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Test Methods

Optimal ligament lengths in impact fracture toughness estimation by the essential work of fracture method

Valeria Pettarin*, Patricia M. Frontini, Guillermo E. Eliçabe

Institute of Materials Science and Technology (INTEMA), University of Mar del Plata and National Research Council (CONICET), Juan B. Justo 4302, Mar del Plata B7608FDQ, Argentina

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Abstract

This work presents a statistical analysis of the confidence in fracture parameters of polymers estimated through the essential work of fracture (EWF) methodology under impact conditions. Fracture toughness parameters—essential, w_e, and non-essential work of fracture, w_p—are obtained from the experimental relationship between specific total work of fracture w_f and ligament length of the tested samples l: w_e is obtained from the y-intercept and w_p from the slope of w_f versus l in cases where a linear fit is suitable. The distribution of ligament lengths within a fixed number of samples to be tested under impact loading conditions is determined in order to minimize the uncertainty of the estimated parameters when linear reduction of the data is assumed. The statistical approach applied to two different polymeric materials shows that the uniform ligament length distribution along the ligament range is not the optimal one. We propose an optimal distribution of ligament lengths to be tested that depends on the distribution of error standard deviations of the measured work of fracture along the l-axis.

Keywords: Impact; Essential work of fracture; SENB; Statistical analysis

1. Introduction

Notched Charpy and Izod tests are commonly used to evaluate the fracture behaviour of polymers and composites at high strain rates due to their simplicity [1–5]. Nevertheless, impact strength is not a material property independent of testing geometry. Moreover, in the case of ductile polymers the samples are frequently not fully broken, and therefore the impact strength is not a good parameter to characterize their toughness. Fracture mechanics theories can provide the necessary theoretical framework needed to overcome these disadvantages. In response to this need, the J-Integral method and CTOD were developed to evaluate the fracture toughness of ductile materials. However, the adoption of fracture mechanics theories under impact conditions is not simple, i.e. these tests require specialized equipment for accurate crack growth measurements.

For polymers presenting ductile behaviour, better characterization of the impact fracture energy can be obtained using essential work of fracture (EWF) methodology [6–11]. EWF has been specially developed as a sound methodology to characterise the fracture toughness of ductile polymer films, ductile metals, paper sheets and fibrous composites under plane stress condition [12–20]. Mai and co-workers [6] have extended the EWF concept to impact testing of ductile polymers. Since then, some research has been conducted on the validity of impact EWF concept to characterize the toughness of ductile polymers and their blends [21–27].

The simplicity of the EWF approach, analogous to the simplicity of the Charpy test, is the main reason why this method has gained so much popularity for evaluation of the fracture toughness of ductile polymers [28]. Its experimental measurement is fairly easy since it only consists of

* Corresponding author.
E-mail address: pettarin@fi.mdp.edu.ar (V. Pettarin).

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the determination of the total fracture energy of several samples differing in initial ligament length, and the linear regression of the data. It has been stated that the EWF method can be applied to either plane stress or plane strain provided the ligament is fully yielded and the plastic zone is scaled with the square of the ligament length (load-line curves self-similarity) [7,29,30]. Furthermore, the size requirements for valid plane strain measurements are still less stringent that the ones for J-integral or CTOD methods [31,32]. When these conditions are fully satisfied, the impact specific EWF can be considered a material constant independent of specimen geometry for a given sheet thickness in plane stress; and it is also invariant with specimen thickness when plane strain conditions are met.

However, in many cases large experimental scatter is observed when tests are done under impact conditions [33]. This scatter could be explained through the combined effect of low resistant areas and low energies. That is, the percentage error in the specific fracture work is inversely proportional to the ligament length. Besides, under impact conditions, periodic variation in the measured force signal caused by specimen bouncing, stress waves developed on first contact with the hammer and kinetic energy effects constitute an inherent problem of impact testing [34–36] leading to extra errors and variability. As a result, the relative uncertainty in the measurements decreases in a sort of exponential form as l is increased. This difficulty to measure precisely the work of fracture of samples with small ligaments has been already pointed out in literature [24,37,38]. Taking into account this scatter, the usual practice in order to obtain valid EWF data is that the linear regression confidence limit has to be at least 95%. As a result, to achieve an acceptable accuracy in the estimated parameters, especially the y-intercept, it is necessary to test a large number of samples.

