The effects of angular momentum conservation in relativistic heavy ion collisions

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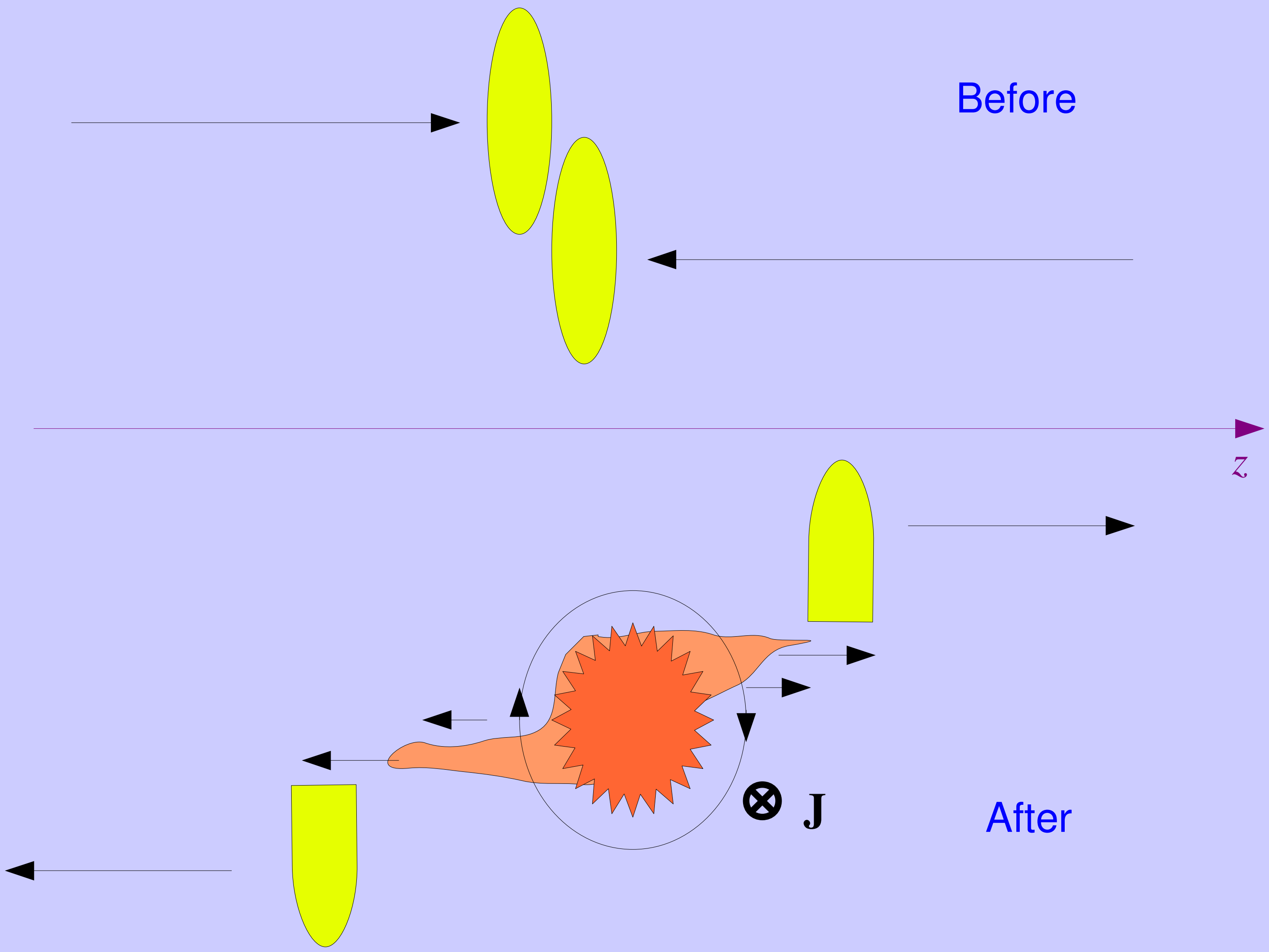
OUTLINE

- Introduction
- Thermodynamics of rotating systems
- Azimuthal anisotropies and polarizations
- Bounds from RHIC measurements
- Predictions for LHC



P. Castorina, D. Kharzeev, H. Satz *Thermal hadronization and Hawking-Unruh radiation in QCD*, arXiv 0704.1426, Sect. 6 ---> AZIMUTHAL ANISOTROPY

Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301 --> LAMBDA POLARIZATION

Rotating system in statistical equilibrium

Grand-canonical partition function with fixed angular momentum – Classical limit (J large)

$$Z = \frac{1}{(2\pi)^3} \int \mathrm{d}^3\phi \, \exp\left[i\mathsf{J}\cdot\phi + \sum_j \frac{\lambda_j}{(2\pi)^3} \int \mathrm{d}^3\mathrm{x} \int \mathrm{d}^3\mathrm{p} \, \mathrm{tr} D^{S_j}(\mathsf{R}_{\hat{\phi}}(\phi)) \mathrm{e}^{-\varepsilon_j/T} \mathrm{e}^{-i\phi\cdot(\mathsf{x}\times\mathsf{p})} \right]$$

F. B., L. Ferroni, *The microcanonical ensemble of the relativistic quantum gas with angular momentum conservation*, to appear.

