Phase shifting speckle interferometry for dynamic phenomena

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Abstract: The paper presents an algorithm able to retrieve the phase in speckle interferometry by a single intensity pattern acquired in a deformed state, provided that the integrated speckle field is resolved in the reference condition in terms of mean intensity, modulation amplitude and phase. The proposed approach, called throughout the paper "one-step", can be applied for studying phenomena whose rapid evolution does not allow the application of a standard phase-shifting procedure, which, on the other hand, must be applied at the beginning of the experiment. The approach was proved by an experimental test reported at the end of the paper.

OCIS codes: (100.5070) Phase retrieval, (120.5050) Phase measurement, (120.6160) Speckle interferometry, (350.4600) Optical engineering.

References and links

1. K. Creath, "Temporal phase measurement methods," in *Interferogram Analysis*, D. W. Robinson and G. T. Reid, eds., (Institute of Physics Publishing, Bristol, Philadelphia, 1993) Chap. 4, pp. 94-140.
2. M. Kujawinska, "Spatial phase measurement methods," in *Interferogram Analysis*, D. W. Robinson and G. T. Reid, eds., (Institute of Physics Publishing, Bristol, Philadelphia, 1993) Chap. 5, pp. 141-193
3. J. M. Huntley, "Automated fringe pattern analysis in experimental mechanics: a review," J. Strain Analysis Eng. Design **33**, 105-125 (1998).
4. P. A. A. M. Somers and N. Bhattacharya, "Maintaining sub-pixel alignment for a single-camera two-bucket shearing speckle interferometer," J. Opt. A: Pure and Appl. Opt. **7**, S385-S391 (2005).
5. L. Yang, W. Steinchen, G. Kupfer, P. Mackel, and F. Vossing, "Vibration analysis by means of digital shearography," Opt. Lasers Eng. **30**, 199-212 (1998).
6. F. J. Casillas, A. Davilla, S. J. Rothberg, and G. Garnica, "Small amplitude estimation of mechanical vibrations using electronic speckle shearing pattern interferometry," Opt. Eng. **43**, 880-887 (2004).
7. S. Liu, D. Thomas, P. R. Samala and L. X. Yang, "Vibration measurement of MEMS by digital laser microinterferometer," Proc. SPIE **5878**, 58780C1-9 (2005).
8. M. Takeda, H. Ina, and S. Kobayashi, "Fourier-transorm method of fringe-pattern analysis for computer-based topography and interferometry," J. Opt. Soc. Am. **72**, 156-160 (1982).
9. G. H. Kaufmann and G. E. Galizzi, "Phase measurement in temporal speckle pattern interferometry: comparison between the phase-shifting and the Fourier transform methods," Appl. Opt. **41**, 7254-7263 (2002).
10. B. Bhaduri, N. Krishna Mohan, M. P. Kothiyal, and R. S. Sirohi, "Use of spatial phase shifting technique in digital speckle pattern interferometry (DSPI) and digital shearography (DS)," Opt. Express **14**, 11598-11607 (2006).
11. M. Kujawinska and D. W. Robinson, "Multichannel phase-stepped holographic interferometry," Appl. Opt. **27**, 312-320 (1988).
12. M. Adachi, Y. Ueyama, and K. Inabe, "Automatic deformation analysis in electronic speckle pattern interferometry using one speckle interferogram of deformed object," Opt. Rev. **4**, 429-432 (1997).
13. C. C. Kao, G. B. Yeh, S. S. Lee, C. K. Lee, C. S. Yang, and K. C. Wu, "Phase-shifting algorithms for electronic speckle pattern interferometry," Appl. Opt. **41**, 46-54 (2002).
14. W. An and T. E. Carlsson, "Speckle interferometry for measurement of continuous deformations," Opt. Lasers Eng. **40**, 529-541 (2003).
15. M. Adachi, J. N. Petzing, and D. Kerr, "Deformation-phase measurement of diffuse objects that have started nonrepeatable dynamic deformation," Appl. Opt. **40**, 6187-6192 (2001).
16. L. Bruno and A. Poggialini, "Phase retrieval in speckle interferometry: a one-step approach," in *Proceedings of Interferometry in Speckle Light – Theory and Applications*, P. Jacquot and J. M. Fournier, eds., (Springer, Heidelberg, Germany, 2000), pp. 461-472.
17. A. E. Arthur, *Regression and the Moore-Penrose Pseudoinverse* (Academic Press, New York, 1972).
18. A. Knight, *Basics of Matlab and Beyond* (Chapman & Hall/CRC, Natick, Massachusetts, 2000).

