Physics

Chiral analysis of the Generalized Form Factors of the nucleon

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We apply the methods of Chiral Perturbation Theory to the analysis of the first moments of the Generalized Parton Distributions in a Nucleon, usually known as *generalized form factors*. These quantities are currently also under investigation in Lattice QCD analyses of baryon structure, providing simulation results at large quark masses to be extrapolated to the "real world" via Chiral Effective Field Theory. We have performed a leading-one-loop calculation in the covariant framework of Baryon Chiral Perturbation Theory (BChPT), predicting both the momentum and the quark-mass dependence for all the vector and axial (generalized) form factors. In particular we discuss the results for the limit of vanishing four-momentum transfer where the GPD-moments reduce to the well known moments of Parton Distribution Functions (PDFs). We fit our results to available lattice QCD data, extrapolating down to the physical point. We conclude by presenting outstanding results from a combined fit to different GPDs-moments.

1 Introduction

In this paper we discuss the findings of references [1] and [2] where the generalized form factors of the nucleon were analyzed in the framework of covariant Baryon Chiral Perturbation Theory (BChPT). In standard SU(2) BChPT the results for form factors typically depend on two variables, the momentum transfer squared and the quark-mass and on a number of low energy constants (LECs). In this work we study the quark-mass dependence of the isoscalar- and isovector-vector as well as the isovector-axial generalized form factor $A_{2,0}(t)$ at zero momentum transfer and predict the physical value of those quantities by determining previously unknown LECs by a fit of the BChPT results to lattice QCD data.

Working in twist-2 approximation, the parity-even part of the structure of the nucleon is encoded via two Generalized Parton Distribution functions (GPDs) $H^q(x, \xi, t)$ and $E^q(x, \xi, t)$ [3].
Moments of GPDs can be interpreted much easier and are connected to well-established hadron structure observables. E.g. the zero-th order (Mellin-) moments in the variable x correspond to the contribution of quark q to the well known Dirac and Pauli form factors $F_1(t)$, $F_2(t)$ of the nucleon:

$$\int_{-1}^{1} dx \, x^0 \, H^q(x, \xi, t) = F_1^q(t) \ , \ \int_{-1}^{1} dx \, x^0 \, E^q(x, \xi, t) = F_2^q(t). \tag{1}$$

Our aim is the application of the methods of ChPT to the analysis of the *first* moments in x of these nucleon GPDs

$$\int_{-1}^{1} dx \, x \, H^q(x, \xi, t) = A_{2,0}^q(t) + (-2\xi)^2 C_{2,0}^q(t) \ , \ \int_{-1}^{1} dx \, x \, E^q(x, \xi, t) = B_{2,0}^q(t) - (-2\xi)^2 C_{2,0}^q(t), \tag{2}$$

where one encounters *three generalized form factors* $A_{2,0}^q(t)$, $B_{2,0}^q(t)$, $C_{2,0}^q(t)$ of the nucleon for each quark flavor q. For the case of 2 light flavours the *generalized isoscalar (u+d) and isovector (u-d) form factors* have been already studied in a series of papers at leading-one-loop order in the non-relativistic framework of Heavy Baryon ChPT (HBChPT) [4]. We will provide the first analysis of these generalized form factors utilizing the methods of covariant BChPT for 2 light flavors [5]. Our BChPT formalism [6] makes use of a variant of Infrared Regularization [7] for the loop diagrams and is constructed in such a way that we *exactly* reproduce the corresponding HBChPT result of the same chiral order in the limit of small
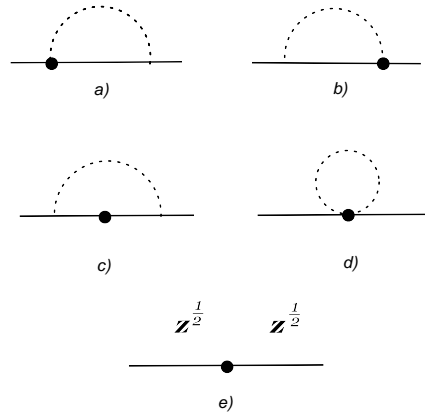
Fig. 1: Loop diagrams contributing to the first moments of the GPDs of a nucleon at leading-one-loop order in BChPT. The solid and dashed lines represent nucleon and pion propagators respectively. The solid dot denotes a coupling to an external tensor field.

pion masses. For the complete analytical expressions of the discussed results and a detailed description of the formalism used for the calculation we refer to [1] and [2].