Saddle-point expansion for *J* and *V* large: introduction of a rotational potential (=angular velocity)

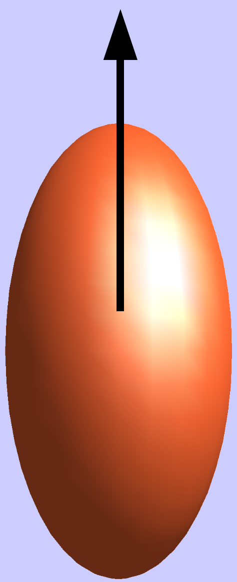
$$\nabla_\phi \left[i\mathsf{J}\cdot\phi + \sum_j \frac{\lambda_j}{(2\pi)^3} \int \mathrm{d}^3\mathrm{x} \int \mathrm{d}^3\mathrm{p} \, \mathrm{tr} D^{S_j}(\mathsf{R}_{\hat{\phi}}(\phi)) \mathrm{e}^{-\varepsilon_j/T} \mathrm{e}^{-i\phi\cdot(\mathsf{x}\times\mathsf{p})} \right] = 0$$

$$\begin{aligned} \mathsf{J} &= \sum_j \frac{\lambda_j}{(2\pi)^3} \int \mathrm{d}^3\mathrm{x} \int \mathrm{d}^3\mathrm{p} \, \mathrm{tr} D^{S_j}(\mathsf{R}_{\hat{\phi}}(\phi))(\mathsf{x}\times\mathsf{p}) \mathrm{e}^{-\varepsilon_j/T} \mathrm{e}^{-i\phi\cdot(\mathsf{x}\times\mathsf{p})} \\ &+ \sum_j \frac{\lambda_j}{(2\pi)^3} \int \mathrm{d}^3\mathrm{x} \int \mathrm{d}^3\mathrm{p} \left[\nabla_\phi \mathrm{tr} D^{S_j}(\mathsf{R}_{\hat{\phi}}(\phi)) \right] \mathrm{e}^{-\varepsilon_j/T} \mathrm{e}^{-i\phi\cdot(\mathsf{x}\times\mathsf{p})} \end{aligned}$$

L

S

Rotating system in statistical equilibrium (2)



If the region is symmetric with respect to the J axis:

$$\hat{\phi} = \hat{\mathsf{J}}$$

Definition of the rotational potential ω

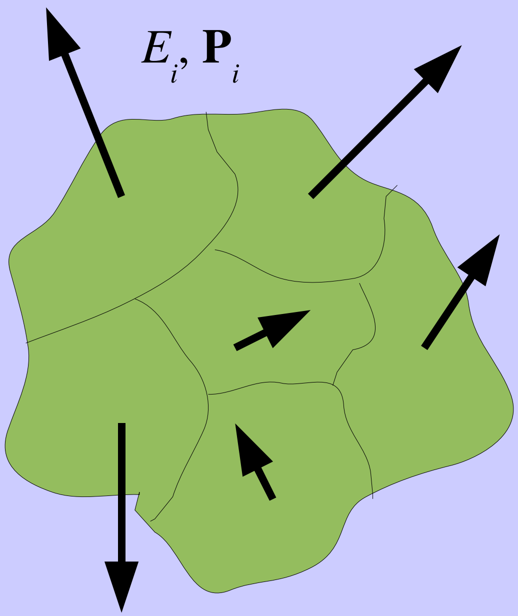
$$\phi \equiv i\omega/T$$

Grand-canonical-rotational partition function:

$$Z_\omega = \exp\left[\sum_j \frac{\lambda_j}{(2\pi)^3}\int \mathrm{d}^3\mathrm{x}\int \mathrm{d}^3\mathrm{p}\,\mathrm{tr}D^{S_j}(\mathsf{R}_{\hat{\mathsf{J}}}(i\omega/T))\mathrm{e}^{-\varepsilon_j/T}\mathrm{e}^{\omega\cdot(\mathsf{x}\times\mathsf{p})}\right]$$

$$S = \frac{U}{T} - \frac{\omega\cdot\mathsf{J}}{T} + \log Z_\omega - \frac{\sum_i \mu_i Q_i}{T}$$

Landau's argument on the equilibrium of macroscopic bodies



$$S = \sum_i S_i\left(\sqrt{E_i^2 - \mathsf{P}_i^{\,2}}\right) \qquad \frac{\partial S_i}{\partial E_i} = \frac{E_i}{M_i}\frac{\partial S_i}{\partial M_i} = \frac{\gamma_i}{T_i}$$

Maximize entropy with constraints

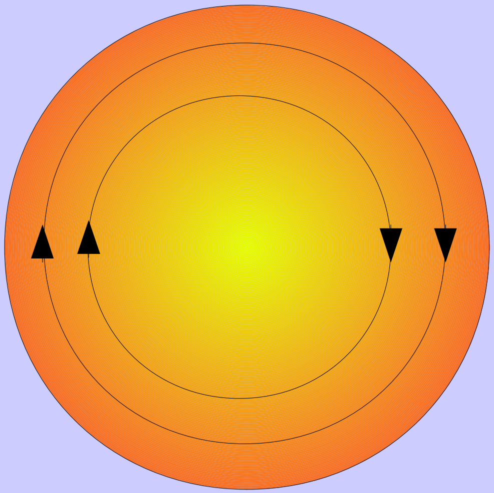
$$\sum_i S_i - \frac{\beta}{T}\cdot\sum_i \mathsf{P}_i - \frac{1}{T}\left(\sum_i E_i - E_0\right) - \frac{\omega}{T}\cdot\left(\sum_i \mathsf{x}_i \times \mathsf{P}_i - \mathsf{J}\right)$$

$$\frac{\gamma_i}{T_i} = \frac{1}{T} \qquad \forall i$$

$$\beta_i = \omega \times \mathsf{x}_i \qquad \forall i$$

$$T_i = \frac{T}{\sqrt{1 - (\omega \times \mathsf{x}_i)^2}}$$

Local temperature



Rotating <u>relativistic</u> system in thermodynamical equilibrium:
outer layers are HOTTER than inner layers

