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A $\wp$-order R-L high-pass filter modeled by local fractional derivative

Kang-jia Wang [a], Cui-ling Li [b,*]

[a] School of Physics and Electronic Information Engineering, Henan Polytechnic University, Jiaozuo, China
[b] Information Engineering School, Huizhou Economics and Polytechnic College, Huizhou, China

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Abstract As an important electronic device, filter is applied to all kinds of electronic products. In this paper, a new $\wp$-order R-L High-pass filter (HPF) modeled by the local fractional derivative (LFD) is proposed for the first time. With the help of the local fractional Laplace transform (LFLT), we obtain the non-differentiable(ND) transfer function, and present the expressions of ND amplitude-frequency characteristic (AFC) and ND phase-frequency characteristics (PFC). The corresponding parameters and properties of the $\wp$-order R-L HPF are also studied. What's interesting is that the $\wp$-order R-L HPF becomesthe ordinary one in the exceptional case at $\wp = 1$. The obtained results in this paper reveal the sufficiency of the local fractional derivative for analyzing the circuit systems in fractal space.

1. Introduction

The fractional calculus, an important branch of mathematics, was born in 1695. It is only in recent decades that researchers have realized that it is more expressive power than ordinary derivatives and can better reflect the changes of things, and the corresponding theoretical and applied studies have increased [1–4]. In [5], Kumar D, et al. proposed a fractional extension of the vibration equation and gave the solution. Singh J studied the fractional rumor spreading dynamical model in the social network and presented the solution by using an iterative scheme in [6]. In [7], the fractional model of nonlinear wave-like equations is studied and the homotopic technique is applied to examine it. A new fractional Drinfeld-Sokolov-Wilson model with exponential memory is analysed by Bhatter S, et al. in [8]. By using Caputo, Caputo-Fabrizio and Atangana-Baleanu fractional operators, the fractional exothermic reactions model with constant heat source in porous media with power, exponential and Mittag–Leffler laws are studied in [9]. In [10], Ghamisi P, et al. studied the segmentation of images based on fractional calculus. He J H studied the fractional oscillators in [11]. Liu S, et al. proposed a new fractal compression method in [12]. In [13], an efficient solution for the fractional equal width equation arising in cold plasma is considered by Goswami A, et al.

Recently, a new definition of the local fractional derivative (LFD) proposed by Yang [20] is used widely to describe many

* Corresponding author.
E-mail address: licl2003@outlook.com (C.-l. Li).
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ND problems that occur in the fractal engineering. For example, the R-C HPF filter described by LFD is proposed in [14]. In [15], Liu J G, et al. studied the nonlinear Korteweg-de Vries equation with space–time fractional derivatives and presented the exact solutions. Yang X J, et al. proposed the local fractional diffusion equation in fractal heat transfer in [16]. The LC-electric circuit modelled by LFD is elaborated in detail in [17]. In [18], the third order modified KdV equation on fractal set is studied. Wang K L, et al. proposed the local fractional Kdv-Burgers Equation and gave the physical insight in [19]. The method of (N + 1)-dimensional local fractional reduced differential transform and its applications are presented in [20]. So, in this paper, we mainly propose a new $\wp$-order R-L HPF model by using the LFD inspired by some recent results in fractal engineering. We arrange the overall structure of the article as follows. In Section 2, we introduce the concepts and their properties of the LFD and LFLT. In Section 3, we define the ND lumped elements in datail.The $\wp$-order HPF is established and studied in Section 4. In Section 5, we mainly analyse ND AFC and ND PFC. Finally, the conclusion is given in Section 6.

