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MEASURING THE SPIN OF SPIRAL GALAXIES

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ABSTRACT

We compute the angular momentum, the spin parameter and the related distribution function for Dark Matter halos hosting a spiral galaxy. We base on scaling laws, inferred from observations, that link the properties of the galaxy to those of the host halo; we further assume that the Dark Matter has the same total specific angular momentum of the baryons. Our main results are: (i) we find that the gas component of the disk significantly contributes to the total angular momentum of the system; (ii) by adopting for the Dark Matter the observationally supported Burkert profile, we compute the total angular momentum of the disk and its correlation with the rotation velocity; (iii) we find that the distribution function of the spin parameter λ peaks at a value of about 0.03, consistent with a no-major-merger scenario for the late evolution of spiral galaxies.

Subject headings: galaxies: halos - galaxies: spiral - galaxies: formation - galaxies: kinematics and dynamics

1. INTRODUCTION

The mechanism of galaxy formation, as currently understood, involves the cooling and condensation of baryons inside the gravitational potential well provided by the Dark Matter (DM); in spirals, a rotationally supported disk is formed, whose structure is governed by angular momentum acquired through tidal interactions during the precollapse phase.

Under the assumption of specific angular momentum conservation, that holds when the baryons and the DM are initially well mixed, the dynamics of the dark halo is directly related to the disk scale length (see Fall & Efstathiou 1980). This tight connection between halo dynamics and disk geometry is quantified by the spin parameter λ (Peebles 1969).

The general procedure for the computation of the angular momentum has been described in detail by Mo, Mao & White (1998); it relies on 3 basic assumptions: (i) the mass of the galactic disk is a universal fraction of the halo's; (ii) the total angular momentum of the disk is also a fixed fraction of the halo's; (iii) the disk is thin, centrifugally supported and stable, with an exponential surface density profile. The theory is applied to a Navarro, Frenk & White (1997; NFW) DM potential.

In the present work, we propose to determine the angular momentum and the spin parameter of disk galaxies by making use of the observed matter distribution in spirals, and of observed scaling relations between halo and disk properties. To this purpose, we adopt a modified set of assumptions: we relax (i), and use instead an empirical relation that links the disk mass to that of its DM halo (Shankar et al. 2005); we retain (ii) and suppose total specific angular momentum conservation during the disk formation, i.e., $J_D/M_D = J_H/M_H$ in terms of the disk and halo masses M_D, M_H and of the related total angular momenta J_D, J_H; as to (iii), we still assume that the disk is centrifugally supported, stable, and distributed according to an exponential surface density profile, but we also take into account the gaseous (HI+He) component. Finally, we perform the computation for a Burkert (1995) halo.

Throughout this work we adopt a flat cosmology with matter density parameter $\Omega_M \approx 0.27$ and Hubble constant $h = H_0/100$ km s^{-1} Mpc^{-1} = 0.71 (Spergel et al. 2003). Correspondingly, given a halo with mass M_H we determine its radius as $R_H = (3\,M_H/4\pi\,\rho_c\,\Omega_M^z\,\Delta_H)^{1/3}$; here $\rho_c = 2.8\times 10^{11}\,h^2\,M_\odot\,\mathrm{Mpc}^{-3}$ is the critical density, $\Omega_M^z = \Omega_M\,(1+z)^3/[(1-\Omega_M)+\Omega_M\,(1+z)^3]$ is the evolved matter density parameter at redshift z, and $\Delta_H = 18\,\pi^2+82\,(\Omega_M^z-1)-39\,(\Omega_M^z-1)^2$ is the density contrast at virialization, that takes on the value $\Delta_H \approx 100$ at $z = 0$.

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2. THE ANGULAR MOMENTUM

The fundamental parameters of the stellar disk and halo mass distributions can be obtained straightforwardly by means of three observational scaling relations, linking the disk mass to the halo mass, to the halo central density, and to the disk scale length.

