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Joint Impact of Hardware Impairments and Imperfect Channel State Information on Multi-Relay Networks

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ABSTRACT In this paper, we investigate the performance of dual-hop (DH) decode-and-forward (DF) multi-relay networks, for which two practical deleterious factors are taken into account, namely hardware impairments (HIs) and imperfect channel state information (ICSI). The communication between the source and the destination is realized with the aid of DF multi-relays, where both hops are assumed to be independent but non-identically distributed α–μ fading. Aiming at improving the system performance, three representative relay selection strategies are considered, in which the best relay is selected according to the link quality of source-to-relay and/or relay-to-destination. To characterize the performance of the proposed strategies, two key performance metrics, namely outage probability (OP) and ergodic capacity (EC), are analyzed insightfully. We first derive closed-form expressions for both exact and asymptotic OPs. Utilizing the derived results, diversity orders achieved at the destinations are obtained. We demonstrate that the OPs of considered networks are limited by HIs and ICSI, and the diversity orders are zeros due to the presence of ICSI. Then, we study the ECs of the proposed relay selection schemes, and upper bounds for the EC and asymptotic expressions for the EC in the high signal-to-noise ratio (SNR) regime are derived. To obtain more insights, the affine expansions for the EC are involved by two metrics of high-SNR slope and high-SNR power offset. It is shown that there are rate ceilings for the EC due to HIs and ICSI, which result in zero high-SNR slopes and finite high-SNR power offsets.

INDEX TERMS $\alpha-\mu$ fading channels, hardware impairments, imperfect CSI, relay selection.

I. INTRODUCTION

A. BACKGROUND

Cooperative communication has been identified as one of the core technologies in the current and future wireless communication networks [1]. With the aid of relays, we can potentially extend the coverage of wireless networks, improve quality of service (QoS) and reduce energy consumption [2]. For these reasons, it thus has attracted considerable interests from both academia [3] and industry [4]. However, when multiple relays are deployed in the wireless networks, it may induce extra inter-relay interference and higher resource consumption as the number of relays increasing [5]. Relay selection has been regarded as a promising countermeasure to improve the spectral efficiency and mitigate interference between relays.

In this context, a variety of relay selection strategies have been investigated, e.g. see [5]–[7] and the reference therein. Among various selection strategies, opportunistic relay selection (ORS) and partial relay selection (PRS) are the two of the most prevalent ones. The pioneering work of ORS was

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originally proposed by Bletsas *et al.* in [8]. Soon after Bletsas' work, researchers proposed different generalization strategies. In [9], Zhao *et al.* proposed an optimal relay selection criterion by selecting a relay with the largest instantaneous end-to-end signal-to-noise ratio (SNR). A multi-source path distributed selection strategy was proposed by Beres *et al.* in [10]. It was demonstrated that the proposed strategy outperforms distributed space-time codes for cooperative networks with more than three relaying nodes. In [11], analytical closed-form expressions for the outage probability (OP) and bit error probability of uncoded threshold-based ORS were derived for arbitrary SNR, as well as arbitrary number of available decode-and-forward (DF) relays. Considering security issue, Liu *et al.* proposed four relay selection strategies for secure communication in cognitive DF relaying networks [12], where new closed-form expressions for secrecy OP (SOP) were derived.

The dominant feature of ORS is that full channel state information (CSI) of both source-to-relay and relay-to-destination links are required, which brings extra signaling overhead and higher power consumption. To mitigate this problem, a PRS strategy was proposed by Krikidis *et al.* in [13], where the best relay is selected according to the CSI of only the source-to-relay or relay-to-destination. Regarding the interference-limited environment, closed-form expressions of OP for PRS strategy were derived by [14], where the best relay is selected based on the CSI of the first hop. Considering imperfect CSI (ICSI), Lee [15] proposed an efficient PRS (EPRS) strategy to improve the system performance, where the candidate relay is determined by both statistical and instantaneous CSI. In [16], Suraweera *et al.* analyzed the impact of outdated CSI on the performance of PRS AF relaying networks. Apart from the above works, three novel PRS schemes for CSI-assisted dual hop (DH) AF relying networks over Nakagami-*m* were proposed by Chen *et al.* in [17], where a relay is selected according to the channel magnitudes.

One common characteristic of the aforementioned literature is ideal hardware and perfect CSI at transceivers. Unfortunately, both assumptions are idealistic for practical applications. In practice, radio frequency (RFs) components suffer from several types of imperfections, such as phase noise [18], in-phase and quadrature-phase imbalance (IQI) [19], amplifier nonlinearity [20] and quantization errors [21]. Although the above impairments may be somehow mitigated by using some appropriate compensation algorithms and calibration methods, there still remain some residual hardware impairments (HIs) due to estimation errors, inaccurate calibration methods and different types of noise [22]. There are some related research contributions on the performance analysis of cooperative communication with HIs (see [23]–[29] and reference therein). In [23], Matthaiou *et al.* analyzed the impact of aggregated transceiver HIs on two-way AF relay networks, where analytical expressions for the OP and symbol error ratio were presented. Considering generic Nakagami-*m* fading channels, Björnson *et al.* in [24] investigated the performance of DH AF and DF relaying networks in the presence of HIs, and derived closed-form expressions for the OP and ergodic capacity (EC). A joint source/relay precoding scheme for MIMO two-way AF relay networks with minimum mean square error (MMSE) criterion was proposed by You *et al.* [25], where the wireless security against eavesdropping attack was improved. Considering multi-antenna systems, [26] analyzed the EC of AF relay networks in presence of hardware impairments. To compensate the performance loss brought by hardware impairments, an optimal beamforming scheme was designed under the condition of sum-power constraint and per-antenna power constraint [27]. Inspired by cognitive spatial modulation systems, authors in [28] derived closed-form expression of the average pairwise error probability and a tight upper bound of the average bit error rate for underlay spectrum-sharing systems with HIs. More particularly, Duy *et al.* [29] studied the effects of ORS and PRS strategies on DH DF relaying networks in presence of HIs and co-channel interference (CCI). However, the major limitation of the previous research contributions is perfect CSI is available at the receivers. In practice, ICSI is inevitable due to quantization error, estimation errors, limited feedback, and short coherence time [30]. Some recent research works have attempted to study the effect of ICSI on wireless communication systems [31], [32]. Assuming channel estimation error is not available, an optimal beamforming vector was obtained by solving a semi-definite programming problem with S-Procedure method [31]. Authors in [32] investigated the combined impact of CCI, ICSI, pilot contamination and antenna correlation on the performance of two-way relay networks. Therefore, it is important to look into the realistic scenario with ICSI.

B. MOTIVATIONS AND RELATED WORKS

The previous research contributions have laid a solid foundation and provided a good understanding for the impact of HIs on single relay networks, while works for investigating joint impact of HIs and ICSI on relay selection strategies are still in their infancy. Some related works have been appeared in [29], [33], [34]. Considering the ORS strategy, Guo *et al.* in [33] investigated the performance of two-way multi-relay networks in the presence of HIs. In [29], Duy *et al.* analyzed the impact of HIs and CCI on DF relaying networks for ORS and PRS strategies, where new exact and asymptotic closed-form expressions for the OP and the EC were derived. The main limitation of [29], [33] is that perfect CSI is assumed at both transmitter and receiver. Recently, the joint impact of HIs and ICSI has been studied in [35]–[38]. The authors in [35] analyzed the joint impact of HIs and ICSI on the outage performance of point-to-point MIMO systems with SIC detections. In [36], the authors studied the performance of a multiuser communication system in rank-1 Rician fading channels, assumed both ICSI and transceiver HIs. The performance evaluation of orthogonal frequency division multiplexing with index modulation (OFDM-IM) under channel estimation error and HIs was investigated in [37]. In [34], Solanki *et al.* analyzed joint impact of HIs and channel estimation error on OP of spectrum sharing DF multiple-relay networks, where the reactive relay selection was adopted. The performance of opportunistic transmission in downlink DF relay network in presence of RF impairment and channel estimation error was investigated in [38].

While how to analyze the performance of cooperative multi-relay networks with HIs and ICSI over various fading channels is still an open research area. The successful attempts have been published in [23]–[26] and leveraged well-known properties of Gamma variables. However, these aforementioned literature is carried on the assumption of homogeneous fading environments.

C. CONTRIBUTIONS

Motivated by the aforementioned discussions, in this treatise, we present a comprehensive investigation on the performance of cooperative DH DF multi-relay networks in the presence of HIs and ICSI, where the general $\alpha-\mu$ fading channel is assumed at both hops since it has been widely used to characterize the nature of non-homogeneous fading environments [39]. Based on different parameter settings, this versatile model includes many types of fading channels. For instance, Rayleigh ($\alpha = 2, \mu = 1$), one-side Gaussian, Weibul ($\alpha = m, \mu = 1$), Nakagami-m ($\alpha = 2, \mu = m$) and Gamma ($\alpha = 1$). Also, $\alpha-\mu$ distribution can capture the large-scale fading channels and/or composite fading channels with the schemes considered in [40], [41]. Moreover, three representative relay schemes are considered insightfully, namely, random relay selection (RRS), ORS and PRS.[1] We aim at quantifying the impact of HIs and ICSI on cooperative multi-relay networks over $\alpha-\mu$ fading channels for these three selection strategies. The main contributions of this paper are summarized as follows:

- Considering two practical and detrimental imperfections, viz., HIs and ICSI, three proactive relay selection strategies are considered, where a single relay is selected prior to the source transmission participates in cooperation. RRS is presented as a benchmark for the purpose of comparison, in which the relay is selected randomly. In ORS, the optimal relay is selected according to the link quality of both source-to-relay and relay-to-destination. To achieve the balance between performance and complexity, PRS is proposed according to the link quality either source-to-relay or relay-to-destination.
- We derive new analytical expressions of OP for RRS, ORS and PRS strategies in closed-form. In order to obtain more insights, the asymptotic behaviors at high SNRs are also explored. It is shown that there are error floors of OP for the proposed strategies due to ICSI. For RRS, the asymptotic OP in the high SNR region is a constant, which only depends on the fading parameters.
- We study the diversity orders at high SNRs for the OP of the three strategies. We demonstrate that for imperfections, the diversity orders for three schemes are zeros due to channel estimation error. For ideal conditions, the diversity orders depend on the number of relays and fading parameters for ORS and PRS, while for RRS, the diversity order is only determined by fading parameters.
- We derive closed-form expressions on upper bound of the EC for the proposed three selection strategies. To obtain more insights, the asymptotic analysis at high SNRs for the EC is carried out. We demonstrate that the EC is limited by the distortion noise and estimation error, which resulted in EC ceilings.
- We analyze the *high-SNR slopes* and *high SNR-power offsets*.[2] As will be shown, three relay selection schemes have the same slope because the slope is unaffected by fading parameters, HIs and channel estimation error, while their capacities may be very different due to sizeable disparities in the power offset. We demonstrate that for non-ideal conditions, owing to these imperfections, the *high-SNR slopes* and *high-SNR power offsets* are zeros and infinities for these three strategies, respectively; for ideal conditions, *high-SNR slopes* are $1/2$ for all three strategies, while *the high SNR power offsets* are constants, which depend on the number of relays and fading parameters.

D. ORGANIZATION AND NOTATIONS

The remainder of this paper is organized as follows: In Section II, the general HI model of DH DF relaying networks is outlined in the presence of HIs and ICSI. In Section III, the analytical closed-form expressions for OPs, high SNR asymptote and diversity orders are derived and analyzed. In Section IV, the EC performance and high SNR analysis are presented. In Section V, a set of numerical results and key findings are articulated to corroborate our theoretical analysis. In Section VI, we conclude the paper.

We use $\mathbb{E}\{\cdot\}$ and $\triangleq$ to denote the expectation and definition operations, respectively. A complex Gaussian random variable with mean μ and variance σ^2 reads as $\mathcal{CN}\{\mu, \sigma^2\}$. Notation $\Gamma\{\cdot\}$ represents the Gamma function, while $n!$ is the factorial operation, while $f_X(\cdot)$ and $F_X(\cdot)$ are the probability density function (PDF) and the cumulative distribution function (CDF) of a random variable, respectively.

II. SYSTEM MODEL

We consider a general relay network. There is one source S, N relays $R = \{R_1, R_2, \cdots, R_N\}$ and one destination D, where all the nodes are equipped with single-antenna.

[1]Relay selection scheme can be performed proactively before transmission or reactively after transmission. Both of them are outage-optimal [42], but proactive relaying scheme can achieve slightly better effective ergodic capacity [43]. Therefore, we adopt proactive relay selection strategy in this paper.

[2]Note that the high SNR slope is also referred to as the number of degrees of freedom or the maximum multiplex gain [44].

