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TRANSFER REACTIONS IN THE SUDDEN LIMIT OF THE PAIRING-ROTOR MODEL

S. LANDOWNE
Physics Division, Argonne National Laboratory Argonne, Illinois 60439, USA
and
Dipartimento di Fisica Teorica, Università di Torino and INFN Sez. di Torino, Torino, Italy

G. POLLAROLO
Dipartimento di Fisica Teorica, Università di Torino
and
INFN Sez. di Torino, Torino, Italy

C.H. DASSO
Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark
and
Sektion Physik der Universität München, D-8046 Garching, BRD

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Abstract: The transfer of multiple pairs of particles in heavy-ion reactions is studied in the sudden limit of the macroscopic pairing-rotor model.

1. Introduction

The possibility that one might be able to observe nuclear reactions in which correlated groups of particles are transferred between the projectile and target has always been an intriguing topic in the field of heavy-ion physics. A novel approach to this subject has been proposed recently wherein the formal analogy between surface and pairing vibrations is exploited to introduce a macroscopic model for the excitation of pair-transfer modes [1]). For normal systems in the vicinity of closed shells the deformations considered have a dynamical character. In the case of superfluid nuclei [2]), on the other hand, the residual interactions are strong enough to generate static deformations in gauge space. The isotropy of the abstract space is thus broken, and one thinks of a system with a symmetry axis oriented along a given angle φ. The finite spread in the number of particles characteristic of the superfluid phase is taken into account by allowing the mass content of the nucleus to change, depending on the point of view from which the system is observed in gauge space. This ansatz can also be interpreted in terms of an effective modulation of the nuclear radius for different orientations in the abstract space. From this premise one can proceed to analyze multi-pair transfer reactions in much the same way as it is done for the inelastic excitation of a rotational band [3]).

Recent studies explore the consequences of the macroscopic pairing-rotor model using classical, semi-classical and quantum mechanical methods [4-7]). The present work investigates the sudden limit of the quantum mechanical formulation, wherein the gauge angle φ is treated as a frozen variable. This limit provides a useful physical insight into the fully numerical quantum mechanical calculation. It also automatically incorporates quantum effects that are either absent or difficult to include in the classical and semi-classical approaches. Finally, the semi-classical limit of the sudden approximation directly reveals the classical structure of the problem in terms of simple formulas.

The next section presents the sudden limit of the macroscopic model for pair-transfer reactions with superfluid systems. An application is made in sect. 3 to a case of multiple-pair transfer in a Ca + Sn collision. The semi-classical limit is derived in sect. 4 and the analytic results are compared to classical and quantum mechanical calculations. The results of this work are summarized in sect. 5.

2. Formulation

Consider a collision of a projectile of charge $Z_1 e$ and mass A_1 with a superfluid target nucleus ($Z_2 e$, A_2). In the macroscopic point of view the projectile only plays the role of an external source which can add pairs of particles to (or remove them from) the target. The aforementioned dependence of the nuclear radius on the orientation in gauge space is parametrized in the intrinsic frame as follows [2]),

$$R_2(\varphi_s) = R_2\left[1 + \frac{2\beta_p}{3A_2}\cos 2\varphi_s\right], \tag{1}$$

where β_p is the pairing deformation parameter and φ_s is a gauge angle measured with respect to the symmetry axis of the sytem.

The characterization of the deformed superfluid system according to eq. (1) makes it possible to consider a dynamical problem in which coordinates in the ordinary space and those of the pairing rotor are connected. This link can be established because changes in the orientation of the symmetry axis in gauge space (specified by the coordinate φ) have effects on the relative motion. Indeed, the short-ranged nuclear interaction V_N, expressed as a function of the distance between the nuclear surfaces, will appear relatively weaker or stronger as the different nuclear sizes in the intrinsic state are sampled. A hamiltonian for this problem can then be written as

$$H = -\frac{\hbar^2}{2\mu}\nabla_r^2 - \frac{\hbar^2}{2I}\frac{\partial^2}{\partial^2\varphi} + \frac{Z_1 Z_2 e^2}{r} + V_N(r - R_1 - R_2(\varphi)), \tag{2}$$

where μ is the reduced mass and $\boldsymbol{r}$ is the position of the projectile with respect to the target*. The second term in the previous expression represents the intrinsic

