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Exchange Effects and SU_4 Invariance in Electromagnetic and Weak Transitions

F. CANNATA and J.-I. FUJITA*

Istituto di Fisica dell'Università, Bologna and
Istituto Nazionale di Fisica Nucleare, Bologna
**Physics Department, Tokyo University of Education, Tokyo*

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We study the consequences of SU_4 symmetry using the hypothesis that the SU_4 algebra is generated by 15 operators:

$$T^\alpha=\int V_4^\alpha(x)d\boldsymbol{x}\,,$$
$$Y_k^\alpha=\int A_k^\alpha(x)d\boldsymbol{x}\,,$$
$$S_k=\int A_k^0(x)d\boldsymbol{x}\,.$$

Here V and A denote the vector and axial vector hadronic currents, with the upper index referring to isospin and the lower one to Lorentz space ($k=1, 2$ and 3).

Particular emphasis is given to the treatment of muon capture. The equality between the vector and axial vector matrix elements and the connection of the vector matrix elements to the photoabsorption cross section are discussed for this process. Finally we draw some conclusions about many-body effects in electromagnetic and weak nuclear transitions from the current algebra and SU_4 invariance for the nuclear Hamiltonian.

§ 1. Introduction

There has been much effort to eliminate the meson degrees of freedom from nuclear wave functions and electromagnetic and weak transition operators. Within the context of a meson theory the problem may be completely solved, so that one may find equivalent transition operators whose matrix elements taken between eigenstates of the nucleonic Hamiltonian (involving only internucleon potentials) describe correctly the transition amplitudes.[1] Ambiguities, however, arise, because, as noted by Bell,[2] correlation effects in the wave functions may be transformed by a unitary transformation into many-body terms in the operators and vice versa, so that effective interactions and correlation functions are not separately well defined concepts.

We study many-body effects in electromagnetic and weak nuclear transitions using current algebra[3] and SU_4 invariance[4] for the nuclear Hamiltonian; in essence we try to find exact relations between matrix elements of the physical electromagnetic and weak currents, between true nuclear states (with mesonic effects included), taking advantage of their algebraic properties.

In § 2 we give briefly the commutators which we use, the current-current and current-density commutators. We construct the SU_4 algebra in terms of these in the way suggested by Radicati[5] and illustrate its usefulness.

In § 3 we study muon capture in particular, concerning ourselves mainly with the equality of the vector and axial vector matrix elements[6),7)] and the relation of the vector matrix elements, in the unretarded dipole approximation to the photoabsorption cross section.[7] This last point seems important, because, as emphasized by Green,[8] for the muon capture of doubly closed shell nuclei Foldy and Walecka[7] related the total capture rates to the photoabsorption cross sections by ignoring the exchange effects. For the electric-dipole transitions it is well known, especially from recent experiments,[9] that the exchange effects are very important; the integrated photoabsorption cross section is about twice of the classical sum rule.*) Fortunately, owing to the Siegert theorem[10] (the low energy limit theorem**)), the unretarded dipole matrix element including the contribution of exchange currents in V_k can be rewritten in terms of V_4. Therefore, the photoabsorption cross section in the Foldy-Walecka formula automatically includes the contribution of exchange currents as far as the unretarded dipole interaction is concerned. However, the situation is more complicated[11] for large momentum transfers. Since the momentum transfer is generally larger for muon capture than the corresponding electromagnetic transition, we need precise knowledge on the exchange currents for finite momentum transfers in order to connect them. Furthermore, there is no simple direct connection between axial vector and vector exchange currents.[12] In this context the authors believe that the relationship among the vector and axial vector matrix elements and the photoabsorption cross section should be again investigated without assuming that the contribution of exchange currents is small.

In § 4 we give some arguments, in a special case of broken SU_4 symmetry, that the SU_4 predictions may still be reasonably accurate.