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19. S. Wolfram, *The Mathematica Book*, 5th edition (Wolfram Media, Champaign, Illinois, 2003).
20. L. Bruno, G. Felice and A. Poggialini, “Design and calibration of a piezoelectric actuator for interferometric applications,” Opt. Lasers Eng. **45**, 1148-1156 (2007).

1. Introduction

The capability to carry out accurate full-field measurements by speckle interferometry mostly relies on the application of phase-shifting procedures [1,2], necessary for resolving the mean intensity, the modulation amplitude and the phase of the speckle pattern, which are “tangled” together in the light intensity. Many algorithms have been developed in the past years for applying the phase-shifting procedures in the several fields of the experimental mechanics [3], but most of them suffer the limitation of being inapplicable for the analysis of dynamic phenomena.

Actually the temporal phase-shifting procedures [1], based on the use of phase-shifters like piezoelectric actuators or proper optical components, are very accurate, reliable and capable of automation, but they require the introduction of known phase variations in each deformed configuration. A first attempt to simplify the temporal phase-shifting consists in reducing the number of steps [4], anyway this approach is strongly restricting in those applications in which the deformations are time-dependent. In the event of periodic phenomena some authors have applied temporal techniques by synchronizing the camera and the phase-shifter with the excitation system of the object under test, this task was obtained by stroboscopic illumination triggered with the acquisition and the phase-shifting devices [5-7].

On the other hand when transient phenomena occur the temporal approach is not suitable, apart from the case of slowly varying problems. In this operating condition a spatial approach [2, 8-10] could be fit for the purpose, although it is not rare that the poor spatial resolution of this family of procedure does not allow an accurate phase retrieval – the operation of windowing typically performed on the experimental data to separate the speckle carrier from the fringe pattern usually implies a loss in resolution, in addition to the problems which arise in managing the boundaries. An improvement of the spatial resolution can be attained if a multi-channel approach is adopted [11], provided that the interferograms are acquired by separate sensors. Otherwise, if the shifted interferograms are focused on the same sensor, the spatial resolution is roughly like that of the conventional spatial approaches.

In recent years some authors have proposed some innovative approaches which require only a single frame of the speckle pattern in the deformed state, provided that the integrated speckle field in the reference condition is resolved [12-14]. After the resolution of the reference condition these approaches are based on a re-estimation in the deformed state of the mean intensity and the modulation amplitude, which are subsequently used to calculate the phase; the ambiguity on the sign of the phase is removed by averaging on the multiple values and/or by statistical considerations. Furthermore in [15] a procedure was proposed able to retrieve the phase from a single step also in the reference condition, but in this case the recording of the intensity changes at each pixel must be observed, in order to evaluate the mean intensity and the modulation amplitude.

In the approach proposed in the present paper the resolution of the speckle pattern in the reference condition (which can be done at the beginning or at the ending of the measurement session) is necessary. Then the phase is calculated directly, by means of the single frame of the deformed object pixel by pixel without any ambiguity on the sign. In the calculation procedure the mean intensity and the modulation amplitude are assumed to be constant during the deformation of the object. Furthermore also the phase variation in a small area around the observed pixel is assumed constant, but this fact does not implies an iterative calculation and the number of the pixels involved in the calculation of the phase is very low – an area of 3x3 pixels turns out to be enough for giving good results.

