

# Characterization of Plasma Accelerators with RF Linac Terminology

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## Abstract

The physics of plasma acceleration is described by using rf linac terminology such as shunt impedance, filling time, transit time factor, etc.. It is shown that some differences between conventional rf accelerators and plasma accelerators make it difficult to import the rf linac terminology directly into the new field. For example, the shunt impedance is of limited use and the filling time is no use in wake-field accelerators with single drive beams or single pump pulses. The beatwave accelerator, a driven oscillator system, has in a sense more similarity to rf linacs than wake-field accelerators. It was shown that plasma wave decay due to collisions and modulational instability seriously deteriorate the quality factor.

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## 1 Introduction

Recent efforts have shown that plasma accelerators can have an acceleration gradient exceeding a GeV/m[1]. A further concern, now that the principle of the acceleration has been proven, is to clarify the performance of those accelerators in other parameters than the gradient, e.g. the current, emittance, and energy spread of the accelerating beams, etc. As the final goal of the

plasma accelerator field is the construction a linear accelerator (linac) based on plasma acceleration which can be used for real applications, and thus we must examine the possible performance in rigorous and understandable terms.

Radio-frequency (rf) linacs have a history of roughly 50 years. Their performance estimation and design procedures are well established and described in textbooks[2]. This paper aims to apply the design criteria and terminology of the rf linacs to plasma accelerators. This will enable us to import procedures developed in the field of rf linacs into the design of laser-plasma accelerators, and will also be useful for rf accelerator physicists as a guide to understanding plasma accelerators.

This paper consists of 8 sections. The next section (section 2) itemizes the differences between plasma accelerators and conventional rf linacs. To clarify the discussion, parameters of a laser wake-field accelerator (LWA) and a plasma wake-field accelerator (PWA) are given in section 3. A physical picture of the plasma wave propagation is then given in section 4. We then move the topic to the decay of the plasma wave, which limits the number of bunches one may accelerate. Section 5 describes the decay of plasma waves due to modulational instability and the collisions. The dependence of these phenomena on the plasma temperature and plasma density is given. Section 6 describes beam loading.

Based on these descriptions, we attempt to usefully import some notions familiar to accelerator physicists, *i.e.* the quality factor, the shunt impedance, filling time, transit time factor, etc. in Section 7. The final section contains some remarks on the preceding results.

## 2 Differences between Plasma Accelerators and RF Linacs

In this paper, we will regard plasma accelerators as a type of traveling wave, constant-impedance linacs. It is useful to begin by considering where the differences exist between plasma accelerators and conventional rf linacs.

The first difference is that the plasma accelerators have their driver inside the accelerating structure, while in an rf linac, electromagnetic waves are fed from outside of an accelerating structure. The drivers of the plasma accelerators are either particle beams (PWA) or pump laser beams (LWA and BWA). The notion of the filling time, the time necessary for drive waves to fill the accelerating structure, is not applicable in wake-field accelerators (PWA and LWA), which are impulsively excited in a time short compared to the wave period. On the other hand, a resonantly excited system such as the BWA takes many periods of the wave to excite to saturation. In this sense the BWA is closer

