

Semileptonic decays of B_c mesons into charmonium states in a relativistic quark modelMikhail A. Ivanov,¹ Juergen G. Körner,² and Pietro Santorelli³¹*Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research, 141980 Dubna, Russia*²*Institut für Physik, Johannes Gutenberg-Universität,
D-55099 Mainz, Germany*³*Dipartimento di Scienze Fisiche, Università di Napoli "Federico II", Italy
Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Italy*

We use the framework of a relativistic constituent quark model to study the semileptonic transitions of the B_c meson into $(\bar{c}c)$ charmonium states where $(\bar{c}c) = \eta_c(^1S_0)$, $J/\psi(^3S_1)$, $\chi_{c0}(^3P_0)$, $\chi_{c1}(^3P_1)$, $h_c(^1P_1)$, $\chi_{c2}(^3P_2)$, $\psi(^3D_2)$. We compute the q^2 -dependence of all relevant form factors and give predictions for their semileptonic B_c decay modes including also their τ -modes. We derive a formula for the polar angle distribution of the charged lepton in the $(l\nu)$ c.m. frame and compute the partial helicity rates that multiply the angular factors in the decay distribution. For the discovery channel $B_c \rightarrow J/\psi(\rightarrow \mu^+\mu^-)l\nu$ we compute the transverse/longitudinal composition of the J/ψ which can be determined by an angular analysis of the decay $J/\psi \rightarrow \mu^+\mu^-$. We compare our results with the results of other calculations.

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I. INTRODUCTION

In 1998 the CDF Collaboration reported on the observation of the bottom-charm B_c meson at Fermilab [1]. The B_c mesons were found in an analysis of the semileptonic decays $B_c \rightarrow J/\psi l\nu$ with the J/ψ decaying into muon pairs. Values for the mass and the lifetime of the B_c meson were given as $M(B_c) = 6.40 \pm 0.39 \pm 0.13$ GeV and $\tau(B_c) = 0.46^{+0.18}_{-0.16}(\text{stat}) \pm 0.03(\text{syst}) \cdot 10^{-12}$ s, respectively. First B_c mesons are now starting to be seen also in the Run II data from the Tevatron [2, 3]. Much larger samples of B_c mesons and more information on their decay properties are expected from the current Run II at the Tevatron and future experiments at the LHC starting in 2007. In particular this holds true for the dedicated detectors BTeV and LHCb which are especially designed for the analysis of B physics where one expects to see up to 10^{10} B_c events per year.

The study of the B_c meson is of great interest due to some of its outstanding features. It is the lowest bound state of two heavy quarks (charm and bottom) with open (explicit) flavor. As far as the bound state characteristics are concerned the B_c meson is quite similar to the $J^{PC} = 0^{-+}$ states η_c and η_b in the charmonium ($c\bar{c}$ -bound state) and the bottomium ($b\bar{b}$ -bound state) sector. However, the η_c and η_b have hidden (implicit) flavor and decay strongly and electromagnetically whereas the B_c -meson decays weakly since it lies below the $B\bar{D}$ -threshold.

The B_c meson and its decays have been widely studied in the literature. The theoretical status of the B_c -meson was reviewed in [4]. The B_c lifetime and decays were studied in the pioneering paper [5]. The exclusive semileptonic and nonleptonic (assuming factorization) decays of the B_c -meson were calculated in a potential model approach [6]. The binding energy and the wave function of the B_c -meson were computed by using a flavor-independent potential with the parameters fixed by the $c\bar{c}$ and $b\bar{b}$ spectra and decays. The same processes were also studied in the framework of the Bethe-Salpeter equation in [7], and, in the relativistic constituent quark model formulated on the light-front in [8]. Three-point sum rules of QCD and NRQCD were analyzed in [9] and [10] to obtain the form factors of the semileptonic decays of $B_c^+ \rightarrow J/\psi(\eta_c)l^+\nu$ and $B_c^+ \rightarrow B_s(B_s^*)l^+\nu$. As shown by the authors of [11], the form factors parameterizing the B_c semileptonic matrix elements can be related to a smaller set of form factors if one exploits the decoupling of the spin of the heavy quarks in the B_c and in the mesons produced in the semileptonic decays. The reduced form factors can be evaluated as an overlap integral of the meson wave-functions which can be obtained, for example, using a relativistic potential model. This was done in [12], where the B_c semileptonic form factors were computed and predictions for semileptonic and non-leptonic decay modes were given.

In [13] we focused on its exclusive leptonic and semileptonic decays which are sensitive to the description of long distance effects. From the semileptonic decays one can obtain results on the corresponding two-body non-leptonic decay processes in the so-called factorization approximation. The calculations have been done within our relativistic constituent quark model based on an effective Lagrangian describing the coupling of hadrons H to their constituent quarks. The relevant coupling strength is determined by the compositeness condition $Z_H = 0$ [14, 15] where Z_H is the wave function renormalization constant of the hadron H .

The relativistic constituent quark model was also employed in a calculation of the exclusive rare decays $B_c \rightarrow D(D^*)\bar{l}$ [16] and of the nonleptonic decays $B_c \rightarrow D_s\bar{D}^0$ and $B_c \rightarrow D_s D^0$ [18]. In the latter case we confirmed that

the nonleptonic decays $B_c \rightarrow D_s \overline{D^0}$ and $B_c \rightarrow D_s D^0$ are well suited to extract the CKM angle γ through amplitude relations, as was originally proposed in [19, 20]. The reason is that the branching fractions into the two channels are of the same order of magnitude.

In this paper we continue the study of B_c decay properties and calculate the branching rates of the semileptonic decays $B_c \rightarrow (\bar{c}c) l\nu$ with $(\bar{c}c) = \eta_c (^1S_0)$, $J/\psi (^3S_1)$, $\chi_{c0} (^3P_0)$, $\chi_{c1} (^3P_1)$, $h_c (^1P_1)$, $\chi_{c2} (^3P_2)$, $\psi (^3D_2)$. We compare our results with the results of [6, 21] where it was shown that these decay rates are quite sizable and may be accessible in RUN II of Tevatron and/or the LHC. Two-particle decays of the B_c -meson into charmonium states have been studied before in [22] by using the factorization of hard and soft contributions. The weak decays of the B_c -meson to charmonium have been studied in the framework of the relativistic quark model based on the quasipotential approach in [23]. In this paper we compute all form factors of the above semileptonic B_c -transitions and give predictions for various semileptonic B_c decay modes including their τ -modes. From a general point of view we would like to remark that the semileptonic decays of the τ -lepton have been studied within perturbative QCD. It has allowed one to determine the strong coupling constant with a high accuracy (see e.g. [24]). We have improved on our previous calculation [13] in that we no longer employ the so-called impulse approximation. In the impulse approximation one assumes that the vertex functions depend only on the loop momentum flowing through the vertex. Dropping the impulse approximation means that the vertex function can also depend on outer momenta according to the flow of momentum through the vertex. A comparison with the results for the decays into the para- and ortho-charmonium states $(\bar{c}c) = \eta_c (^1S_0)$, $J/\psi (^3S_1)$ [13], which was done in the impulse approximation, shows a $\approx 10\%$ downward effect in the rates when the impulse approximation is dropped.

II. BOUND STATE REPRESENTATION OF THE CHARMONIUM STATES

The charmonium states treated in this paper are listed in Table I. We have also included the purported D -wave state $\psi(3836)$ whose quantum numbers have not been established yet. Table I also contains the quark currents used to describe the coupling of the respective charmonium states to the charm quarks. The masses of the charmonium states listed in Table I are taken from the PDG [25].

