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DETERMINATION OF THERMAL PARAMETERS OF POOR CONDUCTORS BY TRANSIENT TECHNIQUES

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SUMMARY

The authors compare the transient hot wire method and the parallel wire method of determining thermal conductivity, using the second to find the thermal diffusivity of two materials. The wires are sandwiched between two samples of the material to be investigated. The influence of pressure is also studied in order to identify the measurement conditions that can be easily achieved. A method is chosen to evaluate thermal parameters and to determine the field of data to be used in relation to sample dimensions.

KEY WORDS Thermal conductivity Diffusivity Parallel wire Insulating materials

INTRODUCTION

Transient techniques are utilized to measure thermal properties of a wide range of materials including rocks, gas, liquids and soils. The hot wire technique is fast (some minutes) and does not require large samples; therefore it is often proposed for *in situ* measurements. The method needs to record the point by point time evolution of temperature, the spatial position of the points and the knowledge of the thermal power applied to the wire. The analysis of the data can be performed using different mathematical models, depending on the scheme chosen; thus we can obtain values of simply thermal conductivity or both thermal conductivity and diffusivity.

THE MATHEMATICAL MODELS

All models considered assume an isotropic material and two-dimensional conduction expressed in cylindrical co-ordinates, with the axis coincident with the heating wire. The temperature is constant and uniform at the beginning; it remains constant as the radius goes to infinity.

The hot wire

With the preceding hypotheses a geometrical straight line suddenly heated with power q per unit of length, causes at radius r and time t the rise in temperature (Carslaw and Jaeger, 1959):

$$T(r,t) = \frac{q}{4\pi k}\int_{r^2/4\alpha t}^{\infty}\frac{e^{-u}}{u}\mathrm{d}u = -\frac{q}{4\pi k}Ei\left(-\frac{r^2}{4\alpha t}\right) \tag{1}$$

Defining the Fourier number as $Fo = \alpha t/r^2$, for high values of Fo, i.e. $Fo > 10$ (Carslaw and Jaeger, 1959; Grazzini *et al.*, 1988), equation (1) is usually written as

$$T(r,t) = (q/4\pi k)[\ln(4Fo/C) + O(1/Fo)] \tag{2}$$

C is a constant whose logarithm gives the Euler constant γ.

It is important to consider that equations (1) and (2) are effective only for $r > 0$. At the same time high Fo can easily be attained by low values of r. Then equation (2) is useful only when referring to wire

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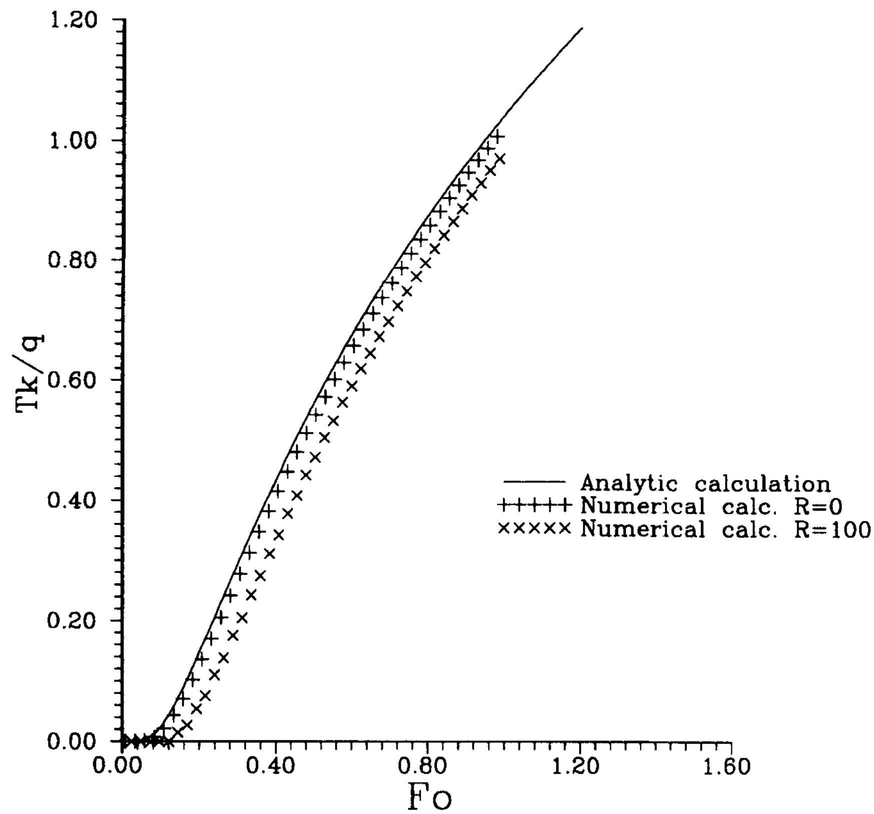