§ 2. SU_4 and current algebra

The free quark model leads to the commutation relations at equal times:

$$\left[\int V_4^3(x)\,d\boldsymbol{x},\ \int A_\mu^j(y)\,d\boldsymbol{y}\right]_{x_0=y_0} = -\varepsilon_{3jk}\int A_\mu^k(y)\,d\boldsymbol{y}\,, \tag{1}$$

$$\left[\int A_l^1(x)\,d\boldsymbol{x},\ \int A_l^2(y)\,d\boldsymbol{y}\right]_{x_0=y_0} = \int V_4^3(x)\,d\boldsymbol{x} \tag{2}$$

and

*) The significance of the experimental results in connection with the Gell-Mann-Goldberger-Thirring sum rule has been discussed by Weise.[9]

**) Siegert's theorem[10] is based on the argument that the impulse approximation can be applied to V_4 but not to V_k:

$$V_4(\boldsymbol{x},0)\cong\sum_i e_i\delta(\boldsymbol{x}-\boldsymbol{x}_i) \quad \text{but} \quad V_k(\boldsymbol{x},0)\neq\sum_i e_i(p_i)_k\delta(\boldsymbol{x}-\boldsymbol{x}_i)/M\,.$$

$$\left[\int A_k^{\pm}(x)\,d\boldsymbol{x},\ \int V_4^3(y)\exp(i\boldsymbol{y}\cdot\boldsymbol{\nu})\,d\boldsymbol{y}\right]_{x_0=y_0} = \mp i\int A_k^{\pm}(y)\exp(i\boldsymbol{y}\cdot\boldsymbol{\nu})\,d\boldsymbol{y}\,. \quad (3)$$

The relations (1), (2) for $l=4$ and (3) for $k=4$ can be proved without invoking the quark model.[13] In the expression

$$\left[\int A_k^{\pm}(x)\,d\boldsymbol{x},\ V_4^3(y)\right]_{x_0=y_0} = \mp i A_k^{\pm}(y)$$

for $k\neq 4$, we might expect to have some contribution of the so-called Schwinger terms, but it is a generally accepted idea that the once-integrated commutation relation is free from Schwinger terms.[14]

Wigner[4] proved that, if the nuclear Hamiltonian H_0 includes potentials of Wigner and Majorana types only, the operators

$$(T^0)^\alpha = \tfrac{1}{2}\sum_i \tau^\alpha(i),$$

$$(S^0)_k = \tfrac{1}{2}\sum_i \sigma_k(i)$$

and

$$(Y^0)_k{}^\alpha = \tfrac{1}{2}\sum_i \sigma_k(i)\tau^\alpha(i),$$

commute with H_0. If the nuclear forces are of sufficiently short range and attractive in relative S-states, the ground states of $A=4n$ nuclei $|0\rangle$ are scalar supermultiplets such that

$$(T^0)^\alpha|0\rangle = (S^0)_k|0\rangle = (Y^0)_k{}^\alpha|0\rangle = 0\,.$$

This idea can be easily extended to the case where the nucleus consists of nucleons and mesons (or quarks) as far as isospin is concerned; this is essentially the philosophy underlying the conserved vector current theory.[15]

We now discuss the SU_4 algebra generated by

$$T^\alpha = \int V_4^\alpha(x)\,d\boldsymbol{x}\,, \quad (4)$$

$$Y_k{}^\alpha = \int A_k{}^\alpha(x)\,d\boldsymbol{x} \quad (5)$$

and

$$S_k = \int A_k^0(x)\,d\boldsymbol{x}\,, \quad (6)$$

where $k=1,2,3$ and $A_k^0(x)$ is the isoscalar axial vector current density.