TABLE I: The charmonium states $^{2S+1}L_J$. We use the notation $\vec{\partial} = \vec{\partial} - \overleftarrow{\partial}$.

quantum number	name	quark current	mass (GeV)
$J^{PC} = 0^{-+}$ (S=0, L=0)	$^1S_0 = \eta_c$	$\bar{q} i \gamma^5 q$	2.980
$J^{PC} = 1^{--}$ (S=1, L=0)	$^3S_1 = J/\psi$	$\bar{q} \gamma^\mu q$	3.097
$J^{PC} = 0^{++}$ (S=1, L=1)	$^3P_0 = \chi_{c0}$	$\bar{q} q$	3.415
$J^{PC} = 1^{++}$ (S=1, L=1)	$^3P_1 = \chi_{c1}$	$\bar{q} \gamma^\mu \gamma^5 q$	3.511
$J^{PC} = 1^{+-}$ (S=0, L=1)	$^1P_1 = h_c(1P)$	$\bar{q} \overleftrightarrow{\partial}^\mu \gamma^5 q$	3.526
$J^{PC} = 2^{++}$ (S=1, L=1)	$^3P_2 = \chi_{c2}$	$(i/2) \bar{q} \left(\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu \right) q$	3.557
$J^{PC} = 2^{--}$ (S=1, L=2)	$^3D_2 = \psi(3836)$	$(i/2) \bar{q} \left(\gamma^\mu \gamma^5 \overleftrightarrow{\partial}^\nu + \gamma^\nu \gamma^5 \overleftrightarrow{\partial}^\mu \right) q$	3.836

Next we write down the Lagrangian describing the interaction of the charmonium fields with the quark currents. We give also the definition of the one-loop self-energy or mass insertions (called mass functions in the following) $\tilde{\Pi}(p^2)$ of the relevant charmonium fields.

We can be quite brief in the presentation of the technical details of our calculation since it is patterned after the calculation presented in [13] which contains more calculational details. We treat the different spin cases ($S = 0, 1, 2$) in turn.

Spin S=0:

$$\mathcal{L}_{S=0}(x) = \frac{1}{2} \phi(x) (\square - m^2) \phi(x) + g \phi(x) J_q(x), \quad \square = -\partial^\alpha \partial_\alpha. \quad (1)$$

$$\Pi(x-y) = i g^2 \langle T \{ J_q(x) J_q(y) \} \rangle_0,$$

$$\begin{aligned}
\tilde{\Pi}(p^2) &= \int d^4x e^{-ipx} \Pi(x) \equiv \frac{3g^2}{4\pi^2} \tilde{\Pi}_0(p^2), \\
Z &= 1 - \tilde{\Pi}'(m^2) = 1 - \frac{3g^2}{4\pi^2} \tilde{\Pi}'_0(m^2) = 0, \\
J_q(x) &= \iint dx_1 dx_2 F_{cc}(x, x_1, x_2) \bar{q}(x_1) \Gamma q(x_2) \quad (\Gamma = i\gamma^5, I), \\
F_{cc}(x, x_1, x_2) &= \delta\left(x - \frac{x_1 + x_2}{2}\right) \Phi_{cc}((x_1 - x_2)^2), \\
\Phi_{cc}(x^2) &= \int \frac{d^4q}{(2\pi)^4} e^{-iqx} \tilde{\Phi}_{cc}(-q^2).
\end{aligned}$$

$\tilde{\Pi}'(m^2)$ is the derivative of the mass function $\tilde{\Pi}(p^2)$.

Spin S=1:

$$\begin{aligned}
\mathcal{L}_{S=1}(x) &= -\frac{1}{2} \phi_\mu(x) (\square - m^2) \phi^\mu(x) + g \phi_\mu(x) J_q^\mu(x), \\
\partial_\mu \phi^\mu(x) &= 0 \quad (\text{leaving three independent components}),
\end{aligned} \tag{2}$$

$$\begin{aligned}
\Pi^{\mu\nu}(x-y) &= -i g^2 \langle T \{ J_q^\mu(x) J_q^\nu(y) \} \rangle_0, \\
\tilde{\Pi}^{\mu\nu}(p) &= \int d^4x e^{-ipx} \Pi^{\mu\nu}(x) = g^{\mu\nu} \tilde{\Pi}^{(1)}(p^2) + p^\mu p^\nu \tilde{\Pi}^{(2)}(p^2), \\
\tilde{\Pi}^{(1)}(p^2) &\equiv \frac{3g^2}{4\pi^2} \tilde{\Pi}_1(p^2), \quad Z = 1 - \frac{3g^2}{4\pi^2} \tilde{\Pi}'_1(m^2) = 0, \\
J_q^\mu(x) &= \iint dx_1 dx_2 F_{cc}(x, x_1, x_2) \bar{q}(x_1) \Gamma^\mu q(x_2), \\
\Gamma^\mu &= \gamma^\mu, \gamma^\mu \gamma^5, \overleftrightarrow{\partial}^\mu \gamma^5.
\end{aligned}$$

The spin 1 polarization vector $\epsilon_\mu^{(\lambda)}(p)$ satisfies the constraints:

$$\begin{aligned}
\epsilon_\mu^{(\lambda)}(p) p^\mu &= 0 && \text{transversality,} \\
\sum_{\lambda=0,\pm} \epsilon_\mu^{(\lambda)}(p) \epsilon_\nu^{\dagger(\lambda)}(p) &= -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2} && \text{completeness,} \\
\epsilon_\mu^{\dagger(\lambda)} \epsilon^{(\lambda')\mu} &= -\delta_{\lambda\lambda'} && \text{orthonormality.}
\end{aligned}$$

Spin S=2:

$$\begin{aligned}
\mathcal{L}_{S=2}(x) &= \frac{1}{2} \phi_{\mu\nu}(x) (\square - m^2) \phi^{\mu\nu}(x) + g \phi_{\mu\nu}(x) J_q^{\mu\nu}(x). \\
\phi^{\mu\nu}(x) &= \phi^{\nu\mu}(x), \quad \partial_\mu \phi^{\mu\nu}(x) = 0, \quad \phi_\mu^\mu(x) = 0, \quad (\text{leaving 5 independent components}),
\end{aligned} \tag{3}$$

$$\Pi^{\mu\nu,\alpha\beta}(x-y) = i g^2 \langle T \{ J_q^{\mu\nu}(x) J_q^{\alpha\beta}(y) \} \rangle_0,$$

$$\tilde{\Pi}^{\mu\nu,\alpha\beta}(p) = \int d^4x e^{-ipx} \Pi^{\mu\nu,\alpha\beta}(x) = \frac{1}{2} (g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha}) \tilde{\Pi}^{(1)}(p^2)$$

$$\begin{aligned}
& + g^{\mu\nu} g^{\alpha\beta} \tilde{\Pi}^{(2)}(p^2) + (g^{\mu\nu} p^\alpha p^\beta + g^{\mu\alpha} p^\nu p^\beta + g^{\mu\beta} p^\nu p^\alpha) \tilde{\Pi}^{(3)}(p^2) + p^\mu p^\nu p^\alpha p^\beta \tilde{\Pi}^{(4)}(p^2), \\
\tilde{\Pi}^{(1)}(p^2) & \equiv \frac{3g^2}{4\pi^2} \tilde{\Pi}_2(p^2), \quad Z = 1 - \frac{3g^2}{4\pi^2} \tilde{\Pi}'_2(m^2) = 0, \\
J_q^{\mu\nu}(x) & = \iint dx_1 dx_2 F_{cc}(x, x_1, x_2) \bar{q}(x_1) \Gamma^{\mu\nu} q(x_2), \\
\Gamma^{\mu\nu} & = \frac{i}{2} \left(\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu \right), \quad \frac{i}{2} \left(\gamma^\mu \gamma^5 \overleftrightarrow{\partial}^\nu + \gamma^\nu \gamma^5 \overleftrightarrow{\partial}^\mu \right).
\end{aligned}$$

The spin 2 polarization vector $\epsilon_{\mu\nu}^{(\lambda)}(p)$ satisfies the constraints:

$$\begin{aligned}
\epsilon_{\mu\nu}^{(\lambda)}(p) & = \epsilon_{\nu\mu}^{(\lambda)}(p) && \text{symmetry,} \\
\epsilon_{\mu\nu}^{(\lambda)}(p) p^\mu & = 0 && \text{transversality,} \\
\epsilon_{\mu\mu}^{(\lambda)}(p) & = 0 && \text{tracelessness,} \\
\sum_{\lambda=0,\pm 1,\pm 2} \epsilon_{\mu\nu}^{(\lambda)} \epsilon_{\alpha\beta}^{\dagger(\lambda)} & = \frac{1}{2} (S_{\mu\alpha} S_{\nu\beta} + S_{\mu\beta} S_{\nu\alpha}) - \frac{1}{3} S_{\mu\nu} S_{\alpha\beta} && \text{completeness,} \\
\epsilon_{\mu\nu}^{\dagger(\lambda)} \epsilon^{(\lambda')\mu\nu} & = \delta_{\lambda\lambda'} && \text{orthonormality,}
\end{aligned}$$

where

$$S_{\mu\nu} = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}.$$

We use the the local representation for the quark propagator when calculating the Fourier-transforms of the mass functions. The local quark propagator is given by

$$S_q(x-y) = \langle T\{q(x) \bar{q}(y)\} \rangle_0 = \int \frac{d^4 k}{(2\pi)^4 i} e^{-ik \cdot (x-y)} \tilde{S}_q(k), \quad \tilde{S}_q(k) = \frac{1}{m_q - \not{k}}. \quad (4)$$

