Baryon Form Factors at threshold

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Scattering and annihilation electromagnetic processes

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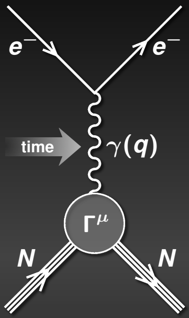
Agenda

- Form Factors: definitions, formulae
- Proton data and Asymptopia
- A dispersive sum rule for asymptotic behaviors
- ISR technique and threshold behaviors in $p\bar{p}$ and neutral channels

Form Factors: definitions, formulae and other facts

Baryon Form Factors definition



- **Electromagnetic current** $(q = p' - p)$

$$j^\mu = \langle N(p')|J^\mu(0)|N(p)\rangle = e\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2(q^2)\right]u(p)$$

- **Dirac and Pauli form factors F_1 and F_2 are real**
- **In the Breit frame**

$$\begin{cases} p = (E, -\vec{q}/2) \\ p' = (E, \vec{q}/2) \\ q = (0, \vec{q}) \end{cases} \qquad \begin{cases} \rho_q = j^0 = e\left[F_1 + \frac{q^2}{4M^2}F_2\right] \\ \vec{j}_q = e\,\bar{u}(p')\vec{\gamma}u(p)\,[F_1 + F_2] \end{cases}$$

- Total charge conservation in the limit $\vec{p}' \to \vec{p}$: $\langle N(p)|J^\mu(0)|N(p)\rangle = e\,F_1(0)$
- Let $\vec{B} = \vec{\nabla}\times\vec{A}$, in the limit $\vec{p}' \to \vec{p}$: $\langle N|\int d^3x\,\vec{S}\cdot\vec{A}|N\rangle = \frac{e}{M}\,[F_1(0)+F_2(0)]\,\vec{S}\cdot\vec{B}$

Sachs form factors

$$G_E = F_1 + \frac{q^2}{4M^2}F_2$$

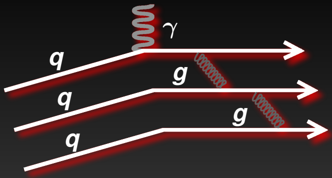
$$G_M = F_1 + F_2$$

Normalizations

$$F_1(0) = Q_N \qquad G_E(0) = Q_N$$

$$F_2(0) = \kappa_N \qquad G_M(0) = \mu_N$$

pQCD asymptotic behavior



- **pQCD**: as $q^2 \to -\infty$, $F_1(q^2)$ and $F_2(q^2)$ must follow counting rules
- Quarks exchange gluons to distribute momentum

Dirac form factor F_1

- Non-spin flip
- Two gluon propagators
- $F_1(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$

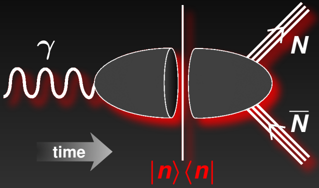
Pauli form factor F_2

- Spin flip
- Two gluon propagators
- $F_2(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-3}$

Sachs form factors G_E and G_M

- $G_{E,M}(q^2) \underset{q^2 \to -\infty}{\sim} (-q^2)^{-2}$
- Ratio: $\frac{G_E}{G_M} \underset{q^2 \to -\infty}{\sim}$ constant

Time-like nucleon form factors



- **Crossing symmetry:**

$$\langle N(p')|J^\mu|N(p)\rangle \longrightarrow \langle \overline{N}(p')N(p)|J^\mu|0\rangle$$

- **Form factors are complex functions of q^2**

Cutkosky rule for nucleons

$$\mathrm{Im}\langle \overline{N}(p')N(p)|J^\mu(0)|0\rangle \sim \sum_n \langle \overline{N}(p')N(p)|J^\mu(0)|n\rangle\langle n|J^\mu(0)|0\rangle \Rightarrow \begin{cases} \mathrm{Im}F_{1,2} \neq 0 \\ \text{for } q^2 > 4m_\pi^2 \end{cases}$$

$|n\rangle$ are on-shell intermediate states: 2π, 3π, 4π, ...

Time-like asymptotic behavior

Phragmèn Lindelöf theorem:

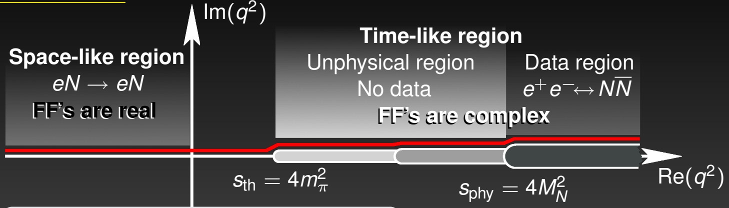
If a function $f(z) \longrightarrow a$ as $z \longrightarrow \infty$ along a straight line, and $f(z) \longrightarrow b$ as $z \longrightarrow \infty$ along another straight line, and $f(z)$ is regular and bounded in the angle between, then $a = b$ and $f(z) \longrightarrow a$ uniformly in this angle.