The total mass of the stellar disk M_D that resides in a halo of given mass M_H is given by the relation (Shankar et al. 2005, see their Fig. 3):

$$M_D \approx 2.3\times 10^{10}\,M_\odot\;\frac{(M_H/3\;10^{11}\;M_\odot)^{3.1}}{1+(M_H/3\;10^{11}\;M_\odot)^{2.2}}\;; \qquad (1)$$

this holds for halo masses between 10^{11} and about $3\times 10^{12}\,M_\odot$, wide enough to include most of the spiral population, except dwarfs.

We model the stellar disk with a thin, exponential surface density profile of the form

$$\Sigma_D(r) = \frac{M_D}{2\pi\;R_D^2}\,e^{-r/R_D}\;. \qquad (2)$$

The characteristic scale-length R_D is estimated through the relation

$$\log\frac{R_D}{\mathrm{kpc}} = 0.633+0.379\log\frac{M_D}{10^{11}M_\odot}+0.069\left(\log\frac{M_D}{10^{11}M_\odot}\right)^2\;, \qquad (3)$$

inferred from dynamical mass determinations by Persic, Salucci & Stel (1996); these scale lengths are consistent with the data by Simard et al. (1999).

For the DM, we adopt a Burkert distribution $\rho_H(r) = \rho_0\;R_0^3/(r+R_0)(r^2+R_0^2)$, with R_0 the core radius and ρ_0 the constant core density. Correspondingly, the total halo mass inside the radius r is given by

$$M_H(<r) = 4\,M_0\,\left[\ln\left(1+\frac{r}{R_0}\right)-\tan^{-1}\left(\frac{r}{R_0}\right)+\frac{1}{2}\ln\left(1+\frac{r^2}{R_0^2}\right)\right]\;, \qquad (4)$$

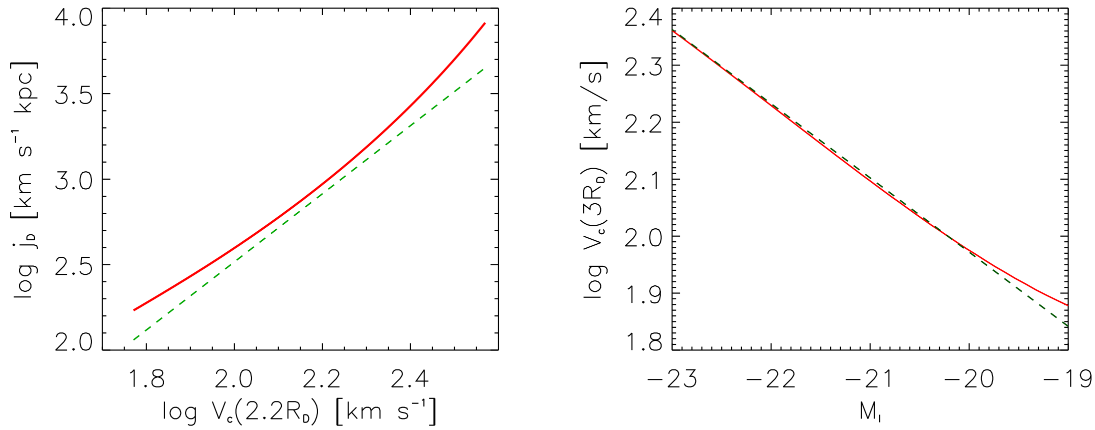

FIG. 1.— Left panel: the specific angular momentum of the disk as a function of the rotation velocity at $2.2\,R_D$. *Solid* line is the result from this work, adopting the Burkert profile; *dashed* line is the best-fit relation from the data collected by Navarro & Steinmetz (2000), see their Figure 3. Right panel: the Tully-Fisher relation. *Solid* line represents the result from this work and *dashed* line illustrates the fit to the data by Giovanelli et al. (1997).

