The Astrophysical Journal, 474:710–718, 1997 January 10

COLLAPSE AND FRAGMENTATION MODELS OF PROLATE MOLECULAR CLOUD CORES. I. INITIAL UNIFORM ROTATION

Leonardo Di G. Sigalotti
International Centre for Theoretical Physics, ICTP, P.O. Box 586, I-34100 Trieste, Italy; and
International School for Advanced Studies, SISSA, Via Beirut 2-4, I-34013 Trieste, Italy

and

Jaime Klapp
Instituto Nacional de Investigaciones Nucleares, ININ, Apartado Postal 18-1027, Mexico 11801 D. F.
Received 1996 April 11; accepted 1996 July 25

ABSTRACT

Studies of the distribution of young stars in well-known regions of star formation indicate the existence of a characteristic length scale ($\sim$0.04 pc), separating the regime of self-similar clustering from that of binary and multiple systems. The evidence that this length scale is comparable to the size of typical molecular cloud clumps, along with the observed high frequency of companions to pre–main-sequence stars, suggest that stars may ultimately form through fragmentation of collapsing molecular cloud cores. Here we use a new hydrodynamic code to investigate the gravitational collapse and fragmentation of protostellar condensations, starting from moderately centrally condensed (Gaussian), prolate configurations with axial ratios of 2:1 and 4:1 and varying thermal energy (α). All the models start with uniform rotation and ratios of the rotational to the gravitational energy $\beta \approx 0.036$. The results indicate that these condensations collapse all the way to form a narrow cylindrical core that subsequently fragments into two or more clumps, even if they are initially close to virial equilibrium ($\alpha + \beta \approx \frac{1}{2}$). The 2:1 clouds formed triple systems for $\alpha \gtrsim 0.36$ and a binary system for $\alpha \approx 0.27$, while the 4:1 clouds all formed binary systems independently of α. The mass and separation of the binary fragments increase with increasing the cloud elongation. The widest binaries formed from clouds with $\alpha \approx 0.36$, and starting from this value, the binary separation decreases with either increasing or decreasing α. In all cases, fragmentation did not result in a net loss of the a/m ratio (specific spin angular momentum per unit mass), as expected from stellar observations. The fragments that formed possess low values of α ($\sim$0.06) and are appreciably elongated, and so they could subfragment before becoming true first protostellar cores.

Subject headings: binaries: general — hydrodynamics — ISM: clouds — stars: formation

1. INTRODUCTION

Recent studies of the spatial distribution of pre–main-sequence (PMS) stars in the Taurus region indicate that on large scales stars are clustered in a self-similar fashion (Gomez et al. 1993). Self-similar clustering of young stars has also been observed in other star-forming regions (Lada, Strom, & Myers 1993; Zinnecker, McCaughrean, & Wilking 1993). This evidence suggests that star formation may generally be a hierarchical process that involves grouping on a wide range of scales. Whereas the clustering of young stars could be explained in terms of preexisting hierarchical structure in dark clouds, where processes involving dissipation of turbulent motions might have allowed regions of enhanced density to condense into a hierarchy of bound clumps (Pouquet, Passot, & Léorat 1991; Elmegreen 1993), much observational evidence indicates that stars may ultimately form inside the smallest clumps in the hierarchy, commonly known as molecular cloud cores (see, e.g., Myers 1983; Beichman et al. 1986; Benson & Myers 1989; Shulz et al. 1991). This general view is also supported by recent detections of close companions to the PMS stars (Ghez, Neugebauer, & Matthews 1993; Leinert et al. 1993; Richichi et al. 1994), which show that on the smallest scales most stars belong to binary and multiple systems. Furthermore, Larson (1995) studied the relation between clustering and binary formation in the Taurus region, finding that there exists an intrinsic scale of $\sim$0.04 pc that divides the regime of hierarchical clustering on larger scales from that of binary and multiple systems on smaller scales. This length scale is comparable to the typical size of molecular cloud cores and has an associated mass of $\sim 1\ M_{\odot}$, supporting the relevance of the Jeans mass in determining stellar masses. Thus, Larson (1995) suggested that the formation of binary and multiple stars within $\sim$0.04 pc may be due to the fragmentation of thermally supported cloud cores of about the Jeans size.

The speckle imaging survey by Ghez et al. (1993) has shown that over a projected linear separation range of 16 to 256 AU, the T Tauri binary star frequency is 4 times greater than that of main-sequence (MS) stars. An excess of young triple and quadruple systems relative to MS stars was also reported. Other surveys using different detection methods have also revealed an excess of PMS binaries (e.g., Reipurth & Zinnecker 1993; Richichi et al. 1994). The prevalence of companions to PMS stars then implies that the bulk of binary and multiple systems had to be formed prior to the PMS phase, i.e., during the protostellar collapse and fragmentation phase. The exact manner in which binary and multiple fragmentation occurs is, however, strongly dependent on the structure and properties of individual precollapse cores.

Mean properties of dense cores have been inferred primarily from observations of the ammonia emission lines. In particular, Benson & Myers (1989) reported a survey of dense cores with sizes of 0.06–0.9 pc, hydrogen densities of 2×10^3–2×10^5 cm^{-3}, and masses from 0.5 to 760 $M_{\odot}$, with most cores having masses on the order of a few $M_\odot$ and temperatures of ≈ 10 K. Evidence for small clumps of stellar mass was also given by Shulz et al. (1991), who reported several cores of 0.02–0.04 pc containing 5–20 $M_\odot$ in NGC 2024. Myers et al. (1991) mapped 16 dense cores using three different molcules and found that many of them are prolate condensations with axial ratios of ~ 0.4–0.5. Strong evidence for rotational motion in dense cores has been given recently by Goodman et al. (1993), who measured linear velocity gradients consistent with solid-body rotation in about 29 of 43 cores studied. They inferred a typical ratio of rotational to gravitational energy of $\beta \sim 0.02$ in the cores with detectable rotation. Prolate shapes and significant rotation are important ingredients in the initial conditions for binary and multiple fragmentation (Bonnell et al. 1991, 1992).

