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# Low-frequency internal friction in clamped-free thin wires

G. Cagnoli [a,b], L. Gammaitoni [a,b,1], J. Kovalik [a], F. Marchesoni [a,c], M. Punturo [a]

[a] *Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, VIRGO Project, I-06100 Perugia, Italy*
[b] *Dipartimento di Fisica, Universita' di Perugia, I-06100 Perugia, Italy*
[c] *Istituto Nazionale di Fisica della Materia, Universita' di Camerino, I-62032 Camerino, Italy*

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**Abstract**

We present a series of internal friction measurements for the normal modes of circular fibres made of different materials, that can suspend the test masses of an interferometric gravity wave detector. For metallic wires, the frequency independent loss angle ranges between $10^{-3}$ and $10^{-4}$. The losses in fused silica are two orders of magnitude lower than those in metals.


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## 1. Introduction

A gravitational wave interferometer, like GEO600 [1], LIGO [2] and VIRGO [3], is an apparatus that is designed to detect the smallest displacement of free masses, which are suspended mechanical pendulums with high quality factors. The quality factor $Q_p$ of an individual pendulum depends mostly upon the internal losses of the suspension wires and the dissipation due to the clamping of the wires. The problem of the wire clamping has already been discussed elsewhere [4]. The internal losses for various wires have already been measured and this paper supplements this previous work [5] ([6–9] for fused silica).

The pendulum $Q_p$ depends on the ratio of energy stored to the energy dissipated in one cycle of the pendulum oscillation [10]. Since most of the energy is stored in the gravitational field which is lossless, while all of the energy is dissipated in the wires, the highest pendulum $Q_p$ occurs when the wires are as thin as possible. Thus, the optimum wire is one that is strong and with a low internal friction.

The quality factor of a mechanical pendulum suspended by $n$ wires is related to the loss angle $\phi_w$ [10] that characterizes the wire material by [11]

$$Q_p = 2l\sqrt{\frac{Mg}{nEI}}\,\frac{1}{\phi_w} \tag{1}$$

where $l$ is the length of the pendulum wire, $M$ is the suspended mass, $E$ is the Young's modulus, and $I = \pi d^4/64$ is the moment of inertia of a wire cross section with diameter $d$.

The results from the experiments presented here show that, at room temperature $T = 300$ K, a good

[1] E-mail: gammaitoni@perugia.infn.it

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model for $\phi_w$ of metallic wires in the frequency range 1–$10^3$Hz is the Zener thermoelastic damping with a frequency independent structure damping term added to it, i.e.,

$$\phi_w(f) = \phi_0 + \Delta \frac{f/f_r}{1 + (f/f_r)^2} \qquad (2)$$

where $\Delta$ is the relaxation strength $\Delta = E\alpha^2 T/c_v$ and $f_r$ is the relaxation frequency $f_r = 2.16\kappa/c_v d^2$. Here, $\alpha$ denotes the linear expansion coefficient, $c_v$ the specific heat and $\kappa$ the thermal conductivity of the wire material [12].

The purpose of this study was to identify materials with low internal friction and to understand their frequency dependence. The mechanical properties of these materials, especially their breaking strength, were also examined in view of possible applications to the present generation of gravitational wave detectors.

## 2. Experimental set-up

In our experiment, the top of each wire with a length of approximately 700 mm was clamped vertically by two pieces of tool steel and then left to hang freely. The normal mode analysis for a thin wire in such a configuration was carried out in Ref. [13]. The clamped-free configuration produces a normal mode spectrum whose first frequency is that of a ‘‘chain’’ that neglects the stiffness of the wire and only has the restoring force of the wire weight

$$f_1 = \frac{1.2}{2\pi}\sqrt{\frac{g}{l}} \qquad (3)$$

and whose higher modes are that of a clamped beam that neglects gravity

$$f_i = \frac{\lambda_i^2}{2\pi l^2}\sqrt{\frac{EI}{m}} \qquad (4)$$

where $m$ is the mass per unit length and $\lambda_i^2 =$ 1.875,4.694,7.855,10.996,14.923 [13].