We note that P.C.A.C. theory[13] relates $(d/dt)\int A_4{}^\alpha(x)\,d\boldsymbol{x}$ to $\int\varphi^\alpha(x)\,d\boldsymbol{x}$, where $\varphi^\alpha(x)$ is the pion field. The quantity $\int A_4{}^\alpha(x)\,d\boldsymbol{x}$ is not a constant of motion; nevertheless the current algebra of the free quark model guarantees that T^α, $Y_k{}^\alpha$ and S_k satisfy the same commutation relations[16] as $(T^0)^\alpha$, $(Y^0)_k{}^\alpha$ and

$(S^0)_k$. It should be noticed that $A_\lambda{}^\alpha(x)$ includes the induced pseudoscalar effect, the two-body Gamow-Teller exchange effects and so on. While the matrix elements of $V_\lambda{}^\alpha$ and $A_\lambda{}^\alpha$ are direct and physically observable quantities through the coupling of hadrons and leptons, the matrix elements of $A_\lambda{}^0$ are not known experimentally so far. We assume that the neutral current $A_\lambda{}^0$ exists,[17] but this is not yet definitely known.

The reason why we are concerned with the algebra generated by T^α, $Y_k{}^\alpha$ and S_k is that it is a direct extension of Wigner's supermultiplet theory.[5] In order to relate the generators defined by (4), (5) and (6) to the quantities appearing in conventional nuclear physics, we must truncate the whole Hilbert space into a model subspace, in which there exist only nonrelativistic nucleons. In this Hilbert space one has effective currents whose single-body parts give

$$(T^1)^\alpha = (T^0)^\alpha , \tag{7}$$

$$(Y^1)_k{}^\alpha = (Y^0)_k{}^\alpha \tag{8}$$

and

$$(S^1)_k = (S^0)_k . \tag{9}$$

We have omitted possible renormalization factors*) in (8) and (9). In the next section we will discuss how the renormalization factor in Eq. (8) may be taken into account.

Now we show the usefulness of these concepts by an application.

Let us consider semileptonic processes in which the hadronic current $\mathcal{J}$ is coupled to the leptonic one. The hadronic structure enters[18] in the expression

$$W_{\lambda\rho} = \overline{\sum}(2\pi)^4\delta^4(p'-p-q)\langle p|J_\rho{}^+(0)|p'\rangle\langle p'|J_\lambda(0)|p\rangle ,$$

which may be written as

$$\overline{\sum}(2\pi/V)\iint \exp(i\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{x}'))\langle p|J_\rho{}^+(\boldsymbol{x}',0)|p'\rangle$$
$$\delta(E_{p'}-E_p-E_q)\langle p'|J_\lambda(\boldsymbol{x},0)|p\rangle d\boldsymbol{x}d\boldsymbol{x}' ,$$

where V represents the normalization volume, and $\overline{\sum}$ the sum over all final states and the average over the initial spin states of nucleus. In these relations p and p' are the total momenta for initial and final nuclear states, respectively. If we sum over all the final hadronic states, we obtain

$$W_{\lambda\rho} = (2\pi/V)\iint \exp(i\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{x}'))\langle p|J_\rho{}^+(\boldsymbol{x}',0)\delta(H-E_p-E_q)J_\lambda(\boldsymbol{x},0)|p\rangle$$
$$\times d\boldsymbol{x}d\boldsymbol{x}' .$$

*) The possibility of interpreting these renormalization constants as a polarization effect related to the highly excited states (nucleon resonances, etc.), neglected in our truncation, has been pointed out by Prof. M. Ericson and Dr. T. E. O. Ericson. One of us (F. C.) is very grateful to them for interesting discussions on this point.

If we particularize J_λ to be V_λ or A_λ, we can define $W_{\lambda\rho}^{(VV)}$ and $W_{\lambda\rho}^{(AA)}$; it is easy to verify that

$$W_{kl}^{(AA)}=\delta_{kl}W_{44}^{(VV)} \tag{10}$$

(for $k, l=1, 2, 3$), by making use of the commutator (3), if $|p\rangle$ is a scalar supermultiplet. One conjecture which we can make is that the exact SU_4 relation is fairly good for sum rules, even if not too good for partial width or individual nuclear matrix elements.*)

In the next section these general arguments are particularized to muon capture.