2. Theory

The light intensity resulting from the interference of two wavefronts at the point $\boldsymbol{P}$ can be mathematically described by the complex intensity $\boldsymbol{I(P)}$ [16]:

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$$\boldsymbol{I}(\boldsymbol{P}) = m(\boldsymbol{P}) + \boldsymbol{A}(\boldsymbol{P}) = m(\boldsymbol{P}) + \left\|\boldsymbol{A}(\boldsymbol{P})\right\| e^{j\varphi(\boldsymbol{P})}, \tag{1}$$

where $m(\boldsymbol{P})$ is the mean intensity, $\|\boldsymbol{A}(\boldsymbol{P})\|$ the modulation amplitude and $\varphi(\boldsymbol{P})$ the phase. The bold type is used for the vector quantity $\boldsymbol{P}$ and for the complex numbers $\boldsymbol{I}(\boldsymbol{P})$ and $\boldsymbol{A}(\boldsymbol{P})$. The physical quantity detected by the light sensor is the light intensity, that is the real part of the complex intensity. If a phase variation occurs, as it happens when a phase-shifting algorithm is applied or when the observed surface undergoes a deformation, the light intensity can be expressed as:

$$\begin{aligned} i(\varphi + \Delta\varphi) &= \mathrm{Re}\left[m + \boldsymbol{A} e^{j(\varphi + \Delta\varphi)} \right] = m + \left\|\boldsymbol{A}\right\| \cos\varphi \cos\Delta\varphi - \left\|\boldsymbol{A}\right\| \sin\varphi \sin\Delta\varphi = \\ &= m + \mathrm{Re}[\boldsymbol{A}] \cos\Delta\varphi - \mathrm{Im}[\boldsymbol{A}] \sin\Delta\varphi \end{aligned}, \tag{2}$$

where $\Delta\varphi$ is the phase variation and i is the light intensity. In Eq. (2) the dependence on the spatial coordinates $\boldsymbol{P}$ is omitted for the sake of brevity. Figure 1 shows graphically the Eq. (1) and (2) in the complex plane, where the vectors represent the complex intensity and the projections on the real axis represent the detected light intensity.

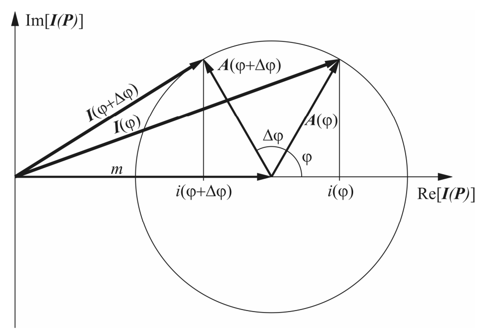

Fig. 1. The complex intensity and the light intensity in the complex plane when a phase variation occurs at a generic point of the speckle interferogram.

A well assessed method used in the conventional temporal phase-shifting method consists in introducing at least three known phase shifts in order to calculate the three unknowns m, $\|\boldsymbol{A}\|$ and φ, and this procedure must be applied before and after the deformation occurs. If we suppose that the mean intensity and modulation amplitude do not change during the deformation of the investigated object, after the speckle field is resolved in the reference condition the light intensity depends only on the phase variation, which is the sole unknown. However, as widely reported in literature, this unknown cannot be directly retrieved from the light intensity, due to the ambiguity on the sign, and at least two acquisitions must be used.

