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Recent Results for the Effects of Distortion in the Inter-Cluster Motion in Light Nuclei and Application to Nuclear Astrophysics

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Abstract Deuteron induced quasi-free scattering and reactions have been extensively investigated in the past few decades. This was done not only for the study of the nuclear structure and processes but also for the important astrophysical implication (Trojan Horse Method, THM). In particular the width of the neutron momentum distribution in deuteron will be studied as a function of the transferred momentum. THM applications will also be discussed because the momentum distribution of the spectator particle inside the Trojan horse nucleus is a necessary input for this method. The impact of the width variation on the extraction of the astrophysical S(E)-factor is discussed as well as the relevance of the s and d wave component in the deuteron wave function.

1 Introduction

The study of processes relevant for astrophysics involving light nuclei has considerably increased in the last decades due to the development of indirect methods trying to measure bare nucleus cross sections at astrophysically relevant energies. Among these methods we cite the Asymptotic Normalization Coefficient determination [1] and the THM. The main features of this latter method are extensively discussed elsewhere [2–5]. Basically the THM allows us to measure the bare-nucleus two-body cross sections (or equivalently, the bare-nucleus astrophysical S(E) factors) by means of quasi-free three-body reactions. In the standard THM, the plane wave impulse approximation (PWIA) is used, where the triple differential cross section for the Trojan Horse (TH) reaction

$$A + a \rightarrow s + c + C \tag{1}$$

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is factorized into two parts (see Eq. (2) below). This is done with the help of direct reactions theory assuming that the TH nucleus, a, has a dominant cluster component (sx). In most applications (see [6–12]), this assumption is trivially fulfilled e.g. $d = (pn)$. Particle s is then considered to be spectator to the binary $A + x \rightarrow c + C$ reaction.

In such an approach, the Fourier transform of the bound-state wave function of the relative motion of the fragment s and particle x in the TH nucleus $a = (sx)$ can be eliminated by a simple procedure allowing us to extract the half-off-energy-shell (HOES) binary reaction cross section (see Eq. (2) below) which can be related with the on-energy-shell astrophysical factor. Nevertheless due to the presence of the other particles the momentum distribution of the spectator s can be distorted thus having an impact on the final result.

In the PWIA the cross section of the three body reaction can be factorized into two terms and is given by [13]

$$\frac{d^3\sigma}{dE_c d\Omega_c d\Omega_C} \propto KF \left(\frac{d\sigma}{d\Omega}_{cm}\right)^{HOES} \cdot |\varphi(\mathbf{p_{xs}})|^{\mathbf{2}}, \tag{2}$$

where

- $[(d\sigma/d\Omega)_{cm}]^{HOES}$ is the HOES differential cross section for the two body A(x,c)C reaction at the relative $A - x$ energy $\mathrm{E_{AX}}$ given in post-collision prescription by

  $$\mathrm{E_{Ax}} = \mathrm{E_{c-C}} - \mathrm{Q_{2b}}, \tag{3}$$

  $\mathrm{Q_{2b}}$ is the two body Q-value of the $A + x \rightarrow c + C$ reaction and $\mathrm{E_{c-C}}$ is the relative energy between the outgoing particles c and C,
- KF is a kinematical factor containing the final state phase-space factor and it is a function of the masses, momenta and angles of the particles [14],
- $\varphi(\mathbf{p_{xs}})$ is the Fourier transform of the $a = (xs)$ bound state wave function $\varphi(\mathbf{r})$,

$$\varphi(\mathbf{p_{xs}}) = (2\pi\hbar)^{-3/2} \int e^{-i\mathbf{p_{xs}}\cdot\mathbf{r}/\hbar} \varphi(\mathbf{r}) d\mathbf{r}. \tag{4}$$

and $\mathbf{p_{xs}} = \frac{m_x \mathbf{p_s} - m_s \mathbf{p_x}}{m_s + m_x}$ is the $s - x$ relative momentum, $\mathbf{p_i}$ and m_i are the momentum and mass of the particle i. If $|\varphi(\mathbf{p_{xs}})|^2$ is known and KF is calculated, it is possible to derive $[(d\sigma/d\Omega)_{cm}]^{HOES}$ from a measurement of $d^3\sigma/dE_c d\Omega_c d\Omega_C$ by using Eq. (2).

$$\left(\frac{d\sigma}{d\Omega}\right)^{HOES} \propto \left[\frac{d^3\sigma}{dE_c d\Omega_c d\Omega_C}\right] \cdot [KF|\varphi(\mathbf{p_{xs}})|^2]^{-1} \tag{5}$$

We refer to [15] for further details on the method.

