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A Comparison of Some Methods of Deriving the Instantaneous Unit Hydrograph

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The geomorphological instantaneous unit hydrograph (IUH) proposed by Gupta et al. (1980) was compared with the IUH derived by commonly used time-area and Nash methods. This comparison was performed by analyzing the effective rainfall-direct runoff relationship for four large basins in Central Italy ranging in area from 934 to 4,147 km^2.

The Nash method was found to be the most accurate of the three methods. The geomorphological method, with only one parameter estimated in advance from the observed data, was found to be little less accurate than the Nash method which has two parameters determined from observations. Furthermore, if the geomorphological and Nash methods employed the same information represented by basin lag, then they produced similar accuracy provided the other Nash parameter, expressed by the product of peak flow and time to peak, was empirically assessed within a wide range of values. It was concluded that it was more appropriate to use the geomorphological method for ungaged basins and the Nash method for gaged basins.

Introduction

Geomorphological models, coupling the principles of hydrologic systems with quantitative geomorphology, have been recently proposed to represent the instantaneous unit hydrograph (IUH) of a given basin. The hydrologic literature deals extensively with a formulation where the response at the basin scale is expressed by averaging the channel network geometry in terms of the Horton-Strahler ordering scheme (Rodríguez-Iturbe and Valdés 1979, Gupta et al. 1980, Wang et al. 1981, Rodríguez-Iturbe et al. 1982). Though this formulation represents an improvement of the classical methods towards the complete IUH synthesis, it is of basic importance to establish if at the present stage the geomorphological method may be considered as a practical tool in representing the IUH for gaged and especially for ungaged basins.

Since the geomorphological IUH requires only an apriori evalutation of either a dynamic parameter (Rodríguez-Iturbe and Valdés 1979) or basin lag (Gupta et al. 1980), its application to ungaged basins is promising. Furthermore, the classical methods have not been very successful in synthesizing the IUH for an ungaged basin for the difficulties in estimation of the involved empirical parameters which exceed one. Therefore it seems appropriate to compare these different formulations in a consistent way. That is, for gaged as well as ungaged basins the same information must be used in determining the IUH by each formulation.

The aim of this paper was to compare the aforementioned methods by analyzing the effective rainfall-direct runoff relationship for four large basins in Central Italy ranging in area from 934 to 4,147 km^2. Specifically these include:

1) The linear geomorphological method proposed by Gupta et al. (1980), which has been further investigated and tested by Corradini et al. (1985).
2) The Clark-Snyder method (Feldman et al. 1982).
3) The Clark method with time-area histogram derived by the channel profile technique (Singh 1984), designated henceforth as TA1 method.
4) The Clark method incorporating time-area histogram derived by the Laurenson (1964) technique, designated henceforth as TA2 method.
5) The Nash method.

Methods for IUH Synthesis

The Geomorphological Method

The probabilistic approach of the geomorphologic IUH has been pioneered by Rodríguez-Iturbe and Valdés (1979), and then reformulated and generalized by Gupta et al. (1980). This approach represents the flow configuration in a basin of any Strahler order W by a path structure such that the number of paths S_f is less than or equal to 2^{W-1}. A given path s is composed of the one overland region (r) and one or more channels (c) of different orders; it can be expressed in terms of states (x_i, i=1,2,...,j and $j \leq W+1$). Each state is associated with a mean holding time $1/K_{x_i}$ and, assuming that its probability density function is exponential, the geomorphologic IUH, $h(t)$, can be written in the form (Gupta et al. 1980)

$$h(t) = \sum_{s \in S_f} \sum_{i=1}^{j} C_{ij}[\exp(-K_{x_i} t)]p(s) , \quad s = \{x_1, x_2, \ldots, x_j\} \quad (1)$$

where $p(s)$ represents the probability that the path s, from amongst all possible paths, will be followed by rainwater. The C_{ij} are coefficients given by

$$C_{ij} = K_{x_1} K_{x_2} \ldots K_{x_j} [(K_{x_1} - K_{x_i}) \ldots$$
$$\ldots (K_{x_{i-1}} - K_{x_i})(K_{x_{i+1}} - K_{x_i}) \ldots (K_{x_j} - K_{x_i})]^{-1} \quad (2)$$