- $$\underbrace{\lim_{q^2 \to -\infty} G_{E,M}(q^2)}_{\text{space-like}} = \underbrace{\lim_{q^2 \to +\infty} G_{E,M}(q^2)}_{\text{time-like}}$$

- $$G_{E,M} \underset{q^2 \to +\infty}{\sim} (q^2)^{-2}$$ **real**

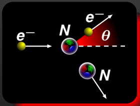
Cross sections and analyticity

q^2-complex plane



Crossing: tot. helicity $= \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$

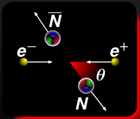
$G_E(4M_N^2) = G_M(4M_N^2)$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2\frac{\theta}{2}}{4E_e^3 \sin^4\frac{\theta}{2}} \left[G_E^2 - \tau\left(1 + 2(1-\tau)\tan^2\frac{\theta}{2}\right) G_M^2 \right] \frac{1}{1-\tau}$$

$$\tau = \frac{q^2}{4M_N^2}$$



Annihilation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta\, \mathcal{C}}{4q^2} \left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2 \right]$$

$\beta = \sqrt{1 - \frac{1}{\tau}}$

$\mathcal{C}$ = Coulomb factor

S and D waves

$$\begin{cases} P_\gamma = -1 & P_{N\overline{N}} = (-1)^L \times (-1) \quad \Rightarrow \quad L = 0, 2 \\ J_\gamma = 1 & (S, L) = (0, 1) \text{ forbidden} \quad \Rightarrow S = 1 \end{cases}$$

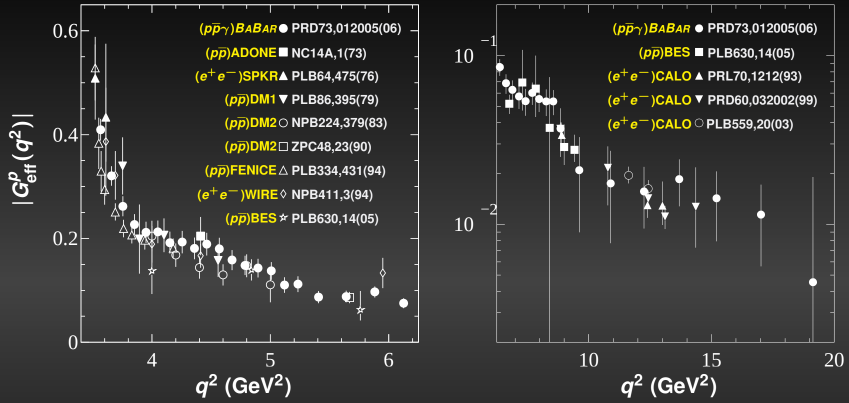
$$G_E = G_S - 2G_D \qquad G_M = \frac{G_S + G_D}{\sqrt{q^2}/2M}$$

At threshold S wave only $\Leftrightarrow G_E = G_M$

$$\begin{cases} G_S = \frac{2G_M\sqrt{q^2}/2M + G_E}{3} \\ G_D = \frac{G_M\sqrt{q^2}/2M - G_E}{3} \end{cases} \Longrightarrow \begin{cases} G_S = G_{M,E} \\ G_D = 0 \end{cases}$$

Proton Form Factor data and Asymptopia

Time-like magnetic proton form factor



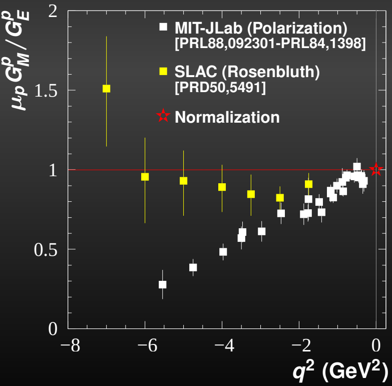
Data obtained assuming $|G^p_M| = |G^p_E| \equiv |G^p_{\text{eff}}|$ (true only at threshold)

$$|G^p_{\text{eff}}|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{16\pi\alpha^2\mathcal{C}}{3}\frac{\sqrt{1-1/\tau}}{4q^2}\left(1+\frac{1}{2\tau}\right)}$$