Fracture toughness could be performed on either compact or single edge notch bend (SENB) specimens [39]. Impact testing with Charpy specimens has been used for several decades as a standard toughness test. It is essentially a high strain rate, three-point bend test of notched specimens. In an effort to obtain flat, plain strain fracture toughness, sharp cracks were employed in Charpy specimens [40,41]. It, therefore, appears that three-point bend configuration is the natural one in impact experiments. The SENB configuration is simple, and therefore less expensive to fabricate. If the test fixture is designed properly, the span can be adjusted continuously allowing specimens with a range of thickness to be tested [42]. One problem with a SENB specimen is that it consumes more material than a compact specimen for the same characteristic dimensions (thickness B, width W, and crack length a) as shown in Fig. 1. This is a major shortcoming when evaluating new materials, where only small samples are available. Hence, there is a need to obtain reliable toughness parameters performing only a limited number of tests.

The present work presents a statistical analysis of the EWF methodology under impact conditions limited to

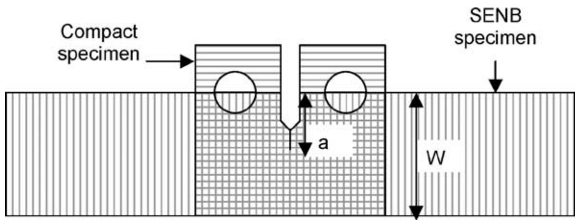

Fig. 1. Comparison of the profiles of compact and SENB specimens with the same in-plane characteristics dimensions (W and a).

the cases where the linear fit is suitable. It aims to determine the form to optimally distribute the ligament lengths to be tested, within a fixed number of them, in order to minimize the uncertainty of the estimated fracture parameters, taking into account that the relative uncertainty in the measurements is not constant along the l-axis.

2. The essential work of fracture method

The EWF Method was developed by Cotterel and Reddel [12], based on the idea of Broberg [43] who proposed that in ductile materials fractured under elasto-plastic conditions, the crack tip region can be divided in two parts: an inner fracture process zone and an outer plastic deformation zone. The total work of fracture is the sum of an essential work of fracture W_e spent in the end regions ahead of the crack tips, i.e. in the fracture process zone, and a non-essential plastic work W_p dissipated in the outer region:

$$W_f = W_e + W_p \tag{1}$$

As the essential term is proportional to the ligament area, it can be written

$$W_f = w_e lt + \beta l^2 t w_p \tag{2}$$

where t is the plate thickness, β is the shape factor, and w_e and w_p are, respectively, the specific essential work of fracture and the specific non-essential work of fracture. By dividing W_f by the ligament area lt, one obtains the specific total work w_f that can be expressed as:

$$w_f = w_e + \beta l w_p = w_e + w'_p l \tag{3}$$

When plane stress conditions prevail for all ligaments, it is further assumed that w_e is a constant dependent on thickness and, if the product $\beta w_p (w'_p)$ is independent of the ligament length l, plotting w_f against ligament length l should give a linear relationship where the ordinate at $l=0$ is the specific essential work of fracture w_e. When ligaments fall in the plane-stress/plane-strain region, the extrapolation to zero ligament length is uncertain and both linear and power curve fitting have been proposed [26]. However, in many cases when ligament ceases to be yielded completely before the plain strain size conditions are violated

$$B \geq l, \quad B \geq 25 w_e/\sigma_y \tag{4}$$

the extrapolation should be still linear since deformation is similar and the specific essential work of fracture is the plain strain value. The latter situation is very common in impact experiments. In fact, many researchers have found experimentally a linear relationship between w_f and l under impact conditions, when the ligament is fully yielded before breaking [3,22,24,25,27,37,44–48]. Therefore, in what follows linear fitting is adopted as a valid approach.