The communication between S and D is established only via one selected relay from the N relays, where the direct link between S and D does not exist due to obstacles and/or severe shadowing.[3] The information transmission between S and D is completed in two time slots: In the first time slot, S transmits the signal to the selected relay R_n, $1 \le n \le N$; In the second time slot, R_n decodes the received signal and forwards to D.

In practice, perfect CSI is supposed to be unavailable due to several factors such as estimation error and feedback error, etc.[4] Some channel estimation algorithms are needed to obtain estimated channel $\hat{g}_i$ of g_i. We assume g_i and $\hat{g}_i$ $i \in \{SR_n, R_nD\}$ are jointly ergodic and stationary process. Utilizing linear MMSE estimator, the channel coefficient can be modeled as $g_i = \hat{g}_i + e_i, i = \{SR_n, R_nD\}$, where e_i is channel estimation error with complex Gaussian distribution $e_i \sim \mathcal{CN}\left(0, \sigma_{e_i}^2\right)$ [39].[5] Considering the HIs model in [24] and channel estimation model in [39], the received signal at R_n, y_{SR_n} and the received signal at D, y_{R_nD} can be given by

$$y_i = \left(\hat{g}_i + e_i\right)\left(s_i + \eta_{t,i}\right) + \eta_{r,i} + \nu_i, i = \{SR_n, R_nD\}, \quad (1)$$

where $P_i = \mathbb{E}_{s_i}\left\{|s_i|^2\right\}$ is the average signal power; $\nu_i \sim \mathcal{CN}(0, N_i)$ represents the complex Gaussian receiver noise; $\eta_{t,i}$ and $\eta_{r,i}$ are the distortion noises from aggregated hardware impairments at the transmitter and the receiver, which are respectively defined as

$$\eta_{t,i} \sim \mathcal{CN}\left(0, \kappa_{t,i}^2 P_i\right), \quad \eta_{r,i} \sim \mathcal{CN}\left(0, \kappa_{r,i}^2 P_i |g_i|^2\right), \quad (2)$$

where $\kappa_{t,i}$ and $\kappa_{r,i}$ are the levels of HIs from the transmitter and the receiver with $\kappa_{t,i}, \kappa_{r,i} \ge 0$, respectively. $\kappa_{t,i}$ and $\kappa_{r,i}$ are related to the error vector magnitude (EVM). In wireless communication, EVM is a common parameter to characterize the quality of RF transceiver and is defined as the magnitude of the mismatch between the desired signal and actual signal RF [46]. For a given channel g, the aggregated distortion at the receiver has power

$$\mathbb{E}_{\eta_t,\eta_r}\left\{\left|g\eta_{t,i} + \eta_{r,i}\right|^2\right\} = P_i|g|^2\left(\kappa_{t,i}^2 + \kappa_{r,i}^2\right). \quad (3)$$

As stated in [24], [47], $\kappa_{t,i}$ is the complex Gaussian distribution, while $\kappa_{r,i}$ is only complex Gaussian distribution on the condition of a channel realization. That is, the real distribution is the product of the complex Gaussian distribution of the distortion noise and fading channel distribution.

Combining (2) with (3), (1) can be re-expressed as

$$y_i = \left(\hat{g}_i + e_i\right)\left(s_i + \eta_i\right) + \nu_i, \quad (4)$$

[3]Although our work focuses on the case of non-direct link between S and D, our results can be extended to that case as well.

[4]In this study, we assume that the feedback to the transmitter is zero delay and error free, which means that the transmitter has full estimated CSI whatever the receiver has.

[5]In fact, estimation error is the function of the SNR [45], e.g. $\sigma_{e_i}^2 \propto 1/(1+\bar{\lambda}_i)$, where $\bar{\lambda}_i, i = \{SR_n, R_nD\}$ is the average transmit SNR. To maintain mathematical tractability, we assume that e_i follows a zero-mean complex Gaussian random variable.

where $\eta_i \sim \mathcal{CN}\left(0, \kappa_i^2 P_i\right)$ is the distortion noise from HIs at transceiver with $\kappa_i \triangleq \sqrt{\kappa_{t,i}^2 + \kappa_{r,i}^2}$. When $\kappa_{t,i} = \kappa_{r,i} = 0$ and $\sigma_{e_{SR_n}}^2 = \sigma_{e_{R_nD}}^2 = 0$, (4) reduces to the ideal conditions as $y_i = g_i s_i + \nu_i$.

Defining estimated channel gain $\rho_i \triangleq |\hat{g}_i|^2$, as in [39], ρ_i follows the approximate $\alpha - \mu$ distribution whose PDF and CDF of the channel gain can be expressed as

$$f_{\rho_i}(x) = \frac{\alpha_i x^{\frac{\alpha_i\mu_i}{2}-1} e^{-\left(\frac{x}{\beta_i}\right)^{\frac{\alpha_i}{2}}}}{2\beta_i^{\frac{\alpha_i\mu_i}{2}}\Gamma(\mu_i)}, \quad x \ge 0, \quad (5)$$

$$F_{\rho_i}(x) = 1 - \sum_{m=0}^{\mu_i-1} \frac{e^{-\left(\frac{x}{\beta_i}\right)^{\frac{\alpha_i}{2}}}}{m!}\left(\frac{x}{\beta_i}\right)^{\frac{\alpha_i m}{2}}, \quad x \ge 0, \quad (6)$$

where $\alpha_i \le 0$ is the nonlinearity power exponent; $\mu_i \le 0$ is related to the number of multipath cluster; $\beta_i \triangleq \mathbb{E}\{x\}\Gamma(\mu_i)/\Gamma\left(\mu_i + 2/\alpha_i\right)$, $\mathbb{E}(x) = \hat{r}_i^2\Gamma(\mu_i + 2/\alpha)/(\mu_i^{2/\alpha_i}\Gamma(\mu_i))$, where $\hat{r}_i$ is defined as the $\hat{r}_i = \sqrt[\alpha_i]{\mathbb{E}(R^{\alpha_i})}$-root mean of the amplitude of random variable.

Thus, the signal-to-noise plus distortion ratio (SNDR) at R_n and D are respectively obtained as

$$\gamma_{SR_n} = \frac{\rho_{SR_n}}{\rho_{SR_n}\kappa_{SR_n}^2 + \sigma_{e_{SR_n}}^2\left(1 + \kappa_{SR_n}^2\right) + \frac{1}{\bar{\lambda}_{SR_n}}}, \quad (7)$$

$$\gamma_{R_nD} = \frac{\rho_{R_nD}}{\rho_{R_nD}\kappa_{R_nD}^2 + \sigma_{e_{R_nD}}^2\left(1 + \kappa_{R_nD}^2\right) + \frac{1}{\bar{\lambda}_{R_nD}}}, \quad (8)$$

where $\kappa_{SR_n} = \sqrt{\kappa_{t,SR_n}^2 + \kappa_{r,SR_n}^2}$ and $\kappa_{R_nD} = \sqrt{\kappa_{t,R_nD}^2 + \kappa_{r,R_nD}^2}$ are the aggregated levels of HIs from transceiver, $\bar{\lambda}_{SR_n} = P_{SR_n}/N_{SR_n}$ and $\bar{\lambda}_{R_nD} = P_{R_nD}/N_{R_nD}$ are the average transmitter SNRs at S and R_n. When $\kappa_{t,i} = \kappa_{r,i} = 0$ and $\sigma_{e_{SR_n}}^2 = \sigma_{e_{R_nD}}^2 = 0$, (7) and (8) tend to the ideal conditions.

According to the criterion of DF protocol [2], the SNDR is the minimum of SNDRs between $S \to R_n$ and $R_n \to D$. Therefore, the end-to-end SNDR is expressed as

$$\gamma^{\mathrm{DF}} = \min\left(\gamma_{SR_n}, \gamma_{R_nD}\right). \quad (9)$$

Then, the end-to-end SNDR for non-ideal conditions can be expressed as

$$\gamma^{\mathrm{DF,ni}} = \min\left(\frac{\rho_{SR_n}}{\rho_{SR_n}\kappa_{SR_n}^2 + \sigma_{e_{SR_n}}^2\left(1 + \kappa_{SR_n}^2\right) + \frac{1}{\bar{\lambda}_{SR_n}}}, \frac{\rho_{R_nD}}{\rho_{R_nD}\kappa_{R_nD}^2 + \sigma_{e_{R_nD}}^2\left(1 + \kappa_{R_nD}^2\right) + \frac{1}{\bar{\lambda}_{SR_n}}}\right). \quad (10)$$

For ideal conditions, the end-to-end SNDR reduces to

$$\gamma^{\mathrm{DF,id}} = \min\left(\bar{\lambda}_{SR_n}\rho_{SR_n}, \bar{\lambda}_{R_nD}\rho_{R_nD}\right). \quad (11)$$

Different from the ideal conditions in (11), the end-to-end SNDR in (10) not only depends on fading parameters, but also depends on distortion noise and estimation error.

III. OUTAGE PROBABILITY ANALYSIS

In this section, by considering HIs and ICSI, we pursue the outage analyses of multi-relay networks over α-μ fading channels for these three selection strategies.

A. RANDOM RELAY SELECTION

For the completeness of analysis, we first consider the RRS as a benchmark for the purpose of comparison. The performance of DH DF multi-relay networks in terms of OP for RRS is investigated.

1) OUTAGE PROBABILITY

Referring to [3], OP is defined as the probability of the end-to-end SNDR below a certain threshold, γ_{th}, as

$$P_{\text{out}}(\gamma_{\text{th}}) \triangleq \Pr\{\gamma \le \gamma_{\text{th}}\}, \tag{12}$$

For RRS, one relay is selected randomly. Thus, the received SNDR of the selected relay R_n in the presence of HIs and ICSI can be formulated as

$$\gamma^{\text{DF,ni}} = \min_{i=SR_n,R_nD}\left(\frac{\rho_i}{\rho_i\kappa_i^2+\sigma_{e_i}^2\left(1+\kappa_i^2\right)+\frac{1}{\bar{\lambda}_i}}\right). \tag{13}$$

For ideal conditions, the effective SNDR is obtained as

$$\gamma^{\text{DF,id}} = \min_{i=SR_n,R_nD}\left(\rho_i\bar{\lambda}_i\right). \tag{14}$$

Based on the above definitions, the following theorem presents the OPs of DH DF multi-relay networks over $\alpha-\mu$ fading channels.

Theorem 1: For $\alpha-\mu$ fading channels, the analytical expressions for the OP are given as[6]

- *Non-ideal conditions ($\kappa_{SR_n} = \kappa_{R_nD} \neq 0$, $\sigma_{e_{SR_n}}^2 = \sigma_{e_{R_nD}}^2 \neq 0$)*

$$P_{out}^{RRS,ni}(\gamma_{th}) = 1-\sum_{m=0}^{\mu_{SR_n}-1}\sum_{l=0}^{\mu_{R_nD}-1}\frac{\theta_n^{\frac{\alpha_{SR_n}m}{2}}\phi_n^{\frac{\alpha_{R_nD}l}{2}}}{m!l!}\times e^{-\left(\theta_n^{\frac{\alpha_{SR_n}}{2}}+\phi_n^{\frac{\alpha_{R_nD}}{2}}\right)}, \tag{15}$$

for $\gamma_{th} < 1/\max\left(\kappa_{SR_n}^2,\kappa_{R_nD}^2\right)$, and $P_{out}^{RRS,ni}(\gamma_{th}) = 1$ for $\gamma_{th} \ge 1/\max\left(\kappa_{SR_n}^2,\kappa_{R_nD}^2\right)$.

- *Ideal conditions ($\kappa_{SR_n} = \kappa_{R_nD} = 0$, $\sigma_{e_{SR_n}}^2 = \sigma_{e_{R_nD}}^2 = 0$)*

$$P_{out}^{RRS,id}(\gamma_{th}) = 1-\sum_{m=0}^{\mu_{SR_n}-1}\sum_{l=0}^{\mu_{R_nD}-1}\frac{e^{-\left(\varphi_n^{\frac{\alpha_{SR_n}}{2}}+\vartheta_n^{\frac{\alpha_{R_nD}}{2}}\right)}}{m!l!}\times\frac{\gamma_{th}^{\frac{\alpha_{SR_n}m+\alpha_{R_nD}l}{2}}}{\left(\beta_{SR_n}\bar{\lambda}_{SR_n}\right)^{\frac{\alpha_{SR_n}m}{2}}\left(\beta_{R_nD}\bar{\lambda}_{R_nD}\right)^{\frac{\alpha_{R_nD}l}{2}}}, \tag{16}$$

with $\theta_n = \frac{c_1\gamma_{th}}{\beta_{SR_n}(a_1-b_1\gamma_{th})}$, $\phi_n = \frac{c_2\gamma_{th}}{\beta_{R_nD}(a_2-b_2\gamma_{th})}$, $\varphi_n = \frac{\gamma_{th}}{\beta_{SR_n}\bar{\lambda}_{SR_n}}$, $\vartheta_n = \frac{\gamma_{th}}{\beta_{R_nD}\bar{\lambda}_{R_nD}}$,[7] where $a_1 = 1$, $a_2 = 1$, $b_1 = \kappa_{SR_n}^2$, $b_2 = \kappa_{R_nD}^2$, $c_1 = \sigma_{e_{SR_n}}^2\left(1+\kappa_{SR_n}^2\right)+\frac{1}{\bar{\lambda}_{SR_n}}$, $c_2 = \sigma_{e_{R_nD}}^2\left(1+\kappa_{R_nD}^2\right)+\frac{1}{\bar{\lambda}_{R_nD}}$.