Data on $R = \mu_p G_E^p / G_M^p$

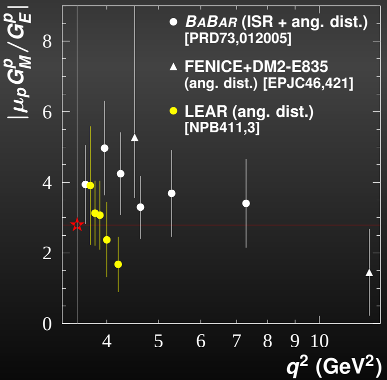
Space-like region

- Old Rosenbluth data in agreement with space-like **scaling** $G_E^p \simeq G_M^p / \mu_p$
- Data from polarization techniques show unexplained **increasing behavior**
- Only polarization data have been used in the dispersive analysis



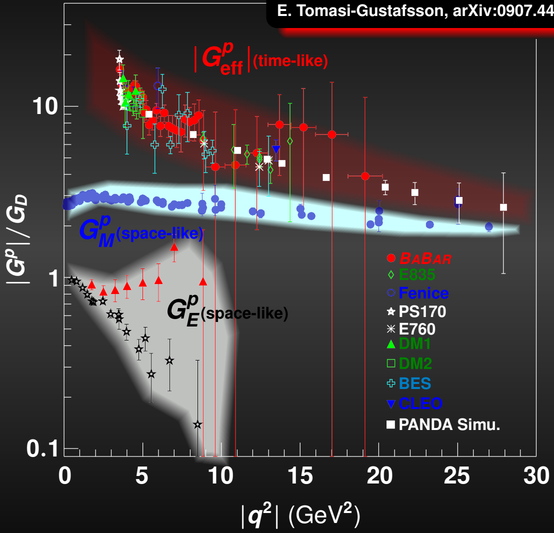
Time-like region

- Only two sets of data from *BABAR* and LEAR obtained studying angular distributions
- **Unique attempts** to perform a time-like $|G_E^p|$-$|G_M^p|$ **separation**
- Only *BABAR* data have been used in the dispersive analysis



Asymptotic behavior

E. Tomasi-Gustafsson and M. P. Rekalo, PLB504,291
E. Tomasi-Gustafsson, arXiv:0907.4442



pQCD

$$G^p_{\rm eff}(q^2) \underset{q^2\to\infty}{\sim} G^p_M(q^2)$$

Phragmèn Lindelöf

$$\lim_{q^2\to\infty} \frac{G^p_{\rm eff}(q^2)}{G^p_M(-q^2)} = 1$$

Negative limits for G^p_E?

$$\lim_{|q^2|\to\infty} G^p_E(q^2) = 0^-$$

$|G_E^p(q^2)|$ and $|G_M^p(q^2)|$ from $\sigma_{p\bar{p}}$ and DR

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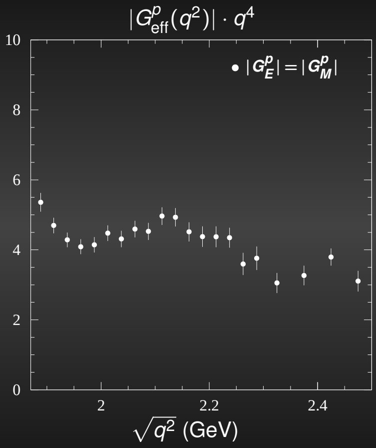

$$|G_{\rm eff}^p(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}}\left(1+\frac{1}{2\tau}\right)^{-1}$$

- Usually what is extracted from the cross section $\sigma(e^+e^- \longrightarrow p\bar{p})$ is the effective time-like form factor $|G_{\rm eff}^p|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$
- Using our parametrization for R and the BABAR data on $\sigma(e^+e^- \longrightarrow p\bar{p})$, $|G_E^p|$ and $|G_M^p|$ may be disentangled

$|G_E^p(q^2)|$ and $|G_M^p(q^2)|$ from $\sigma_{p\bar{p}}$ and DR

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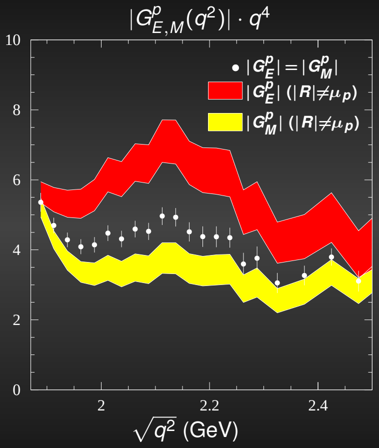