Figure 1. Nondimensional temperature variation at the distance *r* from the heating wire for two models and different values of thermal resistance *R*

temperature. In fact using logarithm properties we write:

$$T(w,t) - T(w,t_0) = (q/4\pi k)\ln(t/t_0) \tag{3}$$

where *w* is the wire radius.

It is evident we can measure only thermal conductivity.

A more complicated model was proposed by Blackwell (1954) who considers the wire mass and the thermal contact of the wire with the sample, assuming the wire as a thermal conductivity of infinity. The solution is valid only for high values of *Fo* referred to the wire radius $w(Fo_w)$:

$$T(r,t) = (q/4\pi k)\{\ln(4Fo_w/C) + 2/Bi + (1/(2Fo_w))[\ln(4Fo_w/C) + 1 - (\alpha k_w/(\alpha_w k))(\ln(4Fo_w/C) + 2/Bi)] + O(1/Fo_w)^2\} \tag{4}$$

where $Bi = \mathrm{Hw/k}$ is the Biot number at the radius of wire and *H* is the heat exchange coefficient between wire and sample.

Using the analytical solution given by Carslaw and Jaeger (1959) for a physical model taking into account the wire mass. Häkansson *et al.* (1988) and Pettersson (1990) also showed how to calculate diffusivity with the 'hot wire method'. This requires long calculation times and cannot include thermal resistance between the wire and the sample.

The parallel wire

Considering one point at some distances from the heating wire the thermocouple must be parallel to the heating wire to avoid perturbation in the cylindrical field; then this experimental configuration is called