Molecular line surveys (e.g., Myers et al. 1991; Fuller & Myers 1992) generally suffered from low resolution, and so they were unable to distinguish between the gas properties of cores with and without embedded sources, suggesting that most cores were consistent with the r^{-2} density profile predicted by the Standard Protostellar Model (Shu 1977). Recent higher resolution observations in the submillimeter continuum of a sample of 21 Myers cores by Ward-Thompson et al. (1994) have shown that starless cores are less centrally peaked than cores with embedded sources and possess density profiles that flatten in their inner regions. The averaged radial flux density profiles for these cores were best fitted by Gaussians, implying that the underlying radial density profile is also a Gaussian function (Boss 1995). Ward-Thompson et al. (1994) identified these condensations as true pre-protostellar objects, i.e., magnetically supported clumps that are undergoing the ambipolar diffusion phase prior to protostellar collapse (Crutcher et al. 1994).

The precollapse cores observed by Ward-Thompson et al. (1994) should then provide the best initial conditions for protostellar collapse. However, very little is known about their rotational properties and internal nonrotational fields, though observations are becoming close to study velocity fields inside cloud cores (Goodman & Barranco 1994). While solid-body rotation may be characteristic of cores contracting under the action of strong magnetic fields, some degree of differential rotation may be expected in cores in which the effects of magnetic braking have been lessened by ambipolar diffusion. Calculations of the collapse of centrally condensed, Gaussian clouds with initial uniform rotation ($\beta \gtrsim 0.16$) and $m = 2$ density variations indicate that binary and multiple protostellar systems may form as a result of rotation-driven fragmentation (Boss 1991; Klapp, Sigalotti, & de Felice 1993). Prolate Gaussian cores with solid-body rotation may also produce binary and eventually higher order systems for $\beta \approx 0.1$, provided that their initial ratio of thermal to gravitational energy $\alpha < 0.4$ (Boss 1993). Uniform rotation, however, results in a less favorable condition for binary and multiple fragmentation in Gaussian cores of nonprolate shape (Sigalotti & Klapp 1994). On the other hand, differential rotation is seen to enhance the possibility for fragmentation in prolate Gaussian cores, allowing binary fragmentation to occur even from configurations initially very close to virial equilibrium (Boss & Myhill 1995). Differential rotation, however, was found to be more important in determining binary fragmentation in clouds with initial power-law density profiles (Myhill & Kaula 1992; Sigalotti & Klapp 1996a), while multiple fragmentation would result, if in addition, these clouds were allowed to start their collapse with internal nonrotational motions (Sigalotti 1994).

This paper presents new three-dimensional calculations of the protostellar collapse and fragmentation of small-size, prolate cores with Gaussian radial density profiles and uniform rotation ($\beta \approx 0.036$). The initial values of α are varied in a range such that the collapse should always result in fragmentation according to the criteria found by Boss (1993). Compared to Boss's calculations, we study the effects of increasing the core elongation along with the initial allowed asymmetry.

2. PROTOSTELLAR CORE MODELS

2.1. *Numerical Methods*

The calculations were made using the isothermal version of a new three-dimensional radiative hydrodynamic code that will be described in full by Sigalotti (1996). This code is an improved extension of a previously implemented and successfully tested scheme (Sigalotti 1993). In its basic form, the code solves the equations of continuity and motion in conservation law form, coupled with the Poisson equation for the gravitational potential. This set of equations is closed by specifying a pressure relation $p \propto \rho^\gamma$, where ρ is the mass density and γ is the adiabatic index. The effects of viscosity and magnetic fields are ignored.

The determining equations are solved using finite-difference methods on a radially moving, spherical-coordinate grid along with a multistep solution procedure for the hydrodynamic equations, in which the convective and source terms are evaluated separately. The spatial differences are based on a volume-centered discretization technique (Mönchmeyer & Müller 1989). This results in a second-order preserving scheme, as was demonstrated through convergence testing in three dimensions. The time integration of the hydrodynamic equations is made explicitly, and temporal second-order accuracy is achieved fully by means of a predictor-corrector treatment of the convective and source terms. A similar spherical coordinate-based code was developed by Boss & Myhill (1992), who showed that over the range of spatial resolution in which three-dimensional calculations can be made, the use of volume-centered differences result in spatial second-order accuracy without the need of implementing directional splitting techniques for advection along the three coordinate directions as advocated by Finn & Hawley (1989). In collapse calculations with Boss & Myhill's code, adequate central resolution is maintained using a grid redefinition procedure so that the second-order properties of the volume-centered method are preserved automatically. With the present code, adequate spatial resolution is obtained by allowing the radial grid to move with matter. Therefore, in order to preserve the second-order accuracy of the volume-centered method, a completely new finite-difference formulation has been introduced to design the advective terms. With this new approach, the hydrodynamical transport algorithm is numerically invariant under grid motion, yielding numerical solutions that are free of systematic errors arising when standard finite differences are applied. This invariance is seen to result in a high degree of accuracy on the pressureless collapse test case. Consistent advection is applied to define the convective fluxes (Norman, Wilson, & Barton

1980), and transport of mass is performed using the van Leer monotonic interpolation scheme. This results in good local conservation of angular momentum during axisymmetric collapse. An artificial viscosity, based on the tensor formulation of Tscharnuter & Winkler (1979), is added to improve numerical stability in zones of shock formation. The code has been tested on a variety of other test cases, including the standard isothermal collapse test case (Sigalotti & Klapp 1996b) and a number of nonisothermal test models (Sigalotti 1996).

