to the time measurements taken for a given fraction of the full length of the inclined plane: 1/4, 1/2, 2/3, 3/4, 1. Each histogram is obtained from 30 measurements of the same quantity, all of which are affected by uncertainty. This graphical representation of the data allows us to observe how the values are distributed (we can get an immediate estimate of the uncertainties and compare the distributions at different positions).

The results of the measurements of the descent times are numbers spaced out by a 0.05 ml pitch (the sensitivity of the burette) that have been grouped in 0.10 ml wide bins[3] in the histograms in figure 6. Thus the number of bins per histogram matches approximately the square root of the number of entries, and the distribution pattern becomes more clear.

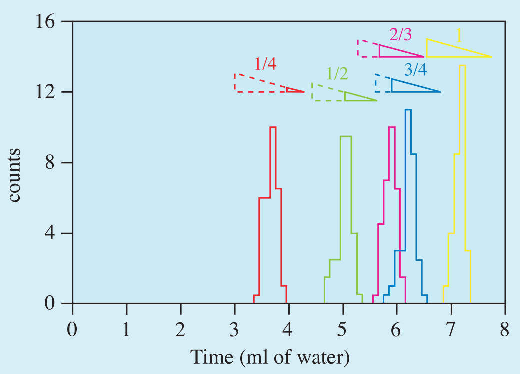

Figure 6. Histograms corresponding to the five series of time measurements for five different starting positions on the inclined plane.

If we assume that the uncertainties are Gaussian-distributed, the barycentre of each histogram, that is the mean value of the $N = 30$ measurements

$$t_{\mathrm{MEAN}} = \frac{t_1 + t_2 + \cdots + t_N}{N}$$

gives our best estimation of the descent time. The standard deviation of the mean

$$\sigma(t_{\mathrm{MEAN}}) = \frac{1}{\sqrt{N}} \times \sqrt{\frac{(t_1 - t_{\mathrm{MEAN}})^2 + \cdots + (t_N - t_{\mathrm{MEAN}})^2}{N - 1}}$$

represents the best estimation of the associated uncertainty [7].

For examination of the proportionality typical of the uniformly accelerated motion (that Galileo describes in part (iii) of the text) we recorded on a Cartesian diagram the squared descent time as a function of the travelled distance, written as a fraction of the full length of the inclined plane (figure 7). The behaviour is consistent with a linear relationship (the interpolating straight line is drawn in red) and the estimation of the uncertainties is consistent with the hypothesis of Gaussian errors[4].

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[3] When a value falls exactly on the boundary between two adjacent bins, we attribute half an entry to both the left and the right bins (for example: the value 3.45 gives a half count in bin 3.40 and half in bin 3.50). That way the barycentre of the histogram corresponds to the mean value of the data.

[4] Remember that, if an error bar in a graph corresponds to a ±1 standard deviation, there is a 68% probability that such a bar intersects the interpolating curve. It means that, for five measurements, the interpolating line will not necessarily lie within the error bars of all the points, but it is more likely that only three or four bars are touched by the line!

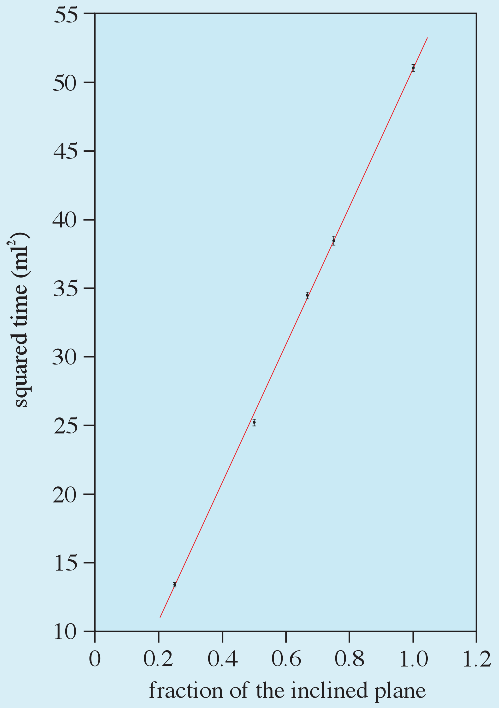

Figure 7. Linear interpolation among the travelled distances and the squared descent times. The error bars are obtained from the standard deviation of the mean of each data series, propagated to the squared time through the formula: $\Delta (t^2) = 2t\sigma(t)$. The estimated uncertainties on the distances are not visible on this graph.

The results are very clear, especially bearing in mind the simplicity of the apparatus and confirms that Galileo's experiment could be actually carried out and, above all, could produce quantitative results especially if using a longer inclined plane.

Teaching comments

These measurements can be carried out with the whole class or with smaller groups of students when studying the laws of motion. We carried out the experiment at the Physics Department of Firenze University with groups of young students (14–18 years). We noticed that the students were very interested in the experiment and the results. We concluded that this historical approach has several advantages.

(1) The teaching value of the historical narrative should be emphasized as the historical narrative helps the student to understand that, starting from Galileo, many devices have been designed and constructed by physicists to answer precise questions and that this is the basis of the experimental approach.

(2) Students enjoy retracing Galileo's steps as he studied the laws of motion, as well as being involved in the design and construction of the inclined plane and water clock, rather than carrying out an experiment that has already been set up, as is often the case.

(3) Teachers of other subjects such as philosophy and history can be involved in reading the original text and in placing Galileo's human and scientific adventure in its historical context.

(4) From the scientific point of view, these measurements can clarify the meaning of the uncertainties, which many students find mystifying. The histograms, obtained from the measurements, enable them to estimate for themselves the most probable value of the quantity to be determined and to assign a level of uncertainty to it. The difference between statistical and systematic uncertainties can also be explained, showing how each one has to be handled.

The students can be divided into groups, and each group can determine the measurement of the descent time for a given distance on the inclined plane. The data analysis can then be carried out as a whole class under the teacher's guidance followed by a discussion about the best way to present the results.

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