2 The Generalized Form Factors of the Nucleon in ChPT

In ChPT one can direcly access the isoscalar (*s*) and isovector (*v*) contribution to the generalized form factors of the nucleon by evaluating the following matrix elements [3]:

$$i\langle p'|\overline{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\,q|p\rangle_{u+d} = \tag{3}$$

$$\overline{u}(p')\left[A^s_{2,0}(\Delta^2)\gamma_{\{\mu}\overline{p}_{\nu\}} - \frac{B^s_{2,0}(\Delta^2)}{2M_N}\,\Delta^\alpha i\sigma_{\alpha\{\mu}\overline{p}_{\nu\}} + \frac{C^s_{2,0}(\Delta^2)}{M_N}\,\Delta_{\{\mu}\Delta_{\nu\}}\right]\frac{1}{2}\,u(p), \tag{4}$$

$$i\langle p'|\overline{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\,q|p\rangle_{u-d} = \tag{5}$$

$$\overline{u}(p')\left[A^v_{2,0}(\Delta^2)\gamma_{\{\mu}\overline{p}_{\nu\}} - \frac{B^v_{2,0}(\Delta^2)}{2M_N}\,\Delta^\alpha i\sigma_{\alpha\{\mu}\overline{p}_{\nu\}} + \frac{C^v_{2,0}(\Delta^2)}{M_N}\,\Delta_{\{\mu}\Delta_{\nu\}}\right]\frac{\tau^a}{2}\,u(p). \tag{6}$$

The brackets $\{\ldots\}$ denote the completely symmetrized and traceless combination of all indices in an operator. $u\,(\overline{u})$ is a Dirac spinor of the incoming (outgoing) nucleon of mass M_N, for which the quark matrix-element is evaluated.

From a powercounting analysis we find that the Feynman diagrams contributing to the first moments of GPDs of a nucleon at leading-one-loop order $[\mathcal{O}(p^2)]$ in covariant ChPT are the ones depicted in Fig.1.

3   Analysis of the results for $A_{2,0}^{(v,s)}(t=0)$

In this section we present an analysis of the generalized isovector- and isoscalar-vector form factors $A_{2,0}^{v,s}(t)$ in the forward limit $t \to 0$. The details of the ChPT calculation as well as a complete analysis of the form factors $B_{2,0}^{v,s}(t)$, $C_{2,0}^{v,s}(t)$ and their connection to the spin physics sector can be found in ref[1, 2].

In the forward limit $t \to 0$ the generalized form factors $A_{2,0}^{s,v}(t=0)$ can be understood as moments of the ordinary Parton Distribution Functions (PDFs) $q(x)$, $\bar{q}(x)$ [3]:

$$\langle x \rangle_{u \pm d} \quad = \quad A_{2,0}^{s,v}(t=0) = \int_0^1 dx\, x \, (q(x) + \bar{q}(x))_{u \pm d} \,. \tag{7}$$

Experimental results are available for $\langle x \rangle$ in proton- and "neutron-" targets, from which one can estimate the isoscalar and isovector quark contributions at the physical point [8] at a regularization scale μ. We choose $\mu = 2$ GeV for our comparisons with phenomenology.

For the PDF-moment $A_{2,0}^v(t=0)$ we obtain to $\mathcal{O}(p^2)$ in BChPT

$$
\begin{aligned}
A_{2,0}^v(0) \quad &\doteq \quad \langle x \rangle_{u-d} \\
&= \quad a_{2,0}^v + \frac{a_{2,0}^v m_\pi^2}{(4\pi F_\pi)^2} \left\{ -(3g_A^2 + 1)\log\frac{m_\pi^2}{\lambda^2} - 2g_A^2 + g_A^2 \frac{m_\pi^2}{M_0^2}\left(1 + 3\log\frac{m_\pi^2}{M_0^2}\right)\right. \\
&\quad \left. -\frac{1}{2}g_A^2 \frac{m_\pi^4}{M_0^4}\log\frac{m_\pi^2}{M_0^2} + g_A^2 \frac{m_\pi}{\sqrt{4M_0^2 - m_\pi^2}}\left(14 - 8\frac{m_\pi^2}{M_0^2} + \frac{m_\pi^4}{M_0^4}\right)\arccos\left(\frac{m_\pi}{2M_0}\right)\right\} \\
&\quad + \frac{\Delta a_{2,0}^v g_A m_\pi^2}{3(4\pi F_\pi)^2}\left\{2\frac{m_\pi^2}{M_0^2}\left(1 + 3\log\frac{m_\pi^2}{M_0^2}\right) - \frac{m_\pi^4}{M_0^4}\log\frac{m_\pi^2}{M_0^2} + \frac{2m_\pi(4M_0^2 - m_\pi^2)^{\frac{3}{2}}}{M_0^4}\right. \\
&\quad \left. \times \arccos\left(\frac{m_\pi}{2M_0}\right)\right\} + 4m_\pi^2 \frac{c_8^{(r)}(\lambda)}{M_0^2} + \mathcal{O}(p^3).
\end{aligned}
$$