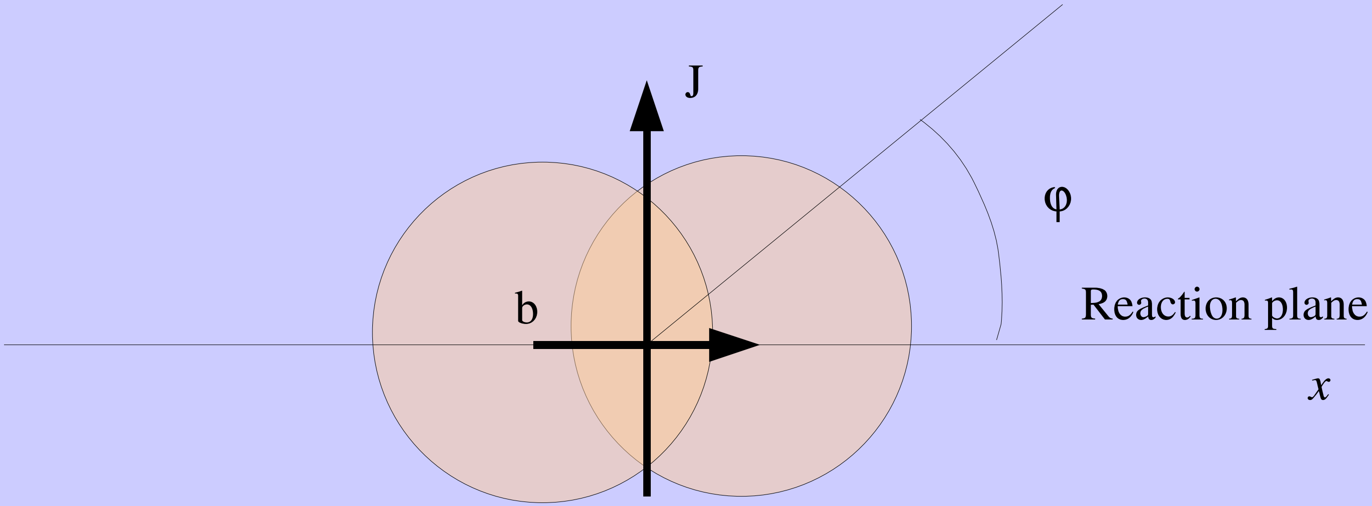
$$T_{loc}(\mathsf{x}) = \frac{T}{\sqrt{1 - (\omega \times \mathsf{x})^2}} \qquad (\omega \times \mathsf{x})^2 < 1$$

BEWARE the distinction between LOCAL temperature (in the local rest frame) and GLOBAL temperature

IF THE SYSTEM DECOUPLES AT THE CRITICAL (LOCAL) TEMPERATURE :

$$T = T_c \sqrt{1 - \omega^2 R_{max}^2} \qquad \omega R_{max} < 1$$

Spectra (primary hadrons)



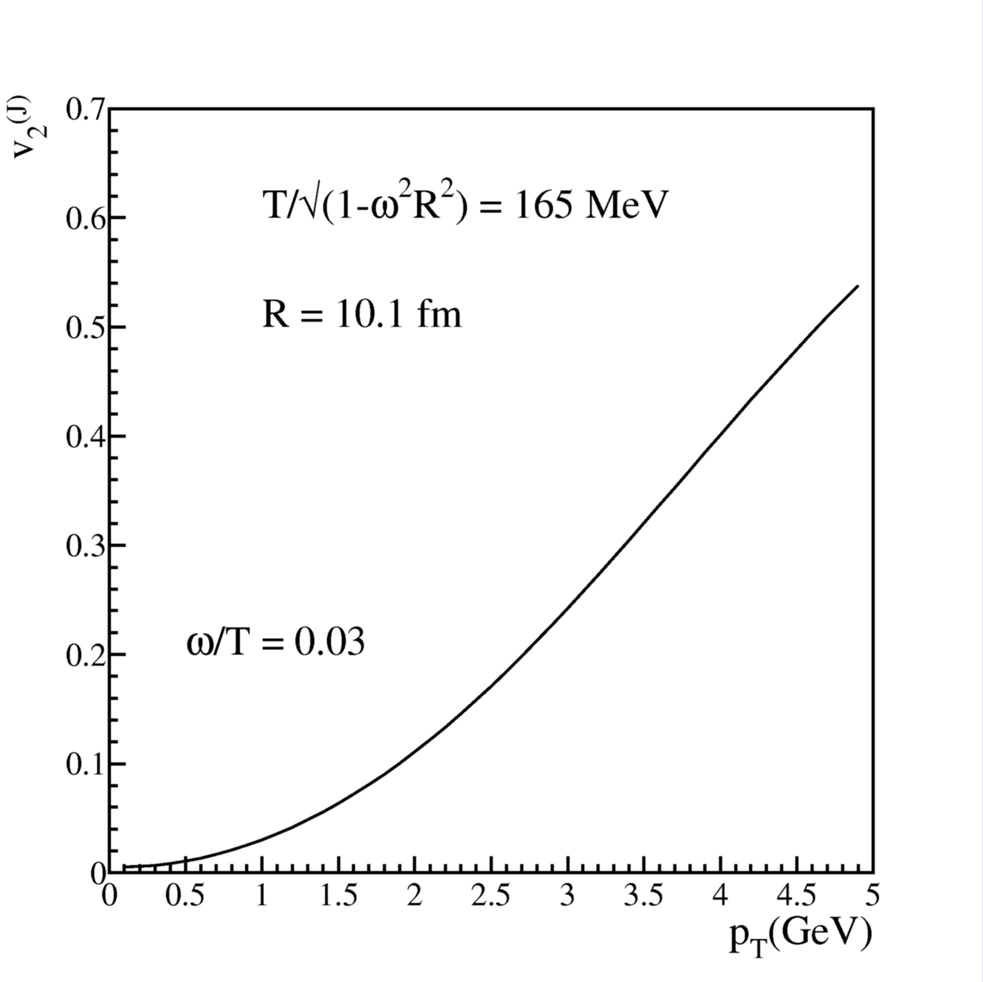
Azimuthal anisotropy

$$\frac{\mathrm{d}n_j}{\mathrm{d}p_T\mathrm{d}\varphi} = \frac{\mathrm{tr}D^{S_j}(\mathsf{R}_{\hat{\mathbf{j}}}(i\omega/T))\lambda_j}{4\pi^3}\int \mathrm{d}^3x \frac{p_T m_T \mathrm{K}_1(m_T\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}/T)}{\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}} \exp\left[\frac{p_T|\omega\times\mathsf{x}|_\perp\cos\varphi}{T}\right]$$

$$\frac{\mathrm{d}n_j}{\mathrm{d}p_T} = \frac{\mathrm{tr}D^{S_j}(\mathsf{R}_{\hat{\mathbf{j}}}(i\omega/T))\lambda_j}{2\pi^2}\int \mathrm{d}^3x \frac{p_T m_T \mathrm{K}_1(m_T\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}/T)}{\sqrt{1-|\omega\times\mathsf{x}|_\parallel^2}} \mathrm{I}_0\left(\frac{p_T|\omega\times\mathsf{x}|_\perp}{T}\right)$$