For the mass functions one needs to calculate the integrals

$$\begin{aligned}
\tilde{\Pi}_0(p^2) & = - \int \frac{d^4 k}{4\pi^2 i} \tilde{\Phi}_{cc}^2(-k^2) \text{Tr} \left[\Gamma \tilde{S}(k-p/2) \Gamma \tilde{S}(k+p/2) \right], \\
\Gamma(P, S) & = i \gamma^5, I. \\
\tilde{\Pi}_1^{\mu\nu}(p) & = \int \frac{d^4 k}{4\pi^2 i} \tilde{\Phi}_{cc}^2(-k^2) \text{Tr} \left[\Gamma^\mu \tilde{S}(k-p/2) \Gamma^\nu \tilde{S}(k+p/2) \right], \\
\Gamma^\mu(V, A, PV) & = \gamma^\mu, \gamma^\mu \gamma^5, 2i k^\mu \gamma^5. \\
\tilde{\Pi}_2^{\mu\nu, \alpha\beta}(p) & = \int \frac{d^4 k}{4\pi^2 i} \tilde{\Phi}_{cc}^2(-k^2) \text{Tr} \left[\Gamma^{\mu\nu} \tilde{S}(k-p/2) \Gamma^{\alpha\beta} \tilde{S}(k+p/2) \right], \\
\Gamma^{\mu\nu}(T, PT) & = i (\gamma^\mu k^\nu + \gamma^\nu k^\mu), i (\gamma^\mu \gamma^5 k^\nu + \gamma^\nu \gamma^5 k^\mu).
\end{aligned}$$

The functional form of the vertex function $\tilde{\Phi}_{cc}(-k^2)$ and the quark propagators $\tilde{S}_q(k)$ can in principle be determined from an analysis of the Bethe-Salpeter and Dyson-Schwinger equations as was done e.g. in [26]. In this paper, however, we choose a phenomenological approach where the vertex functions are modelled by a Gaussian form, the size parameters of which are determined by a fit to the leptonic and radiative decays of the lowest lying charm and bottom mesons. For the quark propagators we use the above local representation Eq. (4).

We represent the vertex function by

$$\tilde{\Phi}_{cc}(-k^2) = e^{s_{cc} k^2}, \quad s_{cc} = \frac{1}{\Lambda_{cc}^2},$$

where Λ_{cc} parametrizes the size of the charmonium state. The quark propagator can easily be calculated using Feynman parametrization. One has

$$\tilde{S}_q(k \pm p/2) = (m_q + \not{k} \pm \not{p}/2) \int_0^\infty d\alpha e^{-\alpha(m_q^2 - (k-p/2)^2)}.$$

We then transform to new α -variables according to

$$\alpha_i \rightarrow 2 s_{cc} \alpha_i,$$

and make use of the identity

$$\int_0^\infty d\alpha_1 d\alpha_2 f(\alpha_1, \alpha_2) = \int_0^\infty dt t \int_0^\infty d\alpha_1 d\alpha_2 \delta(1 - \alpha_1 - \alpha_2) f(t\alpha_1, t\alpha_2).$$

One then obtains

$$\tilde{\Pi}(p) = \left\langle \frac{1}{c_t^2} \int \frac{d^4 k}{\pi^2 i} e^{(k+c_p p)^2/c_t} \frac{1}{4} \text{Tr}(\dots) \right\rangle, \quad (5)$$

where

$$c_t = \frac{1}{2(1+t)s_{cc}}, \quad c_p = \frac{t}{1+t} \left(\alpha - \frac{1}{2} \right)$$

The symbol $\langle \dots \rangle$ stands for the two-fold integral

$$\begin{aligned} \langle \dots \rangle &= \int_0^\infty dt \frac{t}{(1+t)^2} \int_0^1 d\alpha e^{-2s_{cc}z} (\dots), \\ z &= t [m_c^2 - \alpha(1-\alpha)p^2] - \frac{t}{1+t} \left(\alpha - \frac{1}{2} \right)^2 p^2. \end{aligned}$$

It is then convenient to shift the loop momentum according to $k \rightarrow k - c_p p$. The ensuing tensor integrals can be expressed by scalar integrals according to

$$\begin{aligned} \int \frac{d^4 k}{\pi^2 i} f(-k^2) k^\mu k^\nu k^\alpha k^\beta &= \frac{1}{24} (g^{\mu\nu} g^{\alpha\beta} + g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha}) \int \frac{d^4 k}{\pi^2 i} f(-k^2) k^4, \\ \int \frac{d^4 k}{\pi^2 i} f(-k^2) k^\mu k^\nu &= \frac{1}{4} g^{\mu\nu} \int \frac{d^4 k}{\pi^2 i} f(-k^2) k^2. \end{aligned}$$

The remaining scalar integrals can be integrated to give

$$\frac{1}{c_t^2} \int \frac{d^4 k}{\pi^2 i} e^{k^2/c_t} k^{2n} = (-)^n (n+1)! c_t^n. \quad (6)$$

For the mass functions one finally obtains

$$\begin{aligned} \bar{q} i \gamma^5 q &: \tilde{\Pi}(p^2)_P = \langle 2 c_t + m_c^2 + (1/4 - c_p^2) p^2 \rangle \\ \bar{q} q &: \tilde{\Pi}(p^2)_S = \langle 2 c_t - m_c^2 + (1/4 - c_p^2) p^2 \rangle \\ \bar{q} \gamma^\mu q &: \tilde{\Pi}(p^2)_V = \langle c_t + m_c^2 + (1/4 - c_p^2) p^2 \rangle \\ \bar{q} \gamma^\mu \gamma^5 q &: \tilde{\Pi}(p^2)_A = \langle c_t - m_c^2 + (1/4 - c_p^2) p^2 \rangle \\ \bar{q} \overleftrightarrow{\partial}^\mu \gamma^5 q &: \tilde{\Pi}(p^2)_{PV} = 2 c_t \langle 3 c_t + m_c^2 + (1/4 - c_p^2) p^2 \rangle \\ \bar{q} \frac{i}{2} \left(\overleftrightarrow{\partial}^\mu \gamma^\nu + \overleftrightarrow{\partial}^\nu \gamma^\mu \right) q &: \tilde{\Pi}(p^2)_T = 2 c_t \langle c_t + m_c^2 + (1/4 - c_p^2) p^2 \rangle \\ \bar{q} \frac{i}{2} \left(\overleftrightarrow{\partial}^\mu \gamma^\nu \gamma^5 + \overleftrightarrow{\partial}^\nu \gamma^\mu \gamma^5 \right) q &: \tilde{\Pi}(p^2)_{PT} = 2 c_t \langle c_t - m_c^2 + (1/4 - c_p^2) p^2 \rangle \end{aligned}$$