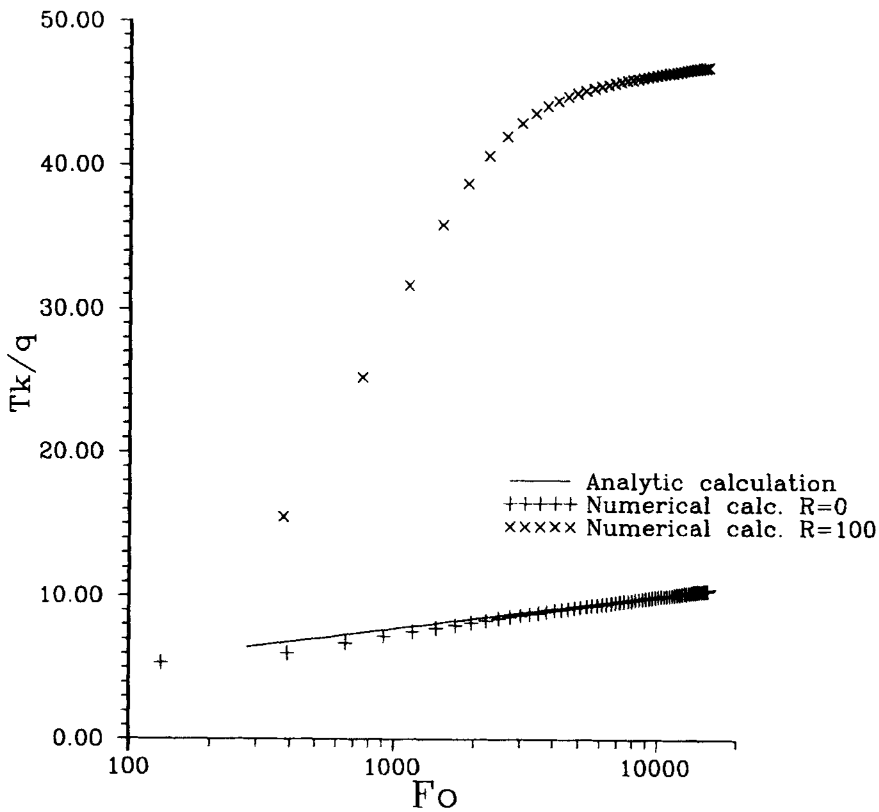

Figure 2. Nondimensional temperature variation at the heating wire for two models and different values of thermal resistance R

'the parallel wire' (Davis *et al.*, 1980). In this case we use equation (1) because the authors do not know any solution taking into account thermal resistance between wire and sample. With sizeable increase calculation time we could consider the mass of the wire (Laurent, 1989), but the mass can be reduced using very thin wire because the position of the wire and the thermocouples is maintained by the sample