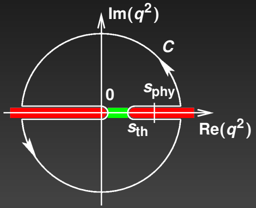
$$|G_M^p(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}} \left(1 + \frac{|R(q^2)|}{2\mu_p\tau}\right)^{-1}$$

- Usually what is extracted from the cross section $\sigma(e^+e^- \longrightarrow p\bar{p})$ is the effective time-like form factor $|G^p_{\text{eff}}|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$
- Using our parametrization for R and the BABAR data on $\sigma(e^+e^- \longrightarrow p\bar{p})$, $|G_E^p|$ and $|G_M^p|$ may be disentangled

Asymptopia and dispersion relations

Assuming $G(q^2) \neq 0$ and using the Cauchy theorem, we have the new DR

New function	Attenuation function	Cauchy theorem
$\phi(z) = \dfrac{f(z)\ln G(z)}{z\sqrt{s_{th} - z}}$	$\int_0^{s_{phy}} f^2(z)\,dz \ll 1$	$\oint_C \phi(z)\,dz = 0$



$$\Downarrow$$

$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)]\ln G(t)}{t\sqrt{s_{th} - t}}\,dt}_{\text{Space-like}} = \underbrace{\int_{s_{th}}^{\infty} \frac{f(s)\ln|G(s)|}{s\sqrt{s - s_{th}}}\,ds}_{\text{Time-like}}$$

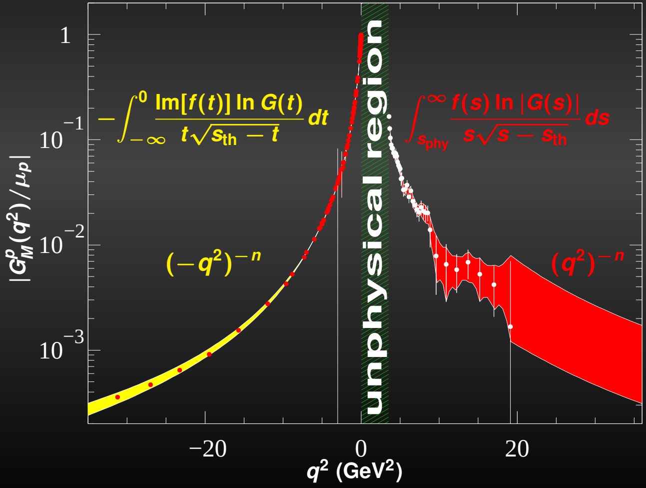
Convergence relation to find the asymptotic power-law behavior of G_M^p

$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)]\ln G(t)}{t\sqrt{s_{th} - t}}\,dt}_{\text{Space-like data + }(-t)^{-n}} = \int_{s_{th}}^{\infty} \frac{f(s)\ln|G(s)|}{s\sqrt{s - s_{th}}}\,ds \approx \underbrace{\int_{s_{phy}}^{\infty} \frac{f(s)\ln|G(s)|}{s\sqrt{s - s_{th}}}\,ds}_{\text{Time-like data + }s^{-n}}$$

***n* is the only free parameter**

Sum rule: result for G_M^p

$$G_M^p(q^2) \underset{|q^2|\to\infty}{\propto} |q^2|^{-(2.27\pm 0.36)}$$



The ISR technique

ISR: Physics Motivations

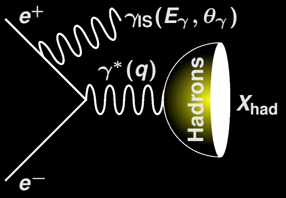
Existing ISR results, obtained by ***BaBar***, show interesting and unexpected behaviors, mainly at thresholds, for

$e^+e^- \to p\bar{p}$ and $e^+e^- \to \Lambda\bar{\Lambda}\,,\ \Sigma^0\bar{\Sigma}^0\,,\ \Lambda\bar{\Sigma}^0$

There are physical limits in reaching the threshold of many of these channels via energy scan (stable hadrons produced at rest can not be detected)

The Initial State Radiation technique provides a unique tool to access threshold regions working at higher resonances

Initial State Radiation



$e^+e^- \longrightarrow \gamma_{IS} X_{had}$

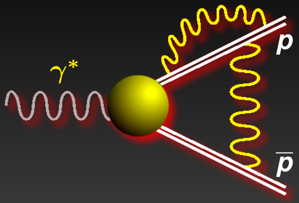
- $$\frac{d^2\sigma}{dE_\gamma\, d\cos\theta_\gamma} = W(E_\gamma,\theta_\gamma)\,\sigma_{e^+e^-\to X_{had}}(s)$$
- $$W(E_\gamma,\theta_\gamma) = \frac{\alpha}{\pi x}\left(\frac{2-2x+x^2}{\sin^2\theta_\gamma} - \frac{x^2}{2}\right)$$