Proof: The detailed proof is provided in Appendix A. □

Although the derived expressions can be expressed in closed-form and efficiently evaluated, it does not provide useful insights into the implications of parameters on OP. To this end, the asymptotic outage behaviors at high SNRs are explored in the following corollary.

Corollary 1: At high SNRs ($\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD} \to \infty$), the asymptotic OPs approach to

- *Non-ideal conditions*

$$P_{out}^{RRS,ni}(\gamma_{th}) = \frac{\theta_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}}{\Gamma\left(\mu_{SR_n}+1\right)}+\frac{\phi_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma\left(\mu_{R_nD}+1\right)}-\frac{\theta_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}\phi_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma\left(\mu_{SR_n}+1\right)\Gamma\left(\mu_{R_nD}+1\right)}, \tag{17}$$

for $\gamma_{th} < 1/\max\left(\kappa_{SR_n}^2,\kappa_{R_nD}^2\right)$, and $P_{out}^{RRS,id}(\gamma_{th}) = 1$ for $\gamma_{th} \ge 1/\max\left(\kappa_{SR_n}^2,\kappa_{R_nD}^2\right)$.

- *Ideal conditions*

$$P_{out}^{RRS,id}(\gamma_{th}) = \frac{\varphi_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}}{\Gamma\left(\mu_{SR_n}+1\right)}+\frac{\vartheta_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma\left(\mu_{R_nD}+1\right)}-\frac{\varphi_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}\vartheta_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma\left(\mu_{SR_n}+1\right)\Gamma\left(\mu_{R_nD}+1\right)}. \tag{18}$$

Proof: Using the similar methodology of [48], (5) can be expanded as Taylor series. When $\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD} \to \infty$, only the first summation term of infinite series is the dominant term. Thus, (5) and (6) can be further respectively simplified as

$$f_{\rho_i}(x) = \frac{\alpha x^{\frac{\alpha_i\mu_i}{2}-1}}{2\beta_i^{\frac{\alpha_i\mu_i}{2}}\Gamma(\mu_i)}+o(x),\ x\ge 0, \tag{19}$$

$$F_{\rho_i}(x) = \frac{1}{\Gamma(\mu_i+1)}\left(\frac{x}{\beta_i}\right)^{\frac{\alpha_i\mu_i}{2}}+o(x),\ x\ge 0. \tag{20}$$

Utilizing the results of (19) and (20), the CDFs of (7) and (8) can be obtained as

$$F_{\gamma_{SR_n}}(\gamma_{th}) \approx \frac{\theta_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}}{\Gamma\left(\mu_{SR_n}+1\right)}, \tag{21}$$

$$F_{\gamma_{R_nD}}(\gamma_{th}) \approx \frac{\phi_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma\left(\mu_{R_nD}+1\right)}. \tag{22}$$

Substituting (21) and (22) into (A.2), we can obtain the result of (17) after some simplifications. Let $\kappa_{SR_n} = \kappa_{R_nD} = 0$, $\sigma_{e_{SR_n}} = \sigma_{e_{R_nD}} = 0$, we can obtain the result of (18). □

[6] All through this paper, we have $\kappa_{SR_n} = \kappa_{R_nD} \neq 0$, $\sigma_{e_{SR_n}}^2 = \sigma_{e_{R_nD}}^2 \neq 0$ and $\kappa_{SR_n} = \kappa_{R_nD} = 0$, $\sigma_{e_{SR_n}}^2 = \sigma_{e_{R_nD}}^2 = 0$ for non-ideal conditions and ideal conditions, respectively.

[7] For convenience, these variable transformations will be extensively used in this paper.

2) DIVERSITY ORDER

To get deeper insights, we examine the diversity orders in term of OP. According to [49], the diversity order is defined to the following standard formula

$$d\left(\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD}\right) = -\lim_{\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD} \to \infty} \frac{P_{\text{out}}^{i}\left(\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD}\right)}{\log\left(\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD}\right)}, \quad i = \{RRS, ORS, PRS\}. \tag{23}$$

Based on the above definition, the diversity orders at high SNR regime for RRS are provided in the following corollary.

Corollary 2: At high SNRs, the diversity orders for the OP are given as

- *Non-ideal conditions*

$$d^{RRS,ni}\left(\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD}\right) = 0, \tag{24}$$

- *Ideal conditions*

$$d^{RRS,id}\left(\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD}\right) = \min\left(\frac{\alpha_{SR_n}\mu_{SR_n}}{2}, \frac{\alpha_{R_nD}\mu_{R_nD}}{2}\right). \tag{25}$$

Proof: For non-ideal conditions, the dominant terms of (17) are the first and second summation as

$$P_{\text{out}}^{\text{RRS,ni}}(\gamma_{th}) = \frac{1}{\Gamma\left(\mu_{SR_n}+1\right)}\theta_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}} + \frac{1}{\Gamma\left(\mu_{R_nD}+1\right)}\phi_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}. \tag{26}$$

Note that (26) is a constant, which is irrelative to $\bar{\lambda}_{SR_n}$ and $\bar{\lambda}_{R_nD}$. Substituting (26) into (23), we can obtain (24).

For ideal conditions, the dominant terms of (18) are the first and second summation as

$$P_{\text{out}}^{\text{RRS,id}}(\gamma_{th}) = \frac{\varphi_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}}{\Gamma\left(\mu_{SR_n}+1\right)} + \frac{\vartheta_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma\left(\mu_{R_nD}+1\right)}, \tag{27}$$

the asymptotic OP can be re-expressed as

$$P_{\text{out}}^{\text{RRS,id}}(\gamma_{th}) \propto \left(\frac{1}{\bar{\lambda}_{SR_n}}\right)^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}} \text{ or } \left(\frac{1}{\bar{\lambda}_{R_nD}}\right)^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}, \tag{28}$$

Substituting (28) into (23), we can obtain (25). □

Remark 1: Corollary 1 and Corollary 2 have provided some insights on the derived analytical results. For non-ideal conditions, there exists a infinite upper bound for the effective SNDR at high SNR by $1/\max\left(\kappa_{SR_n}^2, \kappa_{R_nD}^2\right)$ *as* $P_{out}^{RRS,ni} = 1$ *if* $\gamma_{th} \geq 1/\max\left(\kappa_{SR_n}^2, \kappa_{R_nD}^2\right)$. *When* $\gamma_{th} \leq 1/\max\left(\kappa_{SR_n}^2, \kappa_{R_nD}^2\right)$, *as the average SNR grows infinity, the asymptotic OP is lower-bounded by the floor* $\frac{\theta_n^{\alpha_{SR_n}\mu_{SR_n}/2}}{\Gamma(\mu_{SR_n}+1)} + \frac{\phi_n^{\alpha_{R_nD}\mu_{R_nD}/2}}{\Gamma(\mu_{R_nD}+1)} - \frac{\theta_n^{\alpha_{SR_n}\mu_{SR_n}/2}\phi_n^{\alpha_{R_nD}\mu_{R_nD}/2}}{\Gamma(\mu_{SR_n}+1)\Gamma(\mu_{R_nD}+1)}$, *which results in zero diversity order. In addition, hardware impairments and imperfect CSI degrade the outage performance as the OP increases when the values of distortion noise and channel estimation error increase. For ideal conditions, the asymptotic OP grows without bound as SNR increases, which depends on fading parameters and average SNR. In this case, the diversity order is* $\min\left(\alpha_{SR_n}\mu_{SR_n}/2, \alpha_{R_nD}\mu_{R_nD}/2\right)$. *For RRS, the OPs for non-ideal and ideal conditions are irrelated to the number of relays.*

B. OPPORTUNISTIC RELAY SELECTION (ORS)

To reduce the implementation complexity and to improve the spectral efficiency, the ORS proposed by [8] is provided, where the best relay is selected to retransmit the source signal to the destination. In the following, the performance of DH DF multi-relay networks in terms of OP for ORS is investigated.

1) OUTAGE PROBABILITY

For ORS, the optimal relay R_{n^*} is selected according to the largest SNDRs of both $S \to R$ and $R \to D$.[8] The corresponding mathematical formula is given as

$$n^* = \arg\max_{1\leq n\leq N} \min\left(\gamma_{SR_n}, \gamma_{R_nD}\right). \tag{29}$$

Based on the above definition, the OPs of multi-relay network over α-μ fading channels with transceiver HIs and ICSI are provided in the following theorem.

Theorem 2: For $\alpha - \mu$ *fading channels, the closed-form expressions for the OPs are given as*

- *Non-ideal conditions*

$$P_{out}^{ORS,ni}(\gamma_{th}) = \prod_{n=1}^{N}\left(1 - \sum_{m=0}^{\mu_{SR_n}-1}\sum_{l=0}^{\mu_{R_nD}-1}\frac{\theta_n^{\frac{\alpha_{SR_n}m}{2}}\phi_n^{\frac{\alpha_{R_nD}l}{2}}}{m!l!} \times e^{-\left(\theta_n^{\frac{\alpha_{SR_n}}{2}}+\phi_n^{\frac{\alpha_{R_nD}}{2}}\right)}\right), \tag{30}$$

for $\gamma_{th} < 1/\max\left(\kappa_{SR_n}^2, \kappa_{R_nD}^2\right)$, *and* $P_{out}^{ORS,ni}(\gamma_{th}) = 1$ *for* $\gamma_{th} \geq 1/\max\left(\kappa_{SR_n}^2, \kappa_{R_nD}^2\right)$.

- *Ideal conditions*

$$P_{out}^{ORS,id}(\gamma_{th}) = \prod_{n=1}^{N}\left(1 - \sum_{m=0}^{\mu_{SR_n}-1}\sum_{l=0}^{\mu_{R_nD}-1}\frac{e^{-\left(\varphi_n^{\frac{\alpha_{SR_n}}{2}}+\vartheta_n^{\frac{\alpha_{R_nD}}{2}}\right)}}{m!l!} \times \frac{\gamma_{th}^{\frac{\alpha_{SR_n}m+\alpha_{R_nD}l}{2}}}{\left(\beta_{SR_n}\bar{\lambda}_{SR_n}\right)^{\frac{\alpha_{SR_n}m}{2}}\left(\beta_{R_nD}\bar{\lambda}_{R_nD}\right)^{\frac{\alpha_{R_nD}l}{2}}}\right). \tag{31}$$

Proof: The detailed proof is provided in Appendix B. □

Similarly, the results of **Theorem 2** can be expressed in closed-form and efficiently evaluated, it does not offer useful insights into the implications of system parameters on the outage probability. To this end, the following corollary provides the asymptotic analyses at high SNR regime.

[8] Relay selection operation can be accomplished by a method of distributed timers as presented in [8], [42].

Corollary 3: At high SNRs ($\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD} \to \infty$), the asymptotic OPs approach to

- *Non-ideal conditions*

$$P_{out}^{ORS,ni}(\gamma_{th}) = \prod_{n=1}^{N}\left(\frac{\theta_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}}{\Gamma(\mu_{SR_n}+1)} + \frac{\phi_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma(\mu_{R_nD}+1)} - \frac{\theta_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}\phi_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma(\mu_{SR_n}+1)\Gamma(\mu_{R_nD}+1)}\right), \quad (32)$$

- *Ideal conditions*

$$P_{out}^{ORS,id}(\gamma_{th}) = \prod_{n=1}^{N}\left(\frac{\varphi_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}}{\Gamma(\mu_{SR_n}+1)} + \frac{\vartheta_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma(\mu_{R_nD}+1)} - \frac{\varphi_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}\vartheta_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma(\mu_{SR_n}+1)\Gamma(\mu_{R_nD}+1)}\right). \quad (33)$$

Proof: Follows by substituting (21) and (22) into (B.2). □

For i.i.d. $\alpha-\mu$ fading channels, the expressions for the OP are presented in the following corollary.