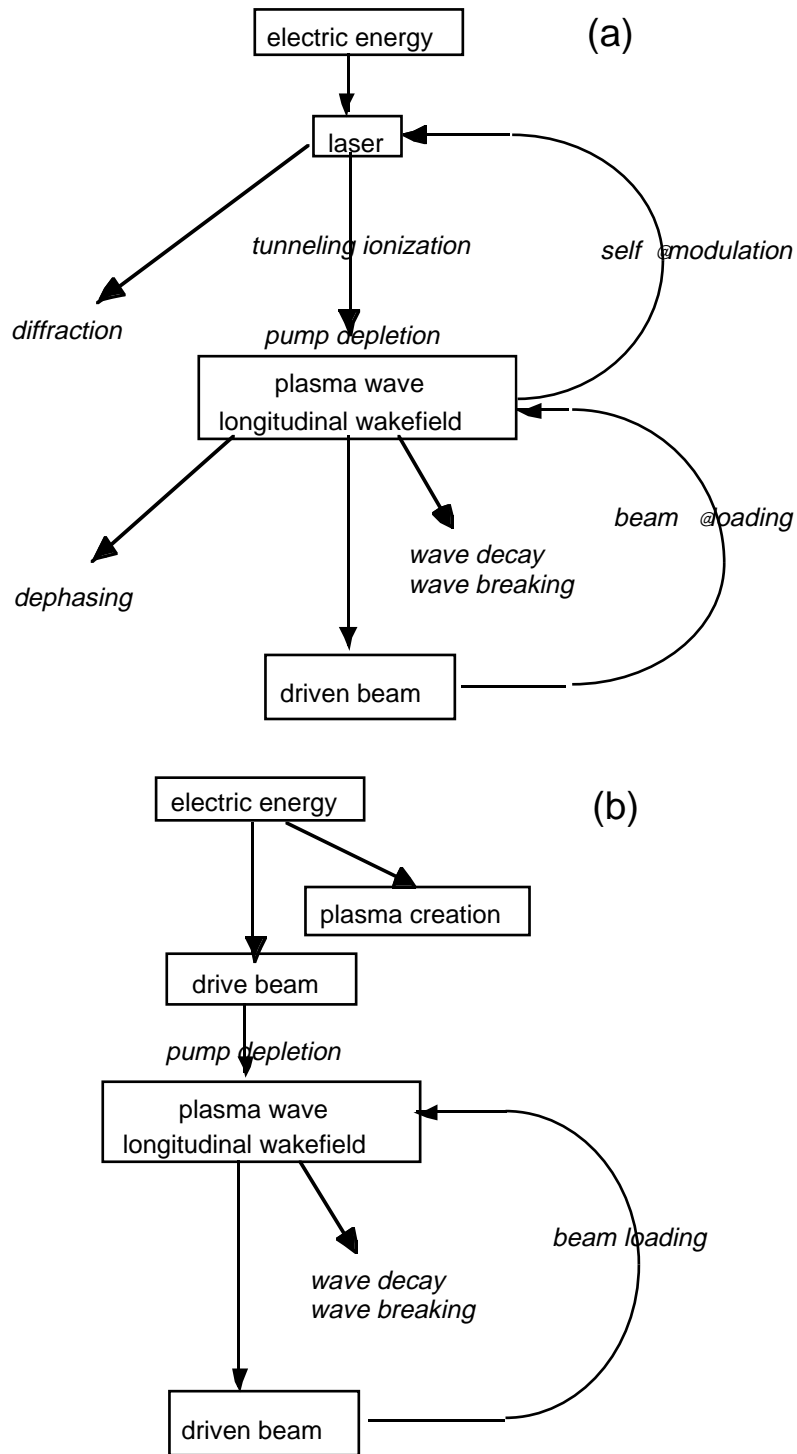


Fig. 1. (a)Processes in an LWA. (b)Processes in a PWA.

to conventional rf linacs than wake-field accelerators. It is useful to consider whether this saturation time corresponds to a filling time of conventional rf linacs.

In order to illuminate this point, it is useful to note that in all cases we are considering, the power being supplied to the accelerating plasma wave is, unlike that of the rf linac, initially of a different form - it is purely electromagnetic, whereas the plasma wave may be dominated by electrostatic components, and is of a different wavelength. Furthermore, this power source is different for the beam-driven acceleration (PWA) and the laser-driven acceleration (LWA and BWA). In the laser-driven accelerations, optical energy is converted into particle-beam energy, a form of wave mode conversion. On the contrary, a PWA excites the plasma wave by a radiative process (plasma wake-fields are referred to in the Russian literature as *Cerenkov emission* of plasmons[4]).

In conventional rf linacs, we have only to consider decay of the rf waves by power absorption to the walls, as the power loss in the wave-guide system is generally negligible. Also, the rf linac phase velocity can be carefully chosen to be synchronous with the beam. Processes involved in plasma acceleration pump transport and wave decay, shown in Fig. 1, are more complicated, however. Fig. 1(a) shows the physical processes involved in an LWA. First, unless the pump laser is guided, it is diffracted, and the diffracting photons can excite plasma waves only weakly. Additionally, an excited plasma wave in turn modulates the laser pulse waveform in self-modulation instability. In the present paper we assume that these effects are avoided in the accelerator design, and proceed to neglect these two processes. After wave excitation, we must consider how much of the wave stored energy can be transferred to the accelerating beam. The relevant processes here are beam loading (accelerating beam wake-field excitation, shown by the second feedback loop in the figure), wave decay, and wave breaking. Moreover, the accelerated driven beam overruns the plasma wave in laser-driven accelerators. This phase slippage dictates the maximum acceleration length of a module, and can be considered as analogous to the transit time factor in an individual rf cavity.

Consideration of the rf cavity brings up one complication that the plasma accelerator does not have but the rf linac does possess - the existence of higher spatial harmonics, or nonsynchronous phase velocity waves, as well as higher order longitudinal modes. Nonsynchronous spatial harmonics do not contribute to the average acceleration of the particle, yet form a portion of the stored energy in the linac. Also, both nonsynchronous spatial harmonics and higher order modes contribute to the beam impedance, or the coupling of the beam charge to its self-wakes.

A further difference between plasma accelerators and traveling wave rf linacs is that the group velocity  $v_g$  of the plasma waves left after wake excitation

is near zero. This means that the energy needed to set up the wave can be considered as "lost" by the driver to the plasma. This resulting power loss could be used to aid in defining, in analogy to the rf linac, a shunt impedance for the drive.

Fig. 1(b) shows the case of a PWA, which is much simpler than Fig. 1(a). Neither diffraction nor phase slippage is typically involved, and the mechanisms of pump energy loss listed on the left of Fig. 1(a) can be safely neglected if proper design work is done, just as in the case of an rf waveguide distribution system. Though tunneling ionization using drive beams is possible under certain conditions[5], it is generally believed that we have to prepare plasmas by some other means in a PWA. The power associated with this process may be important in considering a PWA linac efficiency.

### 3 Parameters of Plasma Accelerators

We now discuss specific plasma accelerator parameter sets to make the discussion concrete, an LWA and a PWA design summarized in Table 1. This table also contains some parameters derived in later sections. In the LWA case, we assume ideal optical guiding, so that the acceleration length is not limited by the laser diffraction. In the parameters of the table, the acceleration length is limited not by the pump depletion but by the phase slippage between the electrons and plasma waves. This is typical of laser-based accelerators in high-density plasmas, but stands in contrast to most rf linacs.

The acceleration gradient of the LWA in the table is based on the linear model[1];

$$eE_{z0} = \frac{2\pi^{1/2}mc^2a_0}{\sigma_{zL} \exp(1)}. \quad (1)$$

Table 1 assumes a nonlinear PWA scheme in a blow-out regime, whose parameters are scaled from those close to those given by simulation as discussed in [3];  $n_b/n_p = 4$  and  $k_p\sigma_r = 0.51$  are able to excite a wake-field  $eE_{z0}/m_e\omega_p c \sim 1$ . The following expression based on a generalized Cerenkov radiation model[4] also gives another good estimation of this wake-field amplitude,

$$eE_{z0} = 4\pi r_e^2 m_e c^2 N_b n_p = \frac{e^2 N_b k_p^2}{4\pi\epsilon_0}. \quad (2)$$

This equation, together with the parameters in Table 1, gives  $eE_{z0} = 2.85\text{GeV/m}$ , which is nearly identical to the linear wave-breaking amplitude  $3.01\text{GeV/m}$ ,

Table 1  
Test parameters of an LWA and a PWA.

