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# Achievable Degrees of Freedom for the Two-Cell Two-Hop MIMO Interference Channel with Half-Duplex Relays

Jin Jin, Xiang-Chuan Gao, Xingwang Li, Shuangzhi Li, and Zhongyong Wang

***Abstract*—We consider the two-cell two-hop multiple-input-multiple-output (MIMO) interference channel with half-duplex relays, where each source group having $M$ single antenna users, communicates with the corresponding destination with $M$ antennas via two relays, each of which has $M$ antennas. For such a channel, by exploiting three time slots, the previously known achievable degrees of freedom (DoF) is $2M/3$ regardless of whether the half-duplex relays have global channel state information (CSI) for the first hop. In this paper, we show that using $n \geq 3$ time slots, the achievable DoF is $(n-1)M/n$, which is higher than the previous result of $2M/3$ DoF for the case of $n \geq 4$. The achievability is shown by a new relaying protocol which combines the alternate transmission strategy with an interference cancellation technique. A major implication of the derived result is that a normalized DoF of one can be achieved asymptotically without requiring global CSI at the source and relay nodes.**

***Index Terms*—Interference alignment (IA), degrees of freedom (DoF), multi-hop, multiple-input-multiple-output (MIMO).**

## I. Introduction

THE degrees of freedom (DoF) characterizations have recently been obtained for a variety of wireless networks, including the relay-assisted two-cell two-hop interference networks [1]-[5]. For the downlink case, the achievable DoF for the two-cell interfering broadcast channels with multiple single-antenna users was studied where the relays are full-duplex [1]. When the relays operate in half-duplex mode, the DoF region of a two-hop multiple-input-multiple-output (MIMO) relay network with two multi-antenna users was analyzed in [2]. On the other hand, for the uplink case, in [3], two multi-antenna relays were used to assist the transmission from two groups consisting of multiple users each equipped with a single antenna to two base stations having multiple antennas. For such an uplink cellular system, the inner bounds of the DoF were derived under both the full-duplex and half-duplex relaying constraints, and the corresponding interference-free relay transmission protocols were provided respectively.

For the half-duplex relaying protocol that achieves the DoF inner bound in [3], it requires three time slots to complete the two-hop transmission when two relays are half-duplex. The first-hop transmission exploits one time slot to allow the two source groups to transmit data symbols to the relays simultaneously, and the second-hop transmission utilizes two time slots to guarantee that both relays alternatively send signals to destinations. Since $2M$ interference-free data symbols are delivered over three time slots, $\frac{2M}{3}$ DoF are achievable under two channel knowledge assumptions for the first hop at the relays: global channel state information (CSI) and no channel knowledge [3]. If relay cooperation is enabled to make the two relays exchange the local CSI for the first hop channel, a relaying strategy, interference shaping, is presented to ensure that the two relays observe the same interference shape sent by two source groups. After each destination receives the virtually shaped signals from the relays, a subtraction approach that properly deals with two observations at each destination during the second and third time slots is applied for decoding. On the other hand, if relay cooperation is disabled, leading to no channel information for the first hop at the relays, a direct decoding method is provided. In general, according to [3], an inner bound of $\frac{2M}{3}$ DoF for the two-cell two-hop MIMO interference channel can be achieved by the existing half-duplex relaying scheme with three time slots.

In this paper, by introducing an interference cancellation method, we propose a new half-duplex relaying protocol which alternatively transmits signals between the source group and the relay. It is shown that the proposed relaying scheme is capable of achieving $\frac{(n-1)M}{n}$ DoF with $n \geq 3$ time slots, meaning that it provides higher or equal values of DoF compared to the existing half-duplex relaying protocol. Additionally, we observe a positive result that as the number of time slots $n$ goes to infinity, our proposed relaying protocol can achieve the cut-set bound without the global channel knowledge at the source and relay nodes.

The rest of the paper is organized as follows. In Section II, the system model is introduced. In Section III, the achievable DoF for the two-cell two-hop MIMO interference channel with half-duplex relays is characterized. This paper is concluded in Section IV.

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J. Jin, X.-C. Gao, S. Li, and Z. Wang are with the School of Information and Engineering, Zhengzhou University, Zhengzhou 450001, China.

X. Li is with the School of Physics and Electronic Information Engineering, Henan Polytechnic University, Jiaozuo 454000, China.