Corollary 4: For i.i.d. $\alpha-\mu$ fading channels, the OPs further reduce to

- *Non-ideal conditions*

$$P_{out}^{ORS,ni}(\gamma_{th}) = \left(1-\sum_{m=0}^{\mu-1}\frac{e^{-\left(\bar{\theta}_n^{\frac{\alpha}{2}}+\bar{\phi}_n^{\frac{\alpha}{2}}\right)}}{(m!)^2}\left(\bar{\theta}_n\bar{\phi}_n\right)^{\frac{\alpha m}{2}}\right)^N, \quad (34)$$

- *Ideal conditions*

$$P_{out}^{ORS,id}(\gamma_{th}) = \left(1-\sum_{m=0}^{\mu-1}\frac{e^{-2\left(\frac{\gamma_{th}}{\beta\bar{\lambda}}\right)^{\frac{\alpha}{2}}}}{(m!)^2}\left(\frac{\gamma_{th}}{\beta\bar{\lambda}}\right)^{\alpha m}\right)^N, \quad (35)$$

where $c'_1 = \sigma_{e_{SR_n}}^2\left(1+\kappa_{SR_n}^2\right)+\frac{1}{\bar{\lambda}}$, $c'_2 = \sigma_{e_{R_nD}}^2\left(1+\kappa_{R_nD}^2\right)+\frac{1}{\bar{\lambda}}$, $\bar{\theta}_n = \frac{c'_1\gamma_{th}}{\beta(a_1-b_1\gamma_{th})}$, $\bar{\phi}_n = \frac{c'_2\gamma_{th}}{\beta(a_2-b_2\gamma_{th})}$.

Proof: Follows by using the similar method of **Theorem 1**. □

2) DIVERSITY ORDER

In the following, the diversity orders at high SNRs for ORS are analyzed.

Corollary 5: At high SNRs, the diversity orders for the OP are given as

- *Non-ideal conditions*

$$d^{ORS,ni}\left(\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD}\right) = 0, \quad (36)$$

- *Ideal conditions*

$$d^{ORS,id}\left(\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD}\right) = \prod_{n=1}^{N}\min\left(\frac{\alpha_{SR_n}\mu_{SR_n}}{2}, \frac{\alpha_{R_nD}\mu_{R_nD}}{2}\right). \quad (37)$$

Proof: For non-ideal conditions, the dominant terms of (32) are the first term and second as

$$P_{out}^{ORS,ni}(\gamma_{th}) = \prod_{n=1}^{N}\left(\frac{1}{\Gamma(\mu_{SR_n}+1)}\theta_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}} + \frac{1}{\Gamma(\mu_{R_nD}+1)}\phi_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}\right). \quad (38)$$

Similarly, (38) is a constant, which is irrelative to $\bar{\lambda}_{SR_n}$ and $\bar{\lambda}_{R_nD}$. Substituting (38) into (23), we can obtain (36).

For ideal conditions, the dominant terms of (33) are

$$P_{out}^{ORS,id}(\gamma_{th}) = \prod_{n=1}^{N}\left(\frac{1}{\Gamma(\mu_{SR_n}+1)}\varphi_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}} + \frac{1}{\Gamma(\mu_{R_nD}+1)}\vartheta_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}\right). \quad (39)$$

Substituting (39) into (23), we can conclude the proof after some simplifications. □

For i.i.d. $\alpha-\mu$ fading channels, the result in (37) can be simplified as

$$d^{ORS,ni}\left(\bar{\lambda}\right) = \frac{N\alpha\mu}{2}. \quad (40)$$

Proof: Follows trivially by $\alpha_{SR_n} = \alpha_{R_nD} = \alpha$, $\mu_{SR_n} = \mu_{R_nD} = \mu$, $\bar{\lambda}_{SR_n} = \bar{\lambda}_{R_nD} = \bar{\lambda}$, $\beta_{SR_n} = \beta_{R_nD} = \beta$. □

Remark 2: From Corollary 3 to Corollary 5, we can observe that, for non-ideal conditions, there exists an upper bound for the effective SNDR at high SNR, and the upper bound depends on the levels of hardware impairments ($\kappa_{SR_n}, \kappa_{R_nD}$). When $\gamma_{th} \le 1/\max\left(\kappa_{SR_n}^2, \kappa_{R_nD}^2\right)$, as the average SNR approaches infinity, the asymptotic OP of i.i.d $\alpha-\mu$ fading channels is lower-bounded by the floor $\left(1-\sum_{m=0}^{\mu-1}\frac{(\bar{\theta}_n\bar{\phi}_n)^{\frac{\alpha m}{2}}}{m!}\exp\left(-\left(\bar{\theta}_n^{\frac{\alpha}{2}}+\bar{\phi}_n^{\frac{\alpha}{2}}\right)\right)\right)^N$, which results in zero diversity order. Moreover, hardware impairments and imperfect CSI degrade the outage performance as the OP grows when the values of distortion and channel estimation error are larger. For ideal conditions, the asymptotic OP of i.i.d. $\alpha-\mu$ increases unlimited as the average grows, and the diversity order is $N\alpha\mu/2$. Finally, we can also conclude that, for non-ideal conditions, the OP is determined by fading parameters, distortion noise, channel estimation error and the number of relays, while for ideal conditions, the OP relies on fading parameters and the number of relays.

C. PARTIAL RELAY SELECTION (PRS)

In some practical systems, only one hop channel information is available to the nodes, such as wireless sensor networks, ad-hoc network, mesh networks, etc [50]. To this end, the PRS was proposed and investigated [13]. Therefore, the performance of DH DF multi-relay networks in terms of OP for PRS is studied.

1) OUTAGE PROBABILITY

For PRS strategy, the optimal relay is selected depending on the link of $S \to R$ or $R \to D$ [12]

$$n^* = \arg \max_{1 \le n \le N} \left(\gamma_{SR_n}\right). \tag{41}$$

We study the OP of multi-relay networks over $\alpha - \mu$ fading channels in presence of HIs and ICSI in the following theorem.

Theorem 3: For $\alpha - \mu$ fading channels, the analytical expression for the OP are given as

- *Non-ideal conditions*

$$P_{out}^{PRS,ni}(\gamma_{th}) = 1 - \sum_{l=0}^{\mu_{R_nD}-1} \frac{e^{-\phi_n^{\frac{\alpha_{R_nD}}{2}}}}{l!} \phi_n^{\frac{\alpha_{R_nD} l}{2}} \times \left(1 - \prod_{n=1}^{N}\left(1 - \sum_{m=0}^{\mu_{SR_n}-1} \frac{e^{-\theta_n^{\frac{\alpha_{SR_n}}{2}}}}{m!} \theta_n^{\frac{\alpha_{SR_n} m}{2}}\right)\right), \tag{42}$$

for $\gamma_{th} < 1/\max\left(\kappa_{SR_n}^2, \kappa_{R_nD}^2\right)$, *and* $P_{out}^{ORS,ni}(\gamma_{th}) = 1$ *for* $\gamma_{th} \ge 1/\max\left(\kappa_{SR_n}^2, \kappa_{R_nD}^2\right)$.

- *Ideal conditions*

$$P_{out}^{PRS,id}(\gamma_{th}) = 1 - \sum_{l=0}^{\mu_{R_nD}-1} \frac{e^{-\vartheta_n^{\frac{\alpha_{R_nD}}{2}}}}{l!} \vartheta_n^{\frac{\alpha_{R_nD} l}{2}} \times \left(1 - \prod_{n=1}^{N}\left(1 - \sum_{m=0}^{\mu_{SR_n}-1} \frac{e^{-\varphi_n^{\frac{\alpha_{SR_n}}{2}}}}{m!} \varphi_n^{\frac{\alpha_{SR_n} m}{2}}\right)\right). \tag{43}$$

Proof: The detailed proof is provided in Appendix D. □

Although the OP of **Theorem 3** can be expressed in closed-form and efficiently evaluated, it does not offer the useful insights into the impacts of parameters on the system performance. We now perform the asymptotic analyses at high SNRs, which result in closed-form expressions for the OP.

Corollary 6: At high SNRs ($\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD} \to \infty$), the asymptotic expressions for the OP approach to

- *Non-ideal conditions*

$$P_{out}^{PRS,ni}(\gamma_{th}) = \frac{\phi_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma\left(\mu_{R_nD}+1\right)} - \prod_{n=1}^{N}\left(\frac{\theta_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}}{\Gamma\left(\mu_{SR_n}+1\right)}\right) \times \left(\frac{\phi_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma\left(\mu_{R_nD}+1\right)} - 1\right), \tag{44}$$

- *Ideal conditions*

$$P_{out}^{PRS,id}(\gamma_{th}) = \frac{\vartheta_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma\left(\mu_{R_nD}+1\right)} - \prod_{n=1}^{N}\left(\frac{\varphi_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}}{\Gamma\left(\mu_{SR_n}+1\right)}\right) \times \left(\frac{\vartheta_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma\left(\mu_{R_nD}+1\right)} - 1\right). \tag{45}$$

Proof: The detailed proof is provided in Appendix E. □

For i.i.d. $\alpha - \mu$ fading channels, the results of (44) and (45) can be simplified as

Corollary 7: For i. i. d. $\alpha - \mu$ fading channels, the asymptotic OPs reduce to

- *Non-ideal conditions*

$$P_{out}^{PRS,ni}(\gamma_{th}) = 1 - \sum_{m=0}^{\mu-1} \frac{e^{-\bar{\phi}_n^{\frac{\alpha}{2}}} \bar{\phi}_n^{\frac{\alpha m}{2}}}{m!}\left(1 - \left(1 - \frac{e^{-\bar{\theta}_n^{\frac{\alpha}{2}}} \theta_n^{\frac{\alpha m}{2}}}{m!}\right)^N\right), \tag{46}$$

- *Ideal condition*

$$P_{out}^{PRS,id}(\gamma_{th}) = 1 - \sum_{m=0}^{\mu-1} \frac{e^{-\left(\frac{\gamma_{th}}{\beta\bar{\lambda}}\right)^{\frac{\alpha}{2}}}}{m!}\left(\frac{\gamma_{th}}{\beta\bar{\lambda}}\right)^{\frac{\alpha m}{2}} \times \left(1 - \left(1 - \frac{e^{-\left(\frac{\gamma_{th}}{\beta\bar{\lambda}}\right)^{\frac{\alpha}{2}}}}{m!}\left(\frac{\gamma_{th}}{\beta\bar{\lambda}}\right)^{\frac{\alpha m}{2}}\right)^N\right). \tag{47}$$

Proof: Following the similar method of **Corollary 4**. □

2) DIVERSITY ORDER

In the following, the diversity orders in the high SNR regime for PRS strategy are analyzed.

Corollary 8: At high SNRs, the diversity orders for the outage probability are given as

- *Non-ideal conditions*

$$d^{PRS,ni}\left(\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD}\right) = 0, \tag{48}$$

- *Ideal conditions*

$$d^{PRS,id}\left(\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD}\right) = \min\left(\prod_{n=1}^{N}\frac{\alpha_{SR_n}\mu_{SR_n}}{2}, \frac{\alpha_{R_nD}\mu_{R_nD}}{2}\right). \tag{49}$$

*Proof: By using the same method of **Corollary 5** and the result of **Corollary 6**.* □

For i.i.d. fading channels, the result in (49) reduces to

$$d^{PRS,ni}\left(\bar{\lambda}\right) = \frac{\alpha\mu}{2}. \tag{50}$$

Proof: The proof is readily obtained by using the same method of (40) and simplifying. □

Remark 3: Corollaries 6-8 have provided some insights on the derived analytical results. For non-ideal conditions, the asymptotic OP at high SNRs is lower-bounded by the floor $\frac{\phi_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma(\mu_{R_nD}+1)} - \prod_{n=1}^{N}\left(\frac{\theta_n^{\frac{\alpha_{SR_n}\mu_{SR_n}}{2}}}{\Gamma(\mu_{SR_n}+1)}\right)\left(\frac{\phi_n^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma(\mu_{R_nD}+1)} - 1\right)$, *which results in zero diversity. This happens because the asymptotic OP is limited by hardware impairments and channel estimation error. For ideal conditions, the diversity order for the OP is a fixed constant, which is the minimum value of* $\prod_{n=1}^{N}\frac{\alpha_{SR_n}\mu_{SR_n}}{2}$ *and* $\frac{\alpha_{R_nD}\mu_{R_nD}}{2}$. *It means that when the number of relays is large, the diversity order is determined by the fading parameters of link between relay and destination.*

In addition, for i.i.d. $\alpha-\mu$ fading, the diversity order for OP is $\frac{\alpha\mu}{2}$, which only depends fading parameters. Finally, it can also seen that the outage performance benefiting from the increase of the number of relays.