* The formulation of the problem for the transfer of an integer number of particles is invariant with respect to the interval $[a, b]$ in which gauge angles are defined as long as $(b-a) = 2\pi$. In eq. (2) we have selected the interval in such a way that the symmetry axis of the rotor and the direction from which angles are measured coincide for $\varphi = 0$.

hamiltonian for the pairing-rotor. Its eigenstates are

$$\Psi_n = e^{i2n\varphi}\frac{1}{\sqrt{2\pi}}; \qquad n = 0, \pm 1, \pm 2, \dots \tag{3}$$

so that

$$-\frac{\hbar^2}{2I}\frac{\partial^2}{\partial^2\varphi}\Psi_n = \frac{\hbar^2(2n)^2}{2I}\Psi_n . \tag{4}$$

The integer n counts the number of pairs which are added to ($n>0$) or removed from ($n<0$) the initial system ($n=0$). In practice, the inertial parameter I may be obtained by fitting the quadratic part of the pair separation energies [8]).

The approach of the present work is to ignore the energy differences due to the different number of pairs. That is, we take the limit $I\to\infty$. The resulting hamiltonian $\bar{H}(\varphi)$ then depends only parametrically on the gauge angle. This is analogous to treating the Coulomb excitation of an ordinary rotor in the sudden limit, where the orientation of the rotor is kept frozen during the collision [9-11]). Two differences are noteworthy, however. In the present case, where the interaction is spherically symmetric, the transfer of particles does not involve transfer of angular momentum. Secondly, although the energy differences implied by eq. (4) are typically larger than those of an ordinary rotor, the collision time determined by the nuclear interaction is much shorter than for Coulomb excitation (see e.g. ref. [11])). Thus it is reasonable to apply the sudden limit to the transfer problem.

Consider then the Schrödinger equation

$$[\bar{H}(\varphi)-E]\chi(\mathbf{r},\varphi)=0 , \tag{5}$$

where E is the energy in the center-of-mass frame. Solving this equation under the usual scattering boundary condition (ignoring the nuclear charges for a moment and introducing the wave number k through $\hbar^2k^2=2\mu E$),

$$\chi(\mathbf{r},\varphi)\to e^{ikz}+f(\vartheta,\varphi)\frac{e^{ikr}}{r} \tag{6}$$

generates an elastic scattering amplitude f as a function of the scattering angle ϑ and the gauge angle φ. The amplitude f_n for transferring n pairs is then given by the matrix element of f between the initial and final intrinsic states

$$f_n(\vartheta)=\int_0^{2\pi}\Psi_n^* f(\vartheta,\varphi)\Psi_0\,d\varphi=\frac{1}{2\pi}\int_0^{2\pi}e^{-i2n\varphi}f(\vartheta,\varphi)\,d\varphi . \tag{7}$$

The corresponding cross section is given by $|f_n|^2$.

To implement this procedure one must expand the total wave function into partial waves. The radial wave equation to be solved for partial wave number l is

$$\left\{\frac{\hbar^2}{2\mu}\left[-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}\right]+\frac{Z_1Z_2e^2}{r}+V_N(r-R_1-R_2(\varphi))\right\}u_l(r,\varphi)=0 , \tag{8}$$

with the asymptotic condition

$$u_l(r,\varphi) \to H_l^{(-)}(r) - S_l(\varphi) H_l^{(+)}(r)\,. \tag{9}$$

Here $H^{(\pm)}$ denote the asymptotic outgoing and ingoing Coulomb partial waves

$$H_l^{(\pm)}(r) = \exp\left[\pm i\left(kr - \eta \ln(2kr) - \frac{1}{2} l\pi + \sigma_l\right)\right], \tag{10}$$

where $\eta = Z_1 Z_2 e^2 \mu / \hbar k$ and σ_l is the Coulomb phase shift which satisfies the recursion relation

$$\sigma_l = \arctan(\eta / l) + \sigma_{l-1}\,. \tag{11}$$

In this way the nuclear partial wave elastic-matrix S_l is generated as a function of φ and the transfer partial wave matrix elements S_{nl} are obtained by

$$S_{nl} = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{e}^{-i2n\varphi} S_l(\varphi)\,\mathrm{d}\varphi = \frac{2}{\pi} \int_0^{\pi/2} \cos 2n\varphi\, S_l(\varphi)\,\mathrm{d}\varphi\,. \tag{12}$$