The mean holding time of an ith order Strahler channel, $x_i = c_i$, and of an ith order overland region, $x_i = r_i$, are computed by

$$\frac{1}{K_{c_i}} = \gamma(\overline{L_i})^{\frac{1}{3}} , \quad 1 \le i \le W \quad (3)$$

$$\frac{1}{K_{r_i}} = \gamma \left(\frac{A_{r_i} A}{2N_i \overline{L_i}} \right)^{\frac{1}{3}} , \quad 1 \le i \le W \quad (4)$$

where γ is an empirical constant, $\bar{L}_i$ the average channel length of order i, A_{r_i} is the ratio of the ith order overland region area to the basin area A, N_i is the number of the ith order overland regions. The constant γ in Eqs. (3-4), on the basis of basin lag K_B, is derived from

$$K_B = \sum_{s \in S_f} \left(\frac{1}{K_{x_1}} + \frac{1}{K_{x_2}} + \ldots + \frac{1}{K_{x_j}} \right) p(s), \quad s \equiv \{x_1, x_2, \ldots, x_j\} \quad (5)$$

The Time-Area Methods

The Clark-Snyder method computes the basin IUH by construction of time-area histogram and routing this through a linear reservoir of storage coefficient K. Furthermore, two constraints for the Snyder parameters C_p and C_t (Wilson 1974, Linsley et al. 1982) computed from basin IUH characteristics are applied. The first parameter C_p in dimensionless form, represents the product of the unit hydrograph peak flow h_p and time to peak t_p, the other C_t is linked to time to peak. The computation procedure, after an apriori assumption of the Snyder parameters C_p° and C_t°, involves:

a) Determining the dimensionless time-area histogram of the basin, with the abscissa expressed as a proportion of the time of concentration T_c.
b) Choosing values of T_c and K.
c) Determining the dimensional time-area diagram and computing the IUH.

d) Computing the parameters C_p and C_t.
e) Comparing C_p and C_t with C_p° and C_t°, respectively. That couple of values of T_c and K which results in equality of computed and assumed Snyder parameters is considered as the correct one and the corresponding IUH is taken for the basin.

This procedure was used here. For consistency with the geomorphologic IUH, the constraint on C_t was modified by requiring that the lag time associated with the computed IUH was equal to the observed basin lag. The dimensionless time-area histogram was determined according to Laurenson (1964).

As to the TA1 and TA2 methods, they use the traditional Clark procedure (Linsley et al. 1982) with T_c and K directly estimated from observed direct runoff hydrographs.

The Nash Method

This approach considers a drainage basin as a cascade of n identical reservoirs in series, each having the storage coefficient K (Nash 1957). The analytical form of the IUH is

$$h(t) \equiv \frac{1}{K(n-1)!} \left(\frac{t}{K}\right)^{n-1} e^{-t/K} \tag{6}$$

where n from a practical point of view only needs to be positive. The parameter n can be easily expressed in terms of the dimensionless product $h_p t_p$ that is the Snyder parameter C_p, through the relationship (see also Rosso 1984)

$$h_p t_p \equiv \frac{1}{(n-1)!} (n-1)^n e^{-1/n} \tag{7}$$

The other parameter K is linked to the basin lag K_B through the relationship $K = K_B/n$ (Chow 1964).

Experimental Data

Experimental Basins

Four Italian basins, located in the Umbria region, were selected for this investigation; their general layout is shown in Fig. 1. These are the Upper Tiber River basin (drainage area of 4,147 km^2) and three of its sub-basins: Chiascio River at Rosciano (area 1,956 km^2), Topino River at Bettona (area 1,220 km^2) and Tiber River at S. Lucia (area 934 km^2). The topography is mainly hilly with elevation above sea level ranging from 200 to 800 m. The mountain peaks on a large portion of the boundary of the Upper Tiber River basin range in elevation from 1,000 to 1,500 m above sea level.