IV. ERGODIC CAPACITY ANALYSIS

In communication systems, EC is another key metric for performance evaluation. We henceforth focus on the ECs of multi-relaying for RRS, ORS and PRS by considering HIs and ICSI, which is defined as [26]

$$C=\min_{i=SR_n,R_nD}\frac{1}{2}\mathbb{E}\left\{\log_2\left(1+\gamma_i\right)\right\},\quad i=\{SR_n,\ R_nD\}, \quad (51)$$

where γ_{SR_n} and γ_{R_nD} represent the SNDR between $S\rightarrow R$ and $R\rightarrow D$, respectively.

A. RANDOM RELAY SELECTION (RRS)

As in Section III A, the RRS is considered as a benchmark for the purpose of comparison, and the performance of DH DF multi-relay networks in terms of EC for the RRS is studied.

1) ERGODIC CAPACITY

For RRS strategy, one relay is chosen randomly. Thus, the EC is presented as

$$C^{\mathrm{RRS}}=\min_{i=SR_n,R_nD}\frac{1}{2}\mathbb{E}\left\{\log_2\left(1+\frac{\rho_i}{\rho_i\kappa_i^2+\sigma_{e_i}^2\left(1+\kappa_i^2\right)+\frac{1}{\bar{\lambda}_i}}\right)\right\}. \quad (52)$$

where the expectation is taken over the fading channels ρ_i and the channel is assumed to be ergodic.

The main challenge is that it is difficult, if not impossible, to derive exact expressions of ECs. To circumvent this problem, approximate expressions for the EC are obtained. To this end, the following theorem provides the approximate expressions for the EC over $\alpha-\mu$ fading channels with HIs and ICSI.

Theorem 4: For $\alpha-\mu$ fading channels, the upper bounds for the EC are given as

- *Non-ideal conditions*

$$C^{RRS,ni}\leq\frac{1}{2}\log_2\left(1+\min_{i=SR_n,R_nD}\frac{\beta_i\epsilon_i}{\beta_i\kappa_i^2\epsilon_i+\Gamma\left(\mu_i\right)\varpi_i}\right), \quad (53)$$

- *Ideal conditions*

$$C^{RRS,id}\leq\frac{1}{2}\log_2\left(1+\min_{i=SR_n,R_nD}\frac{\beta_i\bar{\lambda}_i\epsilon_i}{\Gamma\left(\mu_i\right)}\right), \quad (54)$$

where $\epsilon_i=\Gamma(\mu_i+2/\alpha_i)$, $\varpi_i=\sigma_{e_i}^2\left(1+\kappa_i^2\right)+1/\bar{\lambda}_i$.

Proof: The detailed proof is provided in Appendix F. □

The above theorem shows that the EC can be upper bounded and expressed in closed-form. In the numerical results of Section V, we can observe that, the derived upper bounds hold across the entire SNR regime for ideal/non-ideal conditions. The similar conclusion can be seen in [24], [51]. For non-ideal conditions, the upper bound for the EC depends on the average transmit power, fading parameters, distortion noise and estimation error. For ideal conditions, upper bounded is only determined by the average transmit power, fading parameters. In order to get deeper insights, we now focus on the asymptotic EC analysis in the high-SNR regime.

Corollary 9: At high SNRs ($\bar{\lambda}_{SR_n},\bar{\lambda}_{R_nD}\rightarrow\infty$), the asymptotic ECs are given as

- *Non-ideal conditions*

$$\bar{C}^{RRS,ni}\approx\frac{1}{2}\log_2\left(1+\min_{i=SR_n,R_nD}\frac{\beta_i\epsilon_i}{\beta_i\kappa_i^2\epsilon_i+\Gamma\left(\mu_i\right)\left(\varpi_i-\frac{1}{\bar{\lambda}_i}\right)}\right), \quad (55)$$

- *Ideal conditions*

$$\bar{C}^{RRS,id}\approx\min_{i=SR_n,R_nD}\frac{1}{2}\left(\log_2\left(\frac{\beta_i\bar{\lambda}_i\epsilon_i}{\Gamma\left(\mu_i\right)}\right)\right). \quad (56)$$

Proof: Based on the proof of **Theorem 4**, considering $\bar{\lambda}_{SR_n},\bar{\lambda}_{R_nD}\rightarrow\infty$, we can obtain the asymptotic ergodic capacity by combining (53) and (54). □

Corollary 9 indicates that for non-ideal conditions, there is a rate ceiling for the EC at high SNRs due to the HIs and ICSI, which is irrelative to the transmit power. For ideal conditions, the EC increases logarithmically with the transmit SNR.

2) HIGH-SNR SLOPE AND HIGH-SNR POWER OFFSET

To gather deeper insights, we pursue the asymptotic analyses in the high SNR regime for the EC by invoking the two metrics of *high-SNR slope* and the *high-SNR power offset* [52]

$$\bar{C}=\mathcal{S}_{\infty}\left(\log_2\bar{\lambda}_i-\mathcal{L}_{\infty}\right)+o\left(1\right),\quad i=\{SR_n,R_nD\}, \quad (57)$$

where $\bar{\lambda}_i$ is the average SNR, $\mathcal{S}_{\infty}$ and $\mathcal{L}_{\infty}$ are *high-SNR slope* in bits/s/Hz(3 dB) and the *high-SNR power offset* in 3 dB units, respectively. As stated in [52], the two metrics are defined as

$$\mathcal{S}_{\infty}=\lim_{\bar{\lambda}_i\rightarrow\infty}\frac{\bar{C}^{\mathrm{RRS}}}{\log_2\left(\bar{\lambda}_i\right)}, \quad (58)$$

$$\mathcal{L}_{\infty}=\lim_{\bar{\lambda}_i\rightarrow\infty}\left(\log_2\bar{\lambda}_i-\frac{\bar{C}^{\mathrm{RRS}}}{\mathcal{S}_{\infty}}\right). \quad (59)$$

Corollary 10: The high-SNR slopes and the high-SNR power offsets are given respectively as

- *Non-ideal conditions*

$$\mathcal{S}_{\infty}^{RRS,ni}=0,\quad \mathcal{L}_{\infty}^{RRS,ni}=\infty, \quad (60)$$

- *Ideal conditions*

$$\mathcal{S}_{\infty}^{RRS,id}=\frac{1}{2},\quad \mathcal{L}_{\infty}^{RRS,id}=\min_{i=SR_n,R_nD}\left(\log_2\left(\frac{\Gamma\left(\mu_i\right)}{\epsilon_i\beta_i}\right)\right). \quad (61)$$

Proof: For $\mathcal{S}_{\infty}$, we can obtain the following identities for non-ideal conditions and ideal conditions by substituting (53) and (54) into (58), respectively

$$\mathcal{S}_{\infty}^{\mathrm{RRS,ni}} = \lim_{\bar{\lambda}_i\to\infty}\min_{i=SR_n,R_nD}\frac{\log_2\left(1+\frac{\beta_i\epsilon_i}{\beta_i\kappa_i^2\epsilon_i+\Gamma(\mu_i)\left(\varpi_i-\frac{1}{\bar{\lambda}_i}\right)}\right)}{2\log_2\left(\bar{\lambda}_i\right)}, \tag{62}$$

$$\mathcal{L}_{\infty}^{\mathrm{RRS,id}} = \lim_{\bar{\lambda}_i\to\infty}\min_{i=SR_n,R_nD}\frac{\log_2\left(\frac{\beta_i\bar{\lambda}_i\epsilon_i}{\Gamma(\mu_i)}\right)}{2\log_2\left(\bar{\lambda}_i\right)}. \tag{63}$$

Taking $\bar{\lambda}_i$ into large, we can obtain the results of (60) and (61) after some simplifications.

For $\mathcal{L}_{\infty}$, we can obtain the results of (60) and (61) for non-ideal and ideal conditions by substituting (53), (62), (54), and (63) into (59). □

Remark 4: For non-ideal conditions, the $\bar{C}^{RRS,ni}$ approaches to a fixed constant as the average transmit power growing infinity, which results in zero high-SNR slope *and infinity* high-SNR power offset, *which mean that* high-SNR slope *and infinity* high-SNR power offset *are independent of any parameters due to the distortion noise and estimation error. For ideal conditions, the high-SNR slope is $\frac{1}{2}$, which irrelative to the average transmit power, fading parameters, distortion noise and estimation error. The high-SNR power offset is fixed constant, which only depends on the minimum value of* $\log_2\left(\Gamma(\mu_i)/\epsilon_i\beta_i\right)$.

B. OPPORTUNISTIC RELAY SELECTION (ORS)

To reduce the implementation complexity and to improve the spectral efficiency caused by multi-relay networks, ORS is considered and the performance of DH DF multi-relay networks in terms of EC for the ORS is studied.

1) ERGODIC CAPACITY

For ORS strategy, the optimal relay is selected according the SNDRs of the links both $S \to R_n$ and $R_n \to D$. The corresponding EC is presented as

$$C^{\mathrm{ORS}} = \max_{1\le n\le N}\min_{i=SR_n,R_nD}\frac{1}{2}\mathbb{E}\left\{\log_2\left(1+\gamma_i\right)\right\}. \tag{64}$$

Similarly, it is difficult to obtain the exact closed-form expression for the EC. To circumvent this problem, the upper bound for the EC of DF relaying networks with HIs and ICSI is investigated in the following theorem.

Theorem 5: For $\alpha-\mu$ fading channels, the upper boundeds for the EC are given as

- *Non-ideal conditions*

$$C^{ORS,ni} \le \frac{1}{2}\log_2\left(1+\max_{1\le n\le N}\min_{i=SR_n,R_nD}\frac{\beta_i\epsilon_i}{\beta_i\kappa_i^2\epsilon_i+\Gamma(\mu_i)(\varpi_i)}\right), \tag{65}$$

- *Ideal conditions*

$$C^{ORS,id} \le \frac{1}{2}\log_2\left(1+\max_{1\le n\le N}\min_{i=SR_n,R_nD}\frac{\beta_i\bar{\lambda}_i\epsilon_i}{\Gamma(\mu_i)}\right). \tag{66}$$

Proof: The detailed proof is provided in Appendix G. □

The above theorem shows that HIs and ICSI have detrimental effects on the EC, and the upper bound does not grow unboundedly with the average transmit SNR. For ideal conditions, the upper bound grows logarithmically with the SNR increases. In the follow, we now focus on the asymptotic EC analyses in the high SNR regime in the following corollary.

Corollary 11: At high SNRs ($\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD} \to \infty$), the asymptotic ergodic capacities are obtained as

- *Non-ideal conditions*

$$\bar{C}^{ORS,ni} \approx \frac{1}{2}\log_2\left(1+\max_{1\le n\le N}\min_{i=SR_n,R_nD}\frac{\beta_i\epsilon_i}{\beta_i\kappa_i^2\epsilon_i+\Gamma(\mu_i)\left(\varpi_i-\frac{1}{\bar{\lambda}_i}\right)}\right), \tag{67}$$

- *Ideal conditions*

$$\bar{C}^{ORS,id} \approx \max_{1\le n\le N}\min_{i=SR_n,R_nD}\frac{1}{2}\left(\log_2\left(\frac{\beta_i\bar{\lambda}_i\varpi_i}{\Gamma(\mu_i)}\right)\right). \tag{68}$$

Proof: Combining **Theorem 5** with $\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD} \to \infty$, we can obtain (67) and (68) by using (65) and (66). □

From **Corollary 11** we can observe that for non-ideal conditions, the asymptotic EC tends to a fixed constant, which depends on fading parameters, distortion noise and estimation error. For ideal conditions, the asymptotic EC grows logarithmically with the transmit SNR.

2) HIGH-SNR SLOPE AND HIGH-SNR POWER OFFSET

To gather deeper insights, the *high-SNR slopes* and the *high-SNR power offsets* for the EC are analyzed in the following corollary.

Corollary 12: At high SNRs, the high-SNR slopes and the high-SNR power offsets for the ergodic capacity are given as

- *Non-ideal conditions*

$$\mathcal{S}_{\infty}^{ORS,ni} = 0, \mathcal{L}_{\infty}^{ORS,ni} = \infty, \tag{69}$$

- *Ideal conditions*

$$\mathcal{S}_{\infty}^{ORS,id} = \frac{1}{2},$$
$$\mathcal{L}_{\infty}^{ORS,id} = \max_{1\le n\le N}\min_{i=SR_n,R_nD}\left(\log_2\left(\frac{\beta_i\epsilon_i}{\Gamma(\mu_i)}\right)\right), \tag{70}$$

Proof: By substituting (65) and (66) into (58), respectively. Using the similar method as the proof **Corollary 10**. □

Remark 5: Similarly, for non-ideal conditions, the high-SNR slope is zero and high-SNR power offsets is infinities, which mean that the ECs are not improved by only increasing average transmit power at high SNRs due to the HIs and ICSI. For ideal conditions, the high-SNR slope is 1/2, which irrelative to independent transmit power, fading parameters, distortion noise and estimation error. The high-SNR power offset is a constant, which are determined by the maximize minimum value of $\log_2\left(\beta_i/\epsilon_i\Gamma(\mu_i)\right)$.