- $s = q^2$, q .... X_{had} momentum
- E_γ, θ_γ ..CM γ_{IS} energy, scatt. ang.
- E_{CM} .... CM e^+e^- energy
- $x = 2E_\gamma / E_{CM}$

- **All energies (q^2) at the same time** $\Rightarrow$ **Better control on systematics (greatly reduced point to point)**
- **Detected ISR at large angles** $\Rightarrow$ **full X_{had} angular coverage**
- **CM boost** $\Rightarrow$ **at threshold $\epsilon \neq 0$; energy resolution $\sim$ 1 MeV**

Sommerfeld resummation factor needed?

The Coulomb Factor



$$\sigma_{p\bar{p}} = \frac{4\pi\alpha^2\beta\,\mathcal{C}}{3q^2}\left[|G_M^p(q^2)|^2 + \frac{2M_p^2}{q^2}|G_E^p(q^2)|^2\right]$$

$\mathcal{C}$ describes the $p\bar{p}$ Coulomb interaction as FSI

[Sommerfeld, Sakharov, Schwinger, Fadin, Khoze]

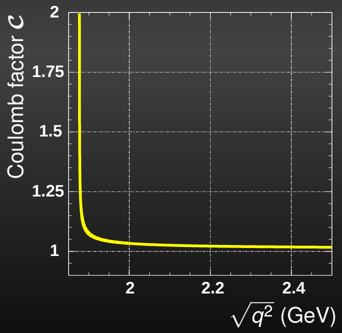
Distorted wave approximation

$$\mathcal{C} = |\Psi_{\text{Coul}}(0)|^2$$

- **S-wave:** $\mathcal{C} = \dfrac{\frac{\pi\alpha}{\beta}}{1 - \exp\left(-\frac{\pi\alpha}{\beta}\right)} \xrightarrow[\beta\to 0]{} \dfrac{\pi\alpha}{\beta}$

- **D-wave:** $\mathcal{C} = 1$

No Coulomb factor for boson pairs (P-wave)



Sommerfeld Enhancement and Resummation Factors

Coulomb Factor $\mathcal{C}$ for S-wave only:

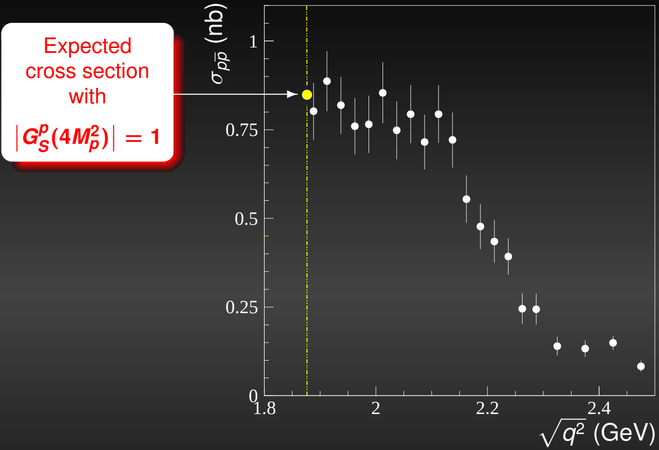
- **Partial wave FF:** $G_S = \dfrac{2G_M\sqrt{q^2/4M^2} + G_E}{3}$ $\quad G_D = \dfrac{G_M\sqrt{q^2/4M^2} - G_E}{3}$
- **Cross section:** $\sigma(q^2) = 2\pi\alpha^2\beta\,\dfrac{4M^2}{(q^2)^2}\left[\mathcal{C}\,|G_S(q^2)|^2 + 2|G_D(q^2)|^2\right]$

$$\mathcal{C} = \mathcal{E} \times \mathcal{R}$$

- **Enhancement factor:** $\mathcal{E} = \pi\alpha/\beta$
- **Step at threshold:** $\sigma_{p\bar{p}}(4M_p^2) = \dfrac{\pi^2\alpha^3}{2M_p^2}\,\dfrac{\cancel{\beta}}{\cancel{\beta}}\,|G_S^p(4M_p^2)|^2$

- **Resummation factor:** $\mathcal{R} = 1/[1 - \exp(-\pi\alpha/\beta)]$
- **Few MeV above threshold:** $\mathcal{C} \simeq 1 \quad \Rightarrow \quad \sigma_{p\bar{p}}(q^2) \propto \beta\,|G_S^p(q^2)|^2$