The mass functions $\tilde{\Pi}_I(p^2)$ enter the compositeness condition in the derivative form

$$Z_I = 1 - \frac{3 g_I^2}{4 \pi^2} \tilde{\Pi}'_I(p^2),$$

where the prime denotes differentiation with respect to p^2 . The differentiation of the mass functions result in

$$\begin{aligned} \tilde{\Pi}'(p^2)_P &= \langle -2 s_{cc} \hat{z} [2 c_t + m_c^2 + (1/4 - c_p^2) p^2] + 1/4 - c_p^2 \rangle \\ \tilde{\Pi}'(p^2)_S &= \langle -2 s_{cc} \hat{z} [2 c_t - m_c^2 + (1/4 - c_p^2) p^2] + 1/4 - c_p^2 \rangle \\ \tilde{\Pi}'(p^2)_V &= \langle -2 s_{cc} \hat{z} [c_t + m_c^2 + (1/4 - c_p^2) p^2] + 1/4 - c_p^2 \rangle \\ \tilde{\Pi}'(p^2)_A &= \langle -2 s_{cc} \hat{z} [c_t - m_c^2 + (1/4 - c_p^2) p^2] + 1/4 - c_p^2 \rangle \\ \tilde{\Pi}'(p^2)_{PV} &= 2 c_t \langle -2 s_{cc} \hat{z} [3 c_t + m_c^2 + (1/4 - c_p^2) p^2] + 1/4 - c_p^2 \rangle \\ \tilde{\Pi}'(p^2)_T &= 2 c_t \langle -2 s_{cc} \hat{z} [c_t + m_c^2 + (1/4 - c_p^2) p^2] + 1/4 - c_p^2 \rangle \\ \tilde{\Pi}'(p^2)_{PT} &= 2 c_t \langle -2 s_{cc} \hat{z} [c_t - m_c^2 + (1/4 - c_p^2) p^2] + 1/4 - c_p^2 \rangle \end{aligned}$$

where

$$\hat{z} = -t \alpha (1 - \alpha) - \frac{t}{1 + t} \left(\alpha - \frac{1}{2} \right)^2.$$

III. THE SEMILEPTONIC DECAYS $B_c \rightarrow (\bar{C}C) + L + \bar{\nu}$

Let us first write down the interaction Lagrangian which we need for the calculation of the matrix elements of the semileptonic decays $B_c \rightarrow (\bar{c}c) + l + \bar{\nu}$. One has

$$\begin{aligned} \mathcal{L}_{\text{int}}(x) &= g_{bc} B_c^-(x) \cdot J_{bc}^+(x) + g_{cc} \phi_{cc}(x) \cdot J_{cc}(x) + \frac{G_F}{\sqrt{2}} V_{bc} (\bar{c} O^\mu b) \cdot (\bar{l} O_\mu \nu), \\ J_{bc}^+(x) &= \iint dx_1 dx_2 F_{bc}(x, x_1, x_2) \cdot \bar{b}(x_1) i \gamma^5 c(x_2), \\ J_{cc}(x) &= \iint dx_1 dx_2 F_{cc}(x, x_1, x_2) \cdot \bar{c}(x_1) \Gamma_{cc} c(x_2), \\ F_{bc}(x, x_1, x_2) &= \delta(x - c_1 x_1 - c_2 x_2) \Phi_{bc}((x_1 - x_2)^2), \\ F_{cc}(x, x_1, x_2) &= \delta\left(x - \frac{x_1 + x_2}{2}\right) \Phi_{cc}((x_1 - x_2)^2), \\ \Phi(x^2) &= \int \frac{d^4 q}{(2\pi)^4} e^{-iqx} \tilde{\Phi}(-q^2). \end{aligned}$$

Here we adopt the notation: $l = e^-, \mu^-, \tau^-, \bar{l} = e^+, \mu^+, \tau^+, O^\mu = \gamma^\mu (1 - \gamma^5)$, $c_1 = m_b/(m_b + m_c)$, $c_2 = m_c/(m_b + m_c)$. The S-matrix element describing the semileptonic decays $B_c \rightarrow (\bar{c}c) + l\bar{\nu}$ is written as

$$\begin{aligned} S_{B_c \rightarrow (\bar{c}c)} &= i^3 g_{bc} g_{cc} \frac{G_F}{\sqrt{2}} V_{bc} \iiint dx dx_1 dx_2 B_c^-(x) \delta(x - c_1 x_1 - c_2 x_2) \Phi_{bc}((x_1 - x_2)^2) \\ &\times \iiint dy dy_1 dy_2 \phi_{cc}(y) \delta\left(y - \frac{y_1 + y_2}{2}\right) \Phi_{cc}((y_1 - y_2)^2) \int dz (\bar{l} O_\mu \nu)_z \\ &\times \langle T \{ \bar{b}(x_1) i \gamma^5 c(x_2) \cdot \bar{c}(y_1) \Gamma_{cc} c(y_2) \cdot \bar{c}(z) O^\mu c(z) \} \rangle_0. \end{aligned}$$

The matrix element is calculated in the standard manner. We have

$$T_{B_c \rightarrow (\bar{c}c)}(p_1, p_2, k_l, k_\nu) = i (2\pi)^4 \delta(p_1 - p_2 - k_l - k_\nu) M_{B_c \rightarrow (\bar{c}c)}(p_1, p_2, k_l, k_\nu),$$

$$\begin{aligned}
M_{B_c \rightarrow (\bar{c}c)}(p_1, p_2, k_l, k_\nu) &= \frac{G_F}{\sqrt{2}} V_{bc} \mathcal{M}^\mu(p_1, p_2) \bar{u}_l(k_l) O^\mu u_\nu(k_\nu), \\
\mathcal{M}^\mu(p_1, p_2) &= -\frac{3 g_{bc} g_{cc}}{4 \pi^2} \int \frac{d^4 k}{\pi^2 i} \tilde{\Phi}_{bc}(-(k+c_2 p_1)^2) \tilde{\Phi}_{cc}(-(k+p_2/2)^2) \\
&\quad \times \frac{1}{4} \text{Tr} \left[i \gamma^5 \tilde{S}_c(k) \Gamma_{cc} \tilde{S}_c(k+p_2) O^\mu \tilde{S}_b(k+p_1) \right]
\end{aligned}$$

where p_1 and p_2 are the B_c and $(\bar{c}c)$ momenta, respectively. The spin coupling structure of the $(\bar{c}c)$ -states is given by

$$\Gamma_{cc} = i \gamma^5, I, \epsilon_\nu^\dagger \gamma^\nu, \epsilon_\nu^\dagger \gamma^\nu \gamma^5, -2 i \epsilon^{\dagger \nu} k_\nu \gamma^5, 2 \epsilon_{\nu\alpha}^\dagger k^\nu \gamma^\alpha, 2 \epsilon_{\nu\alpha}^\dagger k^\nu \gamma^\alpha \gamma^5.$$

The calculation of the transition matrix elements M^μ proceeds along similar lines as in the case of the mass functions. For the scalar vertex functions one has

$$\begin{aligned}
\tilde{\Phi}_{bc}(-(k+c_2 p_1)^2) &= e^{s_{bc}(k+c_2 p_1)^2}, & s_{bc} &= \frac{1}{\Lambda_{bc}^2}, \\
\tilde{\Phi}_{cc}(-(k+p_2/2)^2) &= e^{s_{cc}(k+p_2/2)^2}, & s_{cc} &= \frac{1}{\Lambda_{cc}^2}, \\
\tilde{S}_q(k+p) &= (m_q + \not{k} + \not{p}) \int_0^\infty d\alpha e^{-\alpha(m_q^2 - (k+p)^2)}.
\end{aligned}$$

Again we shift the parameters $\alpha_i (i = 1, 2, 3)$ according to

$$\alpha_i \rightarrow (s_{bc} + s_{cc}) \alpha_i,$$

$$\int_0^\infty d^3 \alpha f(\alpha_1, \alpha_2, \alpha_3) = \int_0^\infty dt t^2 \int_0^\infty d^3 \alpha \delta \left(1 - \sum_{i=1}^3 \alpha_i \right) f(t\alpha_1, t\alpha_2, t\alpha_3)$$

where $d^3 \alpha = d\alpha_1 d\alpha_2 d\alpha_3$. One then obtains

$$\begin{aligned}
\mathcal{M}^\mu(p_1, p_2) &= \left\langle \frac{1}{c_t^2} \int \frac{d^4 k}{\pi^2 i} e^{(k+c_{p_1} p_1 + c_{p_2} p_2)^2 / c_t} \frac{1}{4} \text{Tr}(\dots) \right\rangle, \\
c_t &= \frac{1}{(s_{bc} + s_{cc})(1+t)}, \\
c_{p_1} &= \frac{c_2 w_{bc} + t \alpha_1}{1+t}, & c_{p_2} &= \frac{w_{cc}/2 + t \alpha_2}{1+t}, \\
w_{bc} &= \frac{s_{bc}}{s_{bc} + s_{cc}}, & w_{cc} &= \frac{s_{cc}}{s_{bc} + s_{cc}}.
\end{aligned}$$