The last equality follows from the symmetry $R_2(\varphi + \frac{1}{2}\pi) = R_2(\varphi - \frac{1}{2}\pi)$. It shows explicitly that the amplitudes for pair addition and removal are identical in the present model and that the range of φ can actually be restricted to 90° in the calculations. Finally, the total pair transfer amplitudes are constructed as

$$f_n(\vartheta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1)\left[S_{nl}\, e^{i2\sigma_l} - \delta_{n,0}\right] P_l(\cos\vartheta)\,. \tag{13}$$

3. Application

As an application of this approach, we have considered the collision $^{40}Ca + {}^{116}Sn$ with energies at the Coulomb barrier ($V_b = 120$ MeV) and also well below the barrier. The nuclear interaction is parametrized as

$$V_N = \frac{V_0 + iW}{1 + \mathrm{e}^{[(r - R_1 - R_2(\varphi))/a]}}\,. \tag{14}$$

The strength parameters were chosen as $V_0 = -71$ MeV and $W = -17$ MeV. The radii are given by $R_i = 1.21\, A_i^{1/3}$ fm and the diffuseness is $a = 0.63$ fm. For the calculations shown in fig. 1 we have taken the parameter $\beta_p = 5$. The numerical calculations of $S_l(\varphi)$ were done with the PTOLEMY program [12]).

The resulting cross sections in fig. 1a are obtained at the center-of-mass energy $E = 120$ MeV, while those in fig. 1b are for $E = 105$ MeV. For the higher energy the cross sections peak around $\vartheta = 120°$ with about 1 mb/sr for the one pair transfer. An interesting detail may be seen in the different behavior of the forward-angle cross sections for $n = 1$ and $n = 2$ in fig. 1a. This reflects the fact that there is an interference between the direct transfer of 4 particles and the successive transfer

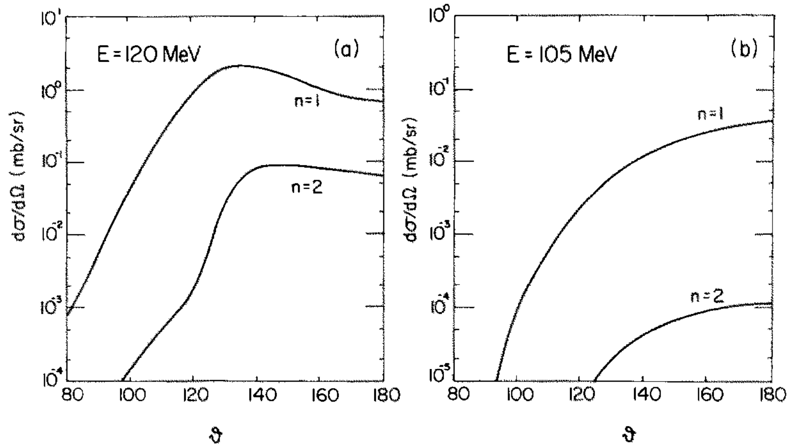

Fig. 1. Angular distributions for the transfer of one ($n = 1$) and two ($n = 2$) pairs in the reaction $^{40}Ca + ^{116}Sn$ calculated in the sudden limit of the macroscopic pairing-rotor model. The cross sections in parts (a) and (b) correspond to center-of-mass energies $E = 120$ MeV and $E = 105$ MeV, respectively. The deformation parameter is $\beta_p = 5$.

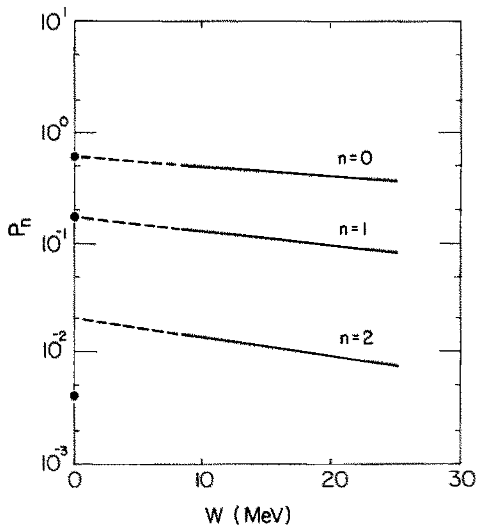

Fig. 2. Probabilities P_n for the transfer of n pairs in the reaction $^{40}Ca + ^{116}Sn$ at a center-of-mass energy $E = 120$ MeV, constructed from the values of the S-matrix element for the near-grazing partial wave $l = 40$. The probabilities were computed for different absorption strengths, as measured by the parameter W in eq. (14) (solid line). The extrapolated values for $W \rightarrow 0$ are compared with the probabilities quoted in ref. [4]) for the same reaction (circles). The deformation parameter is $\beta_p = 18$.