BABAR: $e^+e^- \longrightarrow p\bar{p}$



At the threshold

$$\sigma_{p\bar{p}}(4M_p^2) = \frac{\pi^2\alpha^3}{2M_p^2}\,\frac{\beta_p}{\beta_p}\,|G_S^p(4M_p^2)|^2$$

$$\sigma_{p\bar{p}}(4M_p^2) = 0.85\,|G_S^p(4M_p^2)|^2 \text{ nb}$$

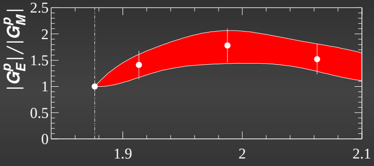
$\Longrightarrow$

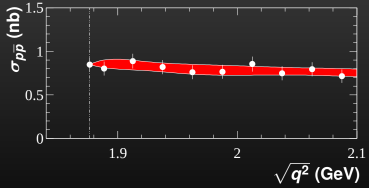
$|G_S^p(4M_p^2)| \equiv 1$

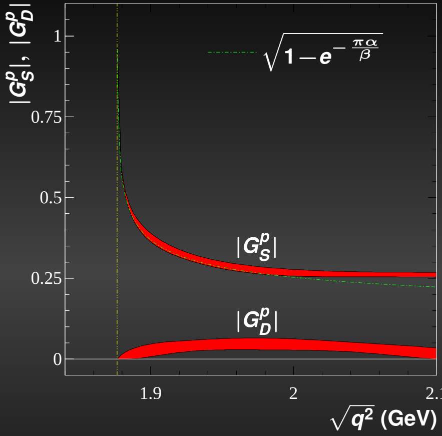
as pointlike fermion pairs!

Extracting $|G_S^p|$ and $|G_D^p|$ using

- data on $\sigma_{p\bar{p}}$
- data on $|G_E^p|/|G_M^p|$
- G_E^p/G_M^p phase $\phi \simeq 0$





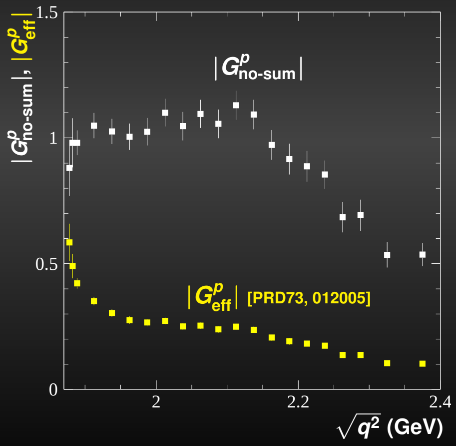


- $|G_S^p| \simeq \sqrt{1-\exp(-\pi\alpha/\beta)}$
- **No need of resummation factor**

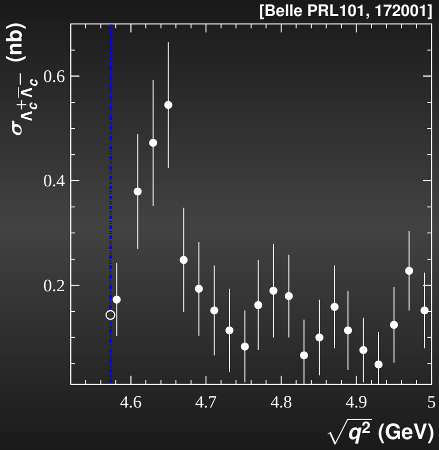
BABAR: $G^p_{\rm eff}$ with and without Sommerfeld factor

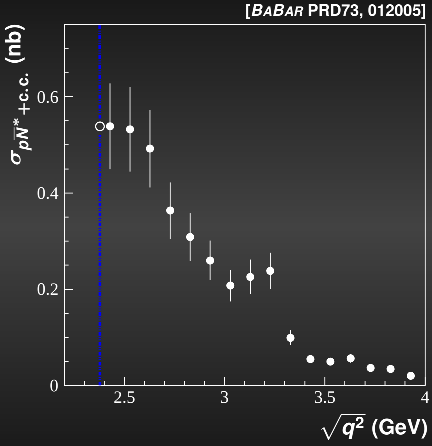
$$|G^p_{\rm eff}|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\mathcal{C}\,\frac{16\pi\alpha^2}{3}\,\frac{\sqrt{1-1/\tau}}{4q^2}\left(1+\frac{1}{2\tau}\right)}$$

$$|G^p_{\text{no-sum}}|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\varepsilon\,\frac{16\pi\alpha^2}{3}\,\frac{\sqrt{1-1/\tau}}{4q^2}\left(1+\frac{1}{2\tau}\right)}$$



$e^+e^- \longrightarrow \Lambda_c^+\overline{\Lambda}_c^-$ and $e^+e^- \longrightarrow p\overline{N}(1440)$+c.c.