The calculations of this paper were made using a spherical grid consisting of $1 + n_r = 41$ initially uniformly spaced radial points (including the origin), $2 + n_\theta = 67$ fairly equidistant points for $0 \leq \theta \leq \pi$, and $n_\phi = 64$ equidistant points for $0 \leq \phi \leq 2\pi$. This resolution gives a total number of $1 + n_r(2 + n_\theta n_\phi) = 166{,}481$ effective cells filling the entire computational volume. Reflection symmetry through the equatorial plane is used, so that only the top hemisphere is represented by the calculations ($1 + n_\theta = 34$ cells for $0 \leq \theta \leq \pi/2$). A constant-volume boundary condition is enforced at the outer surface of the spherical grid by setting a zero radial velocity. Adequate central resolution is achieved by allowing all interior radial points to contract while keeping the outer radius fixed in space and time. With this choice of the numerical techniques, a typical model required $\sim$20,000 time steps, corresponding to about 40 hr of CPU time on a CRAY J916/6 supercomputer.

2.2. *Initial Conditions*

We consider seven protostellar collapse models with initial parameters (Table 1) corresponding to inferred mean properties of molecular cloud cores. The starting point of collapse is taken to be a centrally condensed configuration of prolate shape consistent with the observed elongation of most dense cores (Myers et al. 1991). As in Boss (1993), the initial central condensation is obtained using a Gaussian density profile in radius, with the central density ρ_c being 20 times higher than the boundary density ρ_b. Gaussian profiles seem to be consistent with recent observations of true pre-protostellar clumps (Ward-Thompson et al. 1994). In addition, a moderate density contrast of $\rho_c/\rho_b = 20$ may represent an adequate starting point for the gravitational collapse of magnetically supported clouds that were contracting because of ambipolar diffusion (Tomisaka 1991). More recently, Crutcher et al. (1994) modeled the formation of the protostellar core of Barnard 1 Cloud in Perseus, finding that ambipolar diffusion initiates the collapse of the supercritical core when the central density increased by a factor of ≈ 15 from its initial value. Thus, the models may well apply to low-mass star formation from protostellar condensations that were supported previously by magnetic fields.

A centrally condensed, prolate spheroidal density distribution can be obtained using the exponential profile

$$\rho(x, y, z) = \rho_0 \times \exp\left[-\frac{x^2}{a^2} - \frac{(y^2 + z^2)}{b^2}\right], \tag{1}$$

where a and b are, respectively, the length of the semimajor axis (here aligned with the x-axis of the grid, $\phi = 0$) and of the two semiminor axes (aligned with the y- and z-axis of the grid) of the spheroid, and ρ_0 is the initial central density. We consider two different degrees of prolateness corresponding to an axial ratio of 2:1, resulting from $a \approx 1.16R$ and $b \approx 0.58R$, and of 4:1, resulting from $a \approx 2.31R$ and $b \approx 0.58R$, where R is the outer radius of the spherical grid. We take $R \approx 7.72 \times 10^{16}$ cm for all models, giving a diameter of $\sim$0.05 pc. This value is consistent with the observed size of small clumps of stellar mass (Shulz et al. 1991) and is also comparable to the length scale of $\sim$0.04 pc separating the regime of self-similar clustering from that of binary systems in star-forming regions (Larson 1995). A noise component in the ϕ-coordinate is added by multiplying the density from equation (1) by the factor $[1 + 0.2\mathrm{RAN}(\phi)]$, where $\mathrm{RAN}(\phi)$ is a random number in the interval [0, 1]. The gas is assumed to have a composition of $X = 0.769$, $Y = 0.214$, $Z = 0.017$ and a temperature of 10 K. Furthermore, the evolution is assumed to remain isothermal for $\rho < \rho_T = 3.0 \times 10^{-13}$ g cm^{-3}. When densities higher than ρ_T are reached in the collapse, a polytropic pressure relation (with adiabatic index $\gamma = 7/5$) is used to approximate the nonisothermal effects. Each model starts from rest except for uniform rotation. The initial angular velocity is chosen such that the ratio of the rotational to the absolute value of the gravitational energy is $\beta \approx 0.036$ for all models. Although this value is above that of $\beta \sim 0.02$ inferred for most dense cores in the survey of Goodman et al. (1993), it may also represent rotational motion in a large sample of real protostellar cores.

The entries in Table 1, starting from the second column, give, respectively, the axial ratio of the prolate spheroidal core, the total core mass in solar masses, the initial central density and uniform angular velocity, the specific spin angular momentum J/M, the a/m ratio (defined as cJ/GM^2, where c is the velocity of light and G is the gravitational constant), and the ratio of the thermal to the absolute value of the gravitational energy. Two parameters are varied: the prolate axial ratio (a:b) and the amount of thermal energy α. The importance of including the a/m ratio as an additional parameter hinges on the fact that observed PMS stars have associated $a/m \sim$ 10–100 (de Felice & Yu 1982; de Felice & Sigalotti 1986, 1992), while typical values for rotating protostellar clouds are on the order of $\sim 10^5$, implying that mass and angular momentum must redistrib-

TABLE 1

INITIAL CONDITIONS

Model	a:b	$M/M_\odot$	ρ_0 (g cm^{-3})	ω_0 (rad s^{-1})	$(J/M)_c$ (cm^2 s^{-1})	$(a/m)_c$	α
A1......	2:1	0.90	2.83×10^{-18}	2.10×10^{-13}	4.61×10^{20}	1.16×10^5	0.45
A2......	2:1	1.12	3.54×10^{-18}	2.35×10^{-13}	4.96×10^{20}	1.00×10^5	0.36
A3......	2:1	1.50	4.74×10^{-18}	2.70×10^{-13}	5.94×10^{20}	8.93×10^4	0.27
A4......	4:1	0.93	2.57×10^{-18}	2.00×10^{-13}	4.72×10^{20}	1.15×10^5	0.45
A5......	4:1	1.16	3.21×10^{-18}	2.24×10^{-13}	5.25×10^{20}	1.02×10^5	0.36
A6......	4:1	1.54	4.27×10^{-18}	2.58×10^{-13}	6.10×10^{20}	8.94×10^4	0.27
A7......	4:1	2.28	6.32×10^{-18}	3.15×10^{-13}	7.47×10^{20}	7.40×10^4	0.18

ute in such a way as to ensure a reduction of the a/m ratio by factors of $\sim 10^3$–10^4 during star formation.