2. The LFD and LFLT

Definition 2.1 There is the following definition of LFD for the function $\varepsilon(\upsilon)$ with order $\wp$ $(0 < \wp \leqslant 1)$ [21]:

$$\varepsilon^{(\wp)}(\upsilon_0) = \frac{d^{\wp}\varepsilon(\upsilon)}{d\upsilon^{\wp}}\Big|_{\upsilon=\upsilon_0} = \lim_{\upsilon\to\upsilon_0}\frac{\Delta^{\wp}(\varepsilon(\upsilon)-\varepsilon(\upsilon_0))}{(\upsilon-\upsilon_0)^{\wp}}, \tag{2.1}$$

which for $\forall\varrho > 0, \delta > 0$ and $0 < |\upsilon - \upsilon_0| < \delta$, satisfies the condition that has:

$\left|\varepsilon_{\wp}(\upsilon) - \varepsilon_{\wp}(\upsilon_0)\right| < \varrho^{\wp}$there is $\Delta^{\wp}[\varepsilon(\upsilon) - \varepsilon(\upsilon_0)] \cong \Gamma(1+\wp)[\varepsilon(\upsilon) - \varepsilon(\upsilon_0)]$.

Definition 2.2. We have the definitions of Mittag–Leffler function, sine function and cosine function on Cantor sets with a fractal dimension $\wp$ as follows [21]:

$$E(\upsilon^{\wp}) = \sum_{k=0}^{\infty}\frac{\upsilon^{k\wp}}{\Gamma(1+k\wp)} \tag{2.2}$$

$$sin(\upsilon^{\wp}) = \sum_{k=0}^{\infty}(-1)^{p}\frac{\upsilon^{(2k+1)\wp}}{\Gamma[1+(2k+1)\wp]} \tag{2.3}$$

$$cos(\upsilon^{\wp}) = \sum_{k=0}^{\infty}(-1)^{k}\frac{\upsilon^{2k\wp}}{\Gamma[1+(2k+1)\wp]} \tag{2.4}$$

where $k \in N$, the LFDs of several functions are shown in Table 1.

Table 1 The LFDs of several functions on Cantor sets.

$\varepsilon(\upsilon)$	$\varepsilon^{(\wp)}(\upsilon)$
k	0
$E(k\upsilon^{\wp})$	$kE(k\upsilon^{\wp})$
$\frac{\tau^{k\wp}}{\Gamma(1+k\wp)}$	$\frac{\upsilon^{(k-1)\wp}}{\Gamma(1+(k-1)\wp)}$
$cos(k\upsilon^{\wp})$	$-ksin(k\upsilon^{\wp})$
$sin(k\upsilon^{\wp})$	$kcos(k\upsilon^{\wp})$

Definition 2.3. By noting thatthe LFLT of function $\varepsilon(\upsilon)$ as $\mathbb{L}_{\wp}[\varepsilon(\upsilon)] = \mathbb{R}^{\varepsilon}_{\wp}(\gamma)$, then there is [21]:

$$\mathbb{L}_{\wp}[\varepsilon(\upsilon)] = \mathbb{R}^{\varepsilon}_{\wp}(\gamma) = \frac{1}{\Gamma(1+\wp)}\int_0^{\infty}\varepsilon(\upsilon)E_{\wp}(-\upsilon^{\wp}\gamma^{\wp})(d\upsilon)^{\wp} \tag{2.5}$$

where $\mathbb{L}_{\wp}$ is the LFLT operator. **Theorem 1.** Suppose that the LFLT of function $\varepsilon(\upsilon)$ is taken as $\mathbb{L}_{\wp}[\varepsilon(\upsilon)] = \mathbb{R}^{\varepsilon}_{\wp}(\gamma)$, then there is:

$$\mathbb{L}_{\wp}\left[\varepsilon^{(\wp)}(\upsilon)\right] = \gamma^{\wp}\mathbb{R}^{\varepsilon}_{\wp}(\gamma) - \varepsilon(0). \tag{2.6}$$

We list the LFLTs of several functions on Cantor sets in Table 2.

Table 2 The LFLTs of several functions on Cantor sets.