Elliptic flow

$$v_2^{(J)} = \frac{\int d^3x \frac{K_1(m_T\sqrt{1-|\omega\times x|_\parallel^2}/T)}{\sqrt{1-|\omega\times x|_\parallel^2}} I_2\left(\frac{p_T z\omega}{T}\right)}{\int d^3x \frac{K_1(m_T\sqrt{1-|\omega\times x|_\parallel^2}/T)}{\sqrt{1-|\omega\times x|_\parallel^2}} I_0\left(\frac{p_T z\omega}{T}\right)}$$



An additional contribution to v_2 from hydrodynamics

Polarization

Taking advantage of the formalism developed in:

F. B., L. Ferroni, *The microcanonical ensemble of the relativistic quantum gas with angular momentum conservation*, to appear.

$$\rho = \frac{1}{2}\left[D^S([p]^{-1}\mathsf{R}_{\hat{\mathsf{j}}}(i\omega/T))[p]) + D^S([p]^{\dagger}\mathsf{R}_{\hat{\mathsf{j}}}(i\omega/T)[p]^{\dagger-1})\right]$$

$$\Pi = \frac{\mathrm{tr}(\hat{W}\hat{\rho}/m)}{\mathrm{tr}\hat{\rho}}$$ in the lab frame

Spin 1/2

$$\Pi = (0, \pi_0)$$ in the particle rest frame

$$2\pi_0 \cdot \hat{\mathsf{p}} = \tanh\frac{\omega}{2T}\,\frac{p_y}{p}$$

Longitudinal polarization (helicity) in the particle rest frame

$$2\pi_0 \cdot \hat{\mathsf{j}} = \tanh\frac{\omega}{2T}\left(\frac{\varepsilon}{m} - \frac{p_y^2}{m^2+\varepsilon m}\right)$$

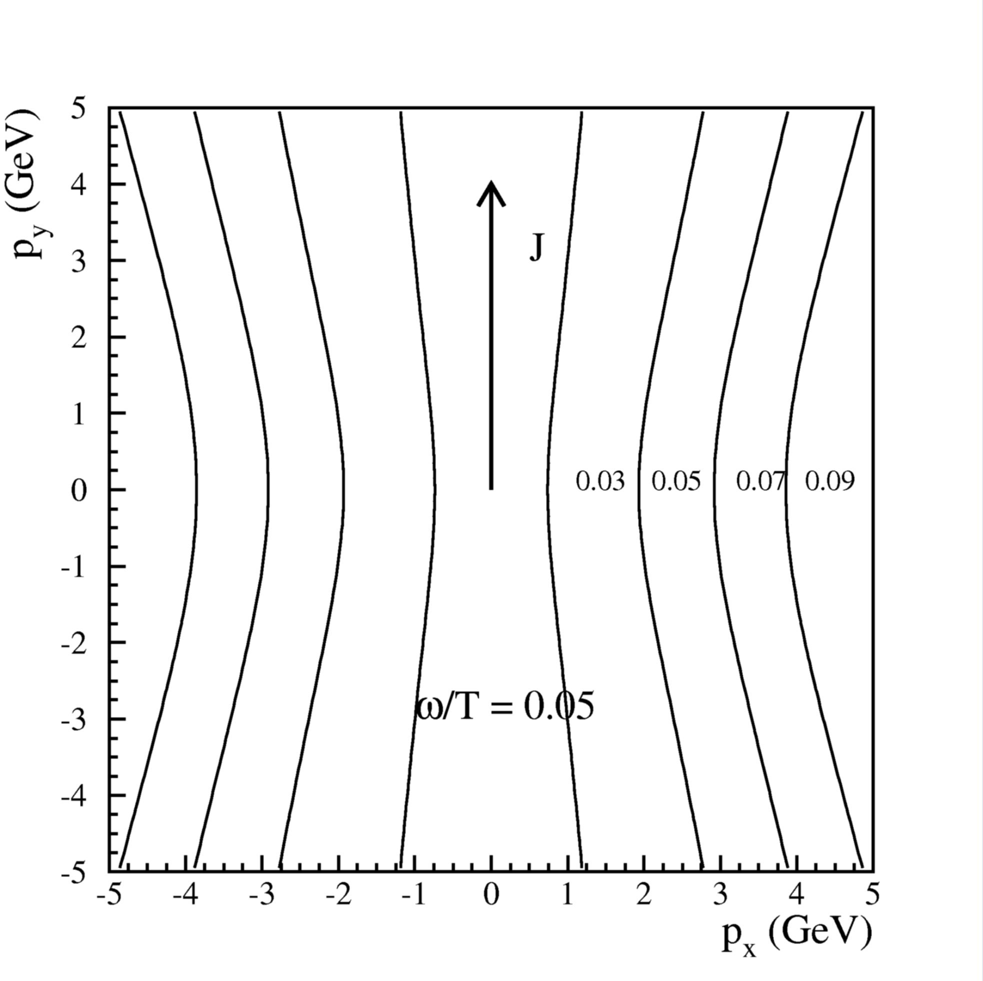
Polarization along the J direction in the particle rest frame

Polarization (2)

Rapidity average

$$\langle 2\pi_0 \cdot \hat{\mathsf{p}} \rangle_y \simeq \tanh \frac{\omega}{2T} \; \sin\varphi$$

$$\langle 2\pi_0 \cdot \hat{\mathsf{j}} \rangle_y \simeq \tanh \frac{\omega}{2T} \; \left(\frac{m_T}{m} - \frac{p_T^2}{m^2 + m_T m} \sin^2\varphi \right)$$