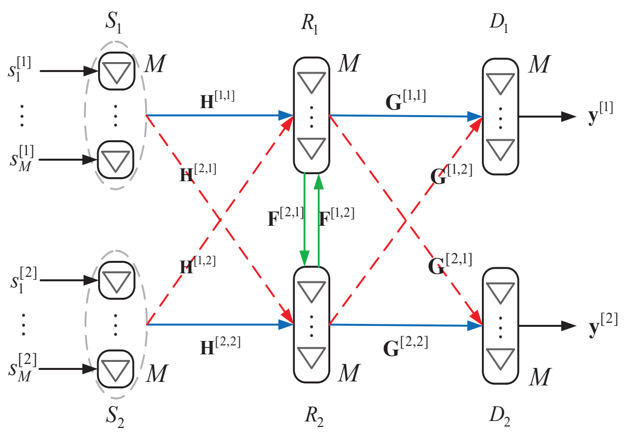


Fig. 1. System model for the symmetric two-cell two-hop MIMO interference channel.

## II. System Model

As shown in Fig. 1, a two-cell two-hop MIMO interference channel consists of two destinations ($D_1$ and $D_2$), two half-duplex relays ($R_1$ and $R_2$), and two source groups ($S_1$ and $S_2$). The destination and relay nodes are equipped with $M$ antennas, and each source group is composed of $M$ single antenna users. Each user in source group $S_i$ transmits data symbol $s_m^{[i]}$ to destination $D_i$ via two relays, where $i \in \{1,2\}$, $m \in \{1,\ldots,M\}$. Note that we assume that there is no direct communication links between the destinations and the source groups as in [3]. Denote $\mathbf{H}^{[j,i]}[n] \in \mathbb{C}^{M\times M}$, $\mathbf{G}^{[i,j]}[n] \in \mathbb{C}^{M\times M}$, $\mathbf{F}^{[2,1]}[n] \in \mathbb{C}^{M\times M}$, and $\mathbf{F}^{[1,2]}[n] \in \mathbb{C}^{M\times M}$ as the channel matrices at time slot $n$, from $S_i$ to $R_j$, from $R_j$ to $D_i$, from $R_1$ to $R_2$, and from $R_2$ to $R_1$, respectively, where $j \in \{1,2\}$. All elements in these channel matrices are independent and identically distributed (i.i.d.) random variables.

Since the relays are half-duplex, each relay can only transmit or receive at a time. Accordingly, there are three possible ways for operating the two relay nodes in one time slot when both relays are active: simultaneous transmission, simultaneous reception, one transmitting relay and one receiving relay. For the existing half-duplex relaying protocol in [3], only the case where the two relays receive simultaneously is taken into account. In this paper, for the two active relays during a certain time slot, besides the simultaneous reception case, we also consider a new case where one relay is receiving and the other is transmitting. When the two relays are active in different modes, the transmitting relay transmits to the destination nodes, while the the receiving relay receives the signals not only from the source group, but also from the transmitting relay. Hence, by activating the two relay nodes as a transmitter as well as a receiver at the same time, the first and second hop transmissions can take place simultaneously within a single time slot. In contrast, in the existing half-duplex relaying scheme, each time slot can only guarantee one hop transmission [3]. Therefore, by employing the alternate transmission mechanism different from that used in [3], our proposed protocol can obtain a normalized DoF of one in an asymptotic manner.

## III. Achievable DoF

In this section, we characterize the achievable DoF for the two-cell two-hop MIMO interference channel with half-duplex relays. The following theorem is the main result of this paper.

*Theorem 1*: For the two-cell two-hop MIMO interference channel where each source group consisting of $M$ single antenna users, transmits to the corresponding destination node with $M$ antennas through two half-duplex relays each equipped with $M$ antennas, $\frac{(n-1)M}{n}$ DoF are achievable without knowledge of global CSI available at the source and relay nodes when $n \geq 3$ time slots are utilized.

### A. Proof Theorem 1

We prove Theorem 1 by providing a new relaying protocol that employs the alternate transmission strategy as well as an interference cancellation method. The procedure used for designing the proposed protocol is illustrated in Fig. 2.