3. Optimal design of the test

In a set of n experimental values of w_f (w_{fi}, $i=1,\ldots,n$) taken at different values of l (l_i, $i=1,\ldots,n$) w_e can be estimated using Eq. (3). The main parameter to be estimated is w_e, but also $w'_p=\beta w_p$ may be of interest in some cases [48]. The estimates of w_e and w'_p using least squares and Eq. (3) are given by

$$\hat{w}_e = \frac{\sum_{j=1}^{n}\left\{w_j w_{f_j}\sum_{i=1}^{n} w_i l_i (l_i - l_j)\right\}}{\sum_{j=1}^{n}\left\{w_j l_j \sum_{i=1}^{n} w_i (l_j - l_i)\right\}} \tag{5}$$

$$\hat{w}'_p = \frac{\sum_{j=1}^{n}\left\{w_j w_{f_j}\sum_{i=1}^{n} w_i (l_j - l_i)\right\}}{\sum_{j=1}^{n}\left\{w_j l_j \sum_{i=1}^{n} w_i (l_j - l_i)\right\}} \tag{6}$$

The weights w_i are the inverse values of the variance of each measurement; i.e. $1/\sigma_i$, $i=1,\ldots,n$.

In an ideal situation in which a large number of measurements are available, the estimates of the intercept and the slope of Eq. (3) can be obtained with high accuracy even when the measurement errors involved were significant. However, each measurement requires a lengthy and expensive preparation and in three-point bending the ligament range is limited due to the configuration. Thus, the experiment should be designed considering a small number of samples that have to be placed along the l-axis in an optimal form such as to minimize the uncertainty of the estimated parameters.

The uncertainties of the estimated parameters, described by Eqs. (5) and (6), are usually characterized by their standard deviations

$$\sigma_{\hat{w}_e} = \left\{\frac{\sum_{i=1}^{n} w_i l_i^2}{\sum_{j=1}^{n} w_j l_j \sum_{i=1}^{n}(l_j - l_i)}\right\}^{1/2} \tag{7}$$

$$\sigma_{\hat{w}'_p} = \left\{\frac{\sum_{i=1}^{n} w_i}{\sum_{j=1}^{n} w_j l_j \sum_{i=1}^{n}(l_j - l_i)}\right\}^{1/2} \tag{8}$$

where $\sigma_{\hat{w}_e}$ and $\sigma_{\hat{w}'_p}$ are the estimated standard deviations of w_e and w'_p.

The problem of selecting optimal locations for a fixed number of measurements can be formulated mathematically as

$$\min_{n_1,n_2,\ldots,n_m} \mathbf{J} = \left\{p_1 \frac{\sigma_{\hat{w}_e}}{\hat{w}_e} + p_2 \frac{\sigma_{\hat{w}'_p}}{\hat{w}'_p}\right\} \tag{9}$$

with:

$$n = \sum_{i=1}^{m} n_i \tag{10}$$

The expression to be minimized is the weighted sum of the relative standard deviations of the estimated parameters. The absolute standard deviations are normalized to the estimated value in order to have the terms in the sum weighted only with the parameters p_1 and p_2. For instance, if only the essential work of fracture w_e is needed, p_2 is set to 0 and there is no need for normalization. However, if both parameters are needed with the same relative accuracy, p_1 must be set equal to p_2 and for convenience equal to 1. Fixing the total number of experimental determinations, n, and the number of intervals along the l-axis, m, the unknowns in this optimisation problem are the number of points, n_i, in which each of the m intervals on the l-axis should be divided.

The problem posed by Eqs. (9) and (10) could be solved numerically using some of the available minimization algorithms. However, if one notes that the unknowns can only take integer values, the complete space of feasible solutions can be easily explored and the solution that minimizes Eq. (9) obtained.

4. Data points

In this work, two types of data are used. Initially, a set of simulated data is used to compute the objective function (Eq. (9)) repeatedly in order to explore the influence of the distribution of tested ligament length on the fracture parameters confidence. Finally, experimental data published before [24] are used with the aim of validating the obtained findings.

4.1. Set of simulated data

The set of simulated measurements is generated using Eq. (3). The parameters used in this equation are $\hat{w}_e=8.3$, $\hat{w}'_p=4$. The clean measurements thus generated are corrupted with random error generated from a uniform distribution. The l-axis is divided in m intervals. Each region is now considered to have a different measurement error. The error standard deviations in each region (σ_i, $i=1,\ldots,m$) are given by:

$$\sigma_i = \left\{\frac{\sum_{j=1}^{n_i}(w^{(i)} - \hat{w}_e - \hat{w}'_p l_j^{(i)})^2}{n_i - 2}\right\}^{1/2} \tag{11}$$

with n_i the number of points in each interval. The error standard deviation value employed for the first interval was $\sigma_1=10$, and $\sigma_i(i=2,\ldots)$ were varied in order to obtain different relationships σ_i/σ_1. Ligament lengths were taken at equal spacing within each interval.