$\varepsilon(\upsilon)$	$\mathbb{L}_{\wp}[\varepsilon(\upsilon)]$
1	$\frac{1}{\gamma^{\wp}}$
$E(k\upsilon^{\wp})$	$\frac{1}{\gamma^{\wp}-k}$
$\frac{\upsilon^{k\wp}}{\Gamma(1+k\wp)}$	$\frac{1}{\gamma^{\wp(k+1)}}$
$cos(k\upsilon^{\wp})$	$\frac{\gamma^{\wp}}{\gamma^{2\wp}+k^2}$
$sin(k\upsilon^{\wp})$	$\frac{k^{\wp}}{\gamma^{2\wp}+k^2}$

3. The ND lumped elements modeled by LFD

3.1. The ND resistor (NDR)

Definition 3.1 For the fractal circuit systems, we define the Ohm's Law of NDR as:

$$i_{\wp,R}(\upsilon) = \frac{u_{\wp,R}(\upsilon)}{R_{\wp}} \tag{3.1}$$

where $R_{\wp}, i_{\wp,R}(\upsilon)$ and $u_{\wp,R}(\upsilon)$ are the ND resistance, ND current and ND voltage of the NDR respectively.

3.2. The ND inductor (NDI)

According to the Faraday law of electromagnetic induction, there is the following relation between the ND voltage and ND magnetic flux $\Psi_{\wp}(\upsilon)$:

$$u_{\wp,L}(\upsilon) = \frac{d^{\wp}\Psi_{\wp,L}(\upsilon)}{d\upsilon^{\wp}}, \tag{3.2}$$

Definition 3.2. We define the ND inductance L_{ζ} of the NDI by LFD as:

$$L_{\wp} = \frac{\Psi_{\wp,L}(\upsilon)}{i_{\wp,L}(\upsilon)}, \tag{3.3}$$

Combing Eqs. (3.2) and (3.3) gives:

$$u_{\wp,L}(\upsilon) = L_{\wp}\frac{d^{\wp}i_{\wp,L}(\upsilon)}{d\upsilon^{\wp}}, \tag{3.4}$$

where $L_{\wp}, i_{\wp,L}(\upsilon)$ and $u_{\wp,L}(\upsilon)$ represent the ND inductance, ND current and ND voltage of the NDI respectively.

4. The ℘-order R-L HPF model within LFD

Fig. 1 plots the ND HPF model described by the LFD, where we have the following expression in accordance with the Kirchhoff Voltage Laws (KVL):

$$u_{\wp,i}(\upsilon) = u_{\wp,R}(\upsilon) + u_{\wp,L}(\upsilon), \tag{4.1}$$

With the series theorem, there is:

$$i_{\wp,R}(\upsilon) = i_{\wp,L}(\upsilon), \tag{4.2}$$

It is clearly that

$$u_{\wp,o}(\upsilon) = u_{\wp,L}(\upsilon), \tag{4.3}$$

By using Eqs. (3.1) and (4.2), we obtain:

$$u_{\wp,R}(\upsilon) = i_{\wp,L}(\upsilon)R_{\wp}, \tag{4.4}$$

We can get the following relation with the help of Eqs. (3.4), (4.1), (4.4):

$$u_{\wp,i}(\upsilon) = R_{\wp} i_{\wp,L}(\upsilon) + L_{\wp}\frac{d^{\wp} i_{\wp,L}(\upsilon)}{d\upsilon^{\wp}}, \tag{4.5}$$

Combining Eqs. (3.4) and (4.3) gives:

$$u_{\wp,o}(\upsilon) = L_{\wp}\frac{d^{\wp} i_{\wp,L}(\upsilon)}{d\upsilon^{\wp}}, \tag{4.6}$$

Applying LFLT to the above equation, it yields:

$$\mathbb{R}_{\wp}^{u_{\wp,i}}(\gamma) = R_{\wp}\mathbb{R}_{\wp}^{i_{\wp,L}}(\gamma) + L_{\wp}\left[\gamma^{\wp}\mathbb{R}_{\wp}^{i_{\wp,L}}(\gamma) - i_{\wp,L}(0)\right], \tag{4.7}$$