2 Experimental Momentum Distributions

Since the extraction of the bare-nucleus S(E) factor uses the momentum distribution of the spectator cluster inside the TH nucleus, it is important to evaluate the impact of the uncertainty of the momentum distribution width on the final result.

In previous works [16–18] it is explained the experimental procedure for obtaining the experimental momentum distribution of the spectator in the TH nucleus (e.g. for the $p - n$ momentum distribution in ^{2}H). It has also been shown that its full width at half maximum, $W(q_t)$, slowly increases with increasing mean transferred momentum. The transferred momentum from the projectile A to the center-of-mass of the final system $B = C + c$, the latter can be written as the Galilean invariant quantity:

$$\mathbf{q}_t = \left(\frac{m_B}{m_A}\right)^{1/2} \mathbf{p}_A - \left(\frac{m_A}{m_B}\right)^{1/2} \mathbf{p}_B. \tag{6}$$

The $W(q_t)$, extracted from several experiments, is measured in the case of different TH ^{2}H,^{3} He,^{6} Li,^{9} Be at different beam energies and kinematic conditions (see [17,18] for further details). This allows us to explore a wide range of the transferred momentum and made it possible to explore the correlation between the momentum distribution width and the transferred momentum. This is extensively reported in [17,18].

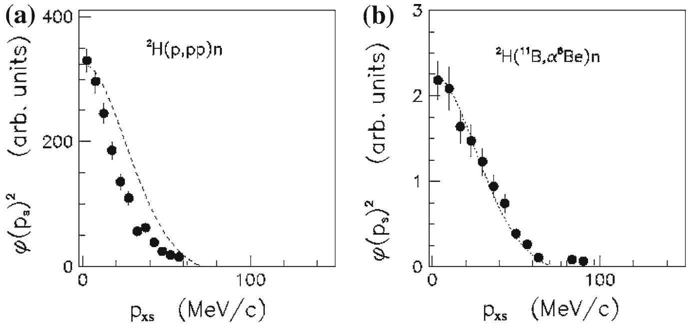

Fig. 1 Experimental momentum distribution for the proton inside deuteron derived according to the method explained in the text for the ^{2}H(p, pp)n, (panel **a**), and ^{2}H(^{11}B, α_0^8Be)n, (panel **b**), reactions. The squared Hulthen function in momentum space is superimposed to data

In this paper we focus on the deuteron case and we compare the experimental momentum distribution of the spectator with the theoretical one at different projectile and ejectile energies and transfer momenta to check whether the PWIA is adequate or not. We will demonstrate that the applicability of the PWIA depends on the kinematical conditions of the experiment and reaction mechanism. We will also verify the importance of including a d-wave component in the momentum distribution.

The experimental results for the neutron-spectator momentum distribution as functions of p_{xs} obtained from the ^{2}H(p, pp)n reaction at the projectile energy $E_0 = 5$ MeV (in the Lab system) [19] are shown in Fig. 1a. In this case $a = d$, $s = n$, $x = p$, $A = p$ and $C = c = p$. The theoretical prediction of the spectator-neutron momentum distribution has been obtained using the Fourier transform of the Hulthen $d = (pn)$ bound-state wave function [20]:

$$\varphi(\mathbf{p_{xs}}) = \frac{1}{\pi}\sqrt{\frac{ab(a+b)}{(a-b)^2}}\left[\frac{1}{a^2 + p_{xs}^2} - \frac{1}{b^2 + p_{xs}^2}\right], \tag{7}$$

with parameters $a = 0.2317$ fm^{-1} and $b = 1.202$ fm^{-1} [20] for the deuteron ground state.

One sees in Fig. 1a that the PWIA is not satisfactory, i.e. the final state interactions $p - n$ are not negligible due to the small relative E_{pn} energies and that the momentum distribution is significatively distorted.

In Fig. 1b similar results are shown for the neutron-spectator momentum distribution from the ^{2}H(^{11}B, α_0^8Be)n reaction at the projectile energy $E_0 = 27$ MeV [15]. In this case $a = d$, $s = n$, $x = p$ and $C = {}^8$Be and $c = \alpha$. Despite the low relative energy E_{Cs} the experimental momentum distribution agrees very well with the theoretical one, i.e. the PWIA works quite well. The momentum distribution shown in Fig. 1b covers the region $p_{xs} < 100$ MeV/c and correspond to the events with the transfer momentum $q_t \approx 210$ MeV/c. High transfer momenta confirm the result [17] that the PWIA works better with increasing of the transfer momentum.