driver parameters		LWA	PWA	unit
Laser wavelength	$\lambda_L$	1		$\mu\text{m}$
Driver energy/pulse	$W_D$	1	27	J
FWHM pulse duration	$2.35\sigma_{zL}/c$	83.2		fs
Drive beam bunch length	$\sigma_{zb}$		360	$\mu\text{m}$
Laser channel radius	$r_L = 2\sigma_{rL}$	33.4		$\mu\text{m}$
Drive beam radius	$r_b = 2\sigma_{rb}$		110	$\mu\text{m}$
Laser pulse power	$P_L$	12		TW
Laser Intensity	$I_L$	$3.4 \times 10^{17}$		$\text{Wcm}^{-2}$
Laser strength parameter	$a_0$	0.5		
Drive beam charge	$eN_b$		9	nC
Drive beam energy			3	GeV
Acceleration gradient	$E_{z0}$	15.5	2.25	$\text{GeVm}^{-1}$
plasma parameters				
Density	$n_p$	$10^{18}$	$10^{15}$	$\text{cm}^{-3}$
Plasma frequency	$\omega_p$	$5.6 \times 10^{13}$	$1.8 \times 10^{12}$	$\text{s}^{-1}$
Plasma temperature	$T_e$	100	1	eV
accelerator parameters				
Pump depletion length	$L_{pmp}$	0.55	2.6	m
Dephasing length	$L_{dphs}$	18.6		mm

and close to the PIC simulation value of 2.25 GeV/m. The overestimation of the field given by eq. (2) is due to the effective plasma density being lowered in the process of beam channel rarefaction.

Table 1 has an asymmetry in driver energy per pulse between LWA and PWA. As these parameter sets represent the reasonable design work already made [3],[7], we have not tried to make them symmetrical. This asymmetry is as much due to working with larger dimensions in the PWA case as it does anything else. The linac for this PWA is easily buildable

In the case of the PWA, the driving beam and thus the plasma wave phase ve-

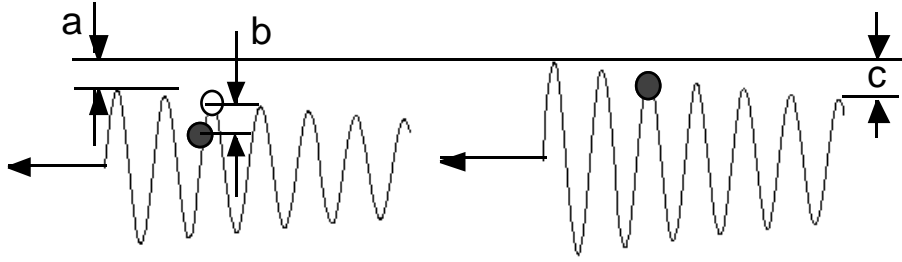


Fig. 2. A plasma wave before (right) and after (left) propagation of certain distance. Amplitude decrease occurs due to (a) pump depletion, (b) dephasing and (c) decay of plasma wave occurs.

locity are ultra-relativistic, as is the accelerating beam. The accelerating phase distribution does not change much and may be chosen to provide a desired combination of high gradient, efficiency, and small energy spread, subject to well-known constraints arising from beam-loading considerations.

The laser energy in the table is already commercially available. Though ideal optical guiding is assumed, this paper clarifies that the optimum acceleration length lies around 20 mm, which is not very far from the experimental results obtained so far[8]. This is the reason why the plasma density assumed in the table is  $10^{18}\text{cm}^{-3}$ . Certainly a less dense plasma eases the pump depletion length and dephasing length, but it instead lowers the acceleration gradient and increases the acceleration length or the guiding length. Reliable techniques for guiding the pump laser pulse a long distance are still under development, but some show considerable promise[9].

Use of hydrogen is assumed in the table. It is assumed that tunnel-ionization creates plasmas in the LWA. Though some data of the plasma temperature created by tunneling are available[10], the conditions are different from those assumed here. The numerical values of plasma temperature in the table have ambiguity.

#### 4 Propagation of Plasma Waves

This section summarizes the evolution of the plasma wave, with an initial concentration on the more complicated case of an LWA. Fig.2 shows an LWA plasma wave before and after propagation of certain distance. The amplitude of the wavefront decreases ("a" of Fig.2), because ionization and wave-excitation consume the laser energy as it proceeds. If the bunches are accelerated to a speed greater than the plasma wave phase velocity, they slip forward from the optimum phase. The effective acceleration rate they obtain decreases, by an amount that is indicated as "b" in Fig.2. The amplitude of the plasma

wave following the wavefront also decays in time, which is shown schematically in Fig.2. The effects of this process are discussed in section 5.

To begin with a simple model for energy and power use in an LWA we assume that the laser or beam energy of the driver is converted to a plasma wave without loss. Also assuming a constant fractional rate of amplitude loss by the driving laser pulse, we have  $E_z(z) = E_{z0} \exp[-z/L_{pmp}]$ . To derive the relationship between the amplitude of the excited wave and this decay, we use an energy balance argument. We begin by writing the wave energy per unit length as

$$W(z) = \frac{1}{2} \epsilon_0 E_z(z)^2 \pi r_w^2 = \frac{1}{2} \epsilon_0 E_{z0}^2 \pi r_w^2 \exp\left[-\frac{2z}{L_{pmp}}\right], \quad (3)$$

where  $r_w$  is the effective wave radius. In the case that the beam is narrower than the plasma skin-depth  $k_p^{-1}$ , it is approximately given by  $r_w \simeq k_p^{-1}$ . If the laser radius in an LWA  $r_L > k_p^{-1}$  or beam radius in a PWA  $r_b > k_p^{-1}$ , then the effective wave radius  $r_w$  takes on the larger value. In this case, however, the accelerating beam, which is usually smaller than the driver, may not efficiently load the wave, as energy may be extracted from the wave over a region  $r < k_p^{-1}$ . For the PWA case we have to even consider this case in the present study, as we are restricting ourselves to the nonlinear blow-out regime, in which case we must have  $r_b > k_p^{-1}$ .