with $M_0 = 1.6\,\rho_0 R_0^3$ being the mass of the core. The core density ρ_0 is determined from the disk mass through the relation obtained from the Universal Rotation Curve (Burkert & Salucci 2000):

$$\log \frac{\rho_0}{\mathrm{g\ cm^{-3}}} = -23.515 - 0.964 \left(\frac{M_D}{10^{11} M_\odot}\right)^{0.31} . \quad (5)$$

The core radius R_0 is set by requiring $M_H(< R_H) = M_H$; we find that the approximate relation

$$\log(R_0/\mathrm{kpc}) \approx 0.65 + 0.56 \log(M_H/10^{11} M_\odot) \quad (6)$$

holds within a few percents.

The total circular velocity of the disk system is

$$V_c^2(r) = V_D^2(r) + V_H^2(r) . \quad (7)$$

For a thin, centrifugally supported disk the circular velocity is given by $V_D^2(r) = (G M_D/2 R_D)\ x^2 B(x/2)$; here $x = r/R_D$ and the quantity $B = I_0 K_0 - I_1 K_1$ is a combination of the modified Bessel functions that accounts for the disk asphericity (Freeman 1970). Moreover, the halo circular velocity is simply $V_H^2(r) = G M_H(< r)/r$, and it is useful to define $V_H = \sqrt{G M_H/R_H}$. Given the relations (1), (3), (5) and (6) linking the basic quantities of the system, the shape and amplitude of the velocity profile depend only on the halo mass.

In order to check our mass model and empirical scaling relations, we compute the I-band Tully-Fisher relation at $r = 3 R_D$. We obtain the B-band luminosity from the stellar disk mass through the relation $\log(L_B/L_\odot) \approx 1.33 + 0.83 \log(M_D/M_\odot)$ by Shankar et al. (2005), then convert the related magnitude in I-band through the mean colour $B-I \approx 2$ (Fukugita, Shimasaku & Ichikawa 1995). In Figure 1 (right) we compare the result with the data by Giovanelli et al. (1997), finding an excellent agreement.

We compute the angular momentum of the disk as

$$J_D = 2\pi \int_0^\infty \Sigma_D(r)\, r\, V_c(r)\, r\, dr = M_D R_D V_H f_R \quad (8)$$

with

$$f_R = \int_0^\infty x^2 e^{-x} \frac{V_c(x R_D)}{V_H} dx ; \quad (9)$$

here $x = r/R_D$ and $M_D = 2\pi \Sigma_0 R_D^2$. Note that J_D depends linearly both on the mass and on the radial extension of the disk, while the DM distribution enters the computation through the integrated velocity profile, encased into the shape factor f_R; the latter slowly varies (by a factor 1.3 at most) throughout our range of halo masses.

In Figure 1 (left) we show the specific angular momentum of the disk as a function of the total circular velocity at $r = 2.2\,R_D$, computed as $j_D = J_D/M_D$ from Eq. (8). Plotted for comparison is also the best-fit relation by Navarro & Steinmetz (2000) from their collection of data; note that these authors adopted a flat rotation curve, so that $f_R = 2$ and $j_D = 2 R_D V_H$.

We derive the halo angular momentum by assuming the conservation of the total specific angular momentum between DM and baryons, an *ansatz* widely supported/adopted in the literature (Mestel 1963; Mo et al. 1998; van den Bosch et al. 2001, 2002; Burkert & D' Onghia 2004; Peirani, Mohayaee & de Freitas Pacheco 2004). The conservation law is written

$$J_H = J_D \frac{M_H}{M_D} . \quad (10)$$

Note that small variations of J_D are magnified by a factor M_H/M_D in the value of J_H, i.e., the latter is rather sensitive to the radial extension of the baryons.