where the symbol $\langle \dots \rangle$ is related to the corresponding symbol $\langle \dots \rangle$ defined in Sec. II Eq. (5). In the present case the symbol $\langle \dots \rangle$ stands for the four-fold integral

$$\langle \dots \rangle = (s_{bc} + s_{cc}) \cdot \int_0^\infty dt \frac{t^2}{(1+t)^2} \int_0^1 d^3 \alpha \delta \left(1 - \sum_{i=1}^3 \alpha_i \right) e^{-(s_{bc} + s_{cc})z} (\dots),$$

where

$$\begin{aligned}
z &= t [(\alpha_2 + \alpha_3) m_c^2 + \alpha_1 m_b^2 - \alpha_1 \alpha_3 p_1^2 - \alpha_2 \alpha_3 p_2^2 - \alpha_1 \alpha_2 q^2] \\
&\quad + \frac{1}{1+t} \left\{ p_1^2 [t w_{bc} c_2 (2\alpha_1 + \alpha_2 - c_2) + t w_{cc} \alpha_1 / 2 - t \alpha_1 (\alpha_1 + \alpha_2) + w_{bc} c_2 (w_{cc}/2 - c_2 + w_{bc} c_2)] \right. \\
&\quad + p_2^2 [t w_{bc} c_2 \alpha_2 + t w_{cc} (\alpha_1/2 + \alpha_2 - 1/4) - t \alpha_2 (\alpha_1 + \alpha_2) + w_{cc}/4 (2 w_{bc} c_2 - 1 + w_{cc})] \\
&\quad \left. + q^2 [t (-w_{bc} c_2 \alpha_2 - w_{cc} \alpha_1 / 2 + \alpha_1 \alpha_2) - w_{bc} w_{cc} c_2 / 2] \right\}.
\end{aligned}$$

One then shifts the loop momentum according to $k \rightarrow k - c_{p_1} p_1 - c_{p_2} p_2$.

Our final results are given in terms of a set of invariant form factors defined by

$$\mathcal{M}^\mu(B_c \rightarrow (\bar{c}c)_{S=0}) = P^\mu F_+(q^2) + q^\mu F_-(q^2), \quad (7)$$

$$\begin{aligned} \mathcal{M}^\mu(B_c \rightarrow (\bar{c}c)_{S=1}) &= \frac{1}{m_{B_c} + m_{cc}} \epsilon_\nu^\dagger \{ -g^{\mu\nu} P q A_0(q^2) + P^\mu P^\nu A_+(q^2) + q^\mu P^\nu A_-(q^2) \\ &\quad + i \varepsilon^{\mu\nu\alpha\beta} P_\alpha q_\beta V(q^2) \}, \end{aligned} \quad (8)$$

$$\begin{aligned} \mathcal{M}^\mu(B_c \rightarrow (\bar{c}c)_{S=2}) &= \epsilon_{\nu\alpha}^\dagger \{ g^{\mu\alpha} P^\nu T_1(q^2) + P^\nu P^\alpha [P^\mu T_2(q^2) + q^\mu T_3(q^2)] \\ &\quad + i \varepsilon^{\mu\nu\delta\beta} P^\alpha P_\delta q_\beta T_4(q^2) \}, \end{aligned} \quad (9)$$

$$P = p_1 + p_2, \quad q = p_1 - p_2.$$

In our results we have dropped an overall phase factors which is irrelevant for the calculation of the decay widths.

The calculation of traces and invariant integrations is done with help of FORM [27]. For the values of the model parameters (hadron sizes Λ_H and constituent quark masses m_q) we use the values of [28]. The numerical evaluation of the form factors is done in FORTRAN.

IV. ANGULAR DECAY DISTRIBUTIONS

Consider the semileptonic decays $B_c^-(p_1) \rightarrow (\bar{c}c)(p_2) + l(k_l) + \bar{\nu}(k_\nu)$ and $B_c^+(p_1) \rightarrow (\bar{c}c)(p_2) + \bar{l}(k_l) + \nu(k_\nu)$. Recalling the expression for the matrix elements, one can write

$$\begin{aligned} M_{B_c^- \rightarrow \bar{c}c}(p_1, p_2, k_l, k_\nu) &= \frac{G_F}{\sqrt{2}} V_{bc} \mathcal{M}_\mu(p_1, p_2) \bar{u}_l^\lambda(\vec{k}_l) O^\mu v_\nu^{\lambda'}(\vec{k}_\nu), \\ M_{B_c^+ \rightarrow \bar{c}c}(p_1, p_2, k_l, k_\nu) &= \frac{G_F}{\sqrt{2}} V_{bc} \mathcal{M}_\mu(p_1, p_2) \bar{u}_\nu^\lambda(\vec{k}_\nu) O^\mu v_l^{\lambda'}(\vec{k}_l), \end{aligned}$$

where p_1 and p_2 are the B_c and $(\bar{c}c)$ momenta, respectively.

The angular decay distribution reads

$$\frac{d\Gamma}{dq^2 d\cos\theta} = \frac{G_F^2}{(2\pi)^3} |V_{bc}|^2 \cdot \frac{(q^2 - \mu^2) |\mathbf{p}_2|}{8 m_1^2 q^2} \cdot L^{\mu\nu} H_{\mu\nu} \quad (10)$$

where μ is the lepton mass and $|\mathbf{p}_2| = \lambda^{1/2}(m_1^2, m_2^2, q^2)/(2m_1)$ is the momentum of the $(\bar{c}c)$ -meson in the B_c -rest frame.

$L^{\mu\nu}$ is the lepton tensor given by

$$L_{\mp}^{\mu\nu} = \frac{1}{8} \text{Tr}(O^\mu \not{k}_\nu O^\nu \not{k}_l) = k_l^\mu k_\nu^\nu + k_l^\nu k_\nu^\mu - g^{\mu\nu} \frac{q^2 - \mu^2}{2} \pm i \varepsilon^{\mu\nu\alpha\beta} k_{l\alpha} k_{\nu\beta}, \quad (11)$$

The lepton tensors $L_-^{\mu\nu}$ and $L_+^{\mu\nu}$ refer to the $(l\bar{\nu})$ and $(\bar{l}\nu)$ cases. They differ in the sign of the parity-odd ε -tensor contribution. The hadron tensor $H_{\mu\nu} = \mathcal{M}_\mu(p_1, p_2) \mathcal{M}_\nu^\dagger(p_1, p_2)$ is given by the corresponding tensor products of the transition matrix elements defined above.

It is convenient to perform the Lorentz contractions in Eq. (10) with the help of helicity amplitudes as described in [29] and [30, 31]. First, we define an orthonormal and complete helicity basis $\epsilon^\mu(m)$ with the three spin 1 components orthogonal to the momentum transfer q^μ , i.e. $\epsilon^\mu(m)q_\mu = 0$ for $m = \pm, 0$, and the spin 0 (time)-component $m = t$ with $\epsilon^\mu(t) = q^\mu/\sqrt{q^2}$.

The orthonormality and completeness properties read

$$\begin{aligned} \epsilon_\mu^\dagger(m) \epsilon^\mu(n) &= g_{mn} \quad (m, n = t, \pm, 0), \\ \epsilon_\mu(m) \epsilon_\nu^\dagger(n) g_{mn} &= g_{\mu\nu} \end{aligned} \quad (12)$$

with $g_{mn} = \text{diag}(+, -, -, -)$. We include the time component polarization vector $\epsilon^\mu(t)$ in the set because we want to include lepton mass effects in the following.

Using the completeness property we rewrite the contraction of the lepton and hadron tensors in Eq. (10) according to

$$\begin{aligned} L^{\mu\nu} H_{\mu\nu} &= L_{\mu'\nu'} g^{\mu'\mu} g^{\nu'\nu} H_{\mu\nu} = L_{\mu'\nu'} \epsilon^{\mu'}(m) \epsilon^{\dagger\mu}(m') g_{mm'} \epsilon^{\dagger\nu'}(n) \epsilon^\nu(n') g_{nn'} H_{\mu\nu} \\ &= L(m, n) g_{mm'} g_{nn'} H(m', n') \end{aligned} \quad (13)$$

where we have introduced the lepton and hadron tensors in the space of the helicity components

$$L(m, n) = \epsilon^\mu(m) \epsilon^{\dagger\nu}(n) L_{\mu\nu}, \quad H(m, n) = \epsilon^{\dagger\mu}(m) \epsilon^\nu(n) H_{\mu\nu}. \quad (14)$$

The point is that the two tensors can be evaluated in two different Lorentz systems. The lepton tensors $L(m, n)$ will be evaluated in the $\bar{l}\nu$ or $l\bar{\nu}$ -c.m. system whereas the hadron tensors $H(m, n)$ will be evaluated in the B_c rest system.