With the zero-state of $i_{\wp,L}(0) = 0$, the above equation can be reduced as:

$$\mathbb{R}_{\wp}^{u_{\wp,i}}(\gamma) = R_{\wp}\mathbb{R}_{\wp}^{i_{\wp,L}}(\gamma) + L_{\wp}\gamma^{\wp}\mathbb{R}_{\wp}^{i_{\wp,L}}(\gamma), \tag{4.8}$$

In a similar manner, the Eq. (4.6) can be changed into the following form:

$$\mathbb{R}_{\wp}^{u_{\wp,o}}(\gamma) = L_{\wp}\gamma^{\wp}\mathbb{R}_{\wp}^{i_{\wp,L}}(\gamma), \tag{4.9}$$

Eqs. (4.8) and (4.9) yield the result:

$$\mathbb{H}_{\wp}(\gamma) = \frac{\mathbb{R}_{\wp}^{u_{\wp,o}}(\gamma)}{\mathbb{R}_{\wp}^{u_{\wp,i}}(\gamma)} = \frac{1}{1 + \frac{R_{\wp}}{L_{\wp}}\gamma^{-\wp}}, \tag{4.10}$$

Then we obtain the following ND transfer function by letting $\sigma_{\wp} = \frac{R_{\wp}}{L_{\wp}}$ and $\gamma = j\varpi$:

$$\mathbb{H}_{\wp}(j\varpi) = \frac{1}{1 + \sigma_{\wp}(j\varpi)^{-\wp}}, \tag{4.11}$$

which gives the expressions of ND amplitude-frequency characteristic(AFC) and ND phase-frequency characteristics(PFC) as:

$$\left|\mathbb{H}_{\wp}(j\varpi)\right| = \frac{1}{\sqrt{1 + \sigma_{\wp}^{2}\varpi^{-2\wp}}}, \tag{4.12}$$

$$\Upsilon_{\wp}(\varpi) = -arctan(\sigma_{\wp}\omega^{-\wp}). \tag{4.13}$$

In engineering application, the ND AFC represents the attenuation of each frequency after the signal passes through the filter and the ND PFC reflects the delay of each frequency component after the signal passes through the filter. Assuming there is ϖ_0 that makes:

$$\left|\mathbb{H}_{\wp}(j\varpi_0)\right| = \frac{\sqrt{2}}{2}\left|\mathbb{H}_{\wp}(j\varpi)\right|_{max}, \tag{4.14}$$

In engineering, we call ϖ_0 the ND cut-off frequency.

From Eq. (4.12), we get the solution of ϖ_0 as:

$$\varpi_0 = \sqrt[\wp]{\sigma_{\wp}}. \tag{4.15}$$

Of special interest is that the ND HPF converts into the ordinary one in the special situation that $\wp = 1$.

5. Analysis of the ND HPF

With the help of Eq. (4.12), we get the ND AFC curves of different fractional orders $\wp$ at $\sigma_{\wp} = 1$ as shown in Fig.2, where it is found that the attenuation of the curves decrease as the angular frequency increases. This means that the smaller the frequency, the greater the attenuation of the signal, on the contrary, the greater the frequency, the smaller the attenuation. This is exactly the characteristic of high pass filter, that is, it allows the frequency higher than a certain intercept to pass through and greatly attenuate the lower frequency. Furthermore, the larger the value of fractional orders $\wp$ is, the faster the curve decays as the angular frequency decreases. In other words, the higher the order, the better the filtering characteristics.And the ND-AFC graph versus the fractional orders $\wp$ when $\sigma_{\wp} = 1$ is shown in Fig.3.