§ 3. Muon capture in scalar supermultiplets

If $|0\rangle$ is a scalar supermultiplet and the Hamiltonian H is SU_4 invariant, then, using the commutator (3), we can prove that

$$\sum_f\left|\langle f|\int A_k^{\pm}(y, 0)\exp(iy\cdot\nu)dy|0\rangle\right|^2$$
$$=\sum_f\left|\langle f|\int V_4^3(y, 0)\exp(iy\cdot\nu)dy|0\rangle\right|^2. \tag{11}$$

From (11) we have further

$$M_V^2=M_A^2, \tag{12}$$

where

$$M_V^2=\sum_f(\nu^2/m_\mu^2)\int d\hat{\nu}(4\pi)^{-1}\left|\langle f|\int V_4^{\pm}(y, 0)\exp(iy\cdot\nu)dy|0\rangle\right|^2 \tag{13}$$

and

$$M_A^2=\sum_f(\nu^2/m_\mu^2)\int d\hat{\nu}(4\pi)^{-1}\left|\langle f|\int A_k^{\pm}(y, 0)\exp(iy\cdot\nu)dy|0\rangle\right|^2, \tag{14}$$

and $\nu=m_\mu-\Delta_\mu-(E_f-E_0)$, where E_f and E_0 represent the energies of the final and initial states, respectively, and Δ_μ takes into account the binding energy of the muon, the difference in mass between neutron and proton and the Coulomb displacement between analogue states. A proof of (12) is obtained by writing

$$M_V^2=\sum_f\int d\hat{\nu}(4\pi)^{-1}\Big|(m_\mu-\Delta_\mu)\langle f|\int V_4^{\pm}(y, 0)\exp(iy\cdot\nu)dy|0\rangle$$
$$-\langle f|\left[H, \int V_4^{\pm}(y, 0)\exp(iy\cdot\nu)dy\right]|0\rangle\Big|^2 \tag{15}$$

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*) This conjecture can be stated more formally; the δ function in Eq. (10) can be expanded in the following way,[19] H_0 being SU_4 invariant:

$$\delta(H-E)=\delta(H_0+H_1-E)=\delta(H_0+\Delta E-E)+(H_1-\Delta E)\delta'(H_0+\Delta E-E)+\cdots\cdots$$

and the second term on the right-hand side is expected to vanish in the RPA.

(and in a similar way M_A^2) and repeating the procedure which leads to (11).

The difference between the conventional theory for muon capture (given, e.g., in Ref. 6)) and the present one lies in the fact that in the conventional theory we have three terms M_V^2, M_A^2 and M_P^2 but in the present theory we have M_V^2 and M_A^2 only. In the Wigner SU_4 limit we have $M_V^2=M_A^2=M_P^2$ and in the present one we have $M_V^2=M_A^2$ only.

Furthermore in the SU_4 limit of the usual theory the muon capture rate λ is given by (from now on we write M_V^2 in the conventional theory[6] as $(M_V^0)^2$)

$$\lambda=(m_\mu^2/2\pi)|\phi|^2_{\text{muon}}\{G_V^2+3G_A^2+G_P^2-2G_PG_A\}(M_V^0)^2, \qquad (16)$$

where ϕ is the muon wave function before the capture,

$$G_V\sim 1.01G, \quad G_A\sim -1.55G \quad \text{and} \quad G_P\sim 0.58G$$

and G is the Fermi constant including the Cabibbo $\cos\theta$ factor.

In the present SU_4 approximation we have

$$\lambda=(m_\mu^2/2\pi)|\phi|^2_{\text{muon}}G^2 4M_V^2, \qquad (17)$$

in which M_V^2 is defined by (13).