Table 1
Conventional properties of used materials

Material	Density, ρ at 23 °C g/cm^3	Elastic modulus, E (tensile) GPa	Notched impact strength, Charpy at 23 °C KJ/m^2
Vestolem P9421	0.898	0.70	20
Lustran ABS-740	1.040	1.93	21

4.2. Set of experimental data

Work of fracture vs. ligament length data points for two commercial polymeric materials: a polypropylene random copolymer, Vestolem P9421 (PP); and an acrylonitrile-butadiene-styrene terpolymer, Lustran ABS-740 (ABS), are used. Conventional properties of these commercial materials are summarized in Table 1. Fracture mechanics evaluation was performed on pre-cracked single-edge-notched-bend (SENB) specimens with razor-sharp notches of different notch depth. The specimen thickness, B, and the span to depth ratio, S/W, were kept equal to $W/2$ and 4, respectively. Specimen thicknesses were the typical ones and equal to 4 and 5 mm for PP and ABS, respectively. Materials were evaluated at room temperature with an impact velocity of 1.8 m/s for the polypropylene copolymer and 3.5 m/s for ABS. It was reported that in both cases geometric similarity was achieved and all the conditions necessary for the EWF method to work were obeyed [24]. Plots of specific total work of fracture, w_f, against ligament length, l, are shown in Fig. 2 for PP and in Fig. 3 for ABS.

5. Results and discussion

As discussed in Section 3, when the total number of experimental determinations, n, and the number of intervals

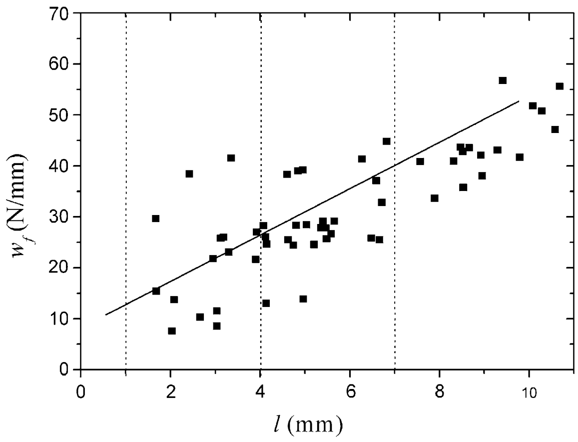

Fig. 2. Specific total work of fracture plotted against ligament length for polypropylene random copolymer (PP).

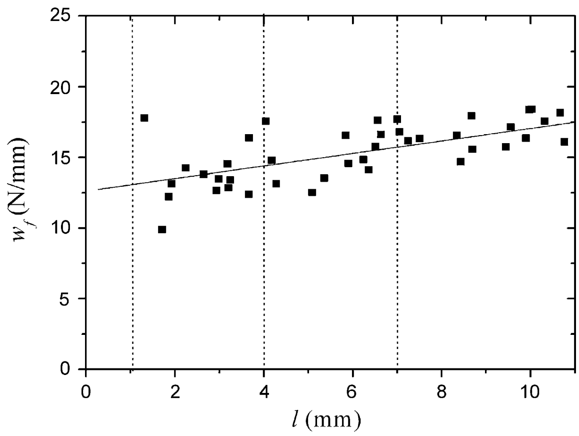

Fig. 3. Specific total work of fracture plotted against ligament length for acrylonitrile-butadiene-styrene (ABS).

along the l-axis, m, are fixed, the unknowns of the optimisation problem are the number of points, n_i, in which each of the m intervals on the l-axis should be divided.

Let us first evaluate the total number of samples to be tested. The influence of the number of data points on the accuracy of the essential work method has been mapped out for different polymeric materials in the ESIS TC4 Experience with the Essential Work of Fracture under plain stress conditions [19,49]. They came to the conclusion that optimum number of specimens to be tested is roughly 30–40 but, as this is rather demanding, a minimum number of 20 is advocated. Regarding data point distribution, they recommended that the specimens should cover the entire ligament length range and found no basis for biasing the data towards the intercept, since the confidence limits on the essential work determined in such situations were not smaller. Even though the preceding discussion is for static cases (under plain stress conditions), in what follows it will be assumed that the total number of samples is fixed and equal to 20, the minimum value recommended.