3. RESULTS

Models A1–A3 provide a sequence of collapse calculations starting from prolate condensations with axial ratios of 2:1 and varying thermal energy. The effects of increasing the degree of prolateness are considered using models A4–A7, which define a similar sequence of calculations starting from condensations with prolate axial ratios of 4:1. Models A1 and A4 are very close to viral equilibrium ($\alpha + \beta \gtrsim 0.48$) and represent cores with relatively high thermal support. The other five models have $\alpha + \beta \lesssim 0.40$, with α in the range of expected values ($0.11 \lesssim \alpha \lesssim 0.36$) for magnetically contracting cloud models at the onset of gravitational collapse (Lizano & Shu 1989; Boss 1995). The parameters in Table 1 provide suitable initial conditions for studying fragmentation during the dynamic collapse phase leading to the formation of the first protostellar core.

All models evolved in similar fashion, ending up with the formation of either a binary or a ternary protostellar system. Because of the moderate initial rotation ($\beta \approx 0.036$), the clouds collapse dynamically, evolving into a general flattened disk about the equatorial plane. Near the center of the cloud, however, the collapse proceeds primarily toward its major axis, making the central core become progressively more prolate as it collapses. This process results in the formation of an intermediate narrow cylindrical configuration that fragments subsequently into at least two clumps. Models A4–A7, initially with a 4:1 axial ratio, formed intermediate narrower cylinders than did models A1–A3, initially with a 2:1 axial ratio. Extremely narrow cylinders may also form from less prolate configurations (i.e., with 2:1 axial ratios), provided that they start collapsing with very low rotation (e.g., Boss 1993). The narrowness of the intermediate cylinders determines their relative elongation and enhances the prospects for binary fragmentation, as was demonstrated by Bonnell et al. (1991), who calculated the collapse of initially homogeneous cylinders with uniform rotation. In their calculations, the assumption of rotation perpendicular to the cloud's elongation resulted in the formation of stable binary systems for a wide range of the space parameter (α, β). The present intermediate cylindrical forms are centrally condensed and possess tapering rather than sharply defined ends, and so they differ from the initial conditions used by Bonnell et al. (1991). Nelson & Papaloizou (1993) investigated the collapse of initially uniform-density, prolate spheroidal clouds with varied axial ratios. They found qualitative differences when comparing the collapse of finite cylinders with that of elongated clouds having tapering ends. In the former case, the collapse results in binary fragmentation with the clumps forming at the ends of the cylinders (e.g., Bastien 1983; Rouleau & Bastien 1990; Bonnell et al. 1991), while in the latter case, fragmentation is seen to occur toward the center with no preference for the clumps to form at the ends of the elongated clouds even for axial ratios of 10:1. Nelson & Papaloizou (1993) interpreted this behavior as due to the collapse of matter onto the major axis being stronger toward the center, where the local mass per unit length was greater. A similar mode of collapse is also seen to occur in the present centrally condensed models, implying that for elongated clouds the outcome of fragmentation depends more strongly on their geometry than on the details of the initial conditions. Models A1 and A2, both with a 2:1 axial ratio and $\alpha \gtrsim 0.36$, produced less narrow cylindrical forms and resulted in a triple protostellar core, i.e., a dominant binary plus a small fragment, while model A3 with $\alpha \approx 0.27$ formed a binary system after having passed through an intermediate narrower cylindrical configuration. Similarly, cases A4–A7 with a 4:1 axial ratio all experienced extremely narrow cylindrical forms and resulted in binary systems regardless of the initial value of α.

The evolution of model A2 is illustrated in the sequence of equatorial density contour plots of Figure 1, which shows the prolate cloud during the initial collapse phases, the intermediate cylindrical configuration, the subsequent binary fragmentation, and the final triple protostellar core. For all models, time is measured in units of the free-fall time for collapse [$t_{\rm ff} = (3\pi/32G\rho_0)^{1/2}$, where ρ_0 is the initial central density]. In models A1 and A2, the triple system arises by subfragmentation of one of the previously formed binary fragments. By way of comparison, Figure 2 shows the time evolution undergone by model A5, with initial conditions similar to case A2 except for the degree of prolateness. In this case, the occurrence of an intermediate narrower cylinder followed by formation of a final binary is clear. Figure 3 depicts the velocity field in the equatorial plane at the same time of Figure 2c ($\sim 1.303 t_{\rm ff}$). Collapse of low angular momentum matter in the direction perpendicular to the elongation of the cylindrical core is evident.

In Table 2 we list the inferred properties of the resulting fragments. The columns, starting from the second, give, respectively, the mass of the fragments in solar masses, their estimated values of α, β, and δ (where δ is the ratio of kinetic nonrotational to gravitational energy), the specific spin angular momentum and the a/m ratio of the original cloud relative to the values of the fragments, the total binary separation distance in terms of the initial cloud radius, the maximum densities of the fragments, and the free-fall time when the integral properties were evaluated. The fragment region is defined by the cell of maximum density plus all surrounding cells with densities higher than a factor $1/f_D$ of the maximum value. The factor f_D was chosen according to the actual shape of the fragments and was found to vary from ≈ 5 to ≈ 44 depending on the model. A method that improves on the accuracy of the rotation properties was also used to calculate the velocity field inside the fragment region (Sigalotti & Klapp 1996b).