Equating

$$W(z) = \int_0^z P(z') dz',$$

we have

$$L_{pmp} = \frac{V_D}{2 E_{z0}^2} \cdot (\text{LWA}) \quad (4)$$

The parameters given in Table 1 give  $L_{pmp} = 0.546$  m in the LWA case. For the PWA, energy balance arguments do not give a depletion length straightforwardly, since the peak deceleration of the driver is  $\sim eE_{z0}/2$ ,

$$L_{pmp} = \frac{\gamma m_e c^2}{eE_{z0}} \cdot (\text{PWA}) \quad (5)$$

For our PWA example, the depletion length is 2.6m.

The phase velocity of the plasma wave in an LWA and a BWA is equal to the



group velocity of the laser, which is given by

$$\frac{v_p}{c} = \left[ 1 - \left( \frac{\lambda_L}{\lambda_p} \right)^2 \right]^{1/2}. \quad (6)$$

The LWA parameters in Table 1 gives  $v_p/c = 0.996$ . Because electrons are easily accelerated to nearly the speed of light, they can outrun an LWA plasma wave, as shown in Fig. 2. If we assume that the plasma frequency is not changed to provide phase adjustment, the electron acceleration gradient decreases gradually as the particles slip from the optimum phase for acceleration. Though the mechanism is quite different, it causes an effect similar to pump depletion, in that the average acceleration is smaller.

The effective strength of the plasma wave which an accelerating particle surfs is given by  $\cos(\omega_p t - k_p z)$ , where  $z/t = v_p$ . The accelerated particles, on the other hand, approximately satisfy the relation  $z = ct$ , and the phase slippage is constant. The phase of the wave then becomes

$$\omega_p t - k_p z = \left( \frac{1}{c} - \frac{1}{v_p} \right) \omega_p z - \frac{1}{2} \left( \frac{\omega_p}{\omega_L} \right)^2 \frac{\omega_p z}{c}.$$

Effective electric field the particle

$$\begin{aligned} E_{zeff}(z) &= E_z(z) \cos \left[ -\frac{1}{2} \left( \frac{\omega_p}{\omega_L} \right)^2 \frac{\omega_p z}{c} \right] \\ &= E_{z0} \exp \left[ -\frac{z}{L_{pm}} - \frac{1}{2} \left( \frac{\omega_p}{\omega_L} \right)^2 \frac{\omega_p z}{c} \right] \\ &= E_{z0} \exp \left[ -\frac{z}{L_{pm}} - \frac{1}{2} \left( \frac{\lambda_L}{\lambda_p} \right)^2 \frac{\pi z}{\lambda_p} \right]. \end{aligned} \quad (7)$$

The gradient becomes zero when the phase  $(\lambda_L/\lambda_p)^2(\pi z/\lambda_p) = \pi/2$ . If we define the dephasing length  $L_{dp}$  by this relation, it is

$$L_{dp} = \frac{\lambda_p}{2} \left( \frac{\lambda_p}{\lambda_L} \right)^2. \quad (8)$$

This limit is 18.6 mm in the present parameters. In the case of the "full" surf (a half wave of slippage, and including pump depletion), the average acceleration in a section of LWA is about half times the peak acceleration.

The energy gain is shown as a function of acceleration length for various  $L_{pm}$  values in Fig. 3. It has a maximum at  $(\lambda_p/2)(\lambda_p/\lambda_L)^2$ , the dephasing length

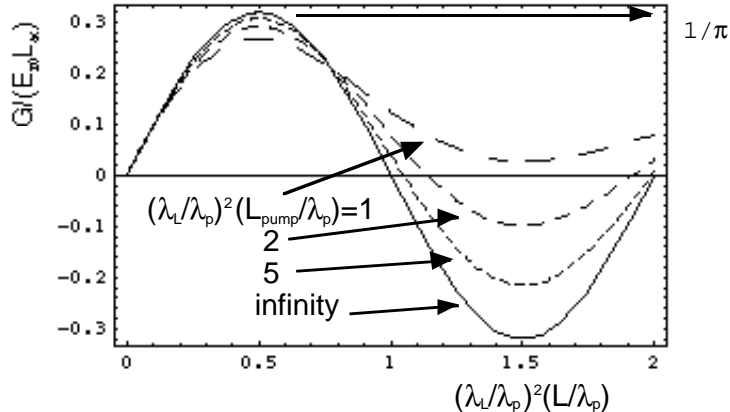


Fig. 3. Energy gain of an LWA as a function of acceleration length for various  $L_{pmp}$  values.

defined above. The parameters in Table 1 gives the energy gain of 183MeV at  $L_{dphs} = 18.6$  mm for the LWA. It should be noted that in the LWA we must take into account the depletion of the pump in calculating the acceleration gradient along the wave axis, while in the nonlinear PWA we do not, as the gradient is strongly dependent only on the charge in the beam, not the energy of the beam, until the energy is nearly fully depleted.

Our view of phase slippage also ignores possible dynamical changes in the driving laser beam, in which the pulse changes its spatio-temporal characteristics by self-focusing, erosion, or other higher order effects. These effects, like the emittance-driven erosion of the driving electron beam head in the PWA, must also be considered in the final analysis of a plasma accelerator design.

## 5 Plasma Wave Decay

The decay of the plasma wave has as yet been neglected in this treatment. We can identify at least three decay processes: collisional damping, modulational instability and Landau damping. Among them, the Landau damping occurs when the plasma electrons have drift velocities near the plasma wave velocity. This can be serious when the plasma electrons are trapped and accelerated to near wave-breaking, as is the case in the experiments using lasers with powers exceeding 20TW[11]. The LILAC, Laser-Injection Laser Accelerator, may also display this behavior[12]. In this regard, if we change our point of view, it is possible to consider the plasma wave decay due to the beam loading as a generalized the Landau damping[13]. It will be discussed in section 7, and thus unnecessary to discuss further this damping here.