The motion of the wire was detected by the wire acting as a shadow between a LED and a split photodiode. The signal from each half of the photodiode was subtracted and amplified. The wires were excited by means of an electrostatic actuator that was a copper plate placed a few millimeters from the wire. Voltages of typically one kilovolt were used. The wires were first excited by white noise in order to find the resonant frequencies. The wires were then excited at their resonant frequencies. The driving signal was switched off and the amplitude of the oscillation free decay was recorded by a personal computer. The slope $\gamma$ of the exponential function which describes the decay, is related to the quality factor, $Q_i$, of the mode with frequency $f_i$ by

$$Q_i = \frac{\pi f_i}{\gamma} \qquad (5)$$

The internal friction of the wire is the inverse of its $Q_i$, that is $\phi_w(f_i) = Q_i^{-1}$

Different wire materials were tested: C85 steel with a diameter of 200 $\mu$m from Wagner, Marval (maraging) steel wire (prepared by De Salvo of INFN, Pisa), sapphire with a diameter of 500 $\mu$m from Saphikon, and fused silica with a diameter between 200 $\mu$m–300 $\mu$m (pulled by the Glasgow group of GEO600 from a 3 mm diameter rod from Heraeus). The silicon carbide sample (supplied by Goodfellow) had a diameter of 100 $\mu$m with a 10 $\mu$m tungsten core. The remaining metals: niobium, Invar (Fe64/Ni36), Cu–Be (Cu98/Be2), Chromalloy (Fe75/Cr20/Al5), Aluchrom (Fe70/Cr25/Al5), titanium, and tungsten were all of diameter 250 $\mu$m and ordered from Goodfellow. The metal samples and the sapphire were all clamped in the tool steel. The 3 mm head of the fused quartz sample was clamped between a V-shaped groove and a flat piece of aluminum.

The fused silica, sapphire and silicon carbide all formed a straight wire. The fused silica was pulled from a straight rod and always kept straight. The sapphire and silicon carbide were spooled and might have been affected by the coiling process.

All the metals were delivered on a spool and may have also been affected by being coiled. The softer metals (Marval, Invar, Chromalloy, Aluchrom, titanium) could all be straightened by pulling the wire with 10–50 N of force. The remaining stiffer wires were straightened by weaving them through a grating and then pulling them through. Although these straightening procedures were performed with the least amount of damage to the wire as possible, it is not clear whether this cold working affected the properties of the wire.

All the wires were wiped clean with ethanol before being measured.

## 3. Experimental results

Figs. 1a–l show the results individually for the various materials studied. Fig. 2 shows all the results