C. PARTIAL RELAY SELECTION (PRS)

As in Section III C, ORS is unavailable in some circumstances, such as wireless sensor networks, ad-hoc network and mesh network. To this end, PRS is an effective way to improve spectral efficiency. Therefore, the performance of DH DF multi-relay networks in terms of EC for PRS is investigated.

1) ERGODIC CAPACITY

Based on the definition, the ECs can be expressed as

$$C^{\rm PRS} = \min\left(\frac{1}{2}\left(\log_2\left(1+\max_{1\le n\le N}\left(\gamma_{SR_n}\right)\right)\right),\ \frac{1}{2}\left(\log_2\left(1+\gamma_{R_nD}\right)\right)\right). \quad (71)$$

The following theorem studies the EC of multi-relay networks over $\alpha-\mu$ fading channels in the presence of HIs and ICSI.

Theorem 6: For $\alpha-\mu$ fading channels, the upper bounds for the EC are given as

- *Non-ideal conditions*

$$C^{PRS,ni} \approx \min_{i=SR_n,R_nD}\left(\frac{1}{2}\log_2\left(1+\max_{1\le n\le N}\frac{\beta_{SR_n}\epsilon_{SR_n}}{\beta_{SR_n}\kappa_{SR_n}^2\epsilon_{SR_n}+\Gamma(\mu_{SR_n})\varpi_{SR_n}}\right),\ \frac{1}{2}\log_2\left(1+\frac{\beta_{R_nD}\epsilon_{R_nD}}{\beta_{R_nD}\kappa_{R_nD}^2\epsilon_{R_nD}+\Gamma\left(\mu_{R_nD}\right)\varpi_{R_nD}}\right)\right), \quad (72)$$

- *Ideal conditions*

$$C^{PRS,id} \approx \min_{i=SR_n,R_nD}\left(\frac{1}{2}\log_2\left(1+\max_{1\le n\le N}\left(\frac{\beta_{SR_n}\epsilon_{SR_n}\bar{\lambda}_i}{\Gamma\left(\mu_{SR_n}\right)}\right)\right),\ \frac{1}{2}\log_2\left(1+\frac{\beta_{R_nD}\epsilon_{R_nD}\bar{\lambda}_i}{\Gamma\left(\mu_{R_nD}\right)}\right)\right). \quad (73)$$

where $\epsilon_{SR_n}=\Gamma\left(\mu_{SR_n}+2/\alpha_{SR_n}\right), \epsilon_{R_nD}=\Gamma\left(\mu_{R_nD}+2/\alpha_{R_nD}\right)$, $\varpi_{SR_n} = \left(\sigma_{e_{SR_n}}^2\left(1+\kappa_{SR_n}^2\right)+1/\bar{\lambda}_i\right)$, $\varpi_{R_nD} = \left(\sigma_{e_{R_nD}}^2\left(1+\kappa_{R_nD}^2\right)+1/\bar{\lambda}_i\right)$

Proof: The detailed proof is provided in Appendix G. □

Theorem 6 shows that ECs are limited by HIs and ICSI. Similarly, in the following, we focus on the asymptotic EC analysis in the high SNR regime.

Corollary 13: At high SNRs ($\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD} \to \infty$), the asymptotic ECs are given as

- *Non-ideal conditions*

$$\bar{C}^{PRS,ni} \approx \min\left(\frac{1}{2}\log_2\left(1+\max_{1\le n\le N}\frac{\beta_{SR_n}\epsilon_{SR_n}}{\beta_{SR_n}\kappa_{SR_n}^2\epsilon_{SR_n}+\Gamma\left(\mu_{SR_n}\right)\left(\varpi_{SR_n}-\frac{1}{\bar{\lambda}_i}\right)}\right),\ \frac{1}{2}\log_2\left(1+\frac{\beta_{R_nD}\epsilon_{SR_n}}{\beta_{R_nD}\kappa_{R_nD}^2\epsilon_{SR_n}+\Gamma\left(\mu_{R_nD}\right)\left(\varpi_{SR_n}-\frac{1}{\bar{\lambda}_i}\right)}\right)\right), \quad (74)$$

- *Ideal conditions*

$$\bar{C}^{PRS,id} \approx \min\left(\frac{1}{2}\max_{1\le n\le N}\left(\log_2\left(\frac{\beta_{SR_n}\epsilon_{SR_n}\bar{\lambda}_i}{\Gamma\left(\mu_{SR_n}\right)}\right)\right),\ \frac{1}{2}\log_2\left(\frac{\beta_{R_nD}\epsilon_{SR_n}\bar{\lambda}_i}{\Gamma\left(\mu_{R_nD}\right)}\right)\right). \quad (75)$$

Proof: The proof is similar to **Corollary 11**. □

Similarly, **Corollary 13** demonstrates there is a rate ceiling for the capacity at high SNR regime for PRS scheme due to the HIs and ICSI. For ideal conditions, the EC grows logarithmically with average transmitter SNR.

2) HIGH-SNR SLOPE AND HIGH-SNR POWER OFFSET

To get deeper insights, the *high-SNR slopes* and the *high-SNR power offsets* for the EC are analyzed in the following corollary.

Corollary 14: For PRS scheme, the high-SNR slopes and high-SNR power offsets for non-ideal and ideal conditions can be expressed as

- *Non-ideal conditions*

$$\mathcal{S}_\infty^{PRS,ni} = 0,\ \mathcal{L}_\infty^{PRS,ni} = \infty, \quad (76)$$

- *Ideal conditions*

$$\mathcal{S}_\infty^{PRS,id} = \frac{1}{2},$$
$$\mathcal{L}_\infty^{PRS,id} = \min\left(\max_{1\le n\le N}\left(\log_2\left(\frac{\beta_{SR_n}^{-1}\Gamma\left(\mu_{SR_n}\right)}{\epsilon_{SR_n}}\right)\right),\ \log_2\left(\frac{{\beta_{R_nD}}^{-1}\Gamma\left(\mu_{R_nD}\right)}{\epsilon_{SR_n}}\right)\right). \quad (77)$$

Proof: By substituting (72) and (73) into (58). We can conclude the proof by using the similar method as the proof **Corollary 10**. □

Remark 6: The above corollary shows that the high-SNR slopes *for non-ideal and ideal conditions are 0 and $\frac{1}{2}$, respectively, which are irrelative to the fading parameters, the number of relays, distortion noise and estimation error. In addition, for non-ideal conditions the* high-SNR power offset *is infinity due to the HIs and ICSI, while for ideal conditions, the* high-SNR power offset *is an positive constant, which only depends on the fading parameters.*

Summaries and comparisons of diversity orders, *high-SNR slopes* and *high-SNR power offsets* for the three selections are provided in Table 1.

V. NUMERICAL RESULTS

In this section, the correctness of the theoretical analysis results is verified by some simulations. Unless otherwise mentioned, the parameters used in the simulations are set as

TABLE 1. Comparison of the three relay selection.

Hardware Type	Relay Selection	Diversity Order	High-SNR Slope	High-SNR Power Offset
Non-ideal	RRS	0	0	∞
	ORS	0	0	∞
	PRS	0	0	∞
Ideal	RRS	$\min\left(\frac{\alpha_{SR_n}\mu_{SR_n}}{2}, \frac{\alpha_{R_nD}\mu_{R_nD}}{2}\right)$	$\frac{1}{2}$	$\min\limits_{i=SR_n,R_nD}\left(\log_2\left(\frac{\beta_i\epsilon_i}{\Gamma(\mu_i)}\right)\right)$
	ORS	$\prod\limits_{n=1}^{N}\min\left(\frac{\alpha_{SR_n}\mu_{SR_n}}{2}, \frac{\alpha_{R_nD}\mu_{R_nD}}{2}\right)$	$\frac{1}{2}$	$\max\limits_{1\leq n\leq N}\min\limits_{i=SR_n,R_nD}\left(\log_2\left(\frac{\beta_i\epsilon_i}{\Gamma(\mu_i)}\right)\right)$
	PRS	$\min\left(\prod\limits_{n=1}^{N}\frac{\alpha_{SR_n}\mu_{SR_n}}{2}, \frac{\alpha_{R_nD}\mu_{R_nD}}{2}\right)$	$\frac{1}{2}$	$\min\left(\max\limits_{1\leq n\leq N}\left(\epsilon_{SR_n}, \epsilon_{R_nD}\right)\right)$

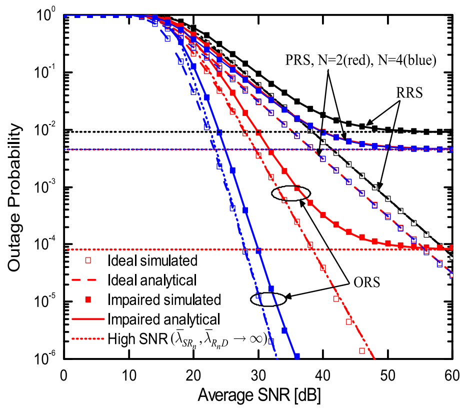

FIGURE 1. Outage probability versus SNR for different *N*.

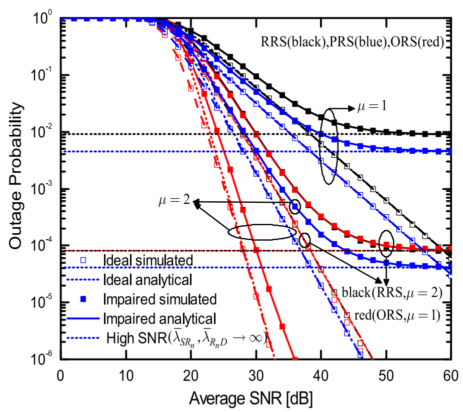

FIGURE 2. Outage probability versus SNR for different μ.

follows: $\alpha_{SR_n} = \alpha_{R_nD} = \alpha = 2$, $\mu_{SR_n} = \mu_{R_nD} = \mu = 1$, $N_{SR_n} = N_{R_nD} = 1$, $\sigma_{e_{SR_n}} = \sigma_{e_{R_nD}} = \sigma_{e_i}$, $\kappa_{SR_n} = \kappa_{R_nD} = \kappa$.

A. OUTAGE PROBABILITY

From Fig. 1-Fig. 5, the OPs for RRS, ORS and PRS are investigated, which correspond to the theoretical analytical in Section III.

Fig. 1 plots the OP of three selections strategies versus the average transmit SNR for different $N = \{1, 2, 4\}$. For comparison, the OPs for ideal conditions are provided. In this simulation, a high-rate system with $\gamma_{th} = 2^5 - 1 = 31$ bit/channel use is considered. We set the levels of impairments ($\kappa = 0.1$)

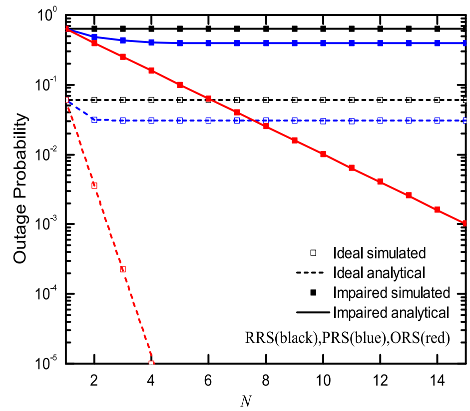

FIGURE 3. Outage probability versus different number of relays *N* (SNR = 30dB).

and estimation error ($\sigma_{e_i} = 0.01$). Fig. 1 confirms the close agreement between the analytical results and simulations, thereby verifying the correctness of our theoretical analyses. Compared with RRS, PRS and ORS have lower OP than RRS one. This implies that the considered PRS and ORS are efficient ways to improve the performance of multi-relay system. In addition, there are error floors for the OP due to the distortion and estimation error. These conclusions are also confirmed by the insights in **Remark 1, Remark 2 and Remark 3**. Finally, we can also observe that these gaps of OP between the ideal and non-ideal hardware for these three selections enlarge with the average transmit SNR. This indicates that distortion noise and estimation error are crucial factors to the high-rate systems.