Once the total number of experimental determinations, n, is fixed, a decision on the number of intervals along the l-axis, m, and their limits must be taken. It is convenient to bear in mind that from the experimental observation it emerges that the percentage error in the specific fracture work is inversely proportional to the ligament length and thus the possible error for very small ligaments is high [37]. This could be explained using the error propagation theory. It states that when some quantities measured with different uncertainties are used to calculate a new quantity, the uncertainties are combined in a new total one. The new calculated quantity, f, can be written as a function of n measured values, $x_1, x_2, \ldots, x_n$:

$$f = f(x_1, x_2, \ldots, x_n) \tag{12}$$

By variation of a single input parameter x_i in the calculation model, the sensitivity $\partial f/\partial x_i$ can be determined on the basis of partial derivation of the function f with respect to the chosen input parameter. The absolute uncertainty of function δf_i related to the uncertainty of the input parameter x_i results from the multiplication of the sensitivity $\partial f/\partial x_i$ with the calculated or estimated absolute uncertainty δx_i. Each input parameter is affected by a different uncertainty, i.e. $x_1+\delta x_1, x_2+\delta x_2, \ldots, x_n+\delta x_n$. Therefore, the total uncertainty of the calculated value f is given by [50]:

$$\delta_f = \left|\frac{\partial f}{\partial x_1}\right|\delta x_1 + \left|\frac{\partial f}{\partial x_2}\right|\delta x_2 + \cdots + \left|\frac{\partial f}{\partial x_n}\right|\delta x_n \tag{13}$$

The work of fracture is a function of different experimental inputs: fracture energy U and fracture surface lB, i.e.

$$w_f = \frac{U}{lB} \tag{14}$$

Based on Eqs. (13) and (14), the relative uncertainty of the work of fracture is given by

$$\frac{\delta w_f}{w_f} = \left(\frac{1}{l} + \frac{1}{B}\right)\delta l + \frac{\delta U}{U} \tag{15}$$

being l the ligament length, B the thickness, δl the uncertainty of l and B, and δU the uncertainty of the calculated energy. It is easy to see from Eq. (15) that when the fracture area is small, i.e. the ligament is small, the uncertainty of the calculated work of fracture, $\delta w_f/w_f$, increases. $\delta w_f/w_f$ calculated from instrument characteristics and experimental points of PP copolymer is shown in Fig. 4 as a function of ligament length l. In this figure, three regions can be recognized clearly: in first place, for low l (identified in Fig. 4 with (1)), the $\delta w_f/w_f$ value is between 30 and 10% approximately with an elevated slope of variation with l; following this, for intermediates values of l (zone (2)), $\delta w_f/w_f$ varies from about 10–5% showing a lower slope

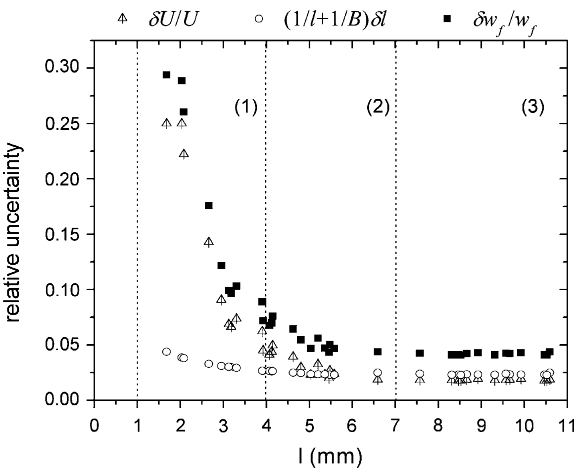

Fig. 4. Distribution of the relative uncertainties of the measured specific total work of fracture along the ligament axis for experimental points of PP.