We now consider, along with the stars, the gaseous component that envelops the disk of spiral galaxies. We derive the total mass of the gas component from the disk luminosity (see above) through the relation

$$M_{\mathrm{gas}} = 2.13 \times 10^6 M_\odot \left(\frac{L_B}{10^6 L_\odot}\right)^{0.81} \left[1 - 0.18 \left(\frac{L_B}{10^8 L_\odot}\right)^{-0.4}\right] \quad (11)$$

by Persic & Salucci (1999), where we have included a factor 1.33 to account for the He abundance. Since the gas mass is on average much less than the halo mass (for increasing halo mass the baryons to DM ratio raises, while the gas to baryon ratio lowers), both the total mass and the rotation curve remain virtually unaltered ($V_{\mathrm{gas}} \sim \sqrt{M_{\mathrm{gas}}/R_{\mathrm{gas}}}$) by the presence of the gaseous component.

However, the gas is much more diffuse than the stars, reaching out several disk scale lengths (see Corbelli & Salucci 2000; Dame 1993); since most of the angular momentum comes from material at large galactocentric distances (van den Bosch et al. 2001), we expect that the gas can add a significant contribution to the total angular momentum (see Eq. [8]), especially in small spirals where the gas to baryon fraction is close to 50%.

The detailed density profile of the gas in spirals is still under debate in the literature. However, we are confident that the main factors entering the computation of the gas angular momentum $J_{\rm gas}$ are just the gas total mass $M_{\rm gas}$ and the radial extension of its distribution; in other words, we expect that the details of the gas profile do not significantly affect the results. In order to check this statement, we considered 3 different gas models: (i) a disk-like distribution (DL), with characteristic scale length αR_D; (ii) a uniform distribution (U) out to a radius βR_D; and (iii) an $M33$-like gaussian distribution (M33; Corbelli et al. 2000). These read

$$\begin{aligned}
\Sigma^{\rm DL}_{\rm gas}(r) &= \frac{M_{\rm gas}}{2\pi\,\alpha^2 R_D^2}\, e^{-r/\alpha R_D} \\
\Sigma^{\rm U}_{\rm gas}(r) &= \frac{M_{\rm gas}}{\pi\,\beta^2 R_D^2}\,\theta(r-\beta R_D) \qquad (12)\\
\Sigma^{\rm M33}_{\rm gas}(r) &= \frac{M_{\rm gas}}{\pi\,(2k_1^2+k_2^2)\,R_D^2}\, e^{-(r/k_1 R_D)-(r/k_2 R_D)^2} \;.
\end{aligned}$$

As fiducial values of the parameters, we adopt $\alpha \approx 3$ in the first expression, $\beta \approx 6$ in the second one (Dame 1993), and $k_1 \approx 11.9$, $k_2 \approx 5.87$ in the last one (Corbelli & Salucci 2000). Each profile has been normalized to the total gas mass $M_{\rm gas}$ as computed from Eq. (11).

In parallel with Eq. (8), the gas component of the angular momentum will be

$$J_{\rm gas} = 2\pi \int_0^\infty \Sigma_{\rm gas}(r)\, r\, V_c(r)\, r\, dr = M_{\rm gas}\, R_D\, V_H\, f_{\rm gas}\;, \qquad (13)$$

where the shape parameter $f_{\rm gas}$ encodes the specific gas distribution. On comparing its values for the 3 models we find differences of less than 15%, and so confidently choose the gaussian profile as a baseline.

We then compute the halo angular momentum as a function of the total baryonic one as

$$J_H = (J_D + J_{\rm gas})\,\frac{M_H}{M_D+M_{\rm gas}}\;. \qquad (14)$$

The gas is dynamically affecting the system mainly through its different spatial distribution with respect to that of the stars, adding an angular momentum component that is significant at large radii compared to R_D.

3. THE SPIN PARAMETER

As already mentioned, the spin parameter is a powerful tool to investigate galaxy formation, as it is strictly related to both the dynamics and the geometry of the system. In this § we compute its values and distribution function basing on the results of § 2.