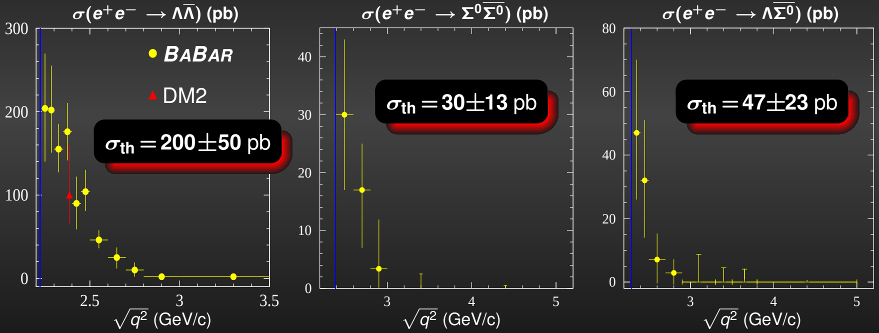
The neutrals puzzle

Neutral Baryons puzzle (BaBar)

[PRD76, 092006]

$$\sigma(e^+e^- \to N^0\overline{N}^0) = \frac{4\pi\alpha^2\beta\mathcal{C}_0}{3q^2}\left[|G_M^{N^0}|^2 + \frac{2M_{N^0}^2}{q^2}|G_E^{N^0}|^2\right] \xrightarrow[\sqrt{q^2}\to 2M_{N^0}]{} \frac{\pi\alpha^2\beta}{2M_{N^0}^2}|G^{N^0}|^2 \to 0$$

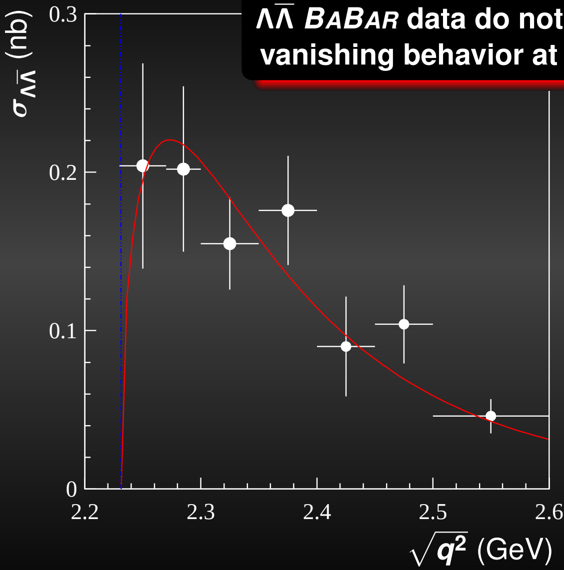
No Coulomb correction at hadron level: $\mathcal{C}_0 = 1$



Threshold values obey U-spin relation: $G^{\Sigma^0} - G^{\Lambda} + \frac{2}{\sqrt{3}}G^{\Lambda\Sigma^0} = 0$

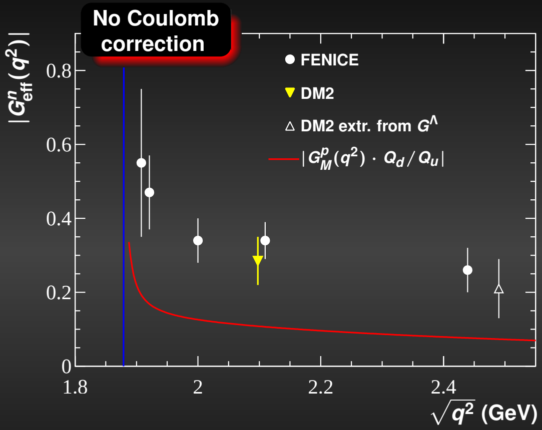
BABAR: $e^+e^- \to \Lambda\bar{\Lambda}$

[PRD76, 092006]



$\Lambda\bar{\Lambda}$ BABAR data do not exclude a vanishing behavior at threshold