In the first time slot, unlike the existing relaying protocol which makes the two source groups convey their own data symbols to the two relay nodes simultaneously, we only let the source group 1, $S_1$, transmit to the relays. The received signal at the $j$-th relay during time slot 1 is given by

$$\mathbf{r}^{[j]}[1] = \mathbf{H}^{[j,1]}[1]\mathbf{s}^{[1]} + \mathbf{z}^{[j]}[1] \tag{1}$$

where $\mathbf{s}^{[1]} = [s_1^{[1]}, s_2^{[1]}, \ldots, s_M^{[1]}]^T$ is the transmit symbol vector with size of $M \times 1$ which is sent by $S_1$, and $\mathbf{z}^{[j]}[1] \in \mathbb{C}^{M\times 1}$ denotes the noise at $R_j$ in this time slot, $j \in \{1,2\}$. Notably, since the noise does not affect the DoF calculation, in the sequel, we ignore the noise term for simplicity as in [3]. After applying a ZF decoder $\mathbf{H}^{[j,1]}[1]^{-1}$ at $R_j$, both relays can easily decode $\mathbf{s}^{[1]}$.

For the second time slot, $R_1$ operates as the transmitting relay, and $R_2$ acts as the receiving relay. As shown in Fig. 2, $R_1$ forwards the decoded symbol vector $\mathbf{s}^{[1]}$ to $D_1$, $D_2$, and $R_2$, while $S_2$ transmits $\mathbf{s}^{[2]}$ to $R_2$, where $\mathbf{s}^{[2]} = [s_1^{[2]}, s_2^{[2]}, \ldots, s_M^{[2]}]^T$ is the $M \times 1$ transmit symbol vector sent by $S_2$. Thus, in time slot 2, the received signal at the receiving relay $R_2$ is given by

$$\mathbf{r}^{[2]}[2] = \mathbf{F}^{[2,1]}[2]\mathbf{s}^{[1]} + \mathbf{H}^{[2,2]}[2]\mathbf{s}^{[2]}. \tag{2}$$

Concatenating the received signals at $R_2$ in time slot 1 and 2, the effective channel input-output relationship is

$$\begin{aligned}\begin{bmatrix}\mathbf{r}^{[2]}[1]\\ \mathbf{r}^{[2]}[2]\end{bmatrix} &= \begin{bmatrix}\mathbf{H}^{[2,1]}[1]\mathbf{s}^{[1]}\\ \mathbf{F}^{[2,1]}[2]\mathbf{s}^{[1]} + \mathbf{H}^{[2,2]}[2]\mathbf{s}^{[2]}\end{bmatrix}\\ &= \underbrace{\begin{bmatrix}\mathbf{H}^{[2,1]}[1] & \mathbf{0}\\ \mathbf{F}^{[2,1]}[2] & \mathbf{H}^{[2,2]}[2]\end{bmatrix}}_{\mathbf{H}_{\text{eff}}^{R_2} \in \mathbb{C}^{2M\times 2M}} \begin{bmatrix}\mathbf{s}^{[1]}\\ \mathbf{s}^{[2]}\end{bmatrix}.\end{aligned} \tag{3}$$

For generic channel matrices $\mathbf{H}^{[2,1]}[1]$, $\mathbf{F}^{[2,1]}[2]$, and $\mathbf{H}^{[2,2]}[2]$, the effective channel matrix $\mathbf{H}_{\text{eff}}^{R_2}$ has full rank of $2M$ almost surely, which is proven in the Appendix. Hence, $R_2$ is able to recover $\mathbf{s}^{[1]}$ and $\mathbf{s}^{[2]}$ by applying a ZF decoder $\mathbf{H}_{\text{eff}}^{R_2\,-1}$. Note that since $R_2$ already obtains $\mathbf{s}^{[1]}$ in time slot 1, an interference cancellation method can be used as another way to decode $\mathbf{s}^{[2]}$ at $R_2$: eliminating the effect of $\mathbf{s}^{[1]}$ from $\mathbf{r}^{[2]}[2]$ first, and then