Polarization along the J direction in the particle rest frame

Scaling laws

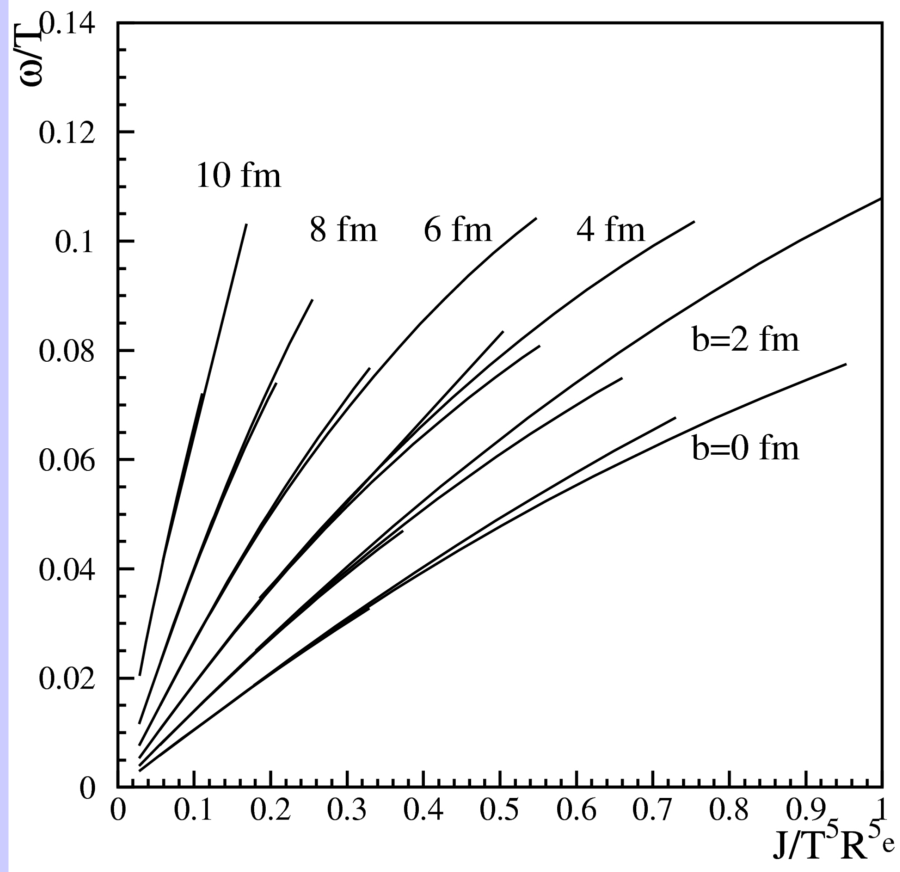
For a given shape, from the saddle-point equation:

$$\omega R_{max} = f\left(\frac{J}{T^4 R_{max}^4}\right) \Rightarrow \omega R_{max} = f'\left(\frac{J}{T_c^4 R_{max}^4}\right)$$

Function *f* is linear for small values of *ωR*, therefore:

$$\frac{\omega}{T} \simeq \alpha \frac{J}{T^5 R_{max}^5} \simeq \alpha' \frac{J}{T_c^5 R_{max}^5}$$

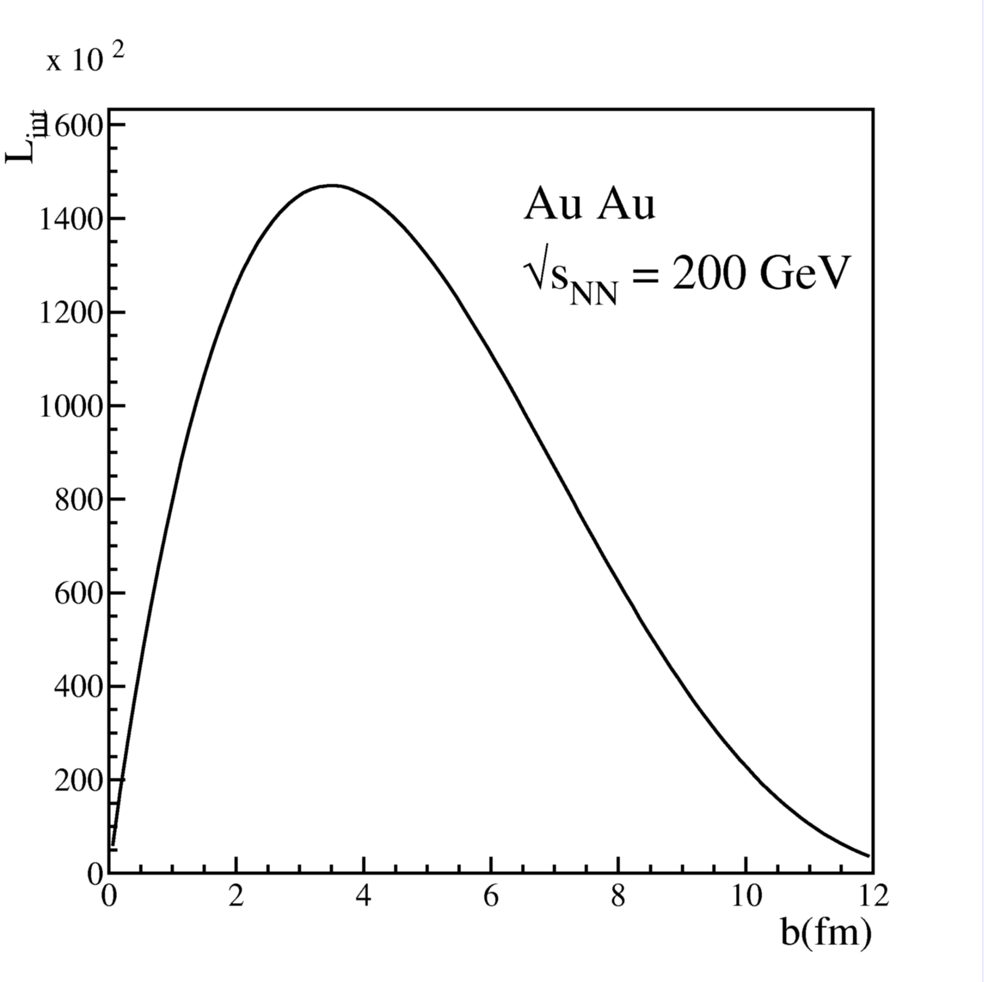
$$\frac{\omega}{T} \simeq \alpha'' \frac{J}{T^5 R_e^5}$$