The modulational instability has been studied by French group both theoret-

ically[14] and experimentally[15]. In the strong field  $v_L/v_{te} > (\omega_{pe}/\omega_{pi})^{1/2}$  as in our case, the decay constant is given by[14]

$$\gamma_{mod} = \left(\frac{3}{2}\right)^{1/4} \omega_{pi} \left(\frac{\omega_{pe}v_{te}}{\omega_{pi}v_L}\right)^{1/2}, \quad (9)$$

where

$$\omega_{pi} = \left(\frac{Z^2 n_i e^2}{m_i \epsilon_0}\right)^{1/2}, v_{te} = \left(\frac{k_B T_e}{m_e}\right)^{1/2}.$$

We employ the practical expression of F. Cheneau in order to derive the collisional decay constant. It is given by  $\gamma_{col} = \frac{1}{2} \nu_{ie}$ , where  $\nu_{ie} \sim \nu_{ee} Z/2$ , and

$$\nu_{ee}[\text{s}^{-1}] = 5.0 \times 10^{-6} n_p \frac{\Lambda}{1.5}. \quad (10)$$

Fig.4 shows the plasma density dependence of these two decay constants under the assumption  $\ln \Lambda = 20$  and  $v_L = 0$ . In the cases of three plasma temperatures. The decay constant due to the collisional instability is an increasing function of the electron temperature, while the collisional decay constant is a decreasing function. Which decay mechanism is dominant depends on plasma density and temperature.

The analyses above apply only to the linear regime. It is, however, likely that the wave decay in an LWA is more significant than in a PWA which is operated in a low-temperature, low-density plasma. When the temperature exceeds 10 eV in an LWA, the decay due to the modulational instability is dominant. The calculated decay constant due to the modulational instability is  $\gamma_{mod} = 2.00 \times 10^{12} \text{s}^{-1}$  at  $T_e = 10 \text{eV}$  and  $\gamma_{mod} = 3.00 \times 10^{12} \text{s}^{-1}$  at  $T_e = 100 \text{eV}$  in a plasma with  $n_p = 10^{18} \text{cm}^{-3}$  of table 1. The laser wavelength is  $9 \mu\text{m}$  and  $84 \mu\text{m}$  in length, respectively, and 500 fs and 280 fs in time. The plasma oscillation period is 111 fs in our parameters. In the LWA experiments, however, the plasma wave decay constant is much smaller than the theoretical prediction in this section[8]. This is almost certainly because the velocity of the electron quiver in the laser and wake-field is much larger than the thermal velocity in the plasma[17]. This condition must lower the collision frequency, violates the assumptions[16] made in the derivation of  $\gamma_{col}$ . More study of this issue is necessary.

Because of the plasma wave decay, it is difficult for an LWA to produce multiple bunches with homogeneous energies, even if the beam loading is absent. A BWA, which is essentially a free-running oscillator akin to an rf cavity is free from this problem. The solution for this problem in the wake-field accelerator case is then to use multiple drivers, albeit only for linear wave

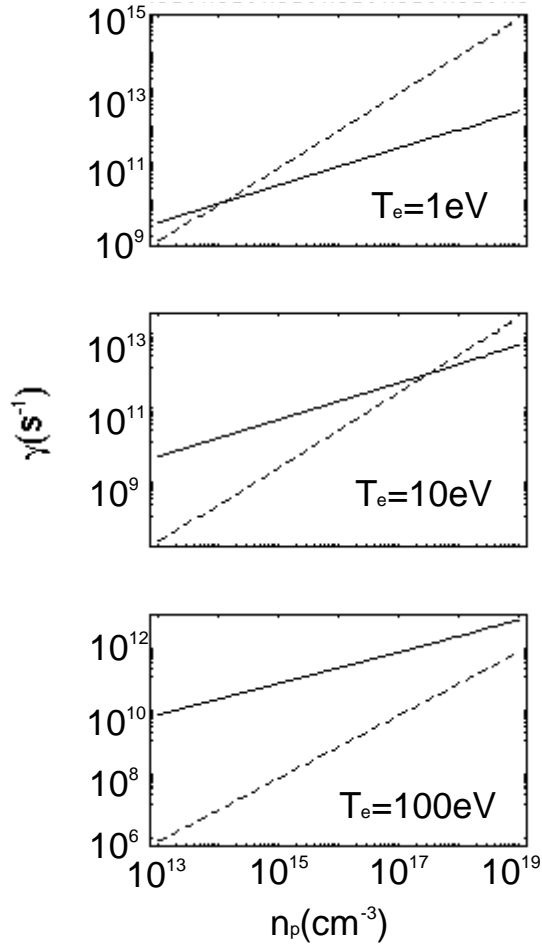


Fig. 4. Decay constant of plasma wave due to the modulational instability (solid lines) and collisions (dashed lines) as a function of plasma density and plasma temperature.

amplitudes. Optimization of the driver waveforms and amplitudes of these multiple drivers in the wake-field accelerators have been discussed[18],[19].

## 6 Beam Loading

Taking the beam loading into account, we write the electric field of an rf linac as  $E_{z\text{eff}}(z) = (E_z - IZ_s)(1 - \exp[-z/L_{pmp}])$ . The term  $IZ_s$  on the right of this expression gives the beam loading. In plasma accelerators, we have not yet clarified the definition of the shunt impedance  $Z_s$ , so we will not use this parameter in this section. This problem of defining the shunt impedance will be discussed in the next section, after the mechanism of beam loading is examined.