The spin parameter is defined as follows:

$$\lambda = \frac{J_H\,|E_H|^{1/2}}{G M_H^{5/2}} \qquad (15)$$

where G is the gravitational constant, and E_H is the total energy of the halo. The latter is computed as $|E_H| = 2\pi\int dr\, r^2\, \rho_H(r)\, V_c^2$ after the virial theorem, and having supposed that all the DM particles orbit on circular tracks.

Bullock et al. (2001) proposed the alternative definition

$$\lambda' = \frac{J_{\rm H}}{\sqrt{2}\,M_H R_H V_H} = \frac{J_H+J_D+J_{\rm gas}}{\sqrt{2}(M_H+M_D+M_{\rm gas})\,R_H V_H}\;, \qquad (16)$$

where the second equality holds after Eq. (14).

While the values of λ and λ' are virtually equivalent in the case of NFW halos (Bullock et al. 2001), we find that for Burkert halos they differ appreciably, with the ratio λ/λ' going from ~ 1.5 to ~ 1.2 when M_H ranges from 10^{11} to $3\times10^{12}\,M_\odot$. In Figure 2 (top panels) we plot both λ and λ' as a function of the halo mass. We highlight the difference in the value of the spin parameter when the gas component is included, especially in low mass halos.

To compute the probability distributions $\mathcal{P}(\lambda)$ and $\mathcal{P}(\lambda')$ of the spin parameters, we make use of the galactic halo mass function, i.e., the number density of halos with mass M_H containing a single baryonic core, as derived by Shankar et al. (2005). A good fit is provided by the Schechter function

$$\Psi(M_H) = \Psi_0\,\left(\frac{M_H}{\bar{M}}\right)^{\alpha}\, e^{-M_H/\bar{M}}\;, \qquad (17)$$

with parameters $\alpha = -1.84$, $\bar{M} = 1.12\times10^{13}\,M_\odot$ and $\Psi_0 = 3.1\times10^{-4}\,M_\odot^{-1}$ Mpc^{-3}; note that within our range of halo masses, this is mostly contributed by spirals. For the computation of $\mathcal{P}(\lambda)$ or $\mathcal{P}(\lambda')$, we randomly picked up a large sample of masses distributed according to Eq. (17), then compute λ or λ' for each using Eqs. (15) and (16), and eventually build up the statistical distributions. Furthermore, during this procedure we have convolved the relations (15) and (16) with a gaussian scatter of 0.15 dex that takes into account the statistical uncertainties in the empirical scaling laws we adopt; these are mostly due to the determination of R_D through Eq. (3), for which we have determined the scatter by using the disk mass estimates of individual spirals reported in Persic & Salucci (1990).

As shown in Figure 2 (bottom panels), we find a distribution peaked around a value of about 0.03 for λ and about 0.02 for λ', when the gas is considered. We stress that this value of λ' is close to the result of the simulations by D'Onghia & Burkert (2004), who on average find $\lambda' = 0.023$ for spirals quietly evolving (i.e., experiencing no major mergers) since $z \approx 3$, see Figure 4 of their paper. In addition, Burkert & D'Onghia (2005) argue that this value of λ' provides a very good fit to the observed relation between the disk scale length and the maximum rotation velocity (see their Figure 1). We also stress that our most probable value for λ is in agreement with the results by Gardner (2001), Vitvitska et al. (2002) and Peirani et al. (2004), who find a peak value of the distribution function at around 0.03 for halos that evolved mainly through smooth accretion. On the other hand, our λ and $P(\lambda)$ differs from that inferred by van den Bosch et al. (2001) for dwarf galaxies, likely because these authors estimate values of R_D higher than ours by factors up to 3 for M_D around $10^9\,M_\odot$.

4. DISCUSSION AND CONCLUSIONS

In this *Letter* we have computed the angular momentum, the spin parameter and the related distribution function for DM halos hosting a spiral galaxy. We have relied on observed scaling relations linking the properties of the baryons to those of their host halos, and have assumed the same total specific angular momentum for the DM and the baryons.