of 2 particles in the $n=2$ amplitude. All such direct and indirect processes are automatically included in the sudden limit approach. Within the macroscopic model, the effective range of the sequential pair transfer is shorter than that of the direct four-particle transfer and thus the latter predominates at very low energies [2,5]. This is borne out by the low-energy cross sections shown in fig. 1b where it is seen that the $n=1$ and the $n=2$ distributions now have the same shape. They are both given by direct excitation mechanisms at this low energy.

The classical and semi-classical probability calculations of ref. [4] were done for the same case considered in fig. 1a but with a larger value of $\beta_p = 18$, no absorptive potential and for one impact parameter corresponding to the grazing partial wave $l=40$. In order to compare to these results we repeated the sudden limit calculations with the larger β_p and made an extrapolation to the no absorption limit. This is illustrated in fig. 2 where the solid curves show the values of $|S_{nl}|^2$ for $l=40$ and different values of the absorption strength parameter W. The dashed-lines show the extrapolation to $W=0$, assuming an exponential behavior. The circles at $W=0$ are the probabilities calculated in ref. [4].

4. Semi-classical limit

It is instructive to study the semi-classical limit of the formalism presented above. The partial wave S-matrix $S_l(\varphi)$ is given in terms of the nuclear phase shift $\delta_l(\varphi)$ as

$$S_l(\varphi) = e^{i2\delta_l(\varphi)} . \tag{15}$$

The simplest semi-classical approximation gives the phase shift in terms of an integral of the nuclear potential along the trajectory corresponding to the partial wave l; namely,

$$2\delta_l(\varphi) = -\frac{1}{\hbar}\int_{-\infty}^{\infty} V_N(r(t), \varphi)\, dt . \tag{16}$$

This expression is valid for low energies or high partial waves such that only the tail of the nuclear interaction is probed. Making use of the exponential decay of V_N and expanding $r(t)$ about the turning point r_0 one obtains

$$2\delta_l(\varphi) = -\frac{1}{\hbar}\sqrt{\frac{2\pi a}{\ddot{r}_0}}\, V_N(r_0, \varphi) \equiv -\frac{\tau}{\hbar} V_N(r_0, \varphi) , \tag{17}$$

where $\ddot{r}_0$ is the radial acceleration at the turning point and τ is the effective collision time. Furthermore, since the φ dependence enters only in the nuclear radius we may write

$$2\delta_l(\varphi) = -\frac{\tau}{\hbar}\, V_N^{(0)}(r_0)\, e^{2\alpha \cos 2\varphi} = 2\delta_l^{(0)}\, e^{2\alpha \cos 2\varphi} , \tag{18}$$

where $V_{\mathrm{N}}^{(0)}$ is the undeformed potential ($\beta_p = 0$), $\delta_l^{(0)}$ is the corresponding phase shift and

$$\alpha = \frac{\beta_p}{3A_2} \frac{R_2}{a}. \tag{19}$$

Thus the transfer S-matrix elements are given by the integral

$$S_{nl} = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{e}^{-i2n\varphi} \exp\left[i2\delta_l^{(0)} \mathrm{e}^{2\alpha\cos 2\varphi}\right] \mathrm{d}\varphi. \tag{20}$$

The classical underpinning of the transfer amplitudes can be seen by evaluating the integral above using the method of steepest descent. Consider the stationary phase condition

$$n = -2\delta_l^{(0)} \mathrm{e}^{2\alpha\cos 2\varphi} 2\alpha \sin 2\varphi \equiv \bar{n}(\varphi). \tag{21}$$

The right-hand side defines the classical number of transferred pairs as a function of the orientation angle in gauge space. This expression is the analogue of the relation between the final transferred spin and the initial orientation angle for the Coulomb excitation of a quadrupole deformed object [9]). To leading order in α eq. (21) reduces to the result for $\bar{n}(\varphi)$ given in ref. [13]).