Hence, also in the hypothesis of constant mean intensity and modulation amplitude, it not possible to evaluate the phase by a single light intensity acquisition in the deformed configuration, unless an additional hypothesis is formulated. In the present paper the phase variation of the considered pixel is assumed equal to those of the neighboring pixels, in this way the phase variation is evaluated by using the light intensity of all these pixels. Theoretically the information of just one neighboring pixel is enough, but the use of a larger number of pixels is strongly recommended. The assumption of constant phase variation of the pixels belonging to a small region is consistent in most of the applications in which speckle interferometry is used. In fact the surface deformation of a component mechanically loaded is usually a continuous and smooth field, thus the components of displacement coded in phase variation by the interferometric set-up are as much continuous and smooth. If this position holds, Eq. (2) can be rewritten for the j-th pixel of the region around the k-th pixel as follows:

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$$i_j(t) = m_j(t_0) + \mathrm{Re}[\boldsymbol{A}_j(t_0)]\cos\Delta\varphi_k(t) - \mathrm{Im}[\boldsymbol{A}_j(t_0)]\sin\Delta\varphi_k(t)\,, \tag{3}$$

where $m_j(t_0)$ and $\boldsymbol{A}_j(t_0)$ are the mean intensity and the modulation amplitude in the initial condition of the j-th pixel, whereas $\Delta\varphi_k(t)$ is the phase variation of the whole region after the deformation. If N pixels fall in this region around the k-th pixel Eq. (3) can be arranged in the following matrix form:

$$\begin{bmatrix} \mathrm{Re}[\boldsymbol{A}_1(t_0)] & -\mathrm{Im}[\boldsymbol{A}_1(t_0)] \\ \vdots & \vdots \\ \mathrm{Re}[\boldsymbol{A}_j(t_0)] & -\mathrm{Im}[\boldsymbol{A}_j(t_0)] \\ \vdots & \vdots \\ \mathrm{Re}[\boldsymbol{A}_N(t_0)] & -\mathrm{Im}[\boldsymbol{A}_N(t_0)] \end{bmatrix}_k \begin{Bmatrix} \cos\Delta\varphi_k(t) \\ \sin\Delta\varphi_k(t) \end{Bmatrix} = \begin{Bmatrix} i_1(t) - m_1(t_0) \\ \vdots \\ i_j(t) - m_j(t_0) \\ \vdots \\ i_N(t) - m_N(t_0) \end{Bmatrix}_k \Rightarrow \quad \mathbf{C}_k\ \mathbf{x}_k = \Delta\mathbf{I}_k\,. \tag{4}$$

These equations can be visualized in the complex plane as shown in Fig. 2, where for each pixel the complex modulation is reported before and after the phase variation $\Delta\varphi_k$ occurs. The real and the imaginary parts of the complex modulation in the reference condition are the two columns of the matrix $\mathbf{C}_k$, the difference between the light intensity in the deformed configuration and the mean intensity in the reference condition is the known vector $\Delta\mathbf{I}_k$ and it is the real part of the complex modulation in the deformed configuration, as shown in Fig. 2.

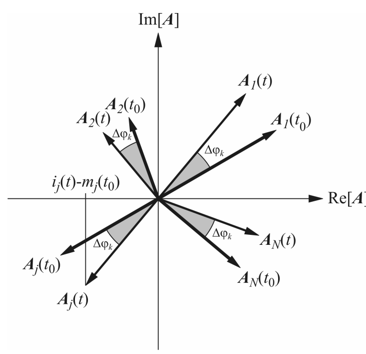

Fig. 2. The complex modulation of the pixels which fall within a small region around the k-th pixel.

The unknown vector of Eq. (4) is evaluated by least square method applied on the overdetermined linear system of equations, as follows:

$$\begin{Bmatrix} \cos\Delta\varphi_k(t) \\ \sin\Delta\varphi_k(t) \end{Bmatrix} = \mathbf{C}_k^{*}\ \Delta\mathbf{I}_k\,, \tag{5}$$

where the notation * indicates the pseudoinverse matrix [17] available in many software environments. Finally the phase of the k-th pixel can be retrieved by applying the two-argument **ArcTan** function implemented in most of the programming languages [18,19] and able to take into account which quadrant the angle is in, and so to overcome the ambiguity on the sign.