Equation (17) seems rather unfamiliar. Let us explain how it can be related to (16). First of all we have to deduce the renormalization constant relating $(Y^1)_k^\alpha$ to $(Y^0)_k^\alpha$. It is seen by taking the matrix element of the commutator (2) between nucleon states that the SU_4 relation is broken in the single nucleon system, which therefore does not belong to a pure representation of this group (this result was emphasized by Redicati.[5]) Therefore we are led to

$$(Y^1)_k^\alpha=g_A(Y^0)_k^\alpha. \qquad (g_A=1.23)$$

Taking this into account, Eq. (17) is transformed into

$$\lambda=(m_\mu^2/2\pi)|\phi|^2_{\text{muon}}(1+3g_A^2)M_V^2. \qquad (18)$$

Numerically Eq. (16) is equivalent[20] to

$$\lambda=(m_\mu^2/2\pi)|\phi|^2_{\text{muon}}(1+3g_A^2)1.08(M_V^0)^2.$$

In impulse approximation we get $M_V^2=1.2(M_V^0)^2$, so that the difference between Eqs. (16) and (18) is not large, even if these relations are derived under different hypotheses.

We turn now to the evaluation of M_V^2. It should be remarked that the quantity $|\langle f|\int V_4^3(\boldsymbol{y},0)\exp(i\boldsymbol{y}\cdot\boldsymbol{\nu})d\boldsymbol{y}|0\rangle|^2$ in Eq. (11), which can be easily related to M_V^2 according to CVC, is the quantity to be directly determined by the inelastic electron scattering data, in principle. However, for practical purposes it is better to connect M_V^2 with the photoabsorption cross section.

First we suppose that ν is small; then we have $\exp(i\boldsymbol{y}\cdot\boldsymbol{\nu})\simeq 1+i\boldsymbol{y}\cdot\boldsymbol{\nu}$. The continuity equation for the vector current leads to

$$(E_f - E_0)\langle f|i\boldsymbol{\nu}\cdot\int \boldsymbol{y}V_4^3(\boldsymbol{y},0)d\boldsymbol{y}|0\rangle = \boldsymbol{\nu}\cdot\langle f|\int \boldsymbol{V}^3(\boldsymbol{x},0)d\boldsymbol{x}|0\rangle\,,$$

so that the unretarded $E1$ matrix element is related to M_V^2. For a finite ν, instead of the Taylor expansion, we introduce a series[11)]

$$\begin{aligned}\exp(i\boldsymbol{y}\cdot\boldsymbol{\nu}) &= j_0(\nu y) + \{3j_1(\nu y)/\nu y\}\, i\boldsymbol{y}\cdot\boldsymbol{\nu} + \cdots \\ &= j_0(\nu y) + i\boldsymbol{y}\cdot\boldsymbol{\nu}\{j_0(\nu y) + j_2(\nu y)\} + \cdots\,.\end{aligned}$$

Here we assume that only the terms containing $j_0(\nu y)$ are to be retained. (This is justified if the single-particle model is adopted.[11)]) Then we obtain

$$\begin{aligned}&\left|\langle f|\int V_4^3(\boldsymbol{y},0)\exp(i\boldsymbol{y}\cdot\boldsymbol{\nu})d\boldsymbol{y}|0\rangle\right|^2 \\ &\quad\cong\left|\langle f|\int V_4^3(\boldsymbol{y},0)\, i\boldsymbol{y}\cdot\boldsymbol{\nu} j_0(\nu y)d\boldsymbol{y}|0\rangle\right|^2 \\ &\quad\equiv\left|j_0(\nu\bar{R})\langle f|\int V_4^3(\boldsymbol{y},0)\boldsymbol{\nu}\cdot\boldsymbol{y}d\boldsymbol{y}|0\rangle\right|^2,\end{aligned}$$

where $\bar{R}$ is an average value and ν-dependent, defined by the theorem of mean value. For comparison we introduce the elastic form factor

$$\begin{aligned}F_{\text{el}} &= \langle 0|\int V_4^3(\boldsymbol{y},0)\exp(i\boldsymbol{y}\cdot\boldsymbol{\nu})d\boldsymbol{y}|0\rangle/(Ze) \\ &\cong\langle 0|\int V_4^3(\boldsymbol{y},0)j_0(\nu y)d\boldsymbol{y}|0\rangle/(Ze) \\ &\equiv j_0(\nu\bar{R}')\,.\end{aligned}$$