Mean annual precipitation over the basins ranges from 700 to 1,600 mm. Higher

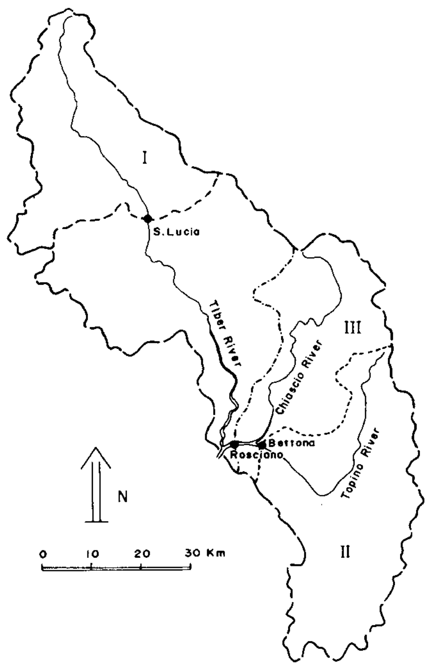

Fig. 1. The Upper Tiber River basin with its sub-basins: Tiber at S. Lucia (I), Topino at Bettona (II), Chiascio at Rosciano (II+III).

monthly precipitation values generally occur during the autumn-winter period (November-December). It is this the period during which floods, caused by widespread rainfall, normally occur.

Rainfall-Runoff Data

Forty effective rainfall-direct runoff events, ten for each basin, were used in this study. These events are summarized in Table 1, where the observed basin lag for each basin is also reported. For a more complete description of the events the reader is referred to Corradini et al. (1985), where the geomorphological parameters estimated by using the map scale 1:200,000 are also given.

Parameter Estimation

For each method, in each basin, optimized parameters were computed. The basin lag K_B of the geomorphological method was derived by averaging the observed lags for all the effective rainfall-direct runoff events on each basin. The parameters T_c and K of the TA1 and TA2 methods were determined by a similar procedure, with T_c and K estimated for each event by using the inflexion point on the recession limb of the direct runoff hydrograph (Wilson 1974). For the Clark-Snyder and Nash methods, the basin lag was used and the other parameter, $h_p t_p$, was optimized by choosing the value producing minimum mean error in peak discharge. Throughout the text each error for a given quantity was computed ignoring algebraic sign.

Table 1 – Some characteristics of effective rainfall-direct runoff events of the Upper Tiber River basin and its sub-basins. Basin lag K_B is also reported.

Basin	Serial number	Date	Effective rainfall Depth mm	Effective rainfall Duration hrs	Direct runoff Peak m^3/s	Direct runoff Time to peak hrs	Direct runoff Duration hrs
Upper Tiber	1	Dec. 12, 1981	2.9	7.0	200.0	19.9	48.0
River	2	Dec. 18, 1981	3.7	8.0	265.4	19.9	48.0
K_B=18.0 hrs	3	Dec. 24, 1981	2.8	5.5	139.0	15.4	62.0
	4	Dec. 29, 1981	11.7	11.0	591.0	22.9	70.0
	5	Nov. 9, 1982	13.1	9.0	702.8	18.9	61.0
	6	Nov. 13, 1982	6.2	8.0	404.9	19.9	57.0
	7	Dec. 1, 1982	4.9	5.4	338.0	20.3	54.0
	8	Dec. 13, 1982	3.2	10.0	188.7	17.9	56.0
	9	Dec. 18, 1982	11.9	7.0	760.3	14.9	53.0
	10	Dec. 21, 1982	17.8	27.0	629.0	26.9	83.0
Chiascio River	1	Dec. 11, 1981	1.3	4.0	45.5	13.9	44.0
at Rosciano	2	Dec. 12, 1981	1.5	3.0	58.5	9.9	43.0
K_B=14.7 hrs	3	Dec. 13, 1981	1.8	3.0	67.0	8.9	47.0
	4	Dec. 29, 1981	3.4	4.0	107.7	17.9	63.0
	5	Nov. 9, 1982	5.3	1.0	165.3	15.9	51.0
	6	Nov. 13, 1982	2.8	4.0	116.7	14.9	48.0
	7	Dec. 1, 1982	2.5	4.0	83.5	16.9	45.0
	8	Dec. 13, 1982	3.2	9.0	87.3	14.9	50.0
	9	Dec. 18, 1982	10.3	7.0	350,0	14.9	48.0
	10	Dec. 21, 1982	17.6	25.0	339.8	25.9	69.0
Topino River	1	Dec. 11, 1981	1.2	5.0	33.6	9.9	44.0
at Bettona	2	Dec. 12, 1981	1.4	3.0	42.0	8.9	44.0
K_B=13.3 hrs	3	Dec. 13, 1981	1.3	1.0	36.0	7.9	42.0
	4	Dec. 24, 1981	1.4	2.0	28.8	7.9	45.0
	5	Dec. 29, 1981	1.7	3.9	32.8	12.8	54.0
	6	Nov. 9, 1982	1.4	1.0	27.0	13.9	47.0
	7	Nov. 13, 1982	2.0	4.9	56.3	14.8	46.0
	8	Dec. 13, 1982	2.7	8.0	50.0	14.9	40.0
	9	Dec. 18, 1982	6.0	6.5	159.6	12.4	42.0
	10	Dec. 21, 1982	13.9	23.0	186.2	21.9	65.0
Tiber River	1	Dec. 11, 1981	2.5	4.0	58.4	10.9	34.0
at S. Lucia	2	Dec. 12, 1981	7.4	9.2	163.0	16.1	41.0
K_B=11.5 hrs	3	Dec. 18, 1981	10.3	11.0	192.0	13.9	37.0
	4	Dec. 19, 1981	5.6	5.1	120.0	9.0	35.0
	5	Nov. 9, 1982	13.4	9.0	220.8	10.9	48.0
	6	Nov. 13, 1982	13.5	11.0	256.0	16.9	37.0
	7	Dec. 1, 1982	12.0	8.6	243.9	12.5	33.0
	8	Dec. 18, 1982	4.3	3.0	78.3	8.9	39.0
	9	Dec. 21, 1982	1.7	3.4	46.2	8.3	31.0
	10	Dec. 22, 1982	3.2	13.2	41.0	11.1	45.0