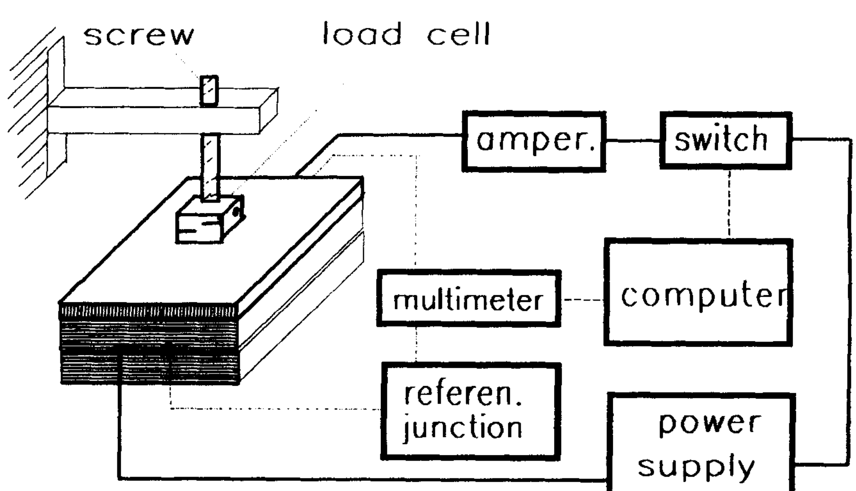

Figure 3. Sketch of experimental apparatus

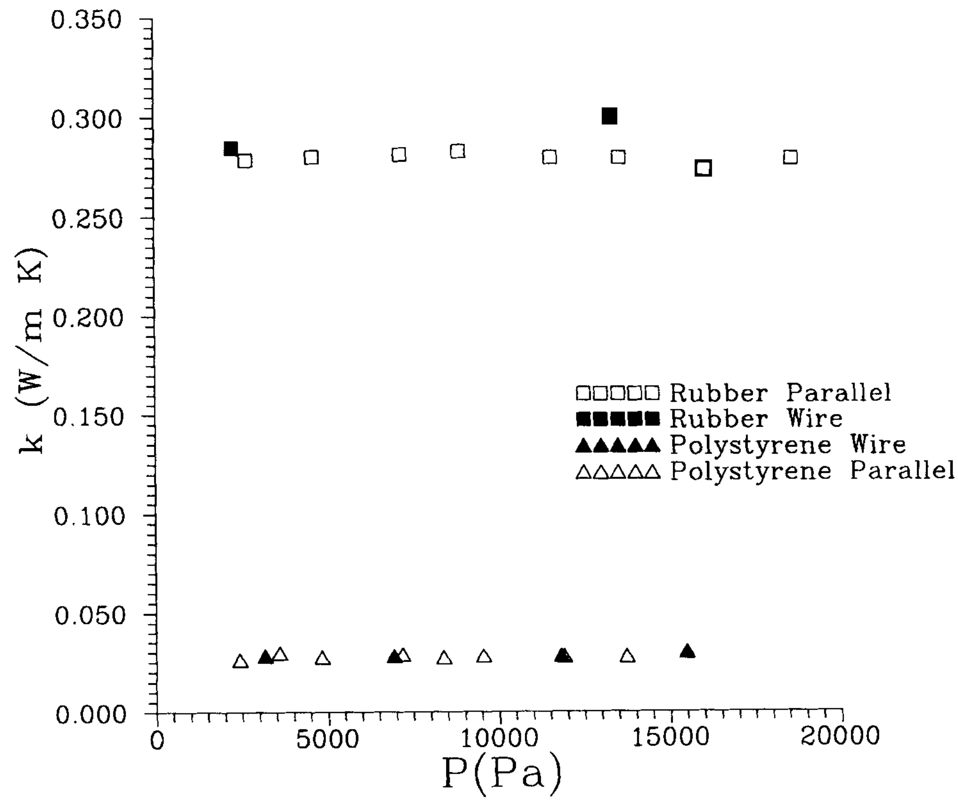

Figure 4. Thermal conductivity versus pressure

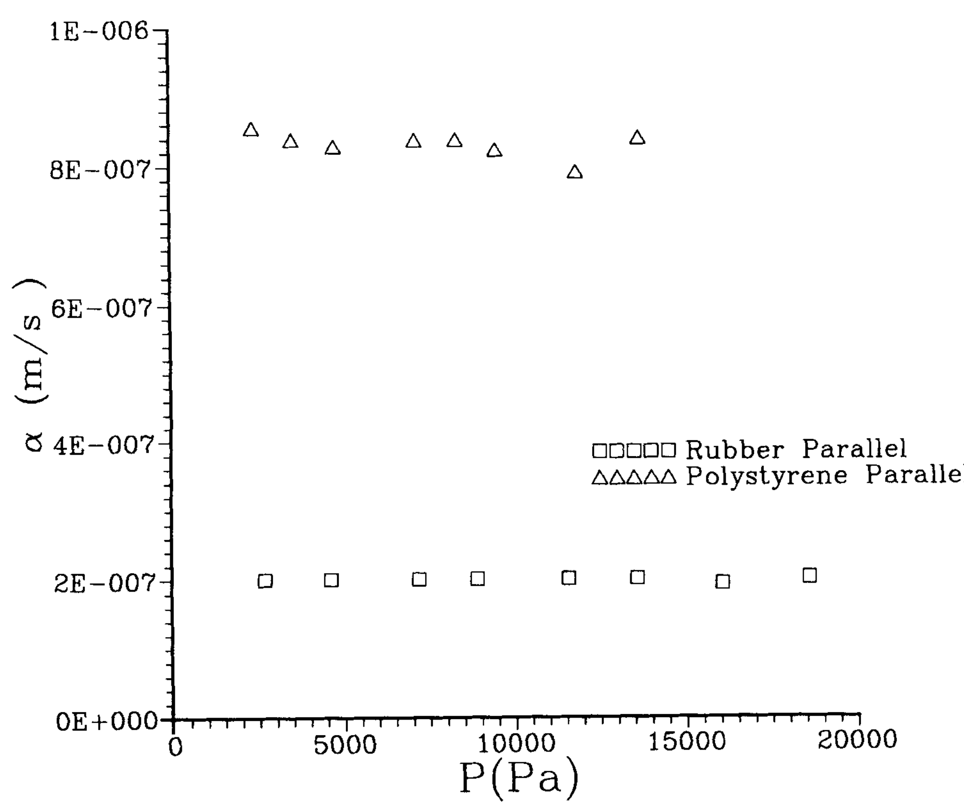

Figure 5. Thermal diffusivity versus pressure

itself. Figures 1 and 2 show the influence of thermal resistance by comparing the results of equation (1) with those of a numerical program that uses finite elements to solve the energy balance (with the physical model proposed by Blackwell). It is easy to see that the main influence is the translation of the curves, which are lower in the case of the parallel wire. The exponential integral *Ei* in equation (1) is evaluated using the polynomials given by Abramowitz and Stegun (1972); the wire temperature is considered as that evaluated at a radius equal to that of the heating wire.