Putting these relations together, we arrive at the formula

$$M_V^2 = (m_\mu^2/2\pi^2\alpha)(E_m/m_\mu)^4\int_0^{E_m}\left(\frac{E_m - E}{E_m}\right)^4\frac{\sigma_\gamma(E)}{E}|F_{\text{el}}|^2 f dE\,, \tag{19}$$

where $f = j_0(\nu\bar{R})/j_0(\nu\bar{R}')$, (if only the unretarded $E1$ part is dominant, $|F_{\text{el}}|^2 f$ in Eq. (19) is replaced by 1) and $E_m = m_\mu - \Delta_\mu + E_0$ is of the order of 100 MeV. The formula (19) looks quite similar to the one derived by Foldy and Walecka[7)] if $f\approx 1$. Of course, whether $f\approx 1$ or not must be carefully examined in individual cases.

What we would like to stress in this section is that, owing to Eq. (11),

$$\sum_f\left|\langle f|\int A_k^{\pm}(\boldsymbol{y},0)\exp(i\boldsymbol{y}\cdot\boldsymbol{\nu})d\boldsymbol{y}|0\rangle\right|^2$$

can be expressed in terms of the directly measurable quantity $|\langle f|\int V_4^3(\boldsymbol{y},0)\times\exp(i\boldsymbol{y}\cdot\boldsymbol{\nu})d\boldsymbol{y}|0\rangle|^2$, even when the contribution of exchange currents is large.

§ 4. Discussion of SU_4 breaking effects

Most of the previous arguments, which were based on exact SU_4, are valid also when, instead of

$$[H,\ Y_k{}^\alpha]=0\,,$$

we only require

$$\{[H,\,Y_k{}^\alpha]-Y_k{}^\alpha\varDelta^\alpha\}\,|0\rangle\cong 0\,, \tag{20}$$

where $\varDelta^\alpha$ is a constant defined by

$$\varDelta^\alpha\cong\frac{\langle 0|\,(Y_k{}^\alpha)^+[H,\,Y_k{}^\alpha]\,|0\rangle}{\langle 0|\,(Y_k{}^\alpha)^+Y_k{}^\alpha|0\rangle}\,. \tag{21}$$

Even if $[H,Y_k{}^\alpha]$ is not small, in so far as Eq. (20) holds, the SU_4 relations are valid with only minor modifications. The discovery of Isobaric Analogue States (IAS) for heavy nuclei in the early sixties showed us[21] that

$$[H,\,T^\pm]\neq\text{small}$$

but

$$[H,\,T^\pm]-T^\pm\varDelta_C=\text{small}\,,$$

$\varDelta_C$ being a constant called the single particle Coulomb displacement. In other words, the diagonal Coulomb term ($\varDelta T=0$) is very important, but its non-diagonal terms ($\varDelta T\neq 0$) are less important. If a similar situation is valid in our case, i.e., if Eqs. (20) and (21) hold, we still obtain*) Eq. (11) and a relation similar to Eq. (12) (see Eqs. (22), (23) and (24)) without assuming any detailed knowledge of nuclear structure at least as far as the first-order effects in the breaking of SU_4 are concerned.