Table 2 – Mean percent error in computed peak of direct runoff, ε_{QP}, and time to peak, ε_{tP}, for different computation methods of basin IUH.

Method	Upper Tiber		Chiascio at Rosciano		Topino at Bettona		Tiber at S. Lucia	
	ε_{QP}	ε_{tP}	ε_{QP}	ε_{tP}	ε_{QP}	ε_{tP}	ε_{QP}	ε_{tP}
Geomorphologic	16	8	13	18	16	18	15	13
TA1	14	24	20	31	26	33	11	18
TA2	21	11	8	22	17	24	11	16
Clark-Snyder	19	14	7	19	16	18	11	15
Nash	15	8	9	17	15	17	11	16

Comparison of Methods

A comparison of the hydrograph synthesis methods was performed by using errors in computed peak discharge, errors in computed time to peak, and shape characteristics of the direct runoff hydrographs. The general shape characteristics of the hydrographs were appropriate for each method and the main differences amongst the considered methods can be pointed out through the analysis of peak errors.

The peak errors obtained in synthesizing the direct runoff hydrographs by optimized parameters are summarized in Table 2. The TA1 method provided the worst results indicating that the channel profile method was not appropriate in determining the time-area histogram. The geomorphological method gave errors not much larger than those obtained for the other classical methods, though it has only one parameter estimated from the observed events. On this basis, the Nash method could be preferred in gaged basins in view of its ease of application, whereas in ungaged basins the accuracy of the geomorphological method must be compared with the other methods using the same information for each formulation.

In order to address the problem in ungaged basins, a sensitivity analysis of the Clark-Snyder and Nash methods to the quantity $h_p t_p$ was carried out. The basin lag, for consistency with the geomorphological method, was kept constant. The TA1 and TA2 methods were not considered because they are not explicitly dependent on basin lag. Figs. 2-3 show the peak errors obtained in synthesizing the direct runoff hydrograph for the Clark-Snyder and Nash methods, respectively. It can be seen, that the error in computed peak of direct runoff, ε_{QP}, for the Nash method was less sensitive to $h_p t_p$. Furthermore, for the Upper Tiber River basin and $h_p t_p$ greater than 0.68, the Clark-Snyder method did not satisfy the constraint on the basin lag. This type of problem confirms the major practical utility of the Nash method. For this method the following features can be observed in Fig. 3.