A very thin wire is also useful to easily obtain a high Fourier number and to avoid end effects which become negligible in the measurements if the ratio of length to diameter of the heating wire is greater than 100 (Wechsler, 1966).

THE EXPERIMENTAL APPARATUS

To fabricate the line source a Teflon-insulated constantan wire of 0·075 mm diameter was utilized. This wire, together with an enamelled copper one of the same diameter, gives the two thermocouples used to measure the temperature variation at the point of interest. The reference junction is at ambient temperature, which is the starting condition of our samples. The e.m.f. is revealed by an HP-3478A multimeter having 100 nV resolution; the thermocouple was tested and shows an e.m.f. of 0·036 mV/K in the field of measurements. The DC heating current comes from a stabilized power supply Philips PE1537.

Having measured the electrical resistance of the wire, the current is controlled by an amperometer during the experiments. Time and e.m.f. are taken via computer which also provides the starting point of the measurements. Figure 3 shows a scheme of the apparatus.

The samples we considered are two polystyrene parallelepipeds with dimensions $0{\cdot}05 \times 0{\cdot}14 \times 0{\cdot}30$ m and two of rubber with dimensions $0{\cdot}03 \times 0{\cdot}14 \times 0{\cdot}31$ m. The wire and thermocouple are sandwiched between the two elements and the specimens are pressed by loading to come into close contact with each other. The pressure is obtained using iron pieces of known weight, and applying a load with a screw. The strength is measured by a load cell with a resolution of 0·1 N. In the parallel wire method the distance between the heating wire and the thermocouple ranges from 5 to 130 mm.

The density of the two materials is measured in relation to water using a balance with a precision of 0·0001 g.

THE EVALUATION OF PARAMETERS

For the hot wire method, the thermal conductivity k was evaluated determining the constant A in equations (3) and (4) rewritten as

$$T(w, t) - T(w, t_0) = A \ln(t/t_0) \tag{5}$$

$$T(w, t) - T(w, t_0) = A \ln(t/t_0) + B[\ln(t)/t - \ln(t_0)/t_0] + C(1/t - 1/t_0) \tag{6}$$

fitting it to the data by a least squares method. Usually the best correlation, i.e. the minimum χ^2, was

Table 1. Thermophysical parameters of the two samples at ambient temperature. Mean values in the field of pressure investigated

	Polystyrene		Rubber	
	This work	ASHRAE (1989)	This work	ASHRAE (1989)
Hot wire $k \pm \sigma$ (W m^{-1} K^{-1})	$0{\cdot}0287 \pm 0{\cdot}0013$	0·029	$0{\cdot}288 \pm 0{\cdot}021$	0·163
Parallel wire $k \pm \sigma$ (W m^{-1} K^{-1})	$0{\cdot}0279 \pm 0{\cdot}0022$		$0{\cdot}279 \pm 0{\cdot}004$	
Parallel wire $\alpha \pm \sigma$ E + 7 (m^2 s^{-1})	$8{\cdot}32 \pm 0{\cdot}46$	6·424	$1{\cdot}991 \pm 0{\cdot}052$	0·9954
$\rho \pm \sigma$ (kg m^{-3})	37 ± 2	$29 \div 56$	1170 ± 13	1150
$c \pm \sigma$ (J kg^{-1} K^{-1})	906 ± 100	1220	1199 ± 38	1424