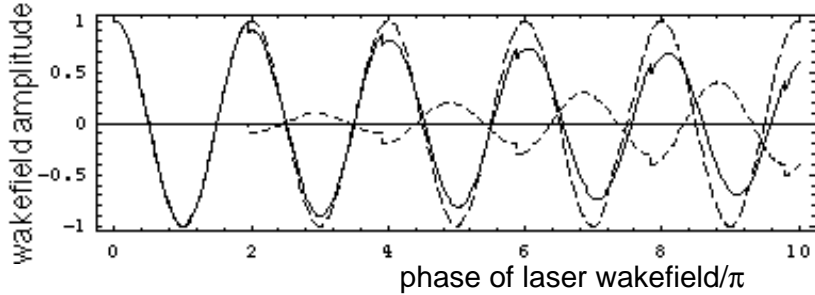


Fig. 5. Wake-field amplitude in the LWA as a function of the phase of the laser wake-field. Dashed line: amplitude in the absence of beam loading, dotted line: wake-field excited by driven beams and solid line: total wake-field.

Fig. 5 shows wake-fields in an LWA; the fields caused by the laser, and that caused by driven bunches and their total resultant longitudinal field. Because the driven beam has the light velocity, the plasma wave excited by the beam also has the light velocity, which is faster than the phase velocity of the plasma wave excited by the laser, in spite of the fact that the plasma frequencies are same in two waves. The negative wake-fields of preceding bunches build up in-phase on the following bunches. One-dimensional simulation of the beam loading taking account of this fact is found in ref.[7]. This effect has no analogue in rf linacs, where the excitation due to the beams is of a single phase velocity at a given location in the device. Likewise, in a PWA, since both the driver and accelerating beam are ultra-relativistic, the fields of driver and driven beams are in phase.

Beam loading calculations have been already given by Katsouleas[20] in the case of linear plasma waves, and Rosenzweig[6] in the case of nonlinear plasma waves. The linear theory predicts that amplitude of the decelerating field caused by the beam loading of  $N$  particles should be equal to the plasma wake-fields induced by the  $N$  particles in its absolute value;

$$eE_{z0} = \frac{8r_e m_e c^2 N}{r_b^2} \left[ 1 - \frac{4}{k_p^2 r_b^2} + 2K_2(k_p r_b) \right] \cos(\omega_p t - cz), \quad (11)$$

where  $r_b$  denotes the beam radius. This equation ignores the functional dependence of the beam-loading field on the radial position, an effect which is strong in linear-regime plasma accelerators, and nearly non-existent in rf linacs.

In the nonlinear blowout regime, which is much more analogous to the rf linac from the view-point of the form of the fields, the beam-loading longitudinal fields are in fact independent of  $r$  inside of the plasma electron rarefaction volume. In this case, the beam loading amplitude is given by eq. (11), by substitution of  $r_b = 2^{-3/2} k_p^{-1}$ , as suggested by eq. (2).

These wake-fields should be much weaker than that excited by the driver,

and are added by linearly superposition to these main accelerating fields. This calculation has been performed to find optimum phases for accelerating, with maximum average gradient and minimum energy spread, analytically for the linear regime[20] and computational for the more complicated case of the non-linear regime[6]. In the case of linear waves, as well one-dimensional nonlinear waves[22], a useful relationship has been deduced between the efficiency  $\eta$  of wave energy extraction for a single monoenergetic bunch and the fraction of the maximum available field  $\eta$  as  $\eta = 1 - \epsilon^2$ . Thus for 20 % energy extraction, the beam can load at 96% of the maximum acceleration available.

In rf linacs with continuous rf excitation (filling), it is common to use a pulse train to effectively load the structure, in that total power due to wall losses can be made small (especially in the case of superconducting linacs) compared to the accelerated beam power. For wake-field accelerators with impulsive excitation, this is not possible. For bunch trains spaced at integer multiples of the plasma wavelength, the average energy gain and energy spread would differ from bunch to bunch. It is possible, however, to accelerate bunches of equal energy gain if the bunch spacing is chosen to be slightly different than this, to choose the phase with a higher fraction of the maximum acceleration in the wave[20]. In this case it is only possible to make the relative energy spread of the bunches the same if the bunch charge is also varied along the train.

## 7 Analogies to Linac Parameters

### 7.1 Quality factor

There are many equivalent possible ways of defining the  $Q$ , the quality factor of a resonant system: through the width of resonance, consideration of stored energy and power loss, the transient approach to equilibrium, and the transient decay of the excitation. As we are interested in non-steady state systems in the time domain, we concentrate here on the decay-based definition. Whatever the linear plasma acceleration scheme is, the evolution of the electric field associated with the linear plasma wave is expressed by the second-order differential equation;

$$\frac{d^2 E_z}{dt^2} + \gamma_w \frac{dE_z}{dt} + E_z \omega_p^2 = f(t), \quad (12)$$

where  $\gamma_w$  is the sum of all of the wave decay constants. The driving term  $f(t)$  on the right hand side of this expression generally describes the wave excitation mechanism, which may be PWA, LWA and BWA.

In the absence of the driving term, the solution of the equation is a decaying exponential with a decay constant, which defines  $Q$

$$Q = \frac{\omega_p}{2\gamma_w}, \quad (13)$$

from the coefficients of eq.(12). This is independent of the exciter. The  $Q$  value of a plasma with  $T_e=100\text{eV}$  and  $n_p = 10^{18}\text{cm}^{-3}$ , the parameters for the LWA in table 1, is only 7.89. This value is surprisingly smaller than those of standard S-band rf linacs, where  $Q \sim 13,000$ [21], but we must recall that our estimate of the decay constant errors is on the high side, by perhaps a large factor.

The definition of  $Q$  given in eq. (13) has a factor of 1/2 in it which is due to consideration of the wave amplitude decay. If one considers the stored energy  $U$  (proportional to the amplitude squared), then the decay constant for governing power loss  $P$  is  $\omega_p/\gamma_w$ , and we obtain the usual stored energy-based definition,  $Q = \omega_p U/P$ .

In the case of the PWA given in Table 1, blowout is assumed. Therefore the wave breaks after one-half of an oscillation, thus rendering the definition of  $Q$ , which assumes slow decay, meaningless. A vivid analogy may be drawn to a pathological mode of running an rf linac - the cavity suffering a catastrophic breakdown after every rf fill.