It must be pointed out that the matrix $\mathbf{C}_k$ depends only on the complex modulation (i.e. the modulation amplitude and the phase) in the reference condition, then the phase of the deformed surface can be retrieved by multiplying the pseudoinverse of the matrix $\mathbf{C}_k$ by the vector $\Delta\mathbf{I}_k$, which is calculated by subtracting the light intensities of the single step acquired in the deformed configuration by the corresponding mean intensity in the reference condition. Obviously this operation must be carried out for each pixel.

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3. Application of the procedure and experimental results

The one-step approach proposed in the present paper was applied to a specimen simulating the typical deformation field on a mechanically stressed debonding. This type of specimen was obtained by cutting a through hole in a thick plate and by sealing this hole on one side by a thin plate and on the other by a pressure meter. The out-of-plane component of displacement was acquired by a Michelson interferometer working by large shear and equipped with a phase-shifter calibrated in-situ by a procedure defined by the authors in a recent paper [20]. The light source is a low-cost laser diode Hitachi model HL6512.

Initially the one-sep approach was tested by varying the shape and the dimension of the region around the considered pixel, which is called in the following kernel due to the convolution function used in the implementation. Figure 3 compares the phase map obtained by a conventional phase-shifting procedure, as that reported in Fig. 3(f), with those obtained by different kernels, which are indicated in the upper-right corner of each map. The minimum number of pixels necessary for the application of the procedure is two, as in the case of Fig. 3(a) where the quality of the map is very poor. By increasing the kernel dimension the quality of the phase maps improves; on the other hand the increasing "speckleness" of the map with the kernel dimension reveals that the proposed algorithm behaves like a sort of spatial filter, thus it suggests to do not use very large kernel which could alter the original information. A good compromise between the quality of the phase maps and filtering effect is attained using as kernel 3x3 matrices, as in the case of Fig. 3(c) and Fig. 3(d).

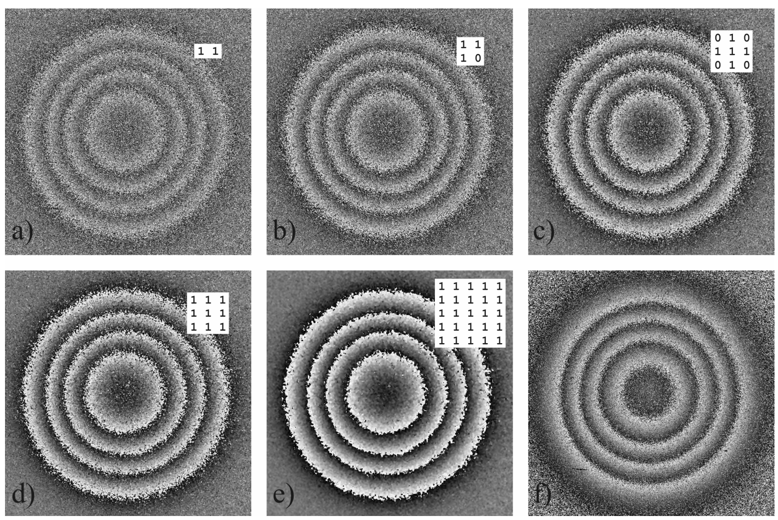

Fig. 3. Phase maps obtained by: a), b), c), d), e) the "one-step" approach with different kernels, the kernel is reported in the upper-right corner of each map; f) a conventional temporal phase-shifting algorithm.

Subsequently the algorithm was applied to a transient phenomenon obtained by acquiring at the frame rate of 30 fps the speckle pattern scattered by the specimen, when a pressure is manually – and irregularly – cycled. The video was acquired by a IEEE 1394 camera Sony model XCD-X710 with a resolution of 1024x768 and a nominal pixel dimension of 4.65x4.65 μm (the speckle size is about half of the pixel size). In the application, after the speckle was resolved in the initial condition by a conventional phase-shifting procedure, a 2.63 *s* video was acquired, corresponding to 79 frames; these were then analyzed one by one by the one-step algorithm. Figure 4 reports three representative frames of the multimedia file (linked to the figure) generated by the processing of the aforementioned video. The region of interest of each frame is an area of 640x640 pixels.