Most LECs in this expression are well known from analyses of chiral extrapolation functions [9]. However, the sizes of $a_{2,0}^v$, $\Delta a_{2,0}^v$ and $c_8^{(r)}(\lambda)$ are only poorly known at this point. The coupling $\Delta a_{2,0}^v$ is related to the spin-dependent analogue of the mean momentum fraction, namely $\langle \Delta x \rangle_{u-d}$ (see Section 4 and [2]). In a first fit (Fit I) to lattice data we constrain $\Delta a_{2,0}^v$ from the phenomenological value of $\langle \Delta x \rangle_{u-d}^{phen.} \approx 0.21$ and perform a 2-parameter fit with the couplings $a_{2,0}^v$, $c_8^{(r)}(1\text{GeV})$ at the regularization scale $\lambda = 1$ GeV. We fit to the LHPC lattice data for this quantity as given in ref.[10], including lattice data up to effective pion masses of $m_\pi \approx 600$ MeV. The resulting values for the fit parameters together with their statistical errors are given in table 1 and the resulting chiral extrapolation function is shown as the solid line in the left hand side of Fig.2. The extrapolation curve tends towards smaller values for small quark-masses, but does not quite reach the phenomenological value at the physical point, which is *not* included in the fit. Since BChPT is based on a systematic perturbative expansion, it does not only provide us with the result at a certain order, but also allows for an estimate of possible higher order effects. From dimensional analysis we know that the leading chiral contribution to $\langle x \rangle_{u-d}$ beyond our calculation takes the form $\mathcal{O}(p^3) \sim \delta_A \frac{m_\pi^3}{\Lambda_\chi^2 M_0} + ...$ Constraining δ_A between values $-1, \ldots, +1$ (the natural scale of all couplings in the observables considered here is below 1) and repeating the fit with this uncertainty term included leads to the grey band indicated in Fig.2. As one can see the phenomenological value for $\langle x \rangle_{u-d}$ lies well within that band of possible next-order corrections, giving us *no indication* that something may be inconsistent with the large values for $\langle x \rangle_{u-d}$ typically found in lattice QCD simulations for large quark-masses.

As stated in the Introduction, the covariant BChPT scheme used in this analysis is able to reproduce exactly the corresponding non-relativistic HBChPT result at the same order by the appropriate truncation in
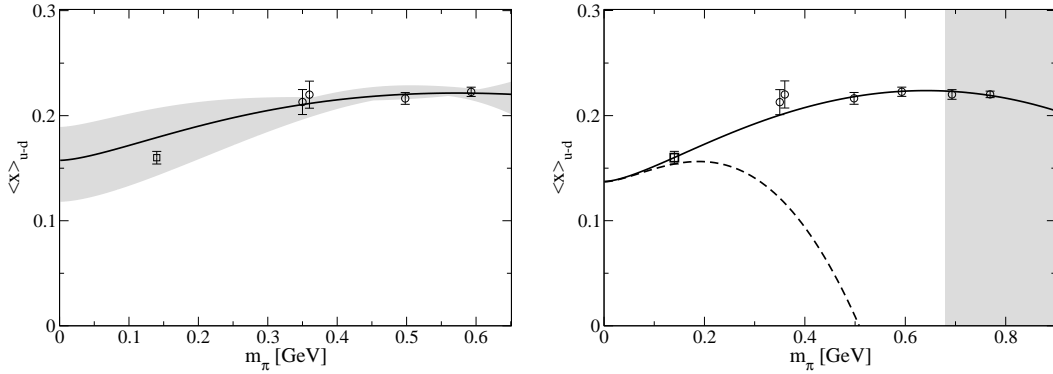
Fig. 2: Left panel. Fit I of the $\mathcal{O}(p^2)$ result of Eq.(3.2) to the LHPC lattice data of ref.[10]. **Right panel**. Fit II of the $\mathcal{O}(p^2)$ BChPT result of Eq.(3.2) to the LHPC lattice data of ref.[10] *and* to the physical point (solid line). The dashed curve shown corresponds to $\mathcal{O}(p^2)$ result in the HBChPT truncation.