The number of transferred pairs $\bar{n}(\varphi)$ calculated in ref. [4]) by integrating the classical equations of motion for the hamiltonian of eq. (2) is shown by the solid curve in fig. 3a. The case is the same as the one mentioned above in connection with fig. 2. The corresponding dashed curve is obtained from eq. (21), using eqs. (17), (18) with the same parameters as in ref. [4]). The overall agreement with the dynamical calculations is quite good. The discrepancy around the maximum indicates that the orbits corresponding to the maximum transfer, which feel the nuclear force more strongly, are distorted at the distance of closest approach. To check this, the classical calculations were repeated for a larger impact parameter corresponding to $l = 50$ [14]). The results in this case are in better agreement with the sudden limit, as shown in fig. 3b.

One can continue to use the steepest descent method and demonstrate the interference effects due to the two roots of eq. (21) for classically allowed values of n and the decay of the amplitudes for the classically forbidden ones. The maximum value of $\bar{n}(\varphi)$ is reached when

$$2\alpha \cos 2\varphi_m = 2(\sqrt{1+\alpha^2} - 1) \equiv x_m \tag{22}$$

and has the value

$$\bar{n}(\varphi_m) = -2\delta_l^{(0)} \mathrm{e}^{x_m} \sqrt{x_m}. \tag{23}$$

It should be noted that typical physical values of $2\delta_l^{(0)}$, under the conditions that the semi-classical limit in eq. (20) is valid, have magnitudes of the order of unity or less. In addition, the value of α should be constrained so that the variation in

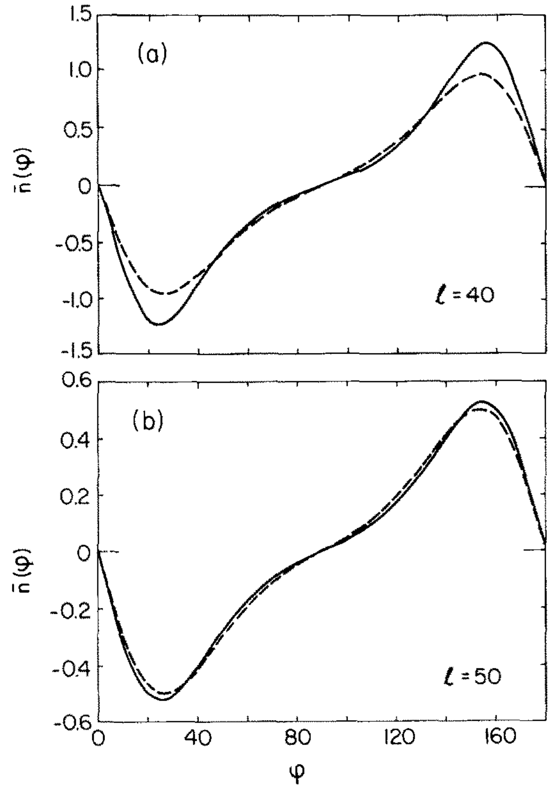

Fig. 3. The function $\bar{n}(\varphi)$ calculated for the reaction $^{40}Ca + ^{116}Sn$ at a center-of-mass energy $E = 120$ MeV. The plots in (a) and (b) correspond to the partial waves $l = 40$ and $l = 50$, respectively. The full-drawn curves were obtained [14]) from classical trajectory calculations, as in ref. [4]). The dashed curves show the sudden limit function given by eq. (21). The deformation parameter is $\beta_p = 18$.

the nuclear radius, according to eq. (1), does not exceed about 10%. This means that usually the one pair transfer will already be classically forbidden.

Since the probabilities for pair transfer are typically small in the limit where eq. (20) is valid, it is appropriate to expand this equation in a power series. Keeping terms to second-order in α one obtains

$$S_{0l} = e^{i2\delta_l^{(0)}}(1 + i2\delta_l^{(0)}\alpha^2 - (2\delta_l^{(0)})^2\alpha^2), \tag{24a}$$

$$S_{1l} = e^{i2\delta_l^{(0)}}\, i2\delta_l^{(0)}\alpha = S_{-1l}, \tag{24b}$$

$$S_{2l} = e^{i2\delta_l^{(0)}}\left(i2\delta_l^{(0)}\frac{\alpha^2}{2} - (2\delta_l^{(0)})^2\frac{\alpha^2}{2}\right) = S_{-2l} \tag{24c}$$

and zero for other values of n. The last expression for $n = 2$ exhibits the direct and sequential components of the amplitude in the first and second terms, respectively. It is a characteristic consequence of the ansatz of eq. (1) that the ratio of these amplitudes is, to leading order, independent of β_p. The condition for their equality