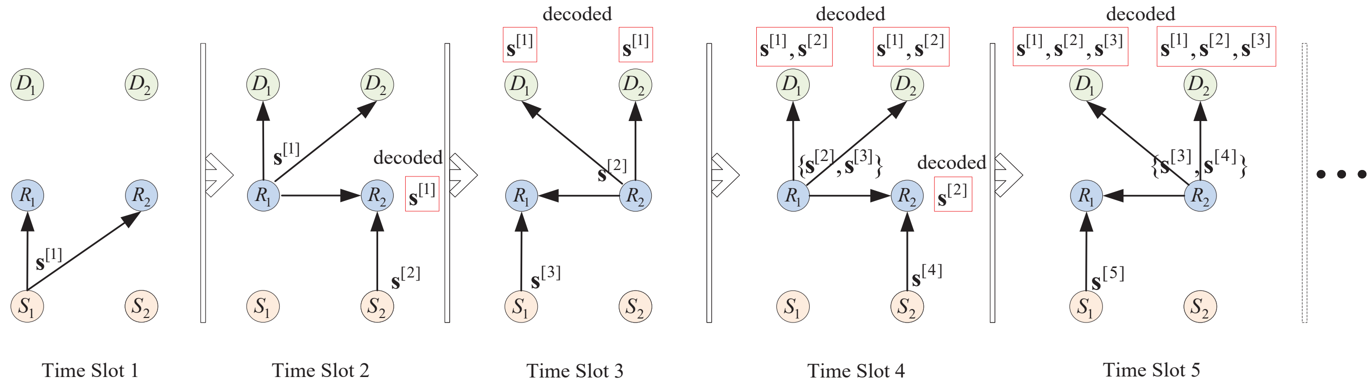


Fig. 2. Illustration of the design procedure for the proposed relaying protocol.

decoding $\mathbf{s}^{[2]}$ using a ZF decoder $\mathbf{H}^{[2,2]}[2]^{-1}$, namely,

$$\begin{aligned}&\mathbf{H}^{[2,2]}[2]^{-1}\left(\mathbf{r}^{[2]}[2]-\mathbf{F}^{[2,1]}[2]\mathbf{s}^{[1]}\right)\\&=\mathbf{H}^{[2,2]}[2]^{-1}\mathbf{H}^{[2,2]}[2]\mathbf{s}^{[2]}=\mathbf{s}^{[2]}.\end{aligned} \tag{4}$$

Consider the received signals at the two destinations in time slot 2, which are given by

$$\mathbf{y}^{[1]}[2]=\mathbf{G}^{[1,1]}[2]\mathbf{s}^{[1]}, \tag{5}$$

$$\mathbf{y}^{[2]}[2]=\mathbf{G}^{[2,1]}[2]\mathbf{s}^{[1]}. \tag{6}$$

By applying the ZF decoders $\mathbf{G}^{[1,1]}[2]^{-1}$ and $\mathbf{G}^{[2,1]}[2]^{-1}$ at $D_1$ and $D_2$, respectively, $D_1$ decodes the intended symbol vector $\mathbf{s}^{[1]}$, and $D_2$ acquires the unintended symbol vector $\mathbf{s}^{[1]}$.

In time slot 3, $R_1$ acts as the receiving relay, and $R_2$ operates as the transmitting relay. As depicted in Fig. 2, $R_2$ conveys the obtained symbol vector $\mathbf{s}^{[2]}$ to $R_1$, $D_1$ and $D_2$, while $S_1$ forwards $\mathbf{s}^{[3]}$ to $R_1$. Remarkably, in time slot $n$, if $n$ is odd, $\mathbf{s}^{[n]}$ is the $M\times 1$ symbol vector transmitted from $S_1$ to $D_1$; otherwise, $\mathbf{s}^{[n]}$ is delivered from $S_2$ to $D_2$ for even $n$. During the third time slot, the received signal at the receiving relay $R_1$ is given by

$$\mathbf{r}^{[1]}[3]=\mathbf{F}^{[1,2]}[3]\mathbf{s}^{[2]}+\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]}. \tag{7}$$

The received signals at the destinations in this time slot are given by

$$\mathbf{y}^{[1]}[3]=\mathbf{G}^{[1,2]}[3]\mathbf{s}^{[2]}, \tag{8}$$

$$\mathbf{y}^{[2]}[3]=\mathbf{G}^{[2,2]}[3]\mathbf{s}^{[2]}. \tag{9}$$

From (8) and (9), it can be seen that $D_1$ attains the undesired symbol vector $\mathbf{s}^{[2]}$ by applying the ZF decoder $\mathbf{G}^{[1,2]}[3]^{-1}$, and $D_2$ is capable of decoding its desired symbol vector $\mathbf{s}^{[2]}$ by applying the ZF decoder $\mathbf{G}^{[2,2]}[3]^{-1}$.