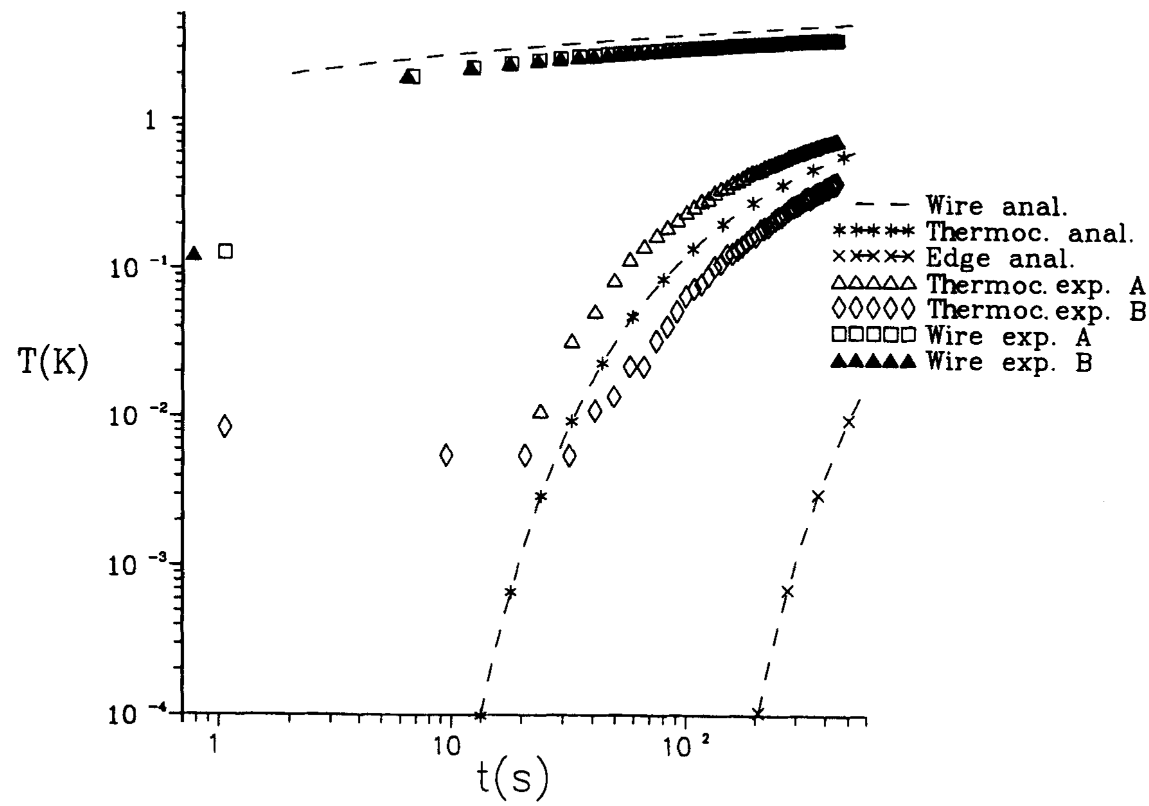

Figure 6. Polystyrene. Analytic calculus of the temperature variation at different point of the sample in comparison with some experimental data

obtained with equation (6) which can represent different physical models (Grazzini *et al.*, 1988).

With the parallel wire approach we can evaluate the diffusivity α using the method reported by Davis *et al.* (1980). From equation (1) the ratio between the temperature at different times gives:

$$\frac{T(r,t)}{T(r,t_0)} = \frac{Ei\left(-\frac{r^2}{4\alpha t}\right)}{Ei\left(-\frac{r^2}{4\alpha t_0}\right)} \tag{7}$$

Through a trial-and-error method we find the value of thermal diffusivity which minimizes the summation of the square of the difference between measured and calculated values, that is the χ^2 (Bevington, 1969). Knowing α and applying the least squares method with equation (1) we can evaluate the thermal conductivity k.

Results obtained with this method are equal, within instrumental error, to those given by the Simplex method (Nelder and Mead, 1965), which is a fast gradient search method to minimize the χ^2 for a nonlinear function of the two variable k and α. In spite of a better approximation in term of χ^2 we have a ten to twenty times greater calculation time.

Figures 4 and 5 show the value of thermal conductivity and diffusivity versus pressure; each point is the mean of at least three measurements. Table 1 relates to the mean values obtained using all data in the pressure range. The standard deviation σ due to repetition of measurements is less than the instrumental error, which is estimated to be about 3%, except when referred to the entire field of pressures investigated.