In order to examine this point in more detail, let us see again the relation (11):

$$\begin{aligned}
&\sum_f\left|\langle f|\int A_k{}^\pm(y,0)\exp(iy\cdot\nu)\,dy|0\rangle\right|^2\\
&=\sum_f\left|\langle f|\int V_4{}^3(y,0)\exp(iy\cdot\nu)\,dy|0\rangle\right|^2\\
&\quad+\langle 0|\,Y_k{}^\pm\int A_k{}^\mp(y,0)\exp(-iy\cdot\nu)\,dy\int V_4{}^3(y,0)\exp(iy\cdot\nu)\,dy|0\rangle\\
&\quad-\langle 0|\int V_4{}^3(y,0)\exp(-iy\cdot\nu)\,dy\,Y_k{}^\mp\int V_4{}^3(y,0)\exp(iy\cdot\nu)\,dy\,Y_k{}^\pm|0\rangle\\
&\quad+\langle 0|\,Y_k{}^\mp\int V_4{}^3(y,0)\exp(-iy\cdot\nu)\,dy\int V_4{}^3(y,0)\exp(iy\cdot\nu)\,dy\,Y_k{}^\pm|0\rangle\,.
\end{aligned} \tag{22}$$

For the correction terms use has been made of a closure approximation. Truely this approximation is dangerous in our case since we do not use the single-

*) Equation (11) was first found within the framework of the harmonic oscillator shell model.[6]

nucleon effective operator for the weak and electromagnetic currents, but we deal with the true physical hadronic currents which include, for example, electromagnetic and weak production of pions. Thus, in performing the closure approximation we are really taking into account excitation processes to states which have nothing to do with the states excited by muon capture.

Let us assume that the closure approximation*) is nevertheless good and draw some conclusions about the correction terms under this hypothesis. There are two kinds of terms (i) and (ii) in Eq. (22):

$$\text{(i)}\qquad \begin{cases} \langle 0|\int V_4^3(y,0)\exp(-iy\cdot\nu)\,dy\,Y_k^{\mp}\int V_4^3(y,0)\exp(iy\cdot\nu)\,Y_k^{\pm}|0\rangle\,, \\ \langle 0|\,Y_k^{\mp}\int V_4^3(y,0)\exp(-iy\cdot\nu)\,dy\int V_4^3(y,0)\exp(iy\cdot\nu)\,Y_k^{\pm}|0\rangle \end{cases}$$

and

$$\text{(ii)}\qquad \langle 0|\,Y_k^{\pm}\int A_k^{\mp}(y,0)\exp(-iy\cdot\nu)\,dy\int V_4^3(y,0)\exp(iy\cdot\nu)\,dy|0\rangle\,.$$

If the first-order perturbation theory is valid, then

$$|0\rangle=|0\rangle\!\rangle+(1-|0\rangle\!\rangle\langle\!\langle 0|)\frac{1}{E-H_0}(1-|0\rangle\!\rangle\langle\!\langle 0|)H_1|0\rangle\!\rangle\,,$$

for which

$$Y_k^{\pm}|0\rangle\!\rangle=0\,,$$

$$[H_0,\,Y_k^{\pm}]=0$$

and

$$[H_1,\,Y_k^{\pm}]\neq 0\,.$$

Then we can rewrite

$$\int V_4^3(y,0)\exp(iy\cdot\nu)\,dy\,Y_k^{\pm}|0\rangle$$

as

$$\int V_4^3(y,0)\exp(iy\cdot\nu)\,dy\,(1-|0\rangle\!\rangle\langle\!\langle 0|)\frac{1}{E_0-H_0}(1-|0\rangle\!\rangle\langle\!\langle 0|)\,[Y_k^{\pm},H_1]\,|0\rangle\!\rangle\,.$$

In a similar way one may treat

$$\langle 0|\,Y_k^{\pm}\int A_k^{\mp}(y,0)\exp(-iy\cdot\nu)\,dy\,.$$

If

$$[H_1,\,Y_k^{\pm}]=Y_k^{\pm}\Delta^{\pm}\,,$$

*) The validity of this hypothesis has been discussed briefly in Ref. 20), but in essence the problem is still open (C. W. Kim, private communication).