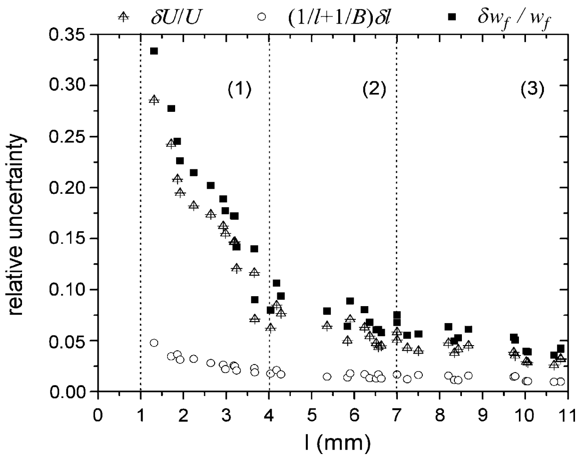

Fig. 5. Distribution of the relative uncertainties of the measured specific total work of fracture along the ligament axis for experimental points of ABS.

of variation with l; finally, for the largest values of l (zone (3)), $\delta w_f/w_f$ seems to be linear and constant with a value below 5%. The same trend was displayed by ABS as clearly seen in Fig. 5. It appears that in three-point bending tests under impact conditions a pattern in the uncertainty of work of fracture measurements could be identified. This could be justified considering that in SENB experiments sample thickness is limited because the dimensions of the testing device do not allow too big or too small samples. According to these observations, we decided to divide the working interval along the l-axis in three sub-intervals, $m=3$, to solve the minimization problem: (1) 1 mm$\leq l \leq$4 mm, (2) 1 mm$< l \leq$7 mm and (3) 7 mm$< l \leq$11 mm.

Table 2 shows the results of minimizing the objective function with the simulated data. The optimal distribution of points along the l-axis (n_1, n_2 and n_3) is shown for every set of possible values of the weights p_1 and p_2, and different combinations of values of the standard deviations σ_1, σ_2 and σ_3. The optimal distribution of data points along the l-axis appeared highly dependent on the distribution of standard deviations and on the weights used in the objective function. Hence, the definition of an optimal data point distribution along the ligament length range to be tested implies the 'a priori' knowledge of the work of fracture error distribution along the l-axis.

First the experimental error standard deviation values for every region for the considered materials are calculated. These values are $\sigma_1=10.7$, $\sigma_2=5.98$, and $\sigma_3=3.8$ for PP, and $\sigma_1=7.36$, $\sigma_2=3.9$, and $\sigma_3=2.08$ for ABS. Therefore, for the experimental analysed cases the same error distribution was determined ($\sigma_2/\sigma_1 \cong 0.5$ and $\sigma_3/\sigma_1 \cong 0.3$ for both materials) suggesting that this distribution is inherent to three point bending experiments under impact conditions. Again, this result seems to be related to the fact that in SENB experiments sample dimensions are restricted. In what follows it will be assumed that error standard

Table 2
Optimal distributions of measurements for different combinations of error variances in the measurements, compared to the case in which the measurements are taken at equal spacing

p_1	p_2	σ_2/σ_1	σ_3/σ_1	Optimal spacing						Equal spacing		
				n_1	n_2	n_3	**J**	$\sigma_{\hat{w}_e}$	$\sigma_{\hat{w}'_p}$	**J**	$\sigma_{\hat{w}_e}$	$\sigma_{\hat{w}'_p}$
1	1	1	1	13	0	7	0.314	2.11	0.38	0.460	2.93	0.46
		0.8	0.6	12	1	7	0.255	1.76	0.30	0.388	2.70	0.41
		0.7	0.5	12	1	7	0.251	1.73	0.30	0.361	2.60	0.38
		0.6	0.4	13	2	5	0.205	1.51	0.25	0.271	2.48	0.37
		0.5	0.3	8	3	9	0.217	1.56	0.24	0.284	2.16	0.30
		0.4	0.2	2	11	7	0.144	1.10	0.14	0.181	2.01	0.29
1	0	1	1	20	0	0	0.191	1.66	0.47	0.298	2.93	0.46
		0.8	0.6	18	2	0	0.123	1.59	0.50	0.215	2.70	0.41
		0.7	0.5	14	2	4	0.124	1.60	0.47	0.197	2.60	0.38
		0.6	0.4	13	3	4	0.113	1.49	0.42	0.125	2.48	0.37
		0.5	0.3	10	2	8	0.121	1.53	0.35	0.164	2.16	0.30
		0.4	0.2	4	10	6	0.108	1.08	0.28	0.140	2.01	0.29
0	1	1	1	9	1	10	0.123	2.60	0.33	0.162	2.93	0.46
		0.8	0.6	8	5	7	0.132	2.81	0.30	0.173	2.70	0.41
		0.7	0.5	7	2	11	0.127	2.65	0.29	0.164	2.60	0.38
		0.6	0.4	6	3	11	0.092	2.32	0.22	0.146	2.48	0.37
		0.5	0.3	4	2	14	0.096	1.91	0.23	0.120	2.16	0.30
		0.4	0.2	1	10	9	0.036	1.09	0.13	0.041	2.01	0.29

deviation distribution is indeed a-priori known in this type of experiment. As a consequence, the optimal distribution of ligament length to be tested seems to be 8, 3, 9 when both w_e and w_p are of interest; 10, 2, 8 when only w_e is desired, and 4, 2, 14 when only w_p is looked for.