We see from Table 2 that the fragments have masses of ~ 0.01–$0.03\ M_\odot$, as is appropriate for newly formed first protostellar cores (Larson 1969). The fragments that formed from the 4:1 clouds are a factor of ~ 2 more massive than those resulting from the 2:1 clouds. The 20% random noise added to the initial density profile provided a sufficient amount of asymmetry to produce systems of unequal mass in models A1 and A2 (initially with $\alpha \gtrsim 0.36$), while model A3 with $\alpha \approx 0.27$ produced a more symmetric binary. Binary systems of roughly equal mass also resulted from models A4–A7 regardless of the initial value of α. Real molecular cores are observed to possess large amounts of asymmetry, and hence they are expected to evolve into protostellar systems with mass ratios $q < 1$. The present results imply that in less prolate clouds even a small amount of asymmetry may lead to systems with $q < 1$, provided that they start collapsing with relatively higher thermal support ($\alpha \gtrsim 0.36$). More prolate condensations, however, are likely to undergo a much stronger cylindrical collapse and there-

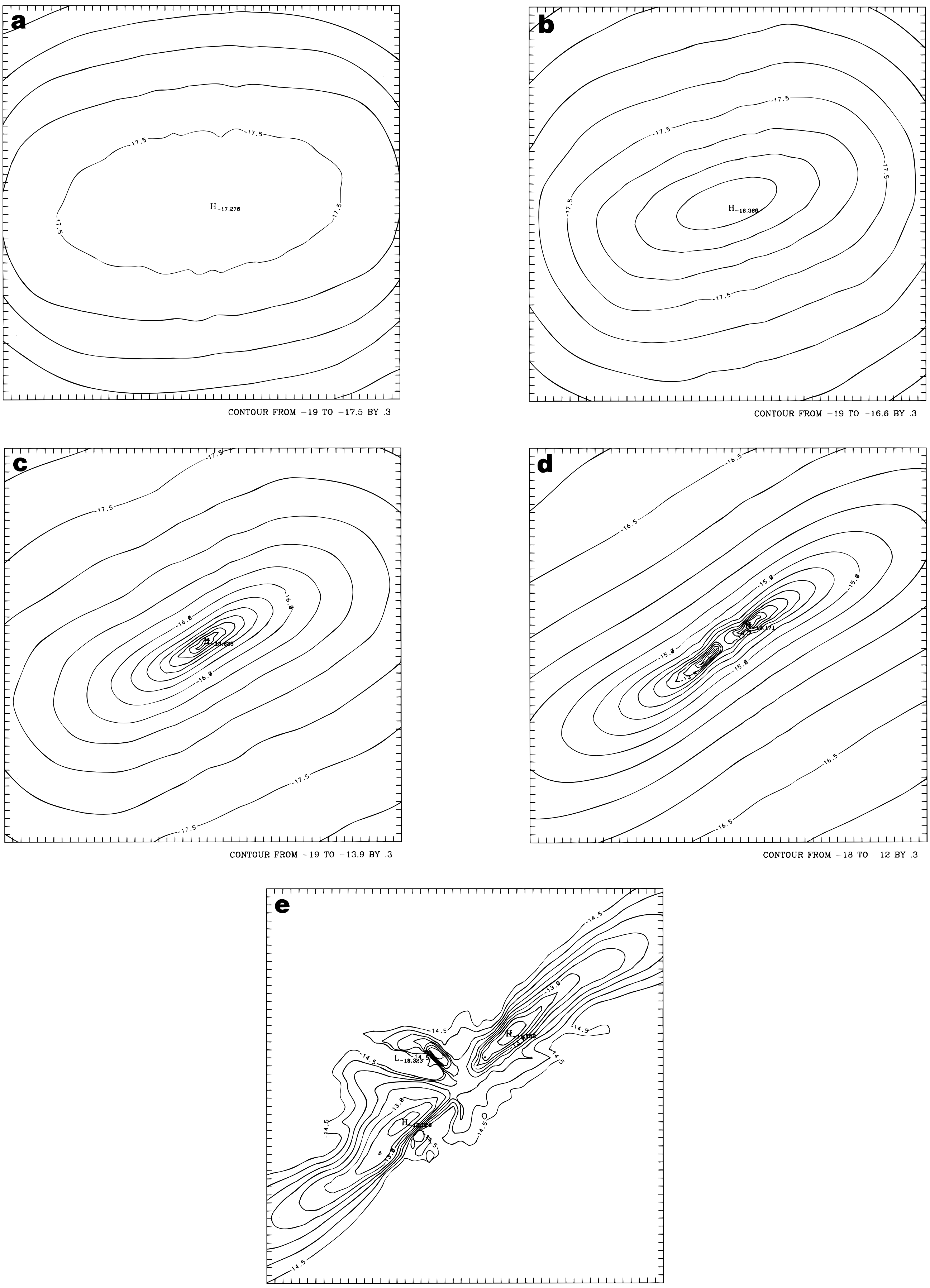

FIG. 1.—Equatorial density contours for the evolution of model A2. The contour lines correspond to the logarithm of the density. The low- and high-density values are labeled. Counterclockwise rotation is assumed. A sequence in time is presented: (*a*) $0.442t_{\rm ff}$, (*b*) $1.029t_{\rm ff}$, (*c*) $1.284t_{\rm ff}$, (*d*) $1.301t_{\rm ff}$, and (*e*) $1.317t_{\rm ff}$. A region of radius 7.0×10^{16} cm is shown in (*a*), of 6.0×10^{16} cm in (*b*), of 3.0×10^{16} cm in (*c*), of 1.0×10^{16} cm in (*d*), and of 3.5×10^{15} cm in (*e*).