A. Hadron tensor

In the B_c rest frame one has

$$\begin{aligned} p_1^\mu &= (m_1, 0, 0, 0), \\ p_2^\mu &= (E_2, 0, 0, -|\mathbf{p}_2|), \\ q^\mu &= (q_0, 0, 0, |\mathbf{p}_2|), \end{aligned} \quad (15)$$

where

$$\begin{aligned} E_2 &= \frac{m_1^2 + m_2^2 - q^2}{2m_1}, & q_0 &= \frac{m_1^2 - m_2^2 + q^2}{2m_1}, \\ E_2 + q_0 &= m_1, & q_0^2 &= q^2 + |\mathbf{p}_2|^2, & |\mathbf{p}_2|^2 + E_2 q_0 &= \frac{1}{2}(m_1^2 - m_2^2 - q^2). \end{aligned}$$

In the B_c -rest frame the polarization vectors of the effective current read

$$\begin{aligned} \epsilon^\mu(t) &= \frac{1}{\sqrt{q^2}}(q_0, 0, 0, |\mathbf{p}_2|), \\ \epsilon^\mu(\pm) &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0), \\ \epsilon^\mu(0) &= \frac{1}{\sqrt{q^2}}(|\mathbf{p}_2|, 0, 0, q_0). \end{aligned} \quad (16)$$

Using this basis one can express the helicity components of the hadronic tensors through the invariant form factors defined in Eqs. (7-9). We treat the three spin cases in turn.

(a) $B_c \rightarrow (\bar{c}c)_{S=0}$ transition:

$$H(m, n) = (\epsilon^{\dagger\mu}(m) \mathcal{M}_\mu) \cdot (\epsilon^{\dagger\nu}(n) \mathcal{M}_\nu)^\dagger \equiv H_m H_n^\dagger \quad (17)$$

The helicity form factors H_m can be expressed in terms of the invariant form factors. One has

$$\begin{aligned} H_t &= \frac{1}{\sqrt{q^2}}(Pq F_+ + q^2 F_-), \\ H_\pm &= 0, \\ H_0 &= \frac{2m_1 |\mathbf{p}_2|}{\sqrt{q^2}} F_+. \end{aligned} \quad (18)$$

(b) $B_c \rightarrow (\bar{c}c)_{S=1}$ transition:

The nonvanishing helicity form factors are given by

$$H_m = \epsilon^{\dagger\mu}(m) \mathcal{M}_{\mu\alpha} \epsilon_2^{\dagger\alpha}(m) \quad \text{for } m = \pm, 0 \quad (19)$$

and

$$H_t = \epsilon^{\dagger\mu}(t) \mathcal{M}_{\mu\alpha} \epsilon_2^{\dagger\alpha}(0) \quad (20)$$

As in Eq. (17) the hadronic tensor is given by $H(m, n) = H_m H_n^\dagger$.

In order to express the helicity form factors in terms of the invariant form factors Eq. (8) one needs to specify the helicity components $\epsilon_2(m)$ ($m = \pm, 0$) of the polarization vector of the $(\bar{c}c)_{S=1}$ state. They are given by

$$\epsilon_2^\mu(\pm) = \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0), \quad (21)$$

$$\epsilon_2^\mu(0) = \frac{1}{m_2}(|\mathbf{p}_2|, 0, 0, -E_2).$$

They satisfy the orthonormality and completeness conditions:

$$\epsilon_2^{\dagger\mu}(r) \epsilon_{2\mu}(s) = -\delta_{rs}, \quad (22)$$

$$\epsilon_{2\mu}(r) \epsilon_{2\nu}^\dagger(s) \delta_{rs} = -g_{\mu\nu} + \frac{p_{2\mu} p_{2\nu}}{m_2^2}.$$

The desired relations between the helicity form factors and the invariant form factors are then

$$\begin{aligned} H_t &= \epsilon^{\dagger\mu}(t) \epsilon_2^{\dagger\alpha}(0) \mathcal{M}_{\mu\alpha} = \frac{1}{m_1 + m_2} \frac{m_1 |\mathbf{p}_2|}{m_2 \sqrt{q^2}} (P \cdot q (-A_0 + A_+) + q^2 A_-), \\ H_\pm &= \epsilon^{\dagger\mu}(\pm) \epsilon_2^{\dagger\alpha}(\pm) \mathcal{M}_{\mu\alpha} = \frac{1}{m_1 + m_2} (-P \cdot q A_0 \mp 2 m_1 |\mathbf{p}_2| V), \\ H_0 &= \epsilon^{\dagger\mu}(0) \epsilon_2^{\dagger\alpha}(0) \mathcal{M}_{\mu\alpha} \\ &= \frac{1}{m_1 + m_2} \frac{1}{2 m_2 \sqrt{q^2}} (-P \cdot q (m_1^2 - m_2^2 - q^2) A_0 + 4 m_1^2 |\mathbf{p}_2|^2 A_+). \end{aligned} \quad (23)$$

(c) $B_c \rightarrow (\bar{c}c)_{S=2}$ transition:

The nonvanishing helicity form factors can be calculated according to

$$H_m = \epsilon^{\dagger\mu}(m) \mathcal{M}_{\mu\alpha_1\alpha_2} \epsilon_2^{\dagger\alpha_1\alpha_2}(m) \quad \text{for } m = \pm, 0 \quad (24)$$

and

$$H_t = \epsilon^{\dagger\mu}(t) \mathcal{M}_{\mu\alpha_1\alpha_2} \epsilon_2^{\dagger\alpha_1\alpha_2}(0) \quad (25)$$

Again the hadronic tensor is given by $H(m, n) = H_m H_n^\dagger$.

For the further evaluation one needs to specify the helicity components $\epsilon_2(m)$ ($m = \pm 2, \pm 1, 0$) of the polarization vector of the $\bar{c}c_{S=2}$. They are given by

$$\begin{aligned} \epsilon_2^{\mu\nu}(\pm 2) &= \epsilon_2^\mu(\pm) \epsilon_2^\nu(\pm), \\ \epsilon_2^{\mu\nu}(\pm 1) &= \frac{1}{\sqrt{2}} (\epsilon_2^\mu(\pm) \epsilon_2^\nu(0) + \epsilon_2^\mu(0) \epsilon_2^\nu(\pm)), \\ \epsilon_2^{\mu\nu}(0) &= \frac{1}{\sqrt{6}} (\epsilon_2^\mu(+) \epsilon_2^\nu(-) + \epsilon_2^\mu(-) \epsilon_2^\nu(+)) + \sqrt{\frac{2}{3}} \epsilon_2^\mu(0) \epsilon_2^\nu(0), \end{aligned} \quad (26)$$

where $\epsilon_2^\mu(r)$ are defined in Eq. (21).