As an important parameter of filter, the ND PFC characteristic is often used to reflect signal delay. Fig.4 plots the curve

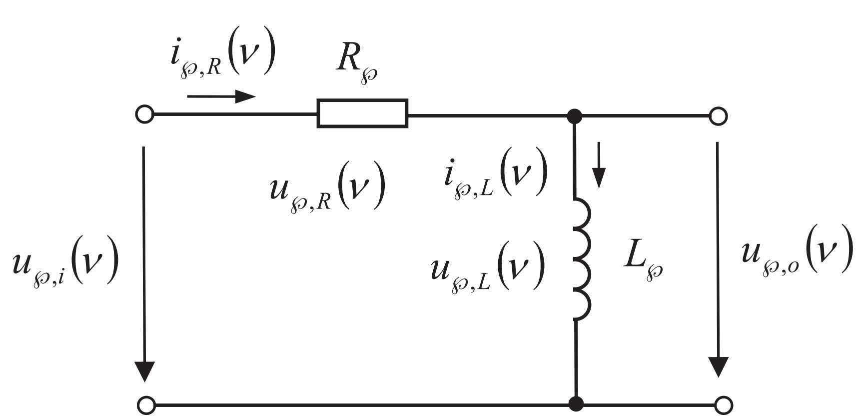

Fig. 1 The ℘-order R-L HPF model within LFD.

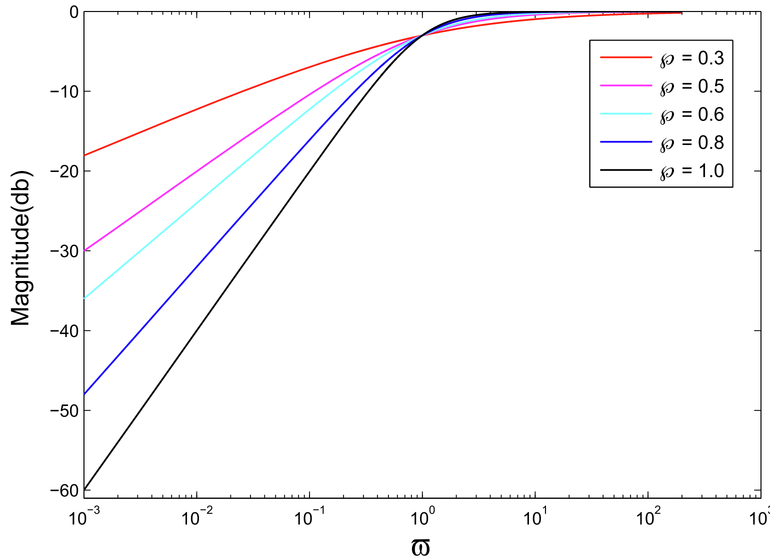

Fig. 2 Curves of the ND AFC with $\wp = 0.3, 0.5, 0.6,0.8, 1.0$ at $\sigma_\wp = 1$.

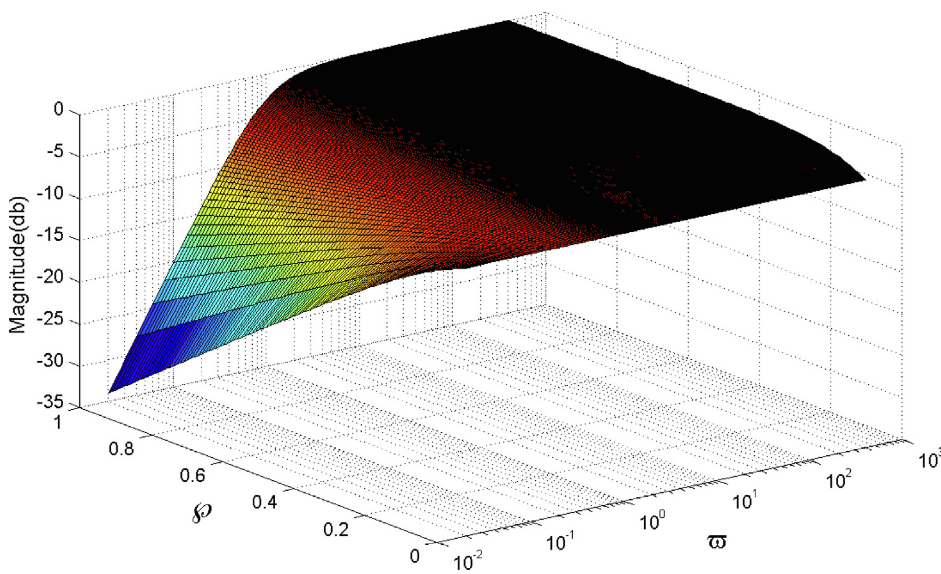

Fig. 3 Three dimensional graph of ND AFC with different fractional orders $\wp$ at $\sigma_\wp = 1$.