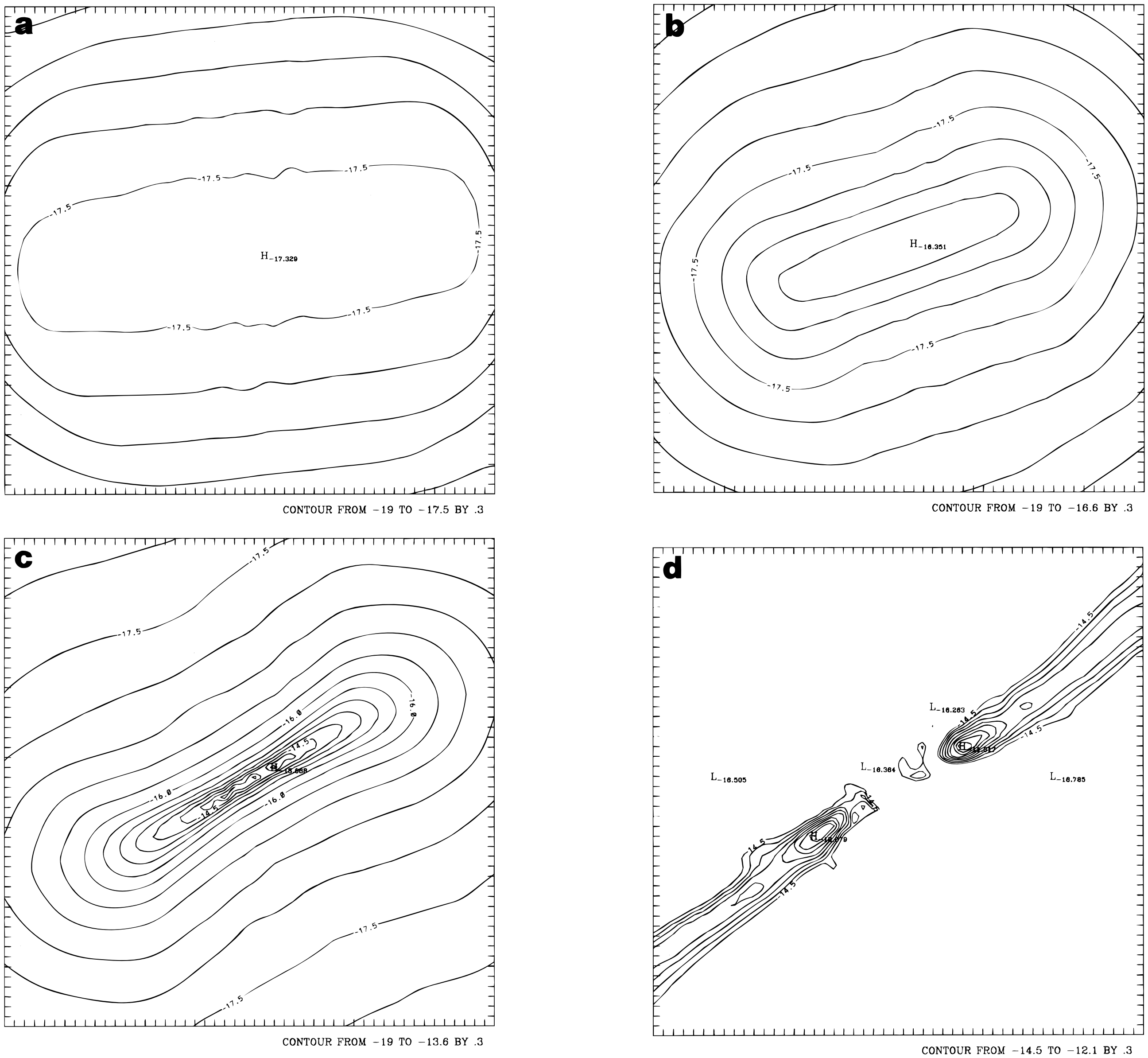

FIG. 2.—Equatorial density contours for the evolution of model A5. The contour lines correspond to the logarithm of the density. The low- and high-density values are labeled. Counterclockwise rotation is assumed. A sequence in time is presented: (*a*) $0.421t_{ff}$, (*b*) $1.060t_{ff}$, (*c*) $1.303t_{ff}$, and (*d*) $1.361t_{ff}$. A region of 7.0×10^{16} cm is shown in (*a*), of 6.0×10^{16} cm in (*b*), of 3.0×10^{16} cm in (*c*), and of 7.0×10^{15} cm in (*d*).

fore result in more symmetric systems independently of the amount of thermal energy. The clumps that formed are centrally condensed and possess elongated shapes with axial ratios of $\sim$0.10–0.45. In addition, they form at low values of α (with typical $\alpha \sim 0.06$) and have sufficient rotation that they could subfragment in the further evolution prior to the formation of the first protostellar core. Significant nonrotational motion is also present in the structure of the newly formed fragments, as evidenced by the relatively high values of δ.

The specific spin angular momentum of the fragments is reduced by factors that range from 14 to 109 compared to the value of the original cloud, with most of them having reduction factors >40. This high efficiency of angular momentum redistribution is a reflection of the fact that the intermediate narrow cylinders (from which the fragments originate) form due to the collapse of essentially nonrotating matter, i.e., matter with very low angular momentum as shown in Figure 3. A completely different trend is, however, observed for the reduction of the a/m ratio. Indeed, the fragments have associated a/m ratios that are comparable or even larger than that of the parent cloud from which they formed, meaning that an efficient reduction of the specific spin angular momentum is not enough to induce a net loss of the a/m ratio. Evidently mass has to be redistributed in a more efficient manner than fragmentation itself is allowed to do in order to guarantee a general decrease of the a/m ratio as demanded by observations of PMS stars. The fragments are close to becoming first protostellar cores, and stating in advance that they will survive as individual entities, they may increase in mass by factors of 10 or more during their subsequent accretion phase prior to the formation of the second protostellar core. Such an increase of mass would then result in a reduction of a/m by factors ranging from 10 to 10^2. Fragmentation during the second collapse phase (Bonnell & Bate 1994a, 1994b) and