With the purpose of validating the proposed distribution, the objective function was calculated using part of the experimental points so as to satisfy ligament length distributions proposed above. The same computation was performed using also part of the experimental points but now to cover the l-axis uniformly (Table 3). It simply emerges that the values of **J** for the proposed distributions are always smaller than the value of **J** for equal spacing of ligament lengths. It could be, therefore, concluded that the uniformly ligament length distribution along the ligament range is not the optimal one, and that the tests could be planned in order to minimize the uncertainty of the desired parameters.

To conclude, we propose to distribute the experimental determinations taking into account that the relative uncertainty in the measurements is not constant along the l-axis, instead of performing a large number of experiments in order to obtain reliable fracture parameters. The distribution of tests along the l-axis proposed in this work reduces the number of measurements needed to obtain a good value of the estimated impact toughness parameters.

6. Conclusions

In this work, the changes in reliability of fracture parameters—arising from the Essential Work of Fracture methodology—with the distribution of tested ligament lengths have been discussed and the following conclusions drawn.

Theoretically, to improve the accuracy of fracture parameters estimated from EWF a large number of measurements is required covering the entire ligament range. However, samples with very small ligaments are

Table 3
Objective function calculated for the points distributions proposed in Table 2 using experimental points, compared to the case in which the measurements are taken at equal spacing, for an error distribution of $\sigma_2/\sigma_1=0.5$ and $\sigma_3/\sigma_1=0.3$

Material	p_1	p_2	Optimal spacing						Equal spacing		
			n_1	n_2	n_3	**J**	$\sigma_{\hat{w}_e}$	$\sigma_{\hat{w}'_p}$	**J**	$\sigma_{\hat{w}_e}$	$\sigma_{\hat{w}'_p}$
PP	1	1	8	3	9	0.297	1.86	0.22	0.357	2.50	0.30
	1	0	10	2	8	0.276	1.57	0.30	0.279		
	0	1	4	2	14	0.076	2.51	0.16	0.078		
ABS	1	1	8	3	9	0.192	0.60	0.08	0.223	0.69	0.09
	1	0	10	2	8	0.047	0.57	0.09	0.057		
	0	1	4	2	14	0.138	0.68	0.05	0.166		

difficult to make and to measure precisely. The reason for this is that the relative uncertainty in the measured work of fracture (w_f) of an experiment is a function of the ligament length: when the fracture area is small, i.e. the ligament is small, the uncertainty of the measured w_f is large, arising from small fracture areas combined with the lack of instrument sensitivity at low energies. As a consequence, there is a need to improve the accuracy of toughness estimated from EWF by selecting the appropriate ligament lengths to be tested within a limited number of samples.

Consistently, the uniform ligament length distribution along the l-axis is not necessarily the optimal one. In this work, it is shown that it is possible to improve the accuracy of EWF parameters by distributing the tests along the l-axis, bearing in mind the distribution of error standard deviations along this axis.

Results demonstrate that the optimal distribution of experimental ligament lengths to be tested depends on the distribution of standard deviations along the l-axis, which is strictly a-priori unknown. However, from experimental data it emerges that the distribution of standard deviation seems to be mainly associated to the SENB impact test itself, appearing to be around to $\sigma_2/\sigma_1=0.5$ and $\sigma_3/\sigma_1=0.3$, when three ranges on the l-axis are selected.

It seems that there is a universal distribution of tests along the l-axis that leads to the estimation of toughness parameters with higher confidence. This distribution also depends on which parameter (w_e or βw_p) is of more interest. For instance, if both parameters are of equal interest, the optimal distribution of experiments along the l-axis is certainly close to 8, 3, and 9 samples with ligament lengths between 1–4, 4–7, and 7–11 mm, respectively.

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