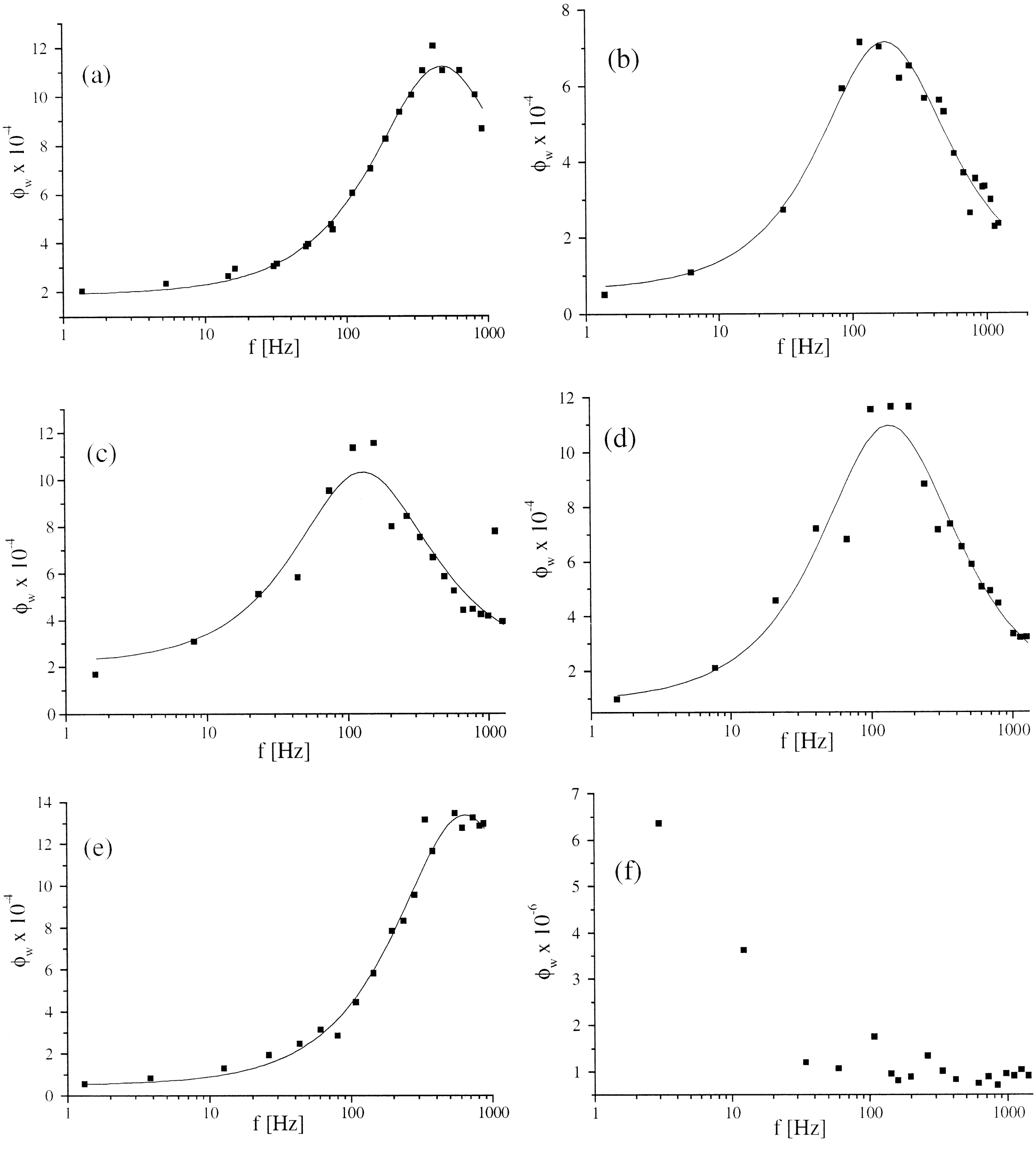


Fig. 1. The internal friction $\phi_w$ versus frequency in thin wires of (a) C85 steel, (b) Marval steel, (c) Aluchrom, (d) Chromalloy, (e) Cu–Be, (f) fused silica, (g) Invar, (h) niobium, (i) sapphire, (j) silicon carbide, (k) titanium, (l) tungsten. The wire diameters, fabrication techniques and/or providers are listed in the text. The curve of Eq. (2) is drawn explicitly when applicable; the fitted values of $\phi_w$, $\Delta$ and $f_r$ are reported in Table 1.