In time slot 4, $R_1$ is the transmitting relay, and $R_2$ is the receiving one, so $R_1$ sends the received signal $\mathbf{r}^{[1]}[3]$ which is a linear combination of $\mathbf{s}^{[2]}$ and $\mathbf{s}^{[3]}$, while $S_2$ transmits $\mathbf{s}^{[4]}$ to $R_2$. Then the received signal at the receiving relay $R_2$ is given by

$$\begin{aligned}\mathbf{r}^{[2]}[4]&=\mathbf{F}^{[2,1]}[4]\mathbf{r}^{[1]}[3]+\mathbf{H}^{[2,2]}[4]\mathbf{s}^{[4]}\\&=\mathbf{F}^{[2,1]}[4]\left(\mathbf{F}^{[1,2]}[3]\mathbf{s}^{[2]}+\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]}\right)\\&\quad+\mathbf{H}^{[2,2]}[4]\mathbf{s}^{[4]}.\end{aligned} \tag{10}$$

Besides, the received signals at the two destination nodes in the fourth time slot are given by

$$\begin{aligned}\mathbf{y}^{[1]}[4]&=\mathbf{G}^{[1,1]}[4]\mathbf{r}^{[1]}[3]\\&=\mathbf{G}^{[1,1]}[4]\left(\mathbf{F}^{[1,2]}[3]\mathbf{s}^{[2]}+\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]}\right),\end{aligned} \tag{11}$$

$$\begin{aligned}\mathbf{y}^{[2]}[4]&=\mathbf{G}^{[2,1]}[4]\mathbf{r}^{[1]}[3]\\&=\mathbf{G}^{[2,1]}[4]\left(\mathbf{F}^{[1,2]}[3]\mathbf{s}^{[2]}+\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]}\right).\end{aligned} \tag{12}$$

Now, we focus on the decoding process at $D_1$. The concatenated received signals at $D_1$ during the third and fourth time slots are given by

$$\begin{aligned}\begin{bmatrix}\mathbf{y}^{[1]}[3]\\\mathbf{y}^{[1]}[4]\end{bmatrix}&=\begin{bmatrix}\mathbf{G}^{[1,2]}[3]\mathbf{s}^{[2]}\\\mathbf{G}^{[1,1]}[4]\left(\mathbf{F}^{[1,2]}[3]\mathbf{s}^{[2]}+\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]}\right)\end{bmatrix}\\&=\underbrace{\begin{bmatrix}\mathbf{G}^{[1,2]}[3]&\mathbf{0}\\\mathbf{G}^{[1,1]}[4]\mathbf{F}^{[1,2]}[3]&\mathbf{G}^{[1,1]}[4]\mathbf{H}^{[1,1]}[3]\end{bmatrix}}_{\mathbf{H}_{\text{eff}}^{D_1}\in\mathbb{C}^{2M\times 2M}}\begin{bmatrix}\mathbf{s}^{[2]}\\\mathbf{s}^{[3]}\end{bmatrix}.\end{aligned} \tag{13}$$

Similar to the proof shown in the Appendix, we can prove that the effective channel matrix $\mathbf{H}_{\text{eff}}^{D_1}$ in (13) has full rank of $2M$ almost surely. This implies that $D_1$ is able to decode the desired symbol vector $\mathbf{s}^{[3]}$ as well as the undesired symbol vector $\mathbf{s}^{[2]}$ by applying a ZF decoder. Note that $\mathbf{s}^{[2]}$ is already acquired at $D_1$ through our relaying design in time slot 3, and thus we provide another manner for $D_1$ to extract the desired symbol vector $\mathbf{s}^{[3]}$: $D_1$ employs the interference cancellation, i.e., $\mathbf{y}^{[1]}[4]-\mathbf{G}^{[1,1]}[4]\mathbf{F}^{[1,2]}[3]\mathbf{s}^{[2]}$ to eliminate the interference effect from the received signal vector $\mathbf{y}^{[1]}[4]$ first and then applies a ZF decoder $\left(\mathbf{G}^{[1,1]}[4]\mathbf{H}^{[1,1]}[3]\right)^{-1}$ to attain $\mathbf{s}^{[3]}$. In a similar fashion, $D_2$ can successfully obtain the undesired symbol vector $\mathbf{s}^{[3]}$ during the fourth time slot.