Wave decay, analogous to the wall losses in rf cavities, is not the only one source of wave power loss, however. There is also the problem of beam loading losses, which as we have already noted is also a familiar effect in multi-bunch train operation of rf linacs. Often these two losses are lumped, and the  $Q$  values defined by each loss mechanism,  $Q_w$  and  $Q_{BL}$ , added in reciprocal to give a quality factor in the beam-loaded case,

$$\frac{1}{Q} = \frac{1}{Q_w} + \frac{1}{Q_{BL}}. \quad (14)$$

The "filling" is the inverse process of wave decay. However, we cannot observe "filling" in wake-field accelerators consisting of a single driver and driven beams. In the case of an LWA with multiple drivers (or driver train), the acceleration gradient is decided by balance between plasma wave decay and excitation by drivers. The finite time which is necessary for the plasma wave to reach its equilibrium (by assumption, not nonlinear saturation) is identical to the decay time. In the case of a BWA, one must look more carefully at the dynamics of resonant excitation, which are also analogous to the standing wave accelerator cavity behavior, but differ in that they entail mode conversion. This subject is outside of the scope of the present paper.

## 7.2 Impedance

Impedance is defined as the transfer function from current to voltage. The PWA converts drive beam current to voltage per unit length so its wake amplitude defines the impedance. Let us now review the physical definition of the wake-fields and impedance in beam physics[23]. Consider two point charges traveling on the  $z$  axis at constant velocity  $c$ . The trailing charge being at a distance  $s = c\tau$  behind the leading one with charge  $q$ . We define the  $\delta$ -function longitudinal wake potential, the potential per unit charge seen by the trailing charge over an interaction length  $L$

$$W(\tau) = -\frac{1}{q} \int_0^L E_z[z = c(t - \tau)] dz. \quad (15)$$

Note that  $W$  has a unit  $V/C = V/(As)$  that of impedance per unit length. The coupling impedance with the beam physics is defined by the Fourier transform of the wake potential over the variable  $\tau$ ,

$$Z(\omega) = \int_{-\infty}^{\infty} W(\tau) \exp(-i\omega\tau) d\tau. \quad (16)$$

and conversely:

$$W(\tau) = \int_{-\infty}^{\infty} Z(\omega) \exp(i\omega\tau) d\omega \quad (17)$$

If the line density of the charge distribution is  $\lambda(\tau)$ , the longitudinal potential is

$$V_z(\tau) = \int_{-\infty}^{\infty} \lambda(\tau') W_z(\tau - \tau'). \quad (18)$$

The total energy loss from a driving charge to the wakefield is then given by

$$\Delta U = \int_{-\infty}^{\infty} I(\tau) V_z(\tau) d\tau = \int_{-\infty}^{\infty} d\tau I(\tau) V_z(\tau), \quad (19)$$



where  $I(\tau) = c\lambda(\tau)$  is the current flow. The total loss factor is defined by

$$k_\ell = \frac{\Delta U}{q^2}. \quad (20)$$

In an rf structure, the energy extracted from the leading bunch goes into a set of resonant modes of the structure and is not fully available for the acceleration of the trailing bunch. To the contrary, one expects that in a single mode, the oscillation of the plasma frequency  $\omega_p$ , will be excited in a plasma accelerator and the wake function of eq.(15) is simply given by  $w(\tau) = W_0 \cos(\omega_p \tau)$ . If we consider a bunch distribution of the simple rectangular shape with width  $T$ , the loss factor of eq.(20) becomes

$$k_\ell = \frac{W_0}{2} \left( \frac{\sin(\omega_p T)}{\omega_p T} \right)^2 \quad (21)$$

which reduces to

$$k_0 = \lim_{T \rightarrow 0} k_\ell$$

in the limit of zero pulse width. This is the standard result - the average bunch energy loss for a beam short compared to the relevant mode wavelength is a factor of 2 smaller than the amplitude of the wake-field behind the bunch.

In a standard rf cavity the total loss factor is related to the impedance  $(Z/Q)_n$  of the  $n$ -th resonant modes of the cavity  $\omega_n$  by the relation  $k_\ell = \sum_n k_{\ell n} = \sum_n (\omega_n/4)(Z/Q)_n$ . For any loading (including a generating beam) in a plasma wave, the total loss factor definition reduces to a single mode and allows the computation of the plasma impedance (or the beam impedance from the accelerator physics viewpoint) from the wake potential of the drive beam as

$$Z_b = \frac{Z_0 [1 - (v_g/c)]}{2} \frac{2k_0}{\omega_p} = \frac{W_0}{\omega_p} = \frac{E_{z0} L}{q\omega_p}. \quad (22)$$

The factor  $[1 - (v_g/c)]$  in the numerator of the right hand side, which is near unity in plasma waves, is due to the possibility that wave energy may be partially catching up to the beam as it traverses the plasma. The beam gives up less energy per unit length to set up its wake-field in this case, and the impedance goes down accordingly.

Combining Eqs. (22) and (21) we have the impedance

$$Z_b = \frac{E_{z0} L}{q\gamma_w}, \quad (23)$$

which is called shunt impedance in linac textbooks. This expression is of general validity, but is not useful because we have not specified the dependence of  $E_{z0}$  on plasma parameters. In this regard, it should be noted that beam wake-fields in the wide-beam regime,  $k_p r_b > 1$ , depend explicitly on the transverse dimensions of the beam. Because of this, one cannot rigorously use the definition of the structure (shunt or beam) impedance. It is in fact simpler to define the impedance in the blow-out regime, or any case where  $k_p r_b < 1$ , because the wake-fields are independent of  $r_b$ , and depend only on the plasma parameters.