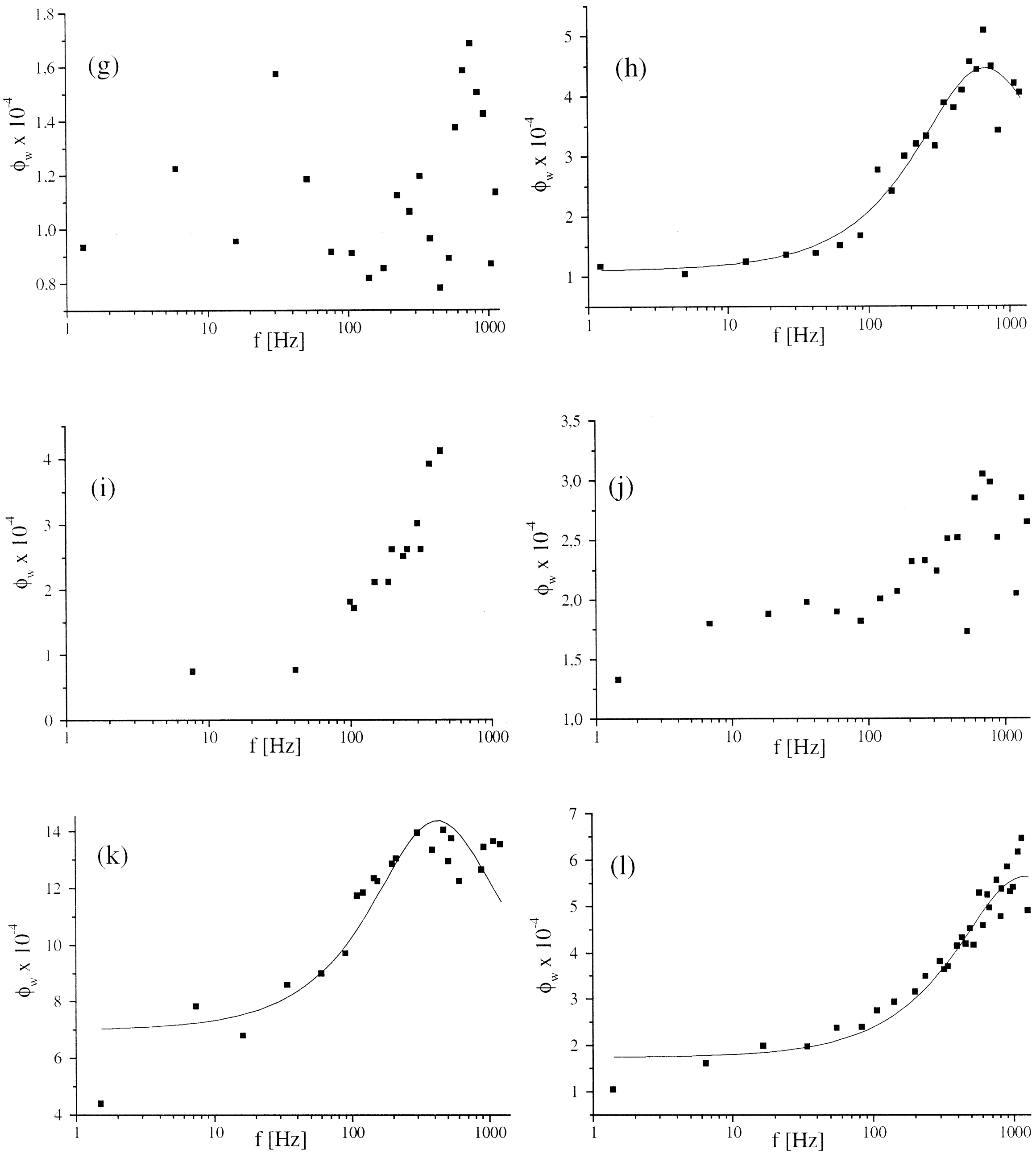


Fig. 1 (continued).

plotted on the same graph. Table 1 lists both the calculated and fitted parameters for the metal samples where the data appears to follow the model. Note that the metal Invar does not follow this model. This is due to the fact that its thermal expansion coefficient, $\alpha$, is an order of magnitude smaller than

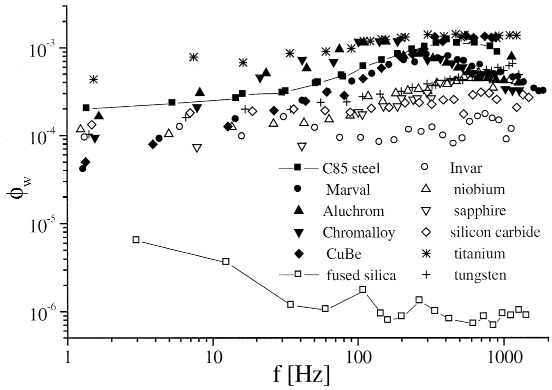


Fig. 2. The internal friction data in Fig. 1 panels (a)–(l) are displayed on the same plot for the sake of comparison.

that of usual metals so that the relaxation strength of the thermoelastic damping (which depends upon $\alpha$ quadratically) is negligible compared to the inherent material loss. The magnitude of the damping strength, $\Delta$, agrees quite well (within 20%) for C85 steel, Aluchrom, titanium and tungsten. For Marval, Cu–Be, Chromalloy and niobium, the measured $\Delta$ is actually smaller than the calculated one, thus suggesting that the parametrization for its relaxation strength used in Eq. (2) may be inadequate.

The relaxation frequencies $f_r$ in general do not agree well with the relevant predictions of Zener's model [12]. Since the relaxation frequency does depend quadratically on the wire diameter, variations in the wire cross-section (especially where the wire is tightly clamped) may have caused this discrepancy.

It is important to note the mechanical properties of the samples. The C85 steel has by far the best yield strength (3 GPa) and handling properties. It has a yield point near 90% of its breaking point and can thus be highly loaded before behaving in an anelastic manner. The Marval steel has a breaking strength that is about 70% that of C85 steel. The other materials have several undesirable properties: titanium, niobium, Invar, Chromalloy and Aluchrom are all very soft and stretch plastically when loaded near 50% of their breaking point; Cu–Be and tungsten are both brittle; tungsten tends to crack when squeezed too tightly, while Cu–Be tends to break when bent too sharply; silicon carbide is also brittle as a thin wire and, finally, sapphire and fused quartz are extremely fragile.