In time slot 5, $R_1$ is the receiving relay, and $R_2$ is the transmitting one. Since $\mathbf{s}^{[2]}$ is already decoded at $R_2$ in time slot 2, $R_2$ first obtains a linear combination containing $\mathbf{s}^{[3]}$ and $\mathbf{s}^{[4]}$ by implementing the interference cancellation, i.e., $\mathbf{r}^{[2]}[4] - \mathbf{F}^{[2,1]}[4]\mathbf{F}^{[1,2]}[3]\mathbf{s}^{[2]} = \mathbf{F}^{[2,1]}[4]\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]} + \mathbf{H}^{[2,2]}[4]\mathbf{s}^{[4]}$, and then forwards this linear combination of $\mathbf{s}^{[3]}$ and $\mathbf{s}^{[4]}$. At the same time, $S_1$ sends $\mathbf{s}^{[5]}$ to $R_1$. Therefore, the received signal at the receiving relay $R_1$ is given by

$$\mathbf{r}^{[1]}[5] = \mathbf{F}^{[1,2]}[5]\big(\mathbf{F}^{[2,1]}[4]\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]} + \mathbf{H}^{[2,2]}[4]\mathbf{s}^{[4]}\big) + \mathbf{H}^{[1,1]}[5]\mathbf{s}^{[5]}. \tag{14}$$

The received signals at the destinations in time slot 5 are given by

$$\mathbf{y}^{[1]}[5] = \mathbf{G}^{[1,2]}[5]\big(\mathbf{F}^{[2,1]}[4]\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]} + \mathbf{H}^{[2,2]}[4]\mathbf{s}^{[4]}\big), \tag{15}$$

$$\mathbf{y}^{[2]}[5] = \mathbf{G}^{[2,2]}[5]\big(\mathbf{F}^{[2,1]}[4]\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]} + \mathbf{H}^{[2,2]}[4]\mathbf{s}^{[4]}\big). \tag{16}$$

Here we explain the decoding approach by focusing on $D_2$. After removing the effect of $\mathbf{s}^{[2]}$ from $\mathbf{y}^{[2]}[4]$, the concatenated input-output relationship seen by $D_2$ is

$$\begin{aligned}&\begin{bmatrix}\mathbf{y}^{[2]}[4] - \mathbf{G}^{[2,1]}[4]\mathbf{F}^{[1,2]}[3]\mathbf{s}^{[2]} \\ \mathbf{y}^{[2]}[5]\end{bmatrix} \\ &= \begin{bmatrix}\mathbf{G}^{[2,1]}[4]\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]} \\ \mathbf{G}^{[2,2]}[5]\big(\mathbf{F}^{[2,1]}[4]\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]} + \mathbf{H}^{[2,2]}[4]\mathbf{s}^{[4]}\big)\end{bmatrix} \\ &= \underbrace{\begin{bmatrix}\mathbf{G}^{[2,1]}[4]\mathbf{H}^{[1,1]}[3] & \mathbf{0} \\ \mathbf{G}^{[2,2]}[5]\mathbf{F}^{[2,1]}[4]\mathbf{H}^{[1,1]}[3] & \mathbf{G}^{[2,2]}[5]\mathbf{H}^{[2,2]}[4]\end{bmatrix}}_{\mathbf{H}_{\text{eff}}^{D_2} \in \mathbb{C}^{2M\times 2M}} \begin{bmatrix}\mathbf{s}^{[3]} \\ \mathbf{s}^{[4]}\end{bmatrix}\end{aligned} \tag{17}$$

where the rank of the effective matrix $\mathbf{H}_{\text{eff}}^{D_2}$ is $2M$, which can be proved in a similar way as that shown in the Appendix. Hence, $D_2$ can obtain the desired symbol vector $\mathbf{s}^{[4]}$ by using a ZF decoder. Notably, Because $\mathbf{s}^{[3]}$ is already recovered at $D_2$ in time slot 4, we present another way for $D_2$ to decode the intended symbol vector $\mathbf{s}^{[4]}$: $D_2$ performs the interference cancellation, i.e., $\mathbf{y}^{[2]}[5] - \mathbf{G}^{[2,2]}[5]\mathbf{F}^{[2,1]}[4]\mathbf{H}^{[1,1]}[3]\mathbf{s}^{[3]}$ to remove the effect of $\mathbf{s}^{[3]}$ from $\mathbf{y}^{[2]}[5]$, and then employs a ZF decoder $\big(\mathbf{G}^{[2,2]}[5]\mathbf{H}^{[2,2]}[4]\big)^{-1}$ to obtain $\mathbf{s}^{[4]}$. Also, $D_1$ is able to acquire the unintended symbol vector $\mathbf{s}^{[4]}$ during the fifth time slot in a similar manner.