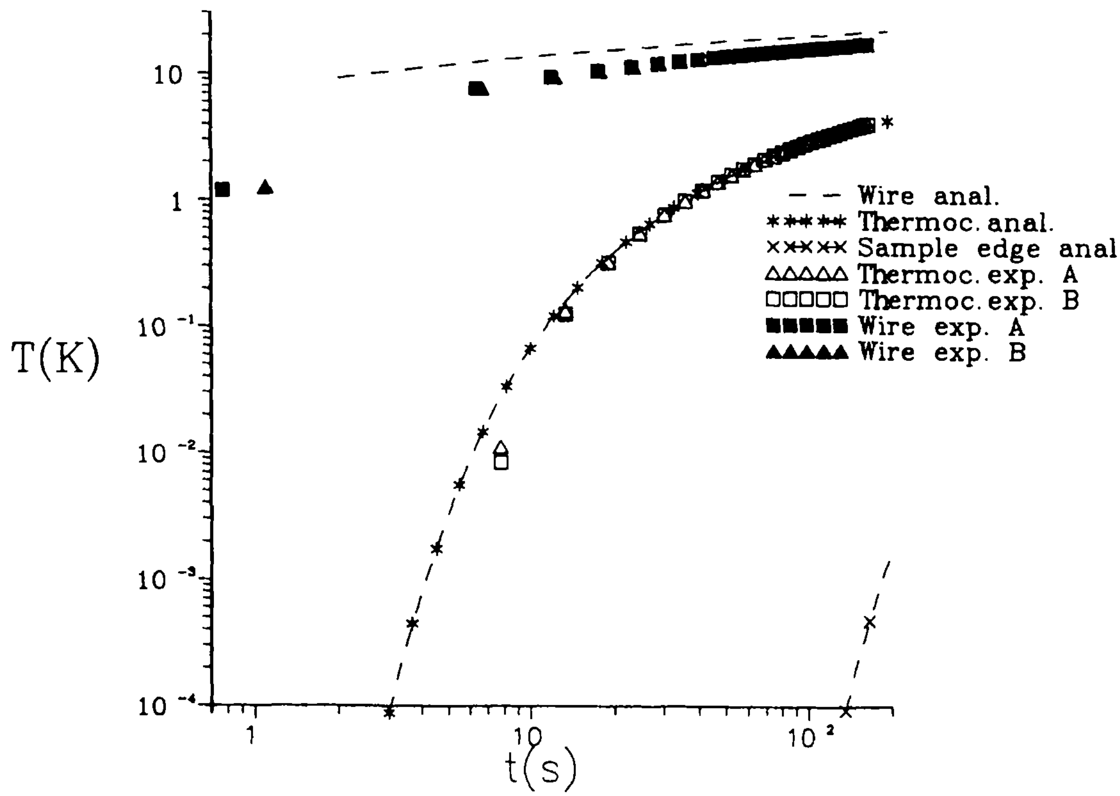

Figure 7. Rubber. Analytic calculus of the temperature variation at different point of the sample in comparison with some experimental data

The same standard deviation was found for a preceding series of experiments performed with a heating wire having a greater diameter.

The mean ambient temperature of all the experiments is about 21 °C.

The data interval to evaluate the parameters was chosen considering $Fo > 100$ as the lower limit in the case of the hotwire and the first stable value for the parallel wire. The highest value for the boot is evaluated, considering the time when the temperature variation at the edge of the sample calculated with equation (1) reaches 0·001 K. We considered this temperature value to be low enough to keep away influence at the measurement point within the instrumental errors. Figures 6 and 7 show the temperature versus time for polystyrene and rubber, comparing some of the experimental data with results of equation (1). The value of specific heat, c, is only indicative, being hardly influenced by the measurement errors.

CONCLUSIONS

The comparison between the hot wire method and the parallel wire method shows that the second is useful in determining both the thermal conductivity and the diffusivity of a material using the same apparatus. This is comparable with that method proposed by Hamid and Hamed (1992) for measuring thermal conductivity. The method is sufficiently rapid and accurate to be utilized for quality control during the production or utilization of insulating materials. The apparatus is quite feasible and simple to use. The values of pressure we tested are easy to obtain and are required only to assure a good thermal contact between the specimens. Current PCs have a calculation capacity larger than that required for the proposed method. The errors are compatible with the usual engineering approximations.

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