In Fig. 2, the effects of fading parameter, μ, on the OP are analyzed. In this simulation, we set estimation error $\sigma_{e_i} = 0.01$, the level of impairments $\kappa = 0.1$ and the threshold $\gamma_{th} = 31$ bit/channel use. The curves represent the exact analytical OP derived in (15), (16), (30), (31), (42) and (43), respectively. The curves represent asymptotic analytical OP derived in (17), (18), (32), (33), (44) and (45), respectively. From Fig. 3, we can conclude that μ strengthens the OP performance, whilst it enlarges the performance gap between the ideal and the non-ideal conditions, which is consistent with result of [39]. Moreover, there are error floors for the OP due to the distortion noise and estimation error. For RRS, the floor is determined by distortion noise, channel estimation error

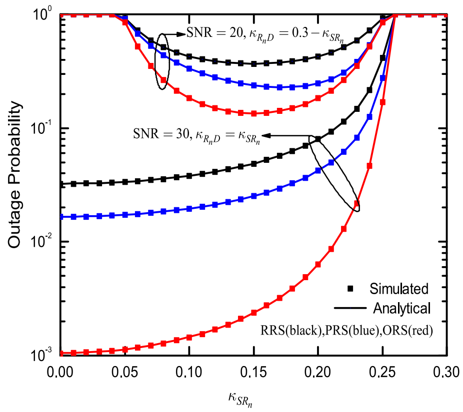

FIGURE 4. **Outage probability versus level of impairments κ_{SR_n}.**

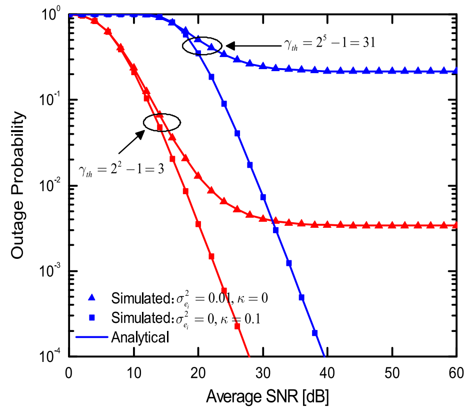

FIGURE 6. **Outage probability of ORS scheme versus SNR for different γ_{th}.**

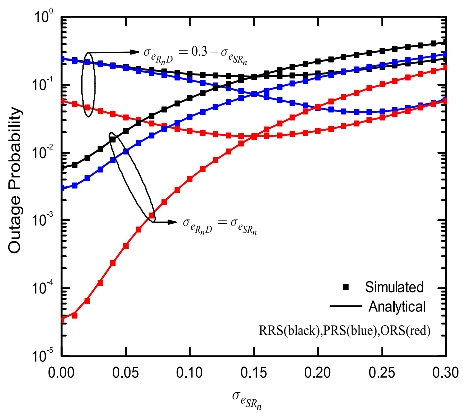

FIGURE 5. **Outage probability versus level of estimation error $\sigma_{e_{SR_n}}$ (SNR = 30).**

and fading parameters, while for ORS and PRS, the floors depend on distortion noise, channel estimation error, fading parameters and the number of relays.

Fig. 3 plots the analytical results of the OP and simulation versus different number of relays for ideal and non-ideal conditions. In this simulation, a high-rate system with $\gamma_{th} = 31$ bit/channel use is considered. We set $\kappa = 0.1$ and $\sigma_{e_i} = 0.1$. It is worth noting that the OP of ORS is the best strategy, and its performance increases rapidly with the number of relays. This implies that ORS is most efficient way to improve the performance of multi-relay system. In addition, we can also observe that for PRS, the OP decreases slowly as the number of relays growing, especially for large N ($N > 4$). This can be explained by the fact in (49) that the diversity order $d^{PRS,id}\left(\bar{\lambda}_{SR_n}, \bar{\lambda}_{R_nD}\right) = \min\left(\prod_{n=1}^{N} \frac{\alpha_{SR_n}\mu_{SR_n}}{2}, \frac{\alpha_{R_nD}\mu_{R_nD}}{2}\right)$, the impact of the number of relays in diversity order will gradually diminish as N increases. For RRS, the OP is irrelative to the number of relay. Finally, there are gaps for the OP between the ideal and non-ideal conditions. This indicates that HIs have deleterious effects on the system performance.

Fig. 4 depicts the OP performance versus different levels of HIs κ_{SR_n}. In this simulation, we set $\sigma_{e_i} = 0.01$. For the case of $\kappa_{R_nD} = \kappa_{SR_n}$, it is clear that the gaps among three relay selection strategies are large, and the differences become smaller as high level of HIs increasing. When $\kappa_{R_nD} = \kappa_{SR_n} > 0.25$, the OPs for the three strategies are always 1 due to heavy HIs. For the case of $\kappa_{R_nD} = 0.3 - \kappa_{SR_n}$,[9] we can observe that the OPs for these strategies decreases and then increases when $0.5 < \kappa_{SR_n} < 0.25$, this is because the system performance with DF relay depends on the level of the weakest hop. We can also observe that the OP is minimized when the weakest hop has a higher hardware quality than the strongest hop. Thus, when $\kappa_{SR_n} < 0.05$ or $\kappa_{SR_n} > 0.25$, the system is always outage due to the poor hardware quality on the weakest hop.

Fig. 5 investigates the effects of channel estimation error on the OP. In this simulation, we set HIs parameter $\kappa = 0.01$. As shown in Fig. 5, for $\sigma_{e_{R_nD}} = \sigma_{e_{SR_n}}$, the outage performance reduces when $\sigma_{e_{SR_n}}$ increases, which means that estimation errors have negative effects on system outage performance. For $\sigma_{e_{R_nD}} = 0.3 - \sigma_{e_{SR_n}}$, there is an optimal estimation error to maximize the OP since that it has serious estimation error on the weakest hop than on the strongest hop. This reason can be explained as the selected relay is the same one by having serious estimation errors on the first hop or on the second hop for RRS and ORS, but for PRS, the selected relay depends on the channel condition on the first hop.

Fig. 6 shows the effect of separate imperfections, HIs κ, or channel estimation error $\sigma_{e_i}^2$, on the outage performance of ORS scheme for different thresholds γ_{th}, i.e., ICSI without HIs $\left(\sigma_{e_i}^2 = 0.01,\ \kappa = 0\right)$ and perfect CSI with HIs $\left(\sigma_{e_i}^2 = 0,\ \kappa = 0.1\right)$. It can be seen that there is a substantial performance loss when γ_{th} is increased to 31. Obviously, an outage floor is presented in the former case, while the diversity order is maintained only in the latter case. Specifically, it can be also seen that ICSI influences the performance more than transceiver HIs.

[9] As stated in [46], the typical value of κ is [0.08, 0.175]. For the purpose of comparison, we let the arrange of summation $\kappa_{R_nD} + \kappa_{SR_n}$ is [0,3].

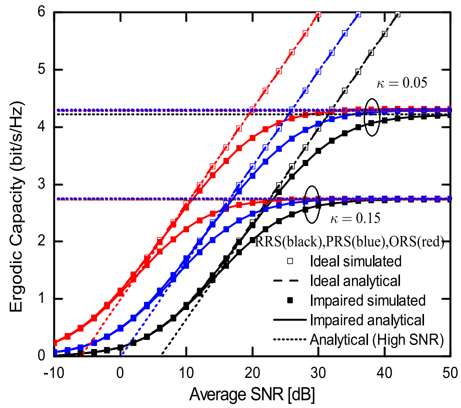

FIGURE 7. Ergodic capacity versus SNR for different κ.

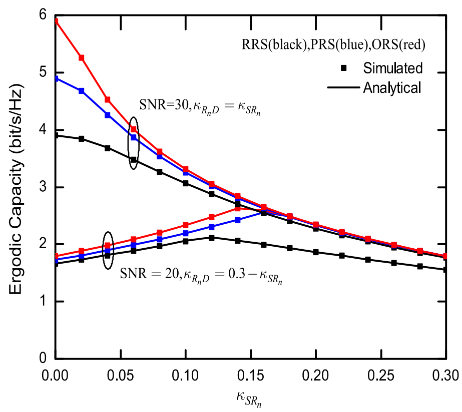

FIGURE 8. Ergodic capacity versus level of impairments κ_{SR_n}.

B. ERGODIC CAPACITY

From Fig. 7 and Fig. 8, the ECs for RRS, ORS and PRS are investigated, which correspond to the theoretical analytical in Section IV.

Fig. 7 shows the EC versus SNR for different HIs κ. This shows that the capacity decreases is limited by the level of HIs κ, especially for the high SNR region, and there is a ceiling for the EC due to the distortion noise and channel estimation error. For ideal conditions, we can see that the curves for different relay selection scheme have the same capacity slope, which is indicated by (61), (70) and (77). This trend is consistent with the results of [53], [54].

Fig. 8 plots the EC versus different HIs κ_{SR_n}. For $\kappa_{R_nD} = \kappa_{SR_n}$, the EC reduces when κ_{SR_n} increases due to the serious impairments. For $\kappa_{R_nD} = 0.3 - \kappa_{SR_n}$, we observe that the EC decline by having serious HIs on the weakest hop than on the strongest hop. This can be explained as the selected relay is the same one by having serious distortion noise on the first hop than on the second hop for RRS and ORS, but for PRS, the selected relay depends on the channel condition of the first hop.

VI. CONCLUSION

This paper investigated the impact of transceiver HIs and ICSI on DH DF multi-relay networks and three relaying selection schemes were proposed. The analytical expressions for the OP and EC for these three strategies were derived. Owing to the estimation error, the diversity orders and *high-SNR slopes* are zeros, and resulted in infinity *high-SNR power offset*. For ideal conditions, the diversity orders and *high-SNR power offset* were constants, which depended on fading parameter and the number of relays, while *high-SNR slopes* were $\frac{1}{2}$. Based on the results, we demonstrated the relay selection scheme effectively improve the system performance.

APPENDIX A
PROOF OF THEOREM 1

Based on (13), the OP of (12) can be expressed as

$$P_{\text{out}}^{\text{RRS,ni}}(\gamma_{th}) = \Pr\left(\gamma_{SR_n} \le \gamma_{th}\right) + \Pr\left(\gamma_{R_nD} \le \gamma_{th}\right) - \Pr\left(\gamma_{SR_n} \le \gamma_{th}\right)\Pr\left(\gamma_{R_nD} \le \gamma_{th}\right), \tag{A.1}$$

when $\gamma_{th} \ge 1/\max\left(\kappa_1^2, \kappa_2^2\right)$, the OP is 1, when $\gamma_{th} < 1/\max\left(\kappa_1^2, \kappa_2^2\right)$, the OP of (A.1) can be rewritten as

$$P_{\text{out}}^{\text{RRS,ni}}(\gamma_{th}) = F_{\gamma_{SR_n}}(\gamma_{th}) + F_{\gamma_{R_nD}}(\gamma_{th}) - F_{\gamma_{SR_n}}(\gamma_{th})\, F_{\gamma_{R_nD}}(\gamma_{th}). \tag{A.2}$$

Utilizing (7), we can obtain

$$F_{\gamma_{SR_n}}(\gamma_{th}) = \Pr\left(\frac{\rho_{SR_n}}{\rho_{SR_n}\kappa_{SR_n}^2 + c_1} \le \gamma_{th}\right), \tag{A.3}$$

by using (6), we can obtain the CDF of (A.3) as

$$F_{\gamma_{SR_n}}(\gamma_{th}) = 1 - \sum_{m=0}^{\mu_{SR_n}-1} \frac{e^{-\left(\frac{c_1\gamma_{th}}{\beta_{SR_n}\left(1-\kappa_{SR_n}^2\gamma_{th}\right)}\right)^{\frac{\alpha_{SR_n}}{2}}}}{m!} \times \left(\frac{c_1\gamma_{th}}{\beta_{SR_n}\left(1-\kappa_{SR_n}^2\gamma_{th}\right)}\right)^{\frac{\alpha_{SR_n}}{2}}. \tag{A.4}$$

Similarly, $F_{\gamma_{R_nD}}(\gamma_{th})$ can be expressed as

$$F_{\gamma_{R_nD}}(\gamma_{th}) = 1 - \sum_{m=0}^{\mu_{R_nD}-1} \frac{e^{-\left(\frac{c_2\gamma_{th}}{\beta_{R_nD}\left(1-\kappa_{R_nD}^2\gamma_{th}\right)}\right)^{\frac{\alpha_{R_nD}}{2}}}}{m!} \times \left(\frac{c_2\gamma_{th}}{\beta_{R_nD}\left(1-\kappa_{R_nD}^2\gamma_{th}\right)}\right)^{\frac{\alpha_{R_nD}}{2}}. \tag{A.5}$$

Substituting (A.4) and (A.5) into (A.2), we can obtain (15).

Then, we obtain the result of (16) by letting $\kappa_{t,i} = \kappa_{r,i} = 0$ and $\sigma_{e_{SR_n}}^2 = \sigma_{e_{R_nD}}^2 = 0$.