Next, we continue the relay transmission strategy exhibited in Fig. 3 in subsequent time slots. For example, in time slot 6, the transmitting relay $R_1$ forwards $\mathbf{r}^{[1]}[5]$ which is a linear combination of $\mathbf{s}^{[3]}$, $\mathbf{s}^{[4]}$ and $\mathbf{s}^{[5]}$, while $S_2$ transmits $\mathbf{s}^{[6]}$ to $R_2$. As can be seen in Fig. 2 and Fig. 3, in time slot $n \geq 3$, before proceeding to the decoding process, both destinations already have knowledge of $n-2$ decoded symbol vectors $\{\mathbf{s}^{[1]}, \ldots, \mathbf{s}^{[n-2]}\}$. Since the received signal at each destination is a linear combination containing some of the decoded symbol vectors as well as the undecoded $\mathbf{s}^{[n-1]}$, a successive interference cancellation approach can be used to eliminate the effect of the known symbol vectors and extract out $\mathbf{s}^{[n-1]}$.

In summary, for all $n \geq 3$, by exploiting the proposed relaying protocol, both destinations are capable of decoding $\mathbf{s}^{[1]}, \mathbf{s}^{[2]}, ..., \mathbf{s}^{[n-1]}$ after the transmission in the $n$-th ($n \geq 3$) time slot while guaranteeing that at least one desired symbol vector is obtained at each destination. As a result, $(n-1)M$ data symbols are conveyed over $n$ time slots, which means that $\frac{(n-1)M}{n}$ DoF can be achieved. In particular, when the number of time slots utilized by the proposed relaying protocol approaches infinity, the achievable normalized DoF is

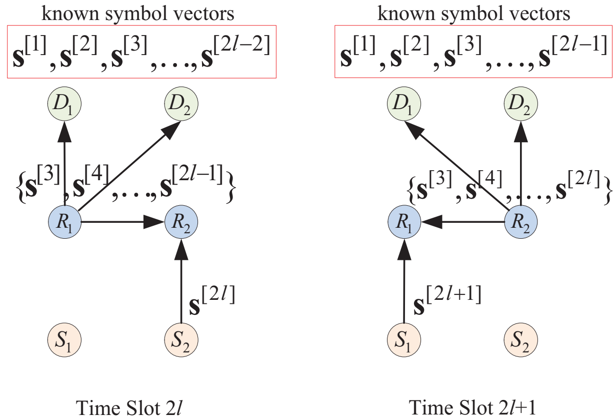


Fig. 3. Illustration of the relay transmission strategy for the proposed relaying protocol in time slots $2l$ and $2l+1$ for $l \geq 3$.

$$DoF_{sum} = \lim_{n\to\infty} \frac{\frac{(n-1)M}{n}}{M} = 1. \tag{18}$$

### B. Channel Estimation and CSI Knowledge

For the existing relaying protocol without relay cooperation in [3], it is assumed that the two destinations can estimate the effective channels, each of which is the product of two channel matrices in different hops of transmission. To this end, the two relay nodes should send precoded pilot symbols using the first hop channel matrices, i.e., $\mathbf{H}^{[j,i]}[1]$, to guarantee the successful estimation of the effective channels, i.e., $\mathbf{G}^{[i,j]}[2]\mathbf{H}^{[j,i]}[1]$ and $\mathbf{G}^{[i,j]}[3]\mathbf{H}^{[j,i]}[1]$ at the destinations. Here, the pilot design for our proposed relaying protocol follows the similar manner as that exploited in [3]. Remarkably, in this paper, the pilot symbols sent by the relays can also be precoded using inter-relay channel matrices (e.g., $\mathbf{F}^{[1,2]}[3]$), so as to ensure the accurate estimation of the effective channels containing the inter-relay channel matrices (e.g., $\mathbf{G}^{[1,1]}[4]\mathbf{F}^{[1,2]}[3]$ in (11)). In this way, the proposed relaying protocol does not need relay cooperation and source group cooperation for sharing the local CSI, but requires local CSI at the relay and destination nodes to achieve the anticipated DoF value. This means that the proposed protocol can asymptotically achieve a normalized DoF of 1 without the global channel knowledge at the source and relay nodes.