The momentum distribution obtained in the study of the ^{2}H(^{11}B, α_0^8Be)n reaction, is also shown in Figs. 2 as black points with their errors. Fig. 2a shows the experimental TH data compared with the momentum distribution calculated by means of the complete form of the s- and d-wave given in [21]. In Fig. 2b, the same

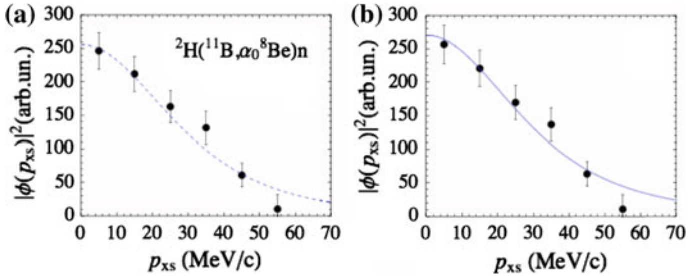

Fig. 2 Experimental neutron momentum distribution (*points*) for the ^{2}H(^{11}B, α_0^8Be)n reaction superimposed onto the theoretical momentum distribution deduced from the exact form given in Eq. (11) (panel **a**, *dashed line*) or obtained from the asymptotic deuteron wave function (panel **b**, *solid line*)

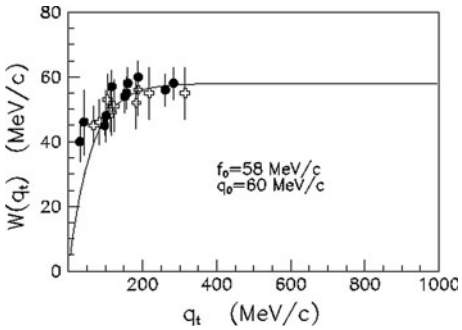

Fig. 3 Width (full width at half maximum) for the momentum distribution of the proton inside deuteron as a function of the transferred momentum q_t. *Open symbols* represent previous results in [18] and references therein, *full dots* are new data from different experiments. The *line* represents an empirical fit described in the text

experimental data are superimposed onto the momentum distribution calculated from the asymptotic form of the s and d waves. It was also shown in [21] that the d-wave contributes to few percent to the overall momentum distribution.

This experimental check ensures that the theoretical momentum distribution weakly depends on the approximation used to deduce it in the momentum range examined here. Moreover, the experimental momentum distribution agrees very well with the theoretical one in the same momentum range. The good agreement between the experimental data and the theoretical function for the p-n motion inside the deuteron represents the experimental evidence that the neutron acted as a "spectator" during the break-up occurred in the ^{2}H(^{11}B,α_0^8Be)n in the examined case.

The experimental momentum distribution full width at half maximum, extracted from several experiments, are reported in Fig. 3 as a function of the transferred momentum, defined above.

Previous data extracted from several studies [18] and references therein are presented as empty crosses while our results from several different experiments and energies are shown as full dots [15,22–25]. Also in this case it is evident how, at low q_t, the W_t smoothly increases until the predicted PWIA asymptotic value (around 60 MeV/c) is reached in the region where q_t is 200 MeV/c. These data strongly confirm the behaviour already discussed in [16,17] for the $\alpha - d$ case in ^{6}Li.

This experimental behavior was fitted by using the following empirical function, as in [17]:

$$W(q_t) = f_0(1 - \exp(-q_t/q_0)) \tag{8}$$

and yields an asymptotic width value of f_0 =58 MeV/c and $q_0 = 60 \pm 12$ MeV/c.

3 Conclusions

What seems clear from this analysis is that as far as the energies of the ejectiles are large enough the momentum distribution of the spectator extracted from the experimental data using the PWIA agrees with the theoretical prediction calculated using the Fourier transform of the bound state wave function of the TH nucleus. We found that the deviation of the theoretical momentum distributions from the experimental ones is correlated with the transfer momentum from the projectile to the ejectiles. When the average transferred momentum is large compared to κ_{xs} the experimental full width at half maximum reaches the theoretical one (the plateau is reached at $q_t \sim (5-6)\kappa_{xs}$). Thus at higher q_t the final-state ejectile-spectator effects are negligible and the reaction is an "ideal" quasi-free reaction. But as soon as the transferred momentum becomes comparable with κ_{xs} the momentum distribution shape changes and its width becomes smaller. This behaviour appears for deuterium in the same way as it was noticed in [18] for other nuclei ^{3}He, ^{6}Li, ^{3}H and ^{9}Be. The application of the THM to these cases should take into account these distortions by adopting the "distorted" full width at half maximum extracted from the experimental data instead of the theoretical one.

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