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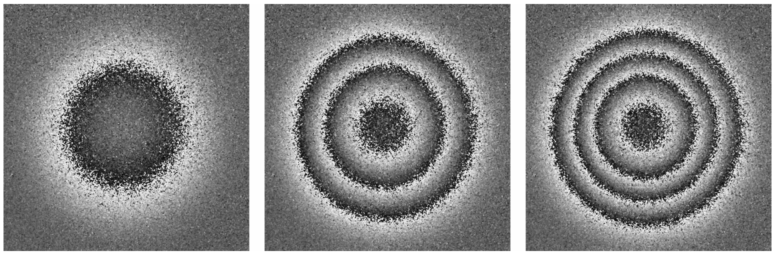

Fig. 4. (2.35 MB) Three different frames of a movie of a transient phenomenon obtained by a video acquired at the standard frame rate of 30 fps. The video, which lasts 2.63 *s*, was built by applying the one-step procedure on 79 frames of 640x640 pixels.

The processing of the speckle patterns was carried out in the *Mathematica*® environment by the two functions reported in Tab. 1. The inputs of first function (**`GlobalPseudoInverse`**) are: the kernel (*`ker`*) which is a matrix like those reported in Fig. 3, the modulation amplitude (*`ia0`*) and the phase (*`phi0`*) in the reference condition. This function generates for each pixel of the image a 2xN matrix, which represents the pseudoinverse on the matrix $\mathbf{C}_k$, where N is the number of pixels identified by the kernel. These data (*`pseudos`*), the kernel (*`ker`*), the mean intensity in the reference condition (*`im0`*) and the frame in the deformed configuration (*`newIm`*) are the inputs of the second function (**`OneStep`**), which evalute pixel by pixel the phase by means of the two-argument built-in **`ArcTan`**.

These functions are not particularly optimized, but substantial improvements can be attained by implementing them by a low-level programming language. If the observed phenomenon does not vary too much rapidly, a real-time implementation of the proposed approach is conceivable.

Table 1. Functions implemented in *Mathematica*® for performing the one-step algorithm.

```
GlobalPseudoInverse[ker_, ia0_, phi0_] := Module[{Mc, x1, Ms, x2},
  Mc = ia0 Cos[phi0]; Ms = -ia0 Sin[phi0];
  x1 = Map[Flatten, ListConvolve[ker, Mc, {-1, 1}, Mc, Times, List], {2}];
  x2 = Map[Flatten, ListConvolve[ker, Ms, {-1, 1}, Ms, Times, List], {2}];
  Map[PseudoInverse[Transpose[#]] &, Transpose[{x1, x2}, {3, 1, 2}], {2}]];
```

```
OneStep[ker_, pseudos_, im0_, newIm_] := Module[{Mi, b},
  Mi = (newIm - im0);
  b = Map[Flatten, ListConvolve[ker, Mi, {-1, 1}, Mi, Times, List], {2}];
  Map[ArcTan @@ (Dot @@ #) &, Transpose[{pseudos, b}, {3, 1, 2}], {2}]];
```

4. Conclusions

The paper presents a novel phase-shifting approach for speckle interferometry able to retrieve the phase information by a single frame in the deformed configuration, provided that the integrated speckle field is resolved, by a conventional phase-shifting procedure, in the reference condition. The approach, implemented in *Mathematica*® environment, is particularly suitable for studying those phenomena rapidly varying in the time. At the present the procedure is not able to work in real-time, but it can be applied at the end of the measuring session on a stored video. In the paper the approach was used for observing the out-of-plane displacements of a debonding cyclically stressed in order to simulate a transient phenomenon.

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