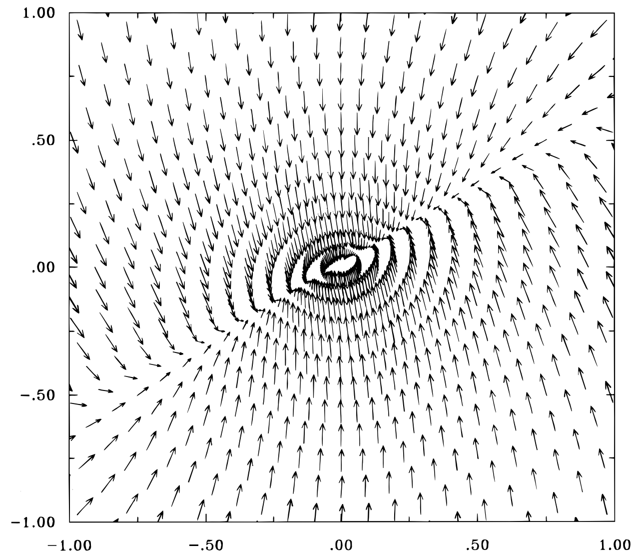

FIG. 3.—Velocity field in the equatorial plane for model A5 at the same time of Fig. 2*c*, when the intermediate narrow cylinder configuration forms. The cloud's rotation axis is in the center of the plot. The numbers on the box sides are given in units of 10^{16} cm. The maximum velocity is 0.62 km s^{-1}. Collapse of low angular momentum mass perpendicular to the elongation of the cylindrical core is evident.

further accretion toward the PMS phase may then increase the reduction factor by at least 1 order of magnitude more, as required to explain star formation (de Felice & Sigalotti 1986, 1992).

The protostellar binaries have separation distances of 99–325 AU, in good agreement with the linear separation range at which PMS binaries are in excess with respect to MS binary stars (Ghez et al. 1993). At comparable initial conditions, the 4:1 clouds resulted in binary separations a factor of ~ 2 larger than the separations of the binaries formed from the 2:1 clouds. For both the 2:1 and the 4:1 model sequences, the widest binary systems occurred for cases A2 and A5, which had $\alpha \approx 0.36$. Starting from this value, the binary separation is seen to decrease with either increasing or decreasing α. This result can be understood in terms of the following argument: centrally condensed configurations tend to collapse in a highly nonhomologous way because the central regions of the cloud, initially at higher densities, contract at a faster rate than does the envelope, where most of the cloud mass resides. If, in addition, the cloud possesses an elongated shape, the collapse will be directed preferentially onto its major axis, making the cloud more prolate. As the collapse proceeds to much higher densities, eventually producing a centrally peaked distribution, the central regions result in a progressively narrower cylindrical-like core of progressively smaller semimajor axis length which, after having accumulated enough mass, may fragment into a binary system. As long as the initial value of α is increased, the corresponding increasing pressure forces become more efficient in retarding the central collapse; the evolution results in narrow cylindrical cores of larger semimajor axis length. These cores may then fragment, producing binary systems of larger separation distances. This trend, however, continues up to a certain critical value of $\alpha(\alpha \sim 0.36)$. Above that value, the pressure forces may hinder the collapse perpendicular to the cloud's elongation, making it proceed on a timescale comparable to that of collapse along the major axis. As a result, the central dense core becomes more spherical, and a small region within it may still contract to higher densities and fragment, forming clumps with smaller separation distances. By increasing α further, fragmentation can be avoided and the evolution may result either in the formation of a single maximum density at the center of the cloud or even in a general cloud expansion after an initial weak infall (e.g., see Boss 1993). The critical value of $\alpha \sim 0.36$ is consistent with the maximum amount of thermal support expected at the beginning of dynamical collapse in magnetically supported, Gaussian cloud models that were undergoing ambipolar diffusion (Lizano & Shu 1989). Rotating precollapse cloud cores with magnetic and thermal support must then have $\alpha + \beta < \frac{1}{2}$, and provided that most of them are rotating slowly with $\beta < 0.04$, the value of $\alpha \sim 0.36$ may represent an upper limit to the thermal support in real molecular cloud cores at the onset of dynamic collapse. If, in addition, an initial axial ratio of 4:1 overestimates the elongation of

TABLE 2

INTEGRAL PROPERTIES OF THE RESULTING FRAGMENTS

Model	$M_f/M_\odot$	α_f	β_f	δ_f	$\frac{(J/M)_c}{(J/M)_f}$	$\frac{(a/m)_c}{(a/m)_f}$	$D_{\rm bin}/R$	ρ_m (g cm^{-3})	$t/t_{\rm ff}$
A1……	0.0104	0.081	0.096	0.085	78	0.90	0.019	1.76×10^{-12}	1.443
	0.0056	0.072	0.403	0.242	42	0.26		9.17×10^{-12}	
	0.0022	0.101	0.150	0.304	19	0.05		2.76×10^{-12}	
A2……	0.0166	0.068	0.050	0.160	49	0.72	0.031	1.59×10^{-12}	1.317
	0.0206	0.055	0.285	1.827	20	0.37		5.60×10^{-13}	
	0.0037	0.081	0.092	0.182	25	0.08		3.98×10^{-13}	
A3……	0.0096	0.058	0.015	0.069	55	0.35	0.028	1.24×10^{-12}	1.196
	0.0103	0.055	0.017	0.064	109	0.76		1.03×10^{-12}	
A4……	0.0253	0.056	0.056	0.211	15	0.41	0.038	6.48×10^{-13}	1.471
	0.0268	0.057	0.030	0.327	20	0.58		6.25×10^{-13}	
A5……	0.0316	0.048	0.027	0.492	53	1.46	0.063	1.22×10^{-12}	1.361
	0.0349	0.054	0.043	0.170	44	1.33		8.21×10^{-13}	
A6……	0.0254	0.066	0.008	0.110	79	1.30	0.050	1.59×10^{-12}	1.229
	0.0244	0.052	0.029	0.083	49	0.77		8.42×10^{-13}	
A7……	0.0365	0.045	0.005	0.072	75	1.20	0.026	5.71×10^{-13}	1.114
	0.0321	0.047	0.031	0.105	14	0.20		4.24×10^{-13}	

typical dense cores, then the widest binary systems composed of first protostellar cores that might form from fragmentation should have separations on the order of ~ 300 AU. More elongated clouds would result in even larger separation distances. These results, however, apply to Gaussian cloud cores starting their collapse with uniform rotation. Differential rotation may favor fragmentation in prolate cloud cores with $\alpha \sim 0.36$ at relatively lower central density contrasts than initially uniformly rotating cores do, therefore resulting in wider binary systems.