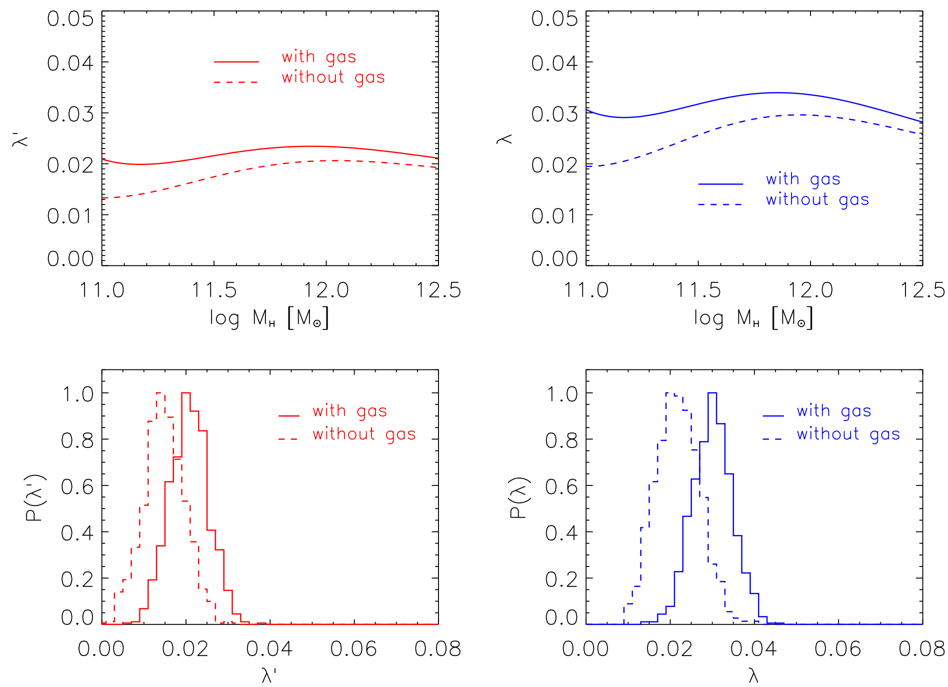

FIG. 2.— The spin parameter and its distribution function. *Top* panels: λ' (*left*) and λ (*right*) as a function of the halo mass, when the gas component is included in the system (*solid* line) and when it is not (*dashed* line). *Bottom* panels: the distribution function of λ' (*left*) and λ (*right*), again with gas and without gas.

Our main findings are: (i) we show that including the gas component beside the stars has a remarkable impact on the total angular momentum; (ii) by adopting for the DM the observationally supported Burkert profile, we compute the total angular momentum of the disk and its relationship with the rotation velocity; (iii) we obtain $\lambda' \approx 0.02$ and $\lambda \approx 0.03$ as most-probable values of the spin parameters.

Simulations based on the ΛCDM framework, performed by various authors (Bullock 2001; D'Onghia & Burkert 2004), have shown that the distribution of the spin parameter λ' for the whole halo catalogue peaks at around 0.035, significantly higher than our empirical value. However, D'Onghia & Burkert (2004) highlight that if one restricts one's attention to halos that hosts spirals and have not experienced major mergers during the late stages of their evolution ($z \lesssim 3$), the average spin parameter λ' turns out to be around 0.023, very close to our observational result.

Moreover, Gardner (2001) and Peirani et al. (2004) showed that the spin parameter λ undergoes different evolutions in halos that have evolved mainly through major mergers or smooth accretion: in the former case λ takes on values around 0.044, while in the latter case λ has lower values around 0.03, well in agreement with our result. Thus our findings point towards a scenario in which the late evolution of spiral galaxies may be characterized by a relatively poor history of major merging events.

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