(i) and (ii) clearly vanish in this approximation.*)

Finally let us take into account the other term in Eq. (15). From the so-called Ahrens-Feenberg approximation,[22]

$$\left[H, \int V_4^{\pm}(y, 0) \exp(iy\cdot\langle\nu\rangle) dy\right] \cong \Delta E_F \int V_4^{\pm}(y, 0) \exp(iy\cdot\langle\nu\rangle) dy$$

and

$$\left[H, \int A_k^{\pm}(y, 0) \exp(iy\cdot\langle\nu\rangle) dy\right] \cong \Delta E_{GT} \int A_k^{\pm}(y, 0) \exp(iy\cdot\langle\nu\rangle) dy ,$$

provided that $\langle\nu\rangle$ is a suitable average of ν, we easily get a relationship between M_V^2 and M_A^2,

$$M_V^2/M_A^2 = (m_\mu - \Delta_\mu - \Delta E_F)^2/(m_\mu - \Delta_\mu - \Delta E_{GT})^2 . \tag{23}$$

In this framework the SU_4 symmetry implies $\Delta E_F = \Delta E_{GT}$. If the dipole mode is assumed to be dominant, we have in the Ahrens-Feenberg approximation

$$\left[H, \int V_4^{\pm}(y, 0) iy\cdot\nu dy\right] \cong \Delta E_F \int V_4^{\pm}(y, 0) iy\cdot\nu dy .$$

So instead of Eq. (23) we obtain a more realistic formula

$$M_V^2/M_A^2 = (m_\mu - \Delta_\mu - \Delta E_F)^4/(m_\mu - \Delta_\mu - \Delta E_{GT})^4 . \tag{24}$$

A formula similar to Eq. (24) has already been obtained and discussed[25] in detail in the framework of conventional nuclear theory; the present argument is based on a different standpoint.

§ 5. Conclusion

The SU_4 symmetry for actual nuclei is broken because of spin-dependent forces, but it is still a useful concept for understanding systematic properties of nuclei, such as sum rules.

In this paper we have tried to develop a nuclear theory on the basis of current algebra and SU_4 invariance along the line proposed by Radicati.[5] Special emphasis is put on possible roles of the exchange current. In §§ 2 and 3, we have derived the relations (10) and (12), $M_V^2 = M_A^2$, and discussed their implication. The M_V^2 is an experimentally measurable quantity if precise experimental data on electron scattering are available; it can also be estimated by relating it to the photoabsorption cross section under the assumption that the unretarded $E1$ interaction is dominant.

It has been believed that owing to the Siegert theorem[10] the exchange corrections to T^α are quite small, and according to meson-theoretic calculations[12] those to Y_k^α are less than 10%. Therefore, the ground state $|0\rangle$, satisfying

*) In general, to study SU_4 impurities, Loewdin's expansion method[23] can be applied.[24]

$T^\alpha|0\rangle = Y_k^\alpha|0\rangle = S_k|0\rangle = 0$ is not expected to differ very much from the conventional one. However, our knowledge on $\int V_4^\alpha(y)\exp(iy\cdot\nu)dy$ or $\int A_k^\alpha(y)\exp(iy\cdot\nu)dy$ for a finite ν is rather poor. Thus, relations such as Eqs. (10) and (12) seem to be useful if we take the viewpoint that the exchange corrections are significant.

In § 4 we have stated the possibility that the relation $[H, Y_k^\alpha] \approx Y_k^\alpha \Delta^\alpha$ is valid for actual nuclei in analogy to the case of $T^\pm$ which was investigated in detail in connection with IAS. We have derived the relation (24), which has been discussed[25] within the framework of the conventional theory.

In summary the present approach seems to be a good starting point for treating the nucleus as a composite system consisting of nucleons and mesons (or quarks); perhaps this is very close to the original idea of Wigner,[4] which was proposed at a very early stage of nuclear physics.

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