of the ND PFC with different fractional orders $\wp = 0.3$, 0.5, 0.6,0.8, 1.0 at $\sigma_\wp = 1$. According to the diagram, the following conclusions can be obtained: When ϖ increases from 0 to ∞, the absolute value of corresponding phase changes decreases gradually and approaches zero from $\frac{\pi}{2}$. And the bigger the order $\wp$ is, the greater the phase changes with the angular frequency.

Recall the ND cut-off frequency mentioned earlier, when the signal angular frequency ϖ is higher than the ND cut-off frequency ϖ_0, the signal can pass through. when the signal angular frequency ϖ is lower than the ND cut-off frequency ϖ_0, the signal output will be greatly attenuated. The ND cut-off frequency is defined as the limit of passband and stopband. In engineering applications, engineers often change the values of $R_\wp$ and $L_\wp$ to get the appropriate ND cut-off frequency.

6. Conclusions

In this paper, we have successfully modeled the $\wp$-order HPF by LFD in fractal space for the first time. The NDTF is obtained by applying the LFLT, and the corresponding ND AFC and ND PFC are presented. We studied the parameters and properties of the $\wp$-order R-L HPF in detail. The results we presented in this paper are expected to open some new perspectives towards the characterization of ND filters via LFDs.

Declaration of Competing Interest

None.

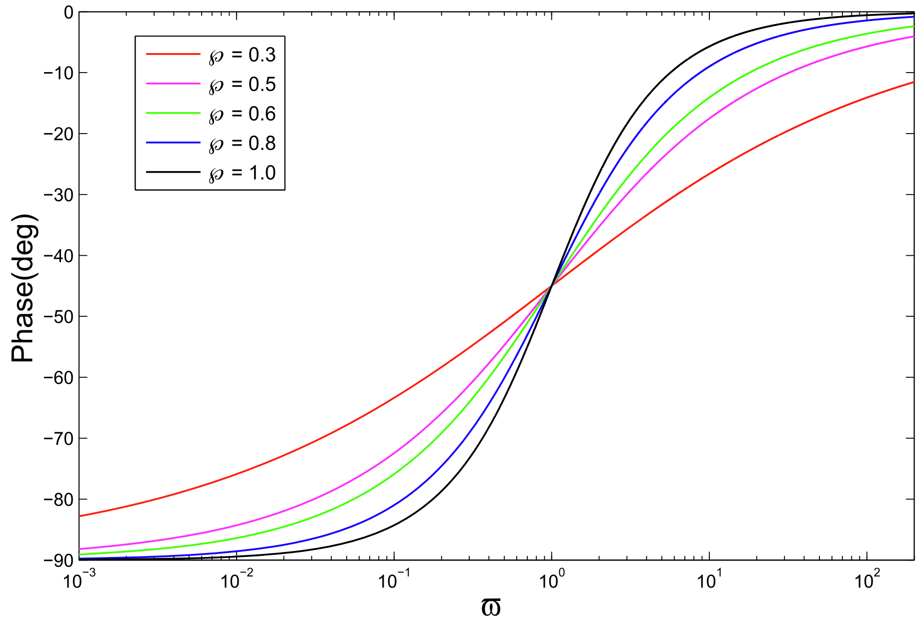

Fig. 4 Curve of the ND PFC with $\wp = 0.3, 0.5, 0.6, 0.8, 1.0$ at $\sigma_{\wp} = 1$.

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