For the blow-out regime, there is one complication in this argument, that the wave response is nonlinear, and so the effective plasma parameters are dependent on the amplitude driving beam current. This amplitude dependence is fairly weak, however, as was discussed in relation of eq. (2). Making use of eq. (2) to relate  $E_{z0}$  to the plasma frequency, we have an expression for the acceleration gradient (voltage per unit length) left by a beam short and narrow compared to plasma skin-depth to be proportional to the current  $I = q\omega_p = eN_b\omega_p$ . Thus the impedance per unit length that an electron beam beam sees in a plasma is

$$Z'_b = \frac{E_{z0}}{eN_b\omega_p} \sim \frac{k_p}{4\pi\epsilon_0 c}. \quad (24)$$

The parameters of the PWA in table 1 give  $Z'_b = 138 \text{ k}\Omega/\text{m}$ . Of course, use of the term "beam impedance" in this case is much more relevant than "shunt impedance", because the blow-out wave exists for only half of an oscillation in the nonlinear PWA. It should also be noted that these results can be approximately applied to the case where  $k_p r_b > 1$ , if one substitutes  $k_p = 2^{3/2}/r_b$ , to obtain the nonstandard shunt impedance which depends on beam radius.

The discussion of the shunt impedance in the case of the LWA is not relevant, unless a pulse train is used to excite the plasma (which is basically the physical situation of the BWA) as purpose of the shunt impedance is to allow calculation of the equilibrium field due to resonant excitation. Wake-field accelerators by the shock excitation of the system, which is never in equilibrium!

### 7.3 Phase slippage and transit-time factor

If the velocity of the plasma wave is different from the velocity of driven bunches, as is the case of laser-driven accelerators, the bunches slip in phase with respect to the plasma wave. The total phase slip in length  $L_{ac}$  for an electron bunch with velocity  $\sim c$  is described by the parameter[24]

$$\delta = \omega_p L_{ac} (1/v_p - 1/c). \quad (25)$$

If we can approximate that the acceleration gradient  $E_{z0}$  is constant and if  $\delta \ll 1$ , we expect the acceleration field as

$$E_z = E_{z0} \frac{\sin(\delta/2)}{\delta/2} = ET, \quad (26)$$

where  $T = \sin(\delta/2)/(\delta/2)$  has a similar form of the transit-time factor in an Alvarez linac. Using this parameter, we can define the effective transit-time factor  $Z_{eff} = T^2 Z_0$ . In the simple, uniform slippage case of the LWA, the square of the transit time factor is obviously the average of  $\cos^2$  over a half period,  $T^2 = 0.5$ . This is the identical result as is obtained in the case of a pure fundamental spatial harmonic rf cavity system. For shorter LWA sections, the transit time factor will be generally larger, as less time is spent in low acceleration phases. Note that this definition will yield a transit time factor which is typically close to unity in the PWA case, and in the case where the density is varied to match the phase of the wave to the accelerated particles in the LWA.

## 8 Remarks

The introduction of linac terminology in the previous section has some utility, not the least of which is to conceptually link the traditional approach to accelerators to the new technology of plasma-based devices. There are a few final points worth discussing in regards to the success of this exercise. The first is that the shunt impedance is of limited use in discussing plasma wake-field accelerators, for the many reasons listed above. The BWA could be usefully described in this way in principle, but in practice, the beatwave is a driven oscillator system in which saturated equilibrium is not determined by losses, but by nonlinear detuning of the resonance. Analogous effects in rf linacs are more rarely encountered, but we could include in this category the phenomena of Lorentz force detuning in superconducting cavities, and dark-current beam loading. It should be reemphasized that the shunt impedance definition is dependent in turn on an accurate determination of the  $Q$  of the system, which for the large amplitude plasma motion considered here is not yet possible.

The concept of the beam impedance, on the other hand, was readily generalizable to the case of plasma accelerators. This impedance allows quick comparison of plasma to rf structures, as well as determination of wave amplitudes driven by PWA drivers, beam loading and efficiency of energy extraction by accelerating beams from the plasma wave, which are related to the number of driven bunches acceleratable. The excitation of plasma waves by the drive beam in the PWA causes energy loss (self-beam loading), with analogous

mechanism in the LWA being pump depletion. Because the LWA is neither resonant, nor based on current, this excitation is not described well by either beam or shunt impedance concepts. Despite this, pump depletion is well understood, so that the accelerating field excited in an LWA can easily be predicted for design purposes.

Single drive pulse wake-field accelerators are very efficient (near unity) at exciting waves, with losses coming into consideration only in the discussion of multibunch train acceleration. While acceleration of bunch trains may be desirable from the point of view of mitigating the beam-beam interaction in linear colliders, in this type of accelerator the efficiency is not enhanced. Thus a major motivation for bunch train acceleration in rf linear colliders is lost. Nevertheless, single bunch beam-loading efficiencies are not too small, with at least 20% easily contemplated. These are not the only efficiency factor one needs to consider in collider design, however[25]. There is also the efficiency from wall-plug electric power to either laser pulses (in a BWA and an LWA) or drive beams (in a PWA).

The wall plug efficiency of  $T^3$  lasers used in an LWA is less than  $10^{-4}$  at present. This is small enough to discourage LWA collider design, but may be significantly improved in the future. The PWA has a much higher efficiency than laser accelerators at the present, because of the possibility of obtaining wall-plug efficiencies in the 40% range using a heavily beam-loaded multi-bunch drive rf linac([3]).

The present paper is the first to attempt discussion of the plasma wave decay from the viewpoint of the accelerator design, which is serious to obtain multiple bunches with homogeneous energy. Although use of the multi-pulse drivers has hitherto been proposed only from the viewpoint to get high acceleration gradient, it is also rather useful to get the homogeneous beam energy.

Previous experiment have attained 300MeV energy gain in less than 20 mm using a 2TW laser[8]. The discussion in Section 6 of this paper indicates a calculated acceleration length of less than 20 mm, but, also predicts, even using a laser with power exceeding 10 TW, an optimum energy gain of around 170MeV. Thus we have a mixed performance in our ability to perform accurate predictions in the proof-of-principle experiments[8]. Improvement of the calculations and terminology given in this paper will clearly be dependent on a careful understanding of experiments, as well as analytical theory and simulations.

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