Angular momentum vs centrality

Simple geometrical model, homogeneous colliding spheres

Angular momentum of the interaction region obtained by subtracting angular momentum of the spectator fragments



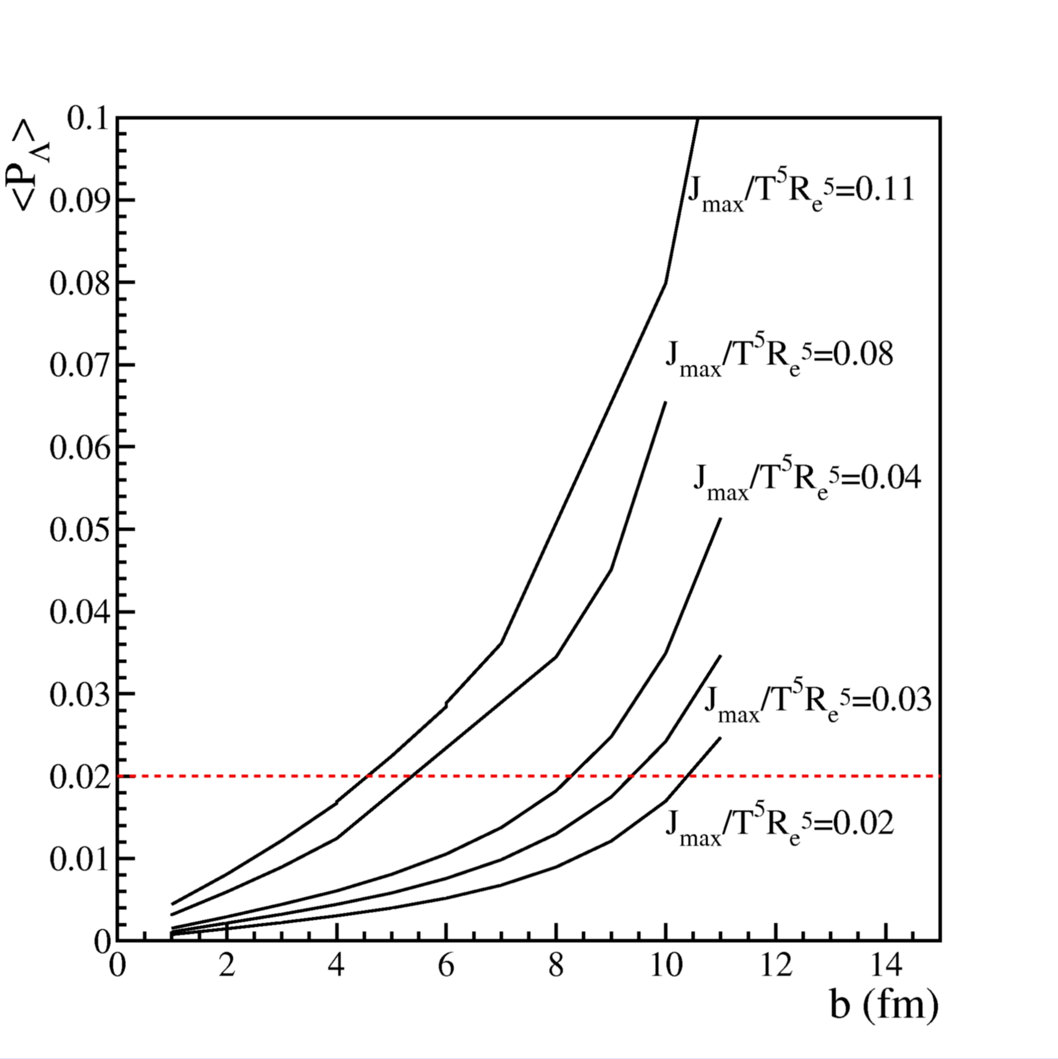
Assuming

- J proportional to L_{int}
- N_p proportional to V

can estimate $\omega(b)$ function

Bounds from RHIC measurements

STAR coll., arXiv 0705.1691



$T_c = 165$ MeV

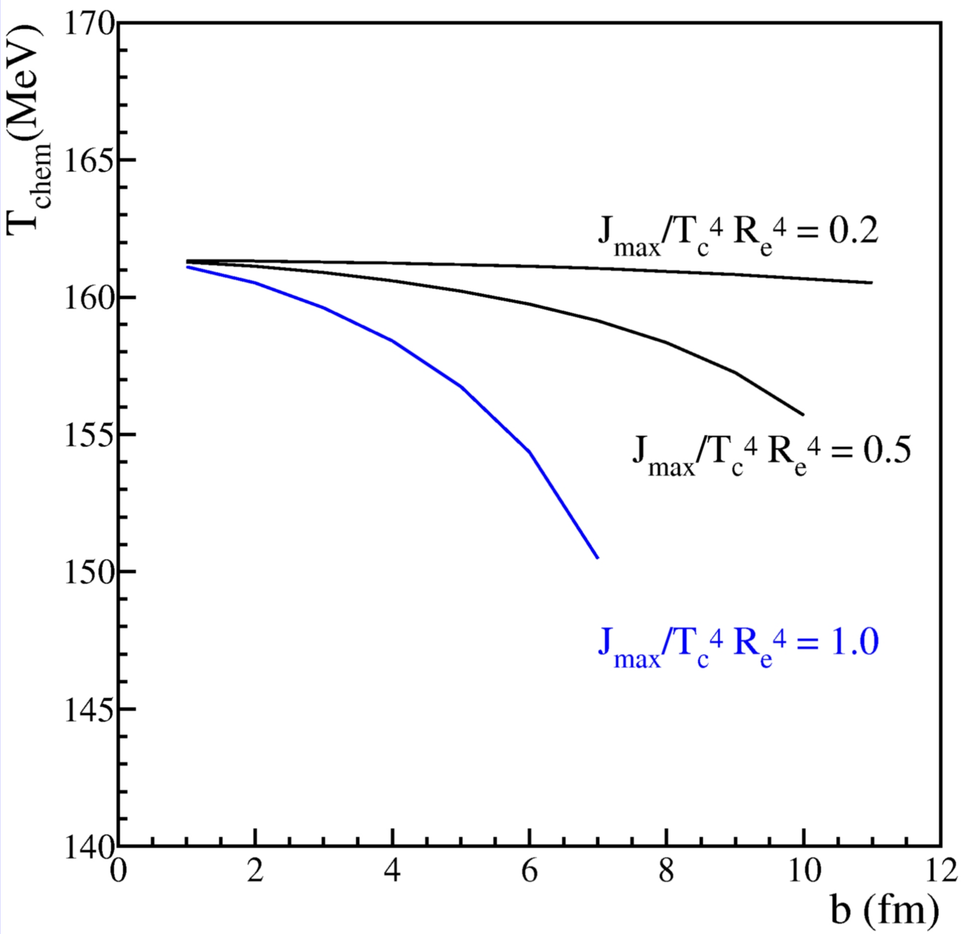
CAVEATS: only primary hadrons! No jets contribution, no distinction between chemical and kinetic freeze-out etc.