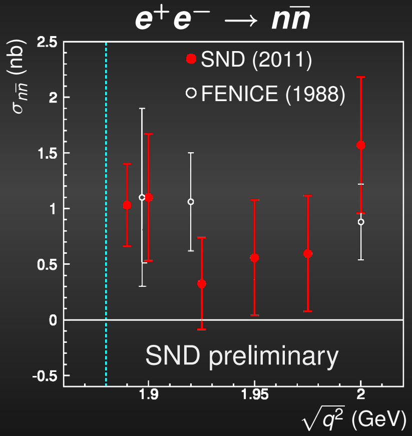
Time-like $|G_M^n|$ measurements



	$\lvert G_{eff}^n / G_{eff}^p \rvert$
Data	$\sim$ 1.5
Naively	$\sim \lvert Q_d / Q_u \rvert$
pQCD	$<$ 1
Soliton models	$\sim$ 1
VMD (Dubnicka)	$\gg$ 1

In this energy range only BESIII can repeat this measurement

$e^+e^- \to n\bar{n}$
Preliminary result from SND at VEPP-2000



- **Scan 2011**
- **Maximum energy: 2 GeV**
- **Efficiency $\sim$ 30%**
- **Above $n\bar{n}$ threshold:**
  $\sigma_{n\bar{n}} = 0.8 \pm 0.2$ **nb**

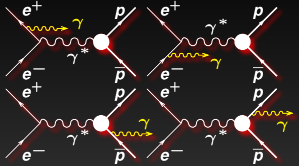
Highlights

- Asymptotic behavior not well understood
- Pointlike behavior not only at threshold
- No Sommerfeld resummation factor
- Neutral baryons puzzle
- More data from BESIII, VEPP-2000 and PANDA

Additional slides

ISR and final state radiation

PRD84,017301

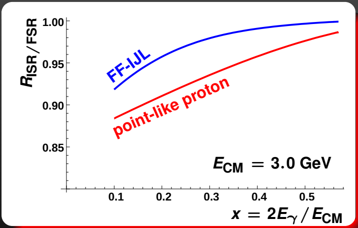


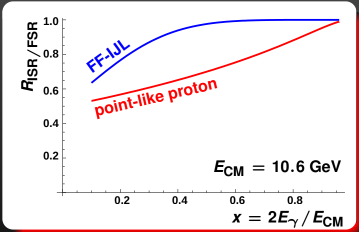
$$\frac{d^2\sigma_{\rm ISR}}{dE_\gamma d\theta_\gamma} = \frac{\alpha^3 E_\gamma}{3E^2_{\rm CM} s}\left(|G^p_M(s)|^2 + \frac{|G^p_E(s)|^2}{2\tau}\right)\mathcal{W}(E_\gamma,\theta_\gamma)$$

$$\frac{d^2\sigma_{\rm FSR}}{dE_\gamma d\theta_\gamma} = \frac{\alpha^3 E_\gamma}{3E^4_{\rm CM}}\,\mathcal{F}\left[E_\gamma,\theta_\gamma, G^p_E(E^2_{\rm CM}), G^p_M(E^2_{\rm CM})\right]$$

No ISR-FSR interference after $d\Phi(p\bar{p})$ integration

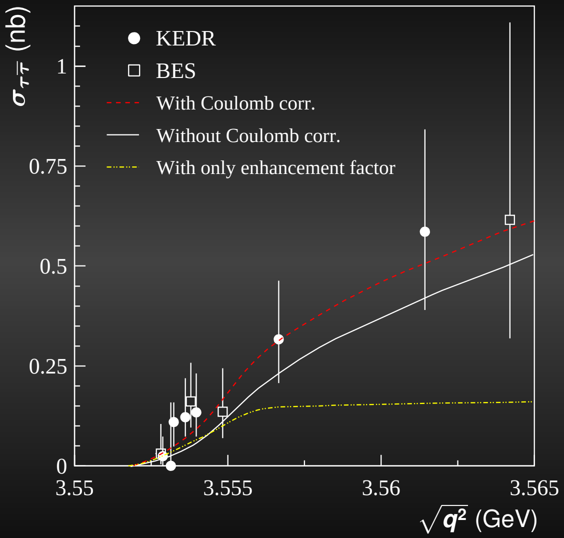
$$R_{\rm ISR/FSR} = \frac{d\sigma_{\rm ISR}/dE_\gamma}{d\sigma_{\rm ISR}/dE_\gamma + d\sigma_{\rm FSR}/dE_\gamma}\,[20^\circ \le \theta_\gamma \le 160^\circ]$$





For large values of x or at small angle θ_γ of photon emission the final state radiation is strongly suppressed

The $e^+e^- \to \tau\overline{\tau}$ case



BaBar: integrated Sommerfeld factor

$$\overline{\mathcal{R}^{-1}} = \frac{1}{\Delta q}\int_0^{\Delta q}\left[1 - e^{-\frac{\pi\alpha}{\beta}}\right] d\sqrt{q^2}$$

$$\Delta q = \sqrt{q^2} - 2M_p$$