$1/(16\pi^2 F_\pi^2 M_0)$. In order to also compare the $\mathcal{O}(p^2)$ HBChPT result of refs.[11] with the $\mathcal{O}(p^2)$ covariant BChPT result of Eq.(8) we perform a second fit (Fit II): We fit the covariant expression for $\langle x \rangle_{u-d}$ of Eq.(8) again to the LHPC lattice data and we constrain the coupling $\Delta a_{2,0}^v$ in such a way, that the resulting chiral extrapolation curve reproduces the phenomenological value of $\langle x \rangle_{u-d}^{phen.} = 0.160 \pm 0.006$ [8] exactly for physical quark masses. The parameter values for this Fit II are again given in table 1, whereas the resulting chiral extrapolation is shown as the solid line in the right hand side of Fig.2. We would like to emphasize that the curve looks very reasonable, connecting the physical point with the lattice data of the LHPC collaboration in a smooth fashion. For the comparison with HBChPT we now utilize the very same values for $a_{2,0}^v$ and $c_8^{(r)}$ of Fit II. The resulting curve based on the $\mathcal{O}(p^2)$ HBChPT truncation is shown as the dashed curve in Fig.2. One observes that this leading-one-loop HBChPT expression agrees with the covariant result between the chiral limit and the physical point, but is not able to extrapolate on towards the lattice data.

We therefore conclude that the smooth extrapolation behaviour of the covariant $\mathcal{O}(p^2)$ BChPT expression for $\langle x \rangle_{u-d}$ of Eq.(8) between the chiral limit and the region of present lattice QCD data is due to an *infinite tower* of $\left(\frac{m_\pi}{M_0}\right)^n$ terms.

The same analysis can also be performed for the isoscalar generalized form factor $A_{2,0}^s(t)$ which in the forward limit reduces to $A_{2,0}^s(0) = \langle x \rangle_{u+d}$. Fig.3 shows a 2-parameter fit of the $\mathcal{O}(p^2)$ covariant BChPT results for this observable [1] to LHPC and QCDSF data of reference [10] and [12]. We want to stress that the experimental value of the quantity $\langle x \rangle_{u+d}$ is not included in the fit. As the plot shows, the obtained chiral extrapolation curve is very satisfying, consistently linking the lattice data at large quark-masses with the phenomenological value. In contrast, the correspondent result in the heavy baryon limit represented

Table 1: Values of the couplings resulting from the two fits to the LHPC lattice data for $\langle x \rangle_{u-d}$ [1]. The errors shown are only statistical and do neither include uncertainties from possible higher order corrections in ChEFT nor from systematic uncertainties connected with the lattice simulation.

	$a_{2,0}^v$	$\Delta a_{2,0}^v$	$c_8^r(1\text{GeV})$
Fit I (4 points - 2 parameter)	0.157 ± 0.006	0.210 (fixed)	-0.283 ± 0.011
Fit II (6+1 points - 3 parameter)	0.141 ± 0.0057	0.144 ± 0.034	-0.213 ± 0.03
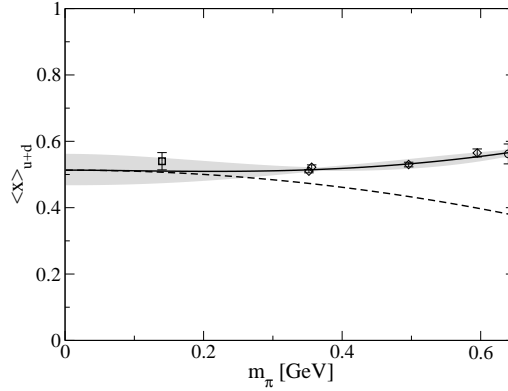
Fig. 3: Two-parameters Fit of the $\mathcal{O}(p^2)$ BChPT result of ref.[1] to the LHPC lattice data of ref.[10] and to the QCDSF data of ref.[12]. We obtain $a_{20}^s = 0.513 \pm 0.006$ and $c_9 = -0.064 \pm 0.005$ as values for the free parameters. The band shown indicate estimate of higher order possible corrections. The dashed line correspond to the respective HBChPT results at this order.

by the dashed line in Fig.3 not even allows for an interpolation between the presently available lattice data and results from experiments.