## 4. Discussion and conclusions

In view of the fluctuation-dissipation theorem, the constant low-frequency loss angle $\phi_w \simeq \phi_0$ can be interpreted as an experimental evidence for a mechanical $1/f$ noise in the lattice structure [11]. However, the microscopic mechanisms underlying such a phenomenon prove elusive [14]. Moreover, the very existence of a frequency independent structure damping has been questioned, for instance, by Granato [15]. He pointed out that very long time-scale relaxation processes might affect internal friction experiments at low frequency, so that their outcome is likely to depend critically on the thermal history of the samples and the preparation procedures followed at each stage of the experiment.

Far from embarking on an exhaustive investigation of this class of effects, we nevertheless carried out a whole series of tests to simulate the assembly

Table 1
Internal friction parameters for various samples. The relaxation strengths and frequencies have been extracted from our data by means of the fitting law (2) and compared with Zener's theoretical prediction reported in the text. Note that for both Aluchrom and Chromalloy a Young's Modulus could not be provided by the manufacturer so that a typical value for a metal of $1.5 \times 10^{11} N/\text{m}^2$ was used.

| Material | $\Delta$ theor. | $f_r$ theor. (Hz) | $\phi_0$ expt. | $\Delta$ expt. | $f_r$ expt. (Hz) |
|---|---|---|---|---|---|
| C85 Steel | $2.2 \times 10^{-3}$ | 648 | $1.9 \pm 0.1 \times 10^{-4}$ | $1.9 \pm 0.4 \times 10^{-3}$ | $480 \pm 20$ |
| Marval Steel | $2.2 \times 10^{-3}$ | 648 | $6.0 \pm 0.1 \times 10^{-5}$ | $1.36 \pm 0.04 \times 10^{-3}$ | $340 \pm 10$ |
| Aluchrom | $2.0 \times 10^{-3}$ | 131 | $2.1 \pm 0.6 \times 10^{-4}$ | $1.6 \pm 0.2 \times 10^{-3}$ | $130 \pm 10$ |
| Cu–Be | $3.5 \times 10^{-3}$ | 889 | $5 \pm 3 \times 10^{-5}$ | $2.57 \pm 0.08 \times 10^{-3}$ | $650 \pm 30$ |
| Chromalloy | $3.1 \times 10^{-3}$ | 168 | $9.0 \pm 5 \times 10^{-5}$ | $2.0 \pm 0.1 \times 10^{-3}$ | $135 \pm 8$ |
| Niobium | $1.15 \times 10^{-3}$ | 807 | $1.1 \pm 0.1 \times 10^{-4}$ | $6.7 \pm 0.4 \times 10^{-4}$ | $680 \pm 50$ |
| Titanium | $1.1 \times 10^{-3}$ | 322 | $7 \pm 0.1 \times 10^{-4}$ | $1.5 \pm 0.2 \times 10^{-3}$ | $420 \pm 40$ |
| Tungsten | $7.4 \times 10^{-4}$ | 233 | $1.7 \pm 0.1 \times 10^{-4}$ | $7.8 \pm 0.4 \times 10^{-4}$ | $1200 \pm 100$ |

procedures foreseen by the VIRGO final design [2]. In particular, the adopted degassing procedure requires that the test masses be suspended in vacuo and then backed at 450 K for a week, at least. Eventually, the antenna will operate at room temperature. Specimens of the metallic wires studied in the present report have been loaded up to 65% of their yield point, then subjected to one or more thermal cycles (one week at 450 K each) and, finally, clipped at one damaged endpoint. Internal friction measurements in the clamped-free configuration of Section 2 proved insensitive to the number of thermal cycles, to the load and amount of cold-work at each cycle and to the unavoidable mechanical manipulations. The appearance of a structure $1/f$ noise at low frequencies is thus confirmed to be a rather robust property of the elastic response in pure metals.

While the losses for the metals do follow the phenomenological model Eq. (2) quite well, the actual magnitudes of the parameters calculated from handbook values for the materials do not necessarily agree with the experimentally determined parameters. Since the frequency independent structure damping, $\phi_0$, must be measured by experiment, it is clear that the only way to actually obtain good parameters for Eq. (2) is to fit experimental data in the frequency range of interest. Once this is done, one obtains a good frequency dependent loss function, $\phi_w(f)$, that can be used in calculations for the thermal noise spectrum of a high $Q_p$ pendulum, for example.

It is also clear that all metals have inherently the same order losses ($10^{-3}$–$10^{-4}$) at frequencies on the order of one to a few tens of Hz. Fused silica is a promising material because of its low loss. Some studies [16] suggest that if the surface of a thin fused silica fibre is damage free, it can have a breaking stress better than that of carbon steel. Further research is being carried out to investigate this phenomenon.

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