4. CONCLUSIONS

We have described numerical solutions for the protostellar collapse and fragmentation of moderately centrally condensed, uniformly rotating ($\beta \approx 0.036$), prolate core models with initial conditions corresponding to inferred properties of molecular cloud cores. The central condensation was obtained using a Gaussian density profile in radius consistently with the precollapse cloud observations of Ward-Thompson et al. (1994). The effects of increasing the degree of prolateness in the initial conditions were assessed by comparing two distinct sequences of calculations for models with prolate axial ratios of 2:1 and 4:1, and varying the amount of the thermal energy α. The models were appropriate for studying the likelihood of binary and multiple fragmentation during the dynamic collapse phase leading to the formation of the first protostellar core. The results from these calculations may be summarized in the following points.

1. All the models underwent a fairly similar evolution: the clouds collapsed and formed intermediate narrow and dense cylindrical configurations before fragmenting into either a binary or a ternary protostellar core. The 2:1 cloud models experienced less narrow cylindrical forms, resulting in a triple system for $\alpha \gtrsim 0.36$ and in a binary system for $\alpha \approx 0.27$. The 4:1 clouds experienced much narrower cylindrical forms, and all formed binary systems regardless of the initial value of α.

2. The 4:1 clouds formed more massive fragments with larger separation distances than did the less prolate 2:1 clouds. In both sequences, the widest binary systems formed from clouds with initial $\alpha \approx 0.36$. Starting from this value, the binary separation distance is seen to decrease with either increasing or decreasing α.

3. The fragments formed with low values of $\alpha_f(\sim 0.06)$, values of β_f ranging from ~ 0.40 to 0.005, and relatively high values of $\delta_f(\gtrsim 0.10)$. They are moderately centrally condensed and possess appreciably elongated shapes. Their structure is almost similar to the initial conditions used in the original collapse calculations, and so they may well subfragment in the further evolution before becoming true first protostellar cores.

4. The amount of rotational (β_f) and nonrotational (δ_f) kinetic energy associated with the fragments was seen to decrease by decreasing the amount of thermal energy in the initial cloud. A tendency for β_f to decrease by increasing the elongation of the initial cloud was observed also.

5. The fragments have their specific spin angular momentum reduced by factors larger than ~ 40 relative to the value of the parent cloud. This high efficiency of spin angular momentum redistribution did not, however, result in an equally efficient redistribution of the a/m ratio. In most cases, the fragments are spinning with an a/m value larger than that of the original cloud. Clearly, a more efficient mechanism than fragmentation is required for mass redistribution in order to ensure a net loss of the a/m ratio for consistency with stellar observations. One likely mechanism is provided by mass accretion during the subsequent evolution phase leading to the formation of the second protostellar core, which may result in reduction factors of 10–10^2. Fragmentation during the second protostellar collapse phase and further accretion to the PMS phase may lead to an additional order-of-magnitude decrease of the a/m ratio.

6. While the initial cloud elongation appears to be a determining factor for binary fragmentation during the dynamic collapse of moderately rotating, Gaussian cores, the results imply that preexisting asymmetric structure may favor the formation of multiple systems of unequal mass in Gaussian cores with moderate prolate shapes and high thermal support ($\alpha \gtrsim 0.36$). Clouds with lower thermal support will preferentially result in binary systems. Gaussian cores with more elongated shapes show a propensity to form binary systems of roughly equal mass for a wide range of the initial conditions. Because of the much stronger cylindrical collapse experienced by these configurations, the formation of binaries with mass ratios significantly different from unity will require starting the collapse with a much larger amount of asymmetry than would be necessary for less prolate clouds.

7. The collapse of centrally condensed (Gaussian), prolate cores with initial uniform rotation has also been studied by Boss (1993). He found that slowly rotating ($0.00012 \lesssim \beta \lesssim 0.12$) clouds with initial axial ratios of 2:1 and relatively high thermal energy ($\alpha \sim 0.39$) may always result in the formation of binary systems. Only when $\alpha \gtrsim 0.46$ was binary fragmentation avoided; the collapse resulted in the formation of a single central core. Although here a direct comparison to the results of Boss (1993) cannot be made because of differences in the details of the initial conditions used, it is worth emphasizing that the great similarity in the basic outcomes indicates that theoretical cloud models resembling observed molecular cloud cores are indeed likely to fragment into binary or higher order protostellar cores. This aspect of the results is particularly important in connection with recent observations of PMS stars, which suggests that fragmentation during protostellar collapse may be the leading explanation for binary and multiple star formation.

We thank Alan P. Boss for providing a number of helpful suggestions and comments. The calculations presented in this paper were performed on the CRAY J916/6-1024 supercomputer of the University of Konstanz, Germany. This work was supported in part by the Ministero dell'Università e della Ricerca Scientifica e Tecnologica of Italy and by the Consejo Nacional de Ciencia y Tecnología (CONACYT) of Mexico. One of us (J. K.) wishes to thank the International Centre for Theoretical Physics (ICTP), Trieste, Italy, and the Bundesministerium für Bildung und Forschung (BmBT) of Germany for partial support.

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