The relation between the helicity form factors H_m and the invariant form factors Eq. (9) read

$$\begin{aligned} H_t &= h(t, 0) = \sqrt{\frac{2}{3}} \frac{m_1^2 |\mathbf{p}_2|^2}{m_2^2 \sqrt{q^2}} \{T_1 + (|\mathbf{p}_2|^2 + E_2 q_0 + m_1 q_0) T_2 + q^2 T_3\}, \\ H_\pm &= h(\pm, \pm) = \frac{m_1 |\mathbf{p}_2|}{\sqrt{2} m_2} (T_1 \mp 2 m_1 |\mathbf{p}_2| T_4), \\ H_0 &= h(0, 0) = \sqrt{\frac{1}{6}} \frac{m_1 |\mathbf{p}_2|}{m_2^2 \sqrt{q^2}} ((m_1^2 - m_2^2 - q^2) T_1 + 4 m_1^2 |\mathbf{p}_2|^2 T_2). \end{aligned}$$

B. Lepton tensor

The helicity components of the lepton tensors $L(m, n)$ are evaluated in the $(l\bar{\nu})$ -c.m. system $\vec{k}_l + \vec{k}_\nu = 0$. One has

$$\begin{aligned} q^\mu &= (\sqrt{q^2}, 0, 0, 0), \\ k_\nu^\mu &= (|\mathbf{k}_1|, |\mathbf{k}_1| \sin \theta \cos \chi, |\mathbf{k}_1| \sin \theta \sin \chi, |\mathbf{k}_1| \cos \theta), \\ k_l^\mu &= (E_l, -|\mathbf{k}_1| \sin \theta \cos \chi, -|\mathbf{k}_1| \sin \theta \sin \chi, -|\mathbf{k}_1| \cos \theta), \end{aligned} \quad (27)$$

with

$$E_l = \frac{q^2 + \mu^2}{2\sqrt{q^2}}, \quad |\mathbf{k}_1| = \frac{q^2 - \mu^2}{2\sqrt{q^2}}.$$

In the $(l\bar{\nu})$ -c.m. frame the longitudinal and time-component polarization vectors are given by

$$\begin{aligned} \epsilon^\mu(t) &= \frac{q^\mu}{\sqrt{q^2}} = (1, 0, 0, 0), \\ \epsilon^\mu(\pm) &= \frac{1}{\sqrt{2}} (0, \mp, -i, 0), \\ \epsilon^\mu(0) &= (0, 0, 0, 1). \end{aligned} \quad (28)$$

Using Eqs. (28) and (11) it is not difficult to evaluate the helicity representation $L(m, n)$ of the lepton tensor.

In this paper we are not interested in the azimuthal χ -distribution of the lepton pair. We therefore integrate over the azimuthal angle dependence of the lepton tensor. Of course, our formalism is general enough to allow for the inclusion of an azimuthal dependence if needed. After azimuthal integration the differential $(q^2, \cos \theta)$ distribution reads

$$\begin{aligned} \frac{d\Gamma}{dq^2 d\cos\theta} &= \frac{3}{8} (1 + \cos^2 \theta) \cdot \frac{d\Gamma_U}{dq^2} + \frac{3}{4} \sin^2 \theta \cdot \frac{d\Gamma_L}{dq^2} \mp \frac{3}{4} \cos \theta \cdot \frac{d\Gamma_P}{dq^2} \\ &+ \frac{3}{4} \sin^2 \theta \cdot \frac{d\tilde{\Gamma}_U}{dq^2} + \frac{3}{2} \cos^2 \theta \cdot \frac{d\tilde{\Gamma}_L}{dq^2} + \frac{1}{2} \frac{d\tilde{\Gamma}_S}{dq^2} + 3 \cos \theta \cdot \frac{d\tilde{\Gamma}_{SL}}{dq^2}, \end{aligned} \quad (29)$$

where we take the polar angle θ to be the angle between the \vec{p}_2 and the \vec{k}_l in the lepton-neutrino c.m. system. The upper and lower signs in front of the parity violating (p.v.) contribution refer to the two cases $l^-\bar{\nu}$ and $l^+\nu$, respectively.

The differential partial helicity rates $d\Gamma_i/dq^2$ and $d\tilde{\Gamma}_i/dq^2$ in Eq. (29) are defined by

$$\begin{aligned} \frac{d\Gamma_i}{dq^2} &= \frac{G_F^2}{(2\pi)^3} |V_{bc}|^2 \frac{(q^2 - \mu^2)^2 |\mathbf{p}_2|}{12 m_1^2 q^2} H_i, \\ \frac{d\tilde{\Gamma}_i}{dq^2} &= \frac{\mu^2}{2 q^2} \frac{d\Gamma_i}{dq^2}, \end{aligned} \quad (30)$$

where we have introduced a standard set of helicity structure functions H_i ($i = U, L, P, S, SL$) given by the following linear combinations of the helicity components of the hadron tensor $H(m, n) = H_m H_n^\dagger$:

$$\begin{aligned}
H_U &= \text{Re} \left(H_+ H_+^\dagger \right) + \text{Re} \left(H_- H_-^\dagger \right) && \text{Unpolarized – transverse} \\
H_L &= \text{Re} \left(H_0 H_0^\dagger \right) && \text{Longitudinal} \\
H_P &= \text{Re} \left(H_+ H_+^\dagger \right) - \text{Re} \left(H_- H_-^\dagger \right) && \text{Parity – odd} \\
H_S &= 3 \text{Re} \left(H_t H_t^\dagger \right) && \text{Scalar} \\
H_{SL} &= \text{Re} \left(H_t H_0^\dagger \right) && \text{Scalar – Longitudinal interference}
\end{aligned}$$

Note that the helicity amplitudes are real such that the complex conjugation symbol can in fact be dropped.

It is evident that the “tilde” rates $\tilde{\Gamma}$ in Eq. (30) do not contribute in the limit of vanishing lepton masses. In the present application this means that they can be neglected for the e^- and μ^- -modes and only contribute to the τ^- -modes.

Integrating over $\cos \theta$ one obtains the differential q^2 distribution

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_U}{dq^2} + \frac{d\Gamma_L}{dq^2} + \frac{d\tilde{\Gamma}_U}{dq^2} + \frac{d\tilde{\Gamma}_L}{dq^2} + \frac{d\tilde{\Gamma}_S}{dq^2}.$$

Finally, integrating over q^2 , one obtains the total rate

$$\Gamma = \Gamma_U + \Gamma_L + \tilde{\Gamma}_U + \tilde{\Gamma}_L + \tilde{\Gamma}_S$$

In Sec. V we list our predictions for the integrated partial helicity rates Γ_i ($i = U, L, P$) and $\tilde{\Gamma}_i$ ($i = U, L, S, SL$).

To save on notation in the following we shall sometimes use a self-explanatory notation for the differential and integrated partial helicity rates. For example, we write U for either the differential or the integrated helicity rates $d\Gamma_U/dq^2$ and Γ_U , respectively, and \tilde{U} for $d\tilde{\Gamma}_U/dq^2$ and $\tilde{\Gamma}_U$.

An interesting quantity is the forward-backward asymmetry A_{FB} of the lepton in the $(l\bar{\nu})$ cm system which is given by

$$A_{FB} = \frac{3}{4} \frac{\pm P + 4\tilde{S}\tilde{L}}{U + \tilde{U} + L + \tilde{L} + \tilde{S}}. \quad (31)$$

In Sec. V we shall give our numerical predictions for the asymmetry A_{FB} for the decay channels under study.

For the discovery channel $B_c \rightarrow J/\psi l \nu$ with the J/ψ decaying into muon pairs the transverse/longitudinal composition of the produced J/ψ is of interest. The transverse/longitudinal composition can be determined by a measurement of the angular orientation of the back-to-back muon pairs in the J/ψ rest frame. The relevant angular distribution reads

$$\begin{aligned}
\frac{d\Gamma}{dq^2 d\cos \theta^*} &= \frac{3}{8} (1 + \cos^2 \theta^*) \left(\frac{d\Gamma_U}{dq^2} + \frac{d\tilde{\Gamma}_U}{dq^2} \right) \\
&\quad + \frac{3}{4} \sin^2 \theta^* \left(\frac{d\Gamma_L}{dq^2} + \frac{d\tilde{\Gamma}_L}{dq^2} + \frac{d\tilde{\Gamma}_S}{dq^2} \right)
\end{aligned} \quad (32)$$

where θ^* is the polar angle of the muon pair relative to the original momentum direction of the J/ψ . We have included lepton mass effects so that the angular decay distribution in Eq. (32) can be also used for the τ -mode in this decay. The transverse and longitudinal contributions of the J/ψ are given by $(U + \tilde{U})$ and $(L + \tilde{L} + \tilde{S})$, respectively.

One can define an asymmetry parameter α^* by rewriting Eq. (29) in terms of its $\cos^2 \theta^*$ dependence, i.e. $d\Gamma \propto 1 + \alpha^* \cos^2 \theta^*$. The asymmetry parameter can be seen to be given by

$$\alpha^* = \frac{U + \tilde{U} - 2(L + \tilde{L} + \tilde{S})}{U + \tilde{U} + 2(L + \tilde{L} + \tilde{S})}. \quad (33)$$

Our predictions for the asymmetry parameter α^* appear in Sec. V.