### C. DoF Comparison

Since the proposed relaying protocol can achieve $\frac{(n-1)M}{n}$ DoF by utilizing $n \geq 3$ time slots, it is clear that the achievable DoF of the proposed protocol is greater than (for $n \geq 4$), or equal to (for $n = 3$), $\frac{2M}{3}$ of DoF achieved by the existing relaying scheme in [3]. In addition, by exploiting the proposed scheme, a normalized DoF of one can be achieved

TABLE I
NORMALIZED DOF COMPARISON BASED ON CSI KNOWLEDGE

| Achievable scheme | Normalized DoF | CSI at source | CSI at relay |
|---|---|---|---|
| Aligned interference neutralization (infinite antenna resources) [6] | 1 | $\mathbf{H}^{[j,i]}$ | $\mathbf{H}^{[j,i]}$ and $\mathbf{G}^{[i,j]}$ |
| Retrospective interference alignment [7] | $\frac{2}{3}$ | delayed $\mathbf{H}^{[j,i]}$ | $\mathbf{H}^{[j,i]}$ and delayed $\mathbf{G}^{[i,j]}$ |
| Existing protocol with relay cooperation [3] | $\frac{2}{3}$ | No | $\mathbf{H}^{[j,i]}$ |
| Existing protocol without relay cooperation [3] | $\frac{2}{3}$ | No | No |
| Proposed protocol (infinite time resources) | 1 | No | No |
| TDMA | $\frac{1}{2}$ | No | No |

asymptotically when infinite time slots are used. Therefore, a detailed comparison of the normalized DoF for the proposed relaying design with infinite time resources and existing relaying schemes is illustrated in Table I. The maximum achievable normalized DoF for the two-hop interference channel with half-duplex relays is limited to 1 (cut-set bound). Notably, the aligned interference neutralization method achieves $2M-1$ DoF in the $2\times2\times2$ MIMO interference networks where each node is equipped with $M$ antennas for the case of full-duplex relaying [6], so that it is able to obtain one normalized DoF for the case of half-duplex relaying as the number of antennas $M$ goes to infinity, i.e., $\lim_{M\to\infty}\frac{2M-1}{2M}=1$. As shown in Table I, to attain the cut-set DoF, the proposed scheme does not require global CSI at the source and relay nodes, while the aligned interference neutralization method in [6] demands global CSI for both hops at the relay and global channel knowledge for the first hop at the source. It means that the proposed relaying protocol has an advantage in the feedback overhead compared to the aligned interference neutralization method.

## IV. CONCLUSION

For the two-cell two-hop MIMO interference channel with two half-duplex relays, $\frac{2M}{3}$ DoF are achievable by the existing half-duplex relaying protocol utilizing three time slots. In this paper, under the assumption that only local CSI is available at the relay and destination nodes, we propose a new relaying protocol which is capable of achieving the DoF of $\frac{(n-1)M}{n}$ by employing $n\geq3$ time slots. This means that the achievable DoF of the proposed scheme is greater than or equal to that of the existing half-duplex relaying design [3]. Furthermore, it is observed that a normalized DoF of one can be achieved in an asymptotic manner by our proposed relaying protocol.

## APPENDIX

According to [8], to prove that $\mathbf{H}_{\text{eff}}^{R_2}$ has full rank of $2M$, it suffices to show that one specialization of $\mathbf{H}_{\text{eff}}^{R_2}$ has full rank. Now, let us specialize to the matrices

$$\mathbf{H}^{[2,1]}[1]:=\mathbf{I}_M,\ \mathbf{F}^{[2,1]}[2]:=\mathbf{I}_M,\ \text{and}\ \mathbf{H}^{[2,2]}[2]:=\mathbf{I}_M$$

where $\mathbf{I}_M$ stands for the identity matrix with size of $M\times M$. By applying these specializations, the effective channel matrix $\mathbf{H}_{\text{eff}}^{R_2}$ becomes

$$\begin{bmatrix}\mathbf{I}_M & \mathbf{0}\\ \mathbf{I}_M & \mathbf{I}_M\end{bmatrix}$$

whose determinant is 1 because the determinant of the above lower triangular matrix directly equals the product of the diagonal entries. Consequently, there exists an inverse of the specialization of $\mathbf{H}_{\text{eff}}^{R_2}$, and thus it is true that $\mathbf{H}_{\text{eff}}^{R_2}$ has full rank of $2M$.

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