Extrapolating to LHC

$$J/(TR)^5 \sim \sqrt{s}_{NN}/(dN/dy)^{5/3} \sim \sqrt{s}_{NN}/\ln \sqrt{s}_{NN}^{5/3}$$

A factor 11   Assume $J_{max}/T^5 R_e^5 = 0.1$

$$J_{max}/T_c^4 R_e^4 = 1.0$$

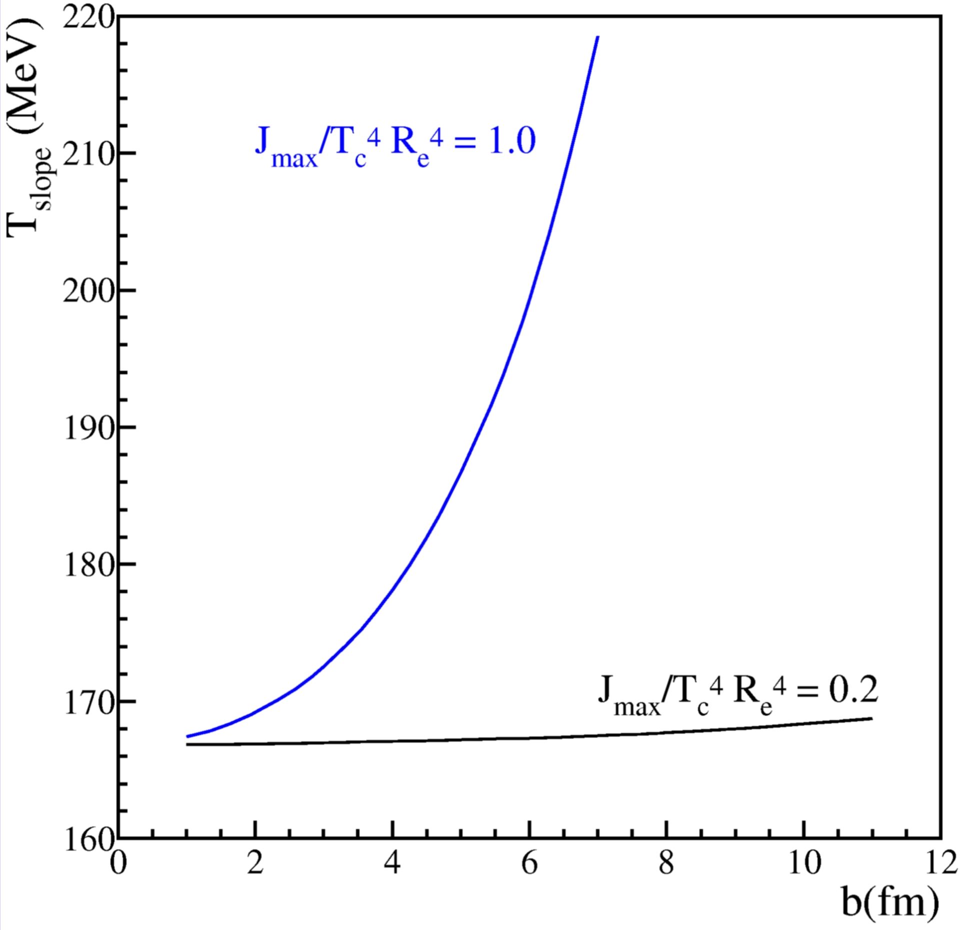


Chemical freeze-out vs b

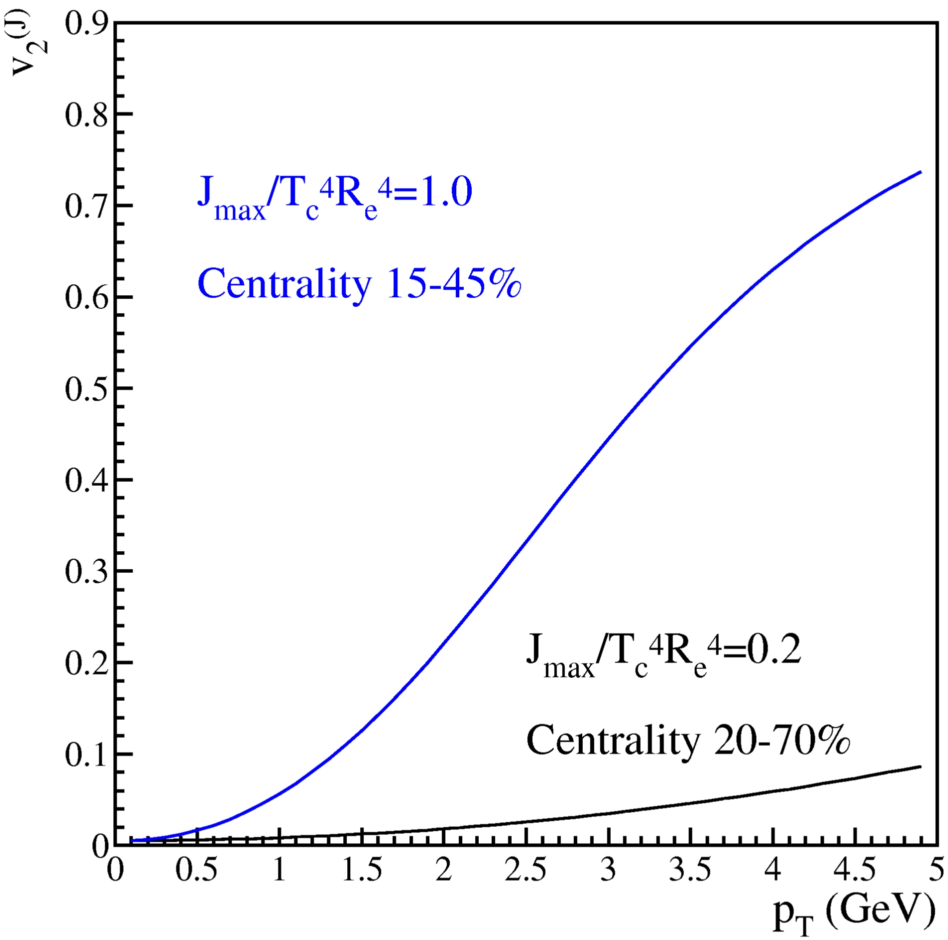
$$T_{chem} = f(J_{max}/T_c^4 R_e^4) \approx f(10 J_{max}/T^5 R_e^5)$$