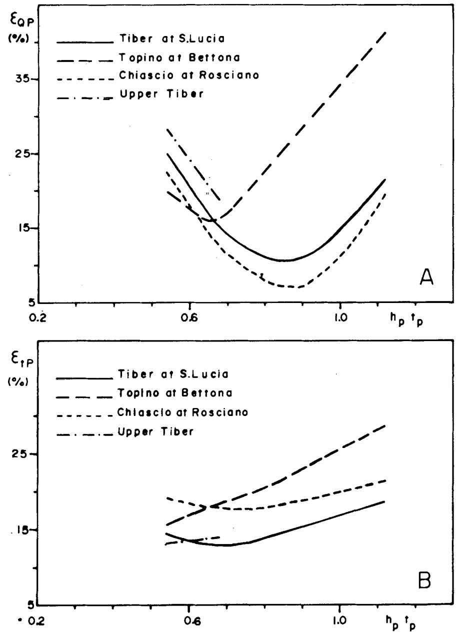

Fig. 2. Clark-Snyder method. A) Mean value of the peak flow error as a function of the product of IUH peak flow and its time to peak. B) Mean value of the time to peak error as a function of the product of IUH peak flow and its time to peak.

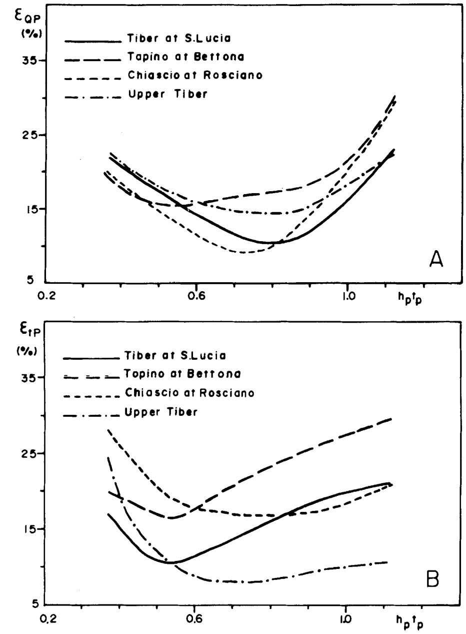

Fig. 3. Nash method. A) Mean value of the peak flow error as a function of the product of the IUH peak flow and its time to peak. B) Mean value of the time to peak error as a function of the product of IUH peak flow and its time to peak.

Mean errors of less than 20 % in computed peak of direct runoff, similar to those obtained by the geomorphological method (see Table 2), were estimated for a wide range of $h_p t_p$. In particular, for the Upper Tiber River basin the computed range was 0.43-1.05 (optimized value 0.80), for the Chiascio River at Rosciano 0.36-1.00 (0.74), for the Topino River at Bettona 0.35-0.96 (0.54), and for the Tiber River at S. Lucia 0.42-1.07 (0.80). Furthermore, if the optimal value of $h_p t_p$ estimated for one of these basins was used in synthesizing the direct runoff hydrographs for the other basins, then errors ε_{QP} and ε_{tP} comparable with those produced by the geomorphological method were obtained. From these results, it is reasonable to argue that the usual technique of deriving $h_p t_p$ from gaged basins, in the vicinity of the problem basin, and using this for the ungaged basin generally produces results with accuracy similar to that obtained by the geomorphological method.

Conclusions

1) For 'large' gaged basins, the geomorphological IUH proposed by Gupta et al. (1980), with only one parameter known in advance, is little less accurate than the IUH associated with other classical methods (such as TA2, Clark-Snyder, Nash) with two parameters optimized by using observed effective rainfall-direct runoff data. Therefore, the Nash method could be preferred in view of its ease of application.

2) For 'large' ungaged basins with basin lag assumed or known a priori, it is reasonable to argue that the Nash method, with parameter $h_p t_p$ determined by transposition from gaged basins in the vicinity of the problem basin, has accuracy similar to that of the geomorphological method. This is especially suggested by the low sensitivity of the results obtained by the Nash method to the product $h_p t_p$. The Clark-Snyder method, with the time-area histogram derived by the Laurenson (1964) technique, can give practical application problems. Furthermore, in the last method the error in computed peak of direct runoff is more sensitive than in the Nash method to $h_p t_p$.

3) Although the Nash method on large ungaged basins can generally give reasonable results as the geomorphological one, it must be considered that in specific cases larger errors are possible. In fact the concept of 'vicinity', or more specifically of basin similarity expressed through gross basin features, is not well defined. Therefore the geomorphological method, which can be applied in a simplified form, should be preferred. In this case geomorphological complexity and computation effort could be considerably reduced by using an appropriate representation of the channel network (Corradini et al. 1985).

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