APPENDIX B
PROOF OF THEOREM 2

Based on (29), the OP of (12) can be expressed as

$$\begin{aligned} P_{\text{out}}^{\text{ORS,ni}}(\gamma_{th}) &= \Pr\left(\max_{1\le n\le N}\min\left(\gamma_{SR_n},\gamma_{R_nD}\right)\le\gamma_{th}\right)\\ &= \prod_{n=1}^{N}\left(\Pr\left(\gamma_{SR_n}\le\gamma_{th}\right)+\Pr\left(\gamma_{R_nD}\le\gamma_{th}\right)\right.\\ &\quad\left.-\Pr\left(\gamma_{SR_n}\le\gamma_{th}\right)\Pr\left(\gamma_{R_nD}\le\gamma_{th}\right)\right), \end{aligned} \tag{B.1}$$

when $\gamma_{th}\ge 1/\max\left(\kappa_1^2,\kappa_2^2\right)$, the OP of (B.1) is 1, when $\gamma_{th}< 1/\max\left(\kappa_1^2,\kappa_2^2\right)$, the OP of (B.1) can be rewritten as

$$\begin{aligned} P_{\text{out}}^{\text{ORS,ni}}(\gamma_{th}) = \prod_{n=1}^{N}&\left(F_{\gamma_{SR_n}}(\gamma_{th})+F_{\gamma_{R_nD}}(\gamma_{th})\right.\\ &\left.-F_{\gamma_{SR_n}}(\gamma_{th})F_{\gamma_{R_nD}}(\gamma_{th})\right). \end{aligned} \tag{B.2}$$

Substituting (A.4) and (A.5) into (B.2), we can obtain (30).

For ideal conditions, we obtain the result of (31) by letting $\kappa_{t,i}=\kappa_{r,i}=0$ and $\sigma_{e_{SR_n}}^2=\sigma_{e_{R_nD}}^2=0$.

APPENDIX C
PROOF OF THEOREM 3

The proof starts by combing (12) with (41) as

$$\begin{aligned} P_{\text{out}}^{\text{PRS,ni}}(\gamma_{th}) &= \Pr\left(\min\left(\gamma_{SR_{n^*}},\gamma_{R_{n^*}D}\right)\le\gamma_{th}\right)\\ &= F_{\gamma_{SR_{n^*}}}(\gamma_{th})+F_{\gamma_{R_{n^*}D}}\\ &\quad+F_{\gamma_{SR_{n^*}}}(\gamma_{th})F_{\gamma_{R_{n^*}D}}(\gamma_{th}). \end{aligned} \tag{C.1}$$

Based on (41), the received SNR is the first hop is given as

$$\gamma_{SR_{n^*}}=\max\left(\gamma_{SR_1},\gamma_{SR_2},\ldots,\gamma_{SR_N}\right), \tag{C.2}$$

then, the CDF of $\gamma_{SR_{n^*}}$ is calculated by

$$F_{\gamma_{SR_{n^*}}}(\gamma_{th})=\prod_{n=1}^{N}\Pr\left(\gamma_{SR_n}\le\gamma_{th}\right), \tag{C.3}$$

where $\Pr\left(\gamma_{SR_n}\le\gamma_{th}\right)=F_{\gamma_{SR_n}}(\gamma_{th})$ is shown in (A.4).

Thus, we can obtain

$$\begin{aligned} F_{\gamma_{SR_{n^*}}}(\gamma_{th}) = \prod_{n=1}^{N}&\left(1-\sum_{m=0}^{\mu_{SR_n}-1}\frac{e^{-\left(\frac{c_1\gamma_{th}}{\beta_{SR_n}\left(1-\kappa_{SR_n}^2\gamma_{th}\right)}\right)^{\frac{\alpha_{SR_n}}{2}}}}{m!}\right.\\ &\left.\times\left(\frac{c_1\gamma_{th}}{\beta_{SR_n}\left(1-\kappa_{SR_n}^2\gamma_{th}\right)}\right)^{\frac{\alpha_{SR_n}m}{2}}\right), \end{aligned} \tag{C.4}$$

In addition, for PRS scheme, the CDF of the received SNDR in the second hop is given as

$$F_{\gamma_{R_{n^*}D}}(\gamma_{th})=F_{\gamma_{R_nD}}(\gamma_{th}), \tag{C.5}$$

where $F_{\gamma_{R_nD}}(\gamma_{th})$ is given in (A.5).

Therefore, $F_{\gamma_{R_{n^*}D}}(\gamma_{th})$ can be obtained as

$$\begin{aligned} F_{\gamma_{R_{n^*}D}}(\gamma_{th}) = 1-\sum_{l=0}^{\mu_{R_nD}-1}&\frac{e^{-\left(\frac{c_2\gamma_{th}}{\beta_{R_nD}\left(1-\kappa_{R_nD}^2\gamma_{th}\right)}\right)^{\frac{\alpha_{R_nD}}{2}}}}{l!}\\ &\times\left(\frac{c_2\gamma_{th}}{\beta_{R_nD}\left(1-\kappa_{R_nD}^2\gamma_{th}\right)}\right)^{\frac{\alpha_{R_nD}l}{2}}. \end{aligned} \tag{C.6}$$

By substituting (C.4) and (C.6) into (C.1), we can obtain (42), the proof is achieved.

For ideal conditions, we can obtain the result of (43) by setting $\kappa_{t,i}=\kappa_{r,i}=0$ and $\sigma_{e_{SR_n}}^2=\sigma_{e_{R_nD}}^2=0$.

APPENDIX D
PROOF OF COROLLARY 4

Based on (C.2), we can obtain the CDF at high SNRs as

$$F_{\gamma_{SR_{n^*}}}(\gamma_{th})=\Pr\left(\gamma_{SR_{n^*}}\le\gamma_{th}\right)=\prod_{n=1}^{N}F_{\gamma_{SR_n}}(\gamma_{th}), \tag{D.1}$$

where $F_{\gamma_{SR_n}}$ is given as (A.4).

In addition, we also obtain

$$F_{\gamma_{R_{n^*}D}}(\gamma_{th}) = \frac{\left(\frac{c_2\gamma_{th}}{\beta_{R_nD}(a_2-b_2\gamma_{th})}\right)^{\frac{\alpha_{R_nD}\mu_{R_nD}}{2}}}{\Gamma\left(\mu_{R_nD}+1\right)}. \tag{D.2}$$

By plugging (D.1) and (D.2) into (C.1), after some manipulate, we conclude the proof.

APPENDIX E
PROOF OF THEOREM 4

Using the result of [55], we can have the following as

$$C^{\text{RRS,ni}}\le\min_{i=SR_n,R_nD}\frac{1}{2}\log_2\left(1+\mathbb{E}\left\{\gamma_i\right\}\right). \tag{E.1}$$

With the help of (7) and (8), the upper bound for the EC is given by

$$\begin{aligned} &C^{\text{RRS,ni}}\\ &\approx\min_{i=SR_n,R_nD}\frac{1}{2}\log_2\left(1+\frac{\mathbb{E}\left\{\rho_i\right\}}{\mathbb{E}\left\{\rho_i\right\}\kappa_i^2+\sigma_{e_i}^2\left(1+\kappa_i^2\right)+\frac{1}{\bar{\lambda}_i}}\right)\\ &=\frac{1}{2}\log_2\left(1+\min_{i=SR_n,R_nD}\frac{\mathbb{E}\left\{\rho_i\right\}}{\mathbb{E}\left\{\rho_i\right\}\kappa_i^2+\sigma_{e_i}^2\left(1+\kappa_i^2\right)+\frac{1}{\bar{\lambda}_i}}\right), \end{aligned} \tag{E.2}$$

For ideal conditions, the upper bound can be obtained as

$$\begin{aligned} C^{\text{RRS,id}} &\approx \min_{i=SR_n,R_nD}\frac{1}{2}\log_2\left(1+\bar{\lambda}_i\mathbb{E}\left\{\rho_i\right\}\right)\\ &=\frac{1}{2}\log_2\left(1+\min_{i=SR_n,R_nD}\left(\bar{\lambda}_i\mathbb{E}\left\{\rho_i\right\}\right)\right), \end{aligned} \tag{E.3}$$

Utilizing the following integral identities [56]

$$\int_0^{\infty}x^m\exp\left(-\beta x^n\right)dx=\frac{\Gamma(r)}{n\beta^r},\ r=\frac{m+1}{n}, \tag{E.4}$$

$$C^{\text{ORS,ni}} \approx \max_{1\le n\le N} \min_{i=SR_n,R_nD} \frac{1}{2}\log_2\left(1+\frac{\mathbb{E}\{\rho_i\}}{\mathbb{E}\{\rho_i\}\kappa_i^2+\sigma_{e_i}^2\left(1+\kappa_i^2\right)+\frac{1}{\bar{\lambda}_i}}\right)$$
$$=\frac{1}{2}\log_2\left(1+\max_{1\le n\le N} \min_{i=SR_n,R_nD} \frac{\mathbb{E}\{\rho_i\}}{\mathbb{E}\{\rho_i\}\kappa_i^2+\sigma_{e_i}^2\left(1+\kappa_i^2\right)+\frac{1}{\bar{\lambda}_i}}\right), \tag{F.2}$$

$$C^{\text{PRS,ni}} = \min\left(\frac{1}{2}\log_2\left(1+\max_{1\le n\le N}\left(\frac{\mathbb{E}\left(\rho_{SR_n}\right)}{\mathbb{E}\left(\rho_{SR_n}\right)\kappa_{SR_n}^2+\sigma_{e_{SR_n}}^2\left(1+\kappa_{SR_n}^2\right)+\frac{1}{\bar{\lambda}_{SR_n}}}\right)\right),\right.$$
$$\left.\frac{1}{2}\log_2\left(1+\frac{\mathbb{E}\left(\rho_{R_nD}\right)}{\mathbb{E}\left(\rho_{R_nD}\right)\kappa_{R_nD}^2+\sigma_{e_{R_nD}}^2\left(1+\kappa_{R_nD}^2\right)+\frac{1}{\bar{\lambda}_{R_nD}}}\right)\right) \tag{G.2}$$

the expectation of the minimum function in (E.3) can be simplified as

$$\mathbb{E}\{\rho_i\} = \frac{\beta_i\Gamma\left(\mu_i+\frac{2}{\alpha_i}\right)}{\Gamma\left(\mu_i\right)}. \tag{E.5}$$

Substituting (E.5) into (E.2) and (E.3), the proof is complete.

APPENDIX F
PROOF OF THEOREM 5

Using the [55], we have

$$C^{\text{ORS,ni}} \le \max_{1\le n\le N} \min_{i=SR_n,R_nD} \frac{1}{2}\log_2\left(1+\mathbb{E}\{\gamma_i\}\right), \tag{F.1}$$

with the help of (7) and (8), the upper bound for the EC in (F.2) is given as shown at the top of this page.

For ideal conditions, upper bound for the EC can be obtained as

$$C^{\text{ORS,id}} \le \max_{1\le n\le N} \min_{i=SR_n,R_nD} \frac{1}{2}\log_2\left(1+\bar{\lambda}_i\mathbb{E}\{\rho_i\}\right)$$
$$=\frac{1}{2}\log_2\left(1+\max_{1\le n\le N} \min_{i=SR_n,R_nD}\left(\bar{\lambda}_i\mathbb{E}\{\rho_i\}\right)\right), \tag{F.3}$$

Substituting (E.5) into (65) and (F.3), the proof is complete.

APPENDIX G
PROOF OF THEOREM 6

By applying the Jensen' inequality, (71) becomes

$$C^{\text{PRS}} \le \min\left(\frac{1}{2}\log_2\left(1+\max_{1\le n\le N}\left(\mathbb{E}\{\gamma_{SR_n}\}\right)\right),\right.$$
$$\left.\frac{1}{2}\left(\log_2\left(1+\left(\mathbb{E}\{\gamma_{R_nD}\}\right)\right)\right)\right), \tag{G.1}$$

By substituting (7) and (8) into (G.1), the EC in (G.2) for PRS is given as shown at the top of the this page

For ideal conditions, the EC is expressed as

$$C^{\text{PRS,id}} \le \min\left(\frac{1}{2}\log_2\left(1+\max_{1\le n\le N}\left(\bar{\lambda}_{SR_n}\mathbb{E}\{\rho_{SR_n}\}\right)\right),\right.$$
$$\left.\frac{1}{2}\log_2\left(1+\bar{\lambda}_{R_nD}\left(\mathbb{E}\{\rho_{R_nD}\}\right)\right)\right). \tag{G.3}$$

Substituting (E.5) into (G.2) and (G.3), the proof is completed.

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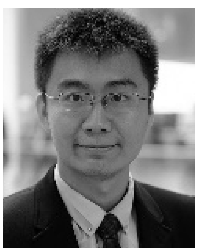

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