4   Combined fit

We have extended the analysis to the first moments of the axial GPDs $\tilde{H}^q(x, \xi, t)$ and $\tilde{E}^q(x, \xi, t)$

$$\int_{-1}^{1} dx\, x\, \tilde{H}^q(x, \xi, t) = \tilde{A}_{2,0}^q(t) \ , \ \int_{-1}^{1} dx\, x\, \tilde{E}^q(x, \xi, t) = \tilde{B}_{2,0}^q(t). \tag{8}$$

Again, in the limit of vanishing four-momentum transfer the isovector form factor $\tilde{A}_{2,0}^v(t \to 0)$ is directly connected to the spin dependent analogue of the mean momentum fraction $\langle \Delta x \rangle_{u-d}$

$$\tilde{A}_{2,0}^v(0) = \langle \Delta x \rangle_{u-d} = \int_0^1 dx\ x\, (q_\downarrow(x) - q_\uparrow(x))\big|_{u-d} \tag{9}$$
$$= \Delta a_{2,0}^v + \mathcal{O}(p^2)$$

where $\Delta a_{2,0}^v$ corresponds to the chiral limit value of $\langle \Delta x \rangle_{u-d}$.

Looking at the $\mathcal{O}(p^2)$ BChPT expression for $\langle \Delta x \rangle_{u-d}$ [2] one can easily observe that each isovector moment ($\langle x \rangle_{u-d}$ and $\langle \Delta x \rangle_{u-d}$) depends on 3 unknown parameters: 2 couplings ($a_{2,0}^v$, $\Delta a_{2,0}^v$) and one counterterm. As the same couplings contribute in both moments, it is hoped that a simultaneous fit of our BChPT results to the lattice data of ref.[10] can considerably reduce the statistical errors. As one can see from Fig.4, the results of this procedure are pretty outstanding, given that the values at the physical pion mass were not included in the fit! The chiral curvature in both observables naturally bends down to the phenomenological value for lighter quark masses, leading to a very satisfactory extrapolation curve. The extrapolated values at the physical pion mass together with the ones known from phenomenology are reported in table 2. As one can easily see, the obtained BChPT values from our $\mathcal{O}(p^2)$ leading-one-loop analysis are clearly consistent with the experimental ones!

We would like to stress that this efficient cross-talk between the ChPT results for $\langle x \rangle_{u-d}$ and $\langle \Delta x \rangle_{u-d}$ occurs only in the covariant framework, while in the non-relativistic approach both observables are completely independent at this order.

We conclude that combined fits of several observables characterized by a common subset of ChEFT couplings are the winning strategy towards the most reliable chiral extrapolations of lattice QCD results.

Table 2: The phenomenological values for the observables $\langle x \rangle_{u-d}$ and $\langle \Delta x \rangle_{u-d}$, together with the extrapolated values obtained from the combined fit shown in fig.4.

	$\langle x \rangle_{u-d}$	$\langle \Delta x \rangle_{u-d}$
PHENOMENOLOGY	$0.16 \pm 0.006[8]$	$0.21 \pm 0.025\ [13]$
EXTRAPOLATED VALUES AT m_π^{phys}	0.16 ± 0.01	0.19 ± 0.01
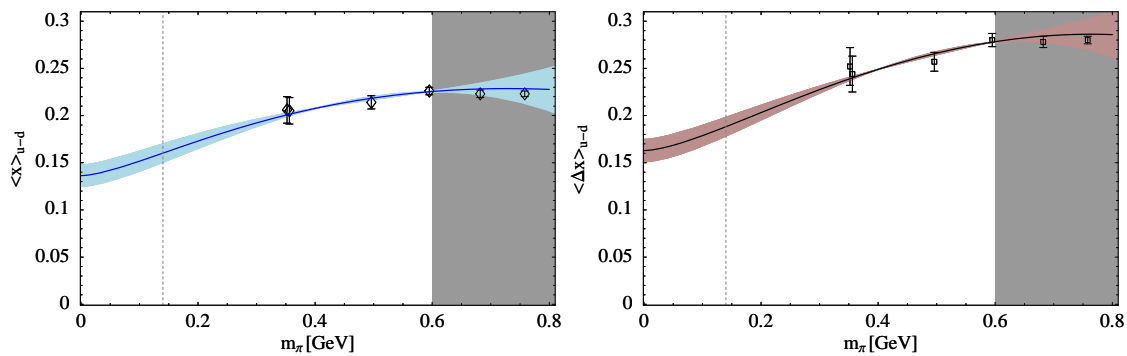
Fig. 4: Combined FIT of the $\mathcal{O}(p^2)$ results of ref. [2] to the lattice data of ref. [10]. Note that the phenomenological values at physical pion mass were not included in the fit. The bands shown indicate estimate of higher order possible corrections.

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