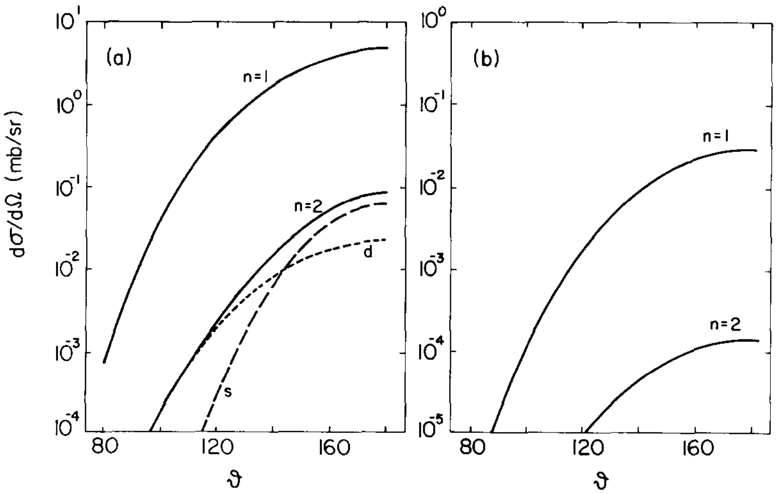

Fig. 4. Estimates for the one- and two-pair transfer cross sections shown in fig. 1 according to the formulas of eqs. (24), (25). The broken lines show the direct (d) and sequential (s) contributions to the two-pair transfer.

is $|V_N^{(0)}(r_0)\tau/\hbar| = 1$, which occurs when the nuclear potential at the turning point is about 1 MeV deep.

The analytic results above may be used to estimate differential cross sections according to

$$\sigma_n(\vartheta) = |S_{nl}|^2 \left(\frac{\eta}{2k}\right)^2 \frac{1}{\sin^4(\frac{1}{2}\vartheta)} \tag{25}$$

where the second factor is the Rutherford cross section and l is related to ϑ by

$$\tan(\tfrac{1}{2}\vartheta) = \eta/l. \tag{26}$$

Using the analytic formulas of eqs. (24), (25) for the case of pure Rutherford trajectories and retaining only the real part of V_N gives the cross sections shown in fig. 4. Comparing them to the numerical results in fig. 1, one sees that the overall magnitudes are qualitatively reproduced. The agreement is best for the lower energy of fig. 4b and, equivalently, at the more forward angles for the higher energy in fig. 4a. The $n = 2$ cross section in fig. 4a is split into its direct (d) and sequential (s) contributions, according to eq. (24c), to show how they dominate in different angular regions.

5. Summary

In this work we have investigated the sudden limit of the pairing-rotor model as it is applied to multi-pair reactions. The main result is that the S-matrix elements for the transfer processes are given in terms of projection integrals of an elastic scattering S-matrix that depends parametrically on the gauge angle. An application to a Ca + Sn collision was carried out to illustrate the technique and also to compare to the resuls of classical calculations. The angular distributions for the two-pair transfer reflect the interference between the direct and sequential amplitudes and show how the direct process dominates as the bombarding energy is lowered below the Coulomb barrier.

At such low energies one can also make a simple semi-classical approximation where the elastic scattering phase shift is proportional to the collision time times the nuclear potential evaluated at the distance of closest approach. This allows one to factorize out the gauge angle dependent part of the phase shift. The transfer S-matrices are then given in terms of integrals of known functions. Evaluating these integrals by the method of steepest descent shows how the pair-transfer probability divides into classically allowed and forbidden regions, in close analogy to the excitation of spin states in an ordinary rotational band. Under typical conditions, however, the transfer of one pair is already classically forbidden at low bombarding energies.

Because of the weak transfer probabilities, one may also evaluate the integrals for the transfer amplitudes using a power series expansion. This yields analytic formulas which are useful for estimating transfer cross sections and for exhibiting the contributions of different order transfer processes.

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