T_c = 165 MeV

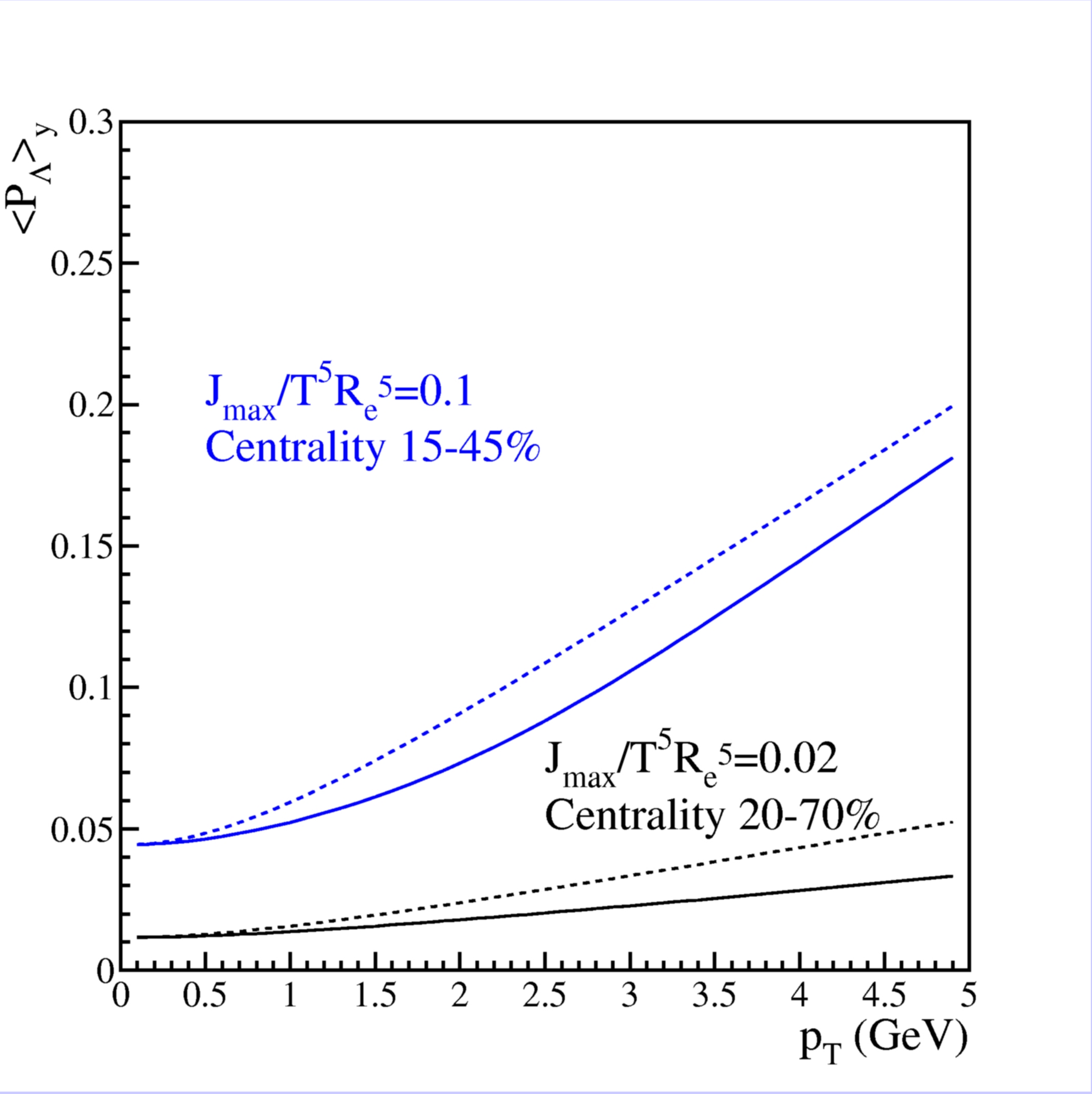
Extra pT broadening vs b



Extra elliptic flow



(primary) Λ polarization along *J*



Spinning (equilibrated) plasma could be observed at LHC

- Lowering of T_{chem} as a function of centrality
- Extra p_T broadening in peripheral collisions
- A large enhancement of elliptic flow
- LAMBDA POLARIZATION VECTOR

$$\pi_0 = \tanh \frac{\omega}{2T} \left(\frac{\varepsilon}{m} \hat{\mathsf{j}} - \frac{\mathsf{p} \cdot \hat{\mathsf{j}}}{m^2 + m\varepsilon} \mathsf{p} \right)$$

CAVEATS: decays, jets production, non-equilibrium (hydrodynamics)