We have only written out single angle decay distributions in this paper. It is not difficult to write down joint angular decay distributions including also azimuthal correlations in our formalism if necessary.

V. NUMERICAL RESULTS

Let us discuss the model parameters and their determination. Since we consider the decay of the B_c -meson into charmonium states only, the adjustable parameters are the constituent masses of charm and bottom quarks and the size parameters of the B_c -meson and charmonium states. The values of quark masses were determined in our previous studies (see, for example, [28]) of the leptonic and semileptonic decays of the low-lying pseudoscalar mesons (π , K , D , D_s , B , B_s and B_c). The values of the charm and bottom quarks were found to be $m_c = 1.71$ GeV and $m_b = 5.12$ GeV. The value of $\Lambda_{bc} = 1.96$ GeV was determined from a fit to the world average of the leptonic decay constant $f_{B_c} = 360$ MeV. The value of $\Lambda_{J/\psi} = 2.62$ GeV was found from a fit to the experimental value of the radiative decay constant $f_{J/\psi} = 405$ MeV which enters in the $J/\psi \rightarrow e^+e^-$ decay width ($f_{J/\psi}^{\text{expt}} = 405 \pm 17$ MeV).

In our calculation we are using free quark propagators with an effective constituent quark mass (see, Eq. 4). This imposes a very simple yet important constraint on the relations between the masses of the bound state and their constituents. One has to assume that the meson mass M_H is less than the sum of the masses of their constituents

$$M_H < m_{q_1} + m_{q_2} \quad (34)$$

in order to avoid the appearance of imaginary parts in physical amplitudes, which are described by the one-loop quark diagrams in our approach. This is satisfied for the low-lying pseudoscalar mesons π , K , D , D_s , B , B_s , B_c and η_c and also for the J/ψ but is no longer true for the excited charmonium states considered here. We shall therefore employ identical masses for all excited charmonium states $m_{cc} = m_{J/\psi} = 3.097$ GeV (except for the η_c) in our matrix element calculations but use physical masses in the phase space calculation. This is quite a reliable approximation because the hyperfine splitting between the excited charmonium states and J/ψ is not large. For example, the maximum relative error is $(m_{\psi(3836)} - m_{J/\psi})/m_{J/\psi} = 0.24$.

The size parameters of the excited charmonium states should be determined from a fit to the available experimental data for the two-photon and the radiative decays as was done for the J/ψ -meson. However, the calculation of the matrix elements involving two photons will be very time consuming because one has to introduce the electromagnetic field into the nonlocal Lagrangians in Eqs. 1-3. This is done by using the path exponential (see, our recent papers [28] and [17]). The gauging of the nonlocal Lagrangian with spin 2 has not yet been done and is a project all of its own. For the time being, we are calculating the widths of the semileptonic decays $B_c \rightarrow (\bar{c}c) + l\nu$ by assuming an identical size parameters for all charmonium states $\Lambda_{cc} = \Lambda_{J/\psi} = 2.62$ GeV.

In order to get a quantitative idea about the invariant form factors we list their $q_{\min}^2 = 0$ and $q_{\max}^2 = (m_1 - m_2)^2$ values in Table II.

We put our values of the decay rates in Table III together with those predicted in other papers. A number of calculations are devoted to the $B_c \rightarrow \eta_c l\nu$ and $B_c \rightarrow J/\psi l\nu$ decays. All of them predict values at the same order of magnitude. A study of the semileptonic decays of the B_c -meson into excited charmonium states was done in [6, 21] within an approach which is quite different from our relativistic quark model. Concerning the electron-modes here is quite good agreement with [6, 21] in the case of $B_c \rightarrow J/\psi e\nu$, $\eta_c e\nu$, $\chi_{c2} e\nu$. Our rates are a factor of 1.5 (1.8) larger for $B_c \rightarrow \chi_{c0} e\nu$, $h_c e\nu$ decays and our rate is a factor 1.6 smaller for $B_c \rightarrow \chi_{c1} e\nu$ decay. Concerning the τ -modes here is quite good agreement with [6, 21] in the case of $B_c \rightarrow \chi_{c0} \tau\nu$, $h_c \tau\nu$ decays but our rates are almost a factor of two smaller for the other modes $B_c \rightarrow \chi_{c1} \tau\nu$, $\chi_{c2} \tau\nu$.

The partial rate for $B_c \rightarrow J/\psi + l + \nu$ is the largest. The partial rates into the P-wave charmonium states are all of the same order of magnitude and are predicted to occur at $\sim 10\%$ of the most prominent decay $B_c \rightarrow J/\psi + l + \nu$. The decays of the B_c into D-wave charmonium state are suppressed. The τ -modes are generally down by a factor of ~ 10 compared to the e -modes except for the transitions $B_c \rightarrow \eta_c$, $B_c \rightarrow J/\psi$ and $B_c \rightarrow \psi(3836)$ where the τ -modes are smaller only by a factor of $\sim 3 - 4$.

In Table IV we list our results for the integrated partial helicity rates Γ_i ($i = U, L, P, \tilde{U}, \tilde{L}, \tilde{S}, \tilde{S}\tilde{L}$). They are needed for the calculation of the forward-backward asymmetry parameter A_{FB} and, in the case of the decay $B_c \rightarrow J/\psi + l + \nu$, for the calculation of the asymmetry parameter α^* determining the transverse/longitudinal composition of the J/ψ in the decay. The partial ‘‘tilde’’ rates $\tilde{\Gamma}_i$ are quite tiny for the e -mode as expected from Eq. (30) but are not negligible for the τ -modes. This shows up in the calculated values for A_{FB} in Table V. For the decays into spin 0 states A_{FB} is proportional to $\tilde{S}\tilde{L}$ and thus tiny for the e -mode but nonnegligible for the τ -modes. For the decay into the other spin states one has $A_{FB}(e^-) = -A_{FB}(e^+)$ but $A_{FB}(\tau^-) \neq -A_{FB}(\tau^+)$ as can easily be appreciated by looking at Eq. (31). The forward-backward asymmetry can amount up to 40 %. The transverse and the longitudinal pieces of the J/ψ in the decay $B_c \rightarrow J/\psi + l + \nu$ are almost equal for both the e - and the τ -modes (see Table IV). According to Eq. (33) this implies that the asymmetry parameter α^* should be close to -33% as is indeed the case as the entries in Table V show. For the other two modes involving spin 1 charmonium states the transverse/longitudinal population is quite different. For the transition $B_c \rightarrow \chi_{c1}$ the transverse mode dominates by a factor of ~ 3 for both the e - and

TABLE II: Predictions for the form factors of the $B_c \rightarrow (\bar{c}c)$ transitions.

	q^2	F_+	F_-
η_c	0	0.61	-0.32
	q_{\max}^2	1.14	-0.61
χ_{c0}	0	0.40	-1.00
	q_{\max}^2	0.65	-1.63

	q^2	A_0	A_+	A_-	V
J/ψ	0	1.64	0.54	-0.95	0.83
	q_{\max}^2	2.50	0.97	-1.76	1.53
χ_{c1}	0	-0.064	-0.39	1.52	-1.18
	q_{\max}^2	0.46	-0.50	2.36	-1.81
h_c	0	0.44	-1.08	0.52	0.25
	q_{\max}^2	0.54	-1.80	0.89	0.365

	q^2	T_1	T_2, GeV^{-2}	T_3, GeV^{-2}	T_4, GeV^{-2}
χ_{c2}	0	1.22	-0.011	0.025	-0.021
	q_{\max}^2	1.69	-0.018	0.040	-0.033
$\psi(3836)$	0	0.052	0.0071	-0.036	0.026
	q_{\max}^2	0.35	0.0090	-0.052	0.038

τ -modes whereas for the transition $B_c \rightarrow h_c$ the longitudinal mode dominates by a factor of ~ 13 and ~ 7 for the e - and τ -modes, respectively.

Taking the central value of the CDF lifetime measurement $\tau(B_c) = 0.46 \cdot 10^{-12}$ s [1] and our predictions for the rates into the different charmonium states one finds branching fractions of $\sim 2\%$ and $\sim 0.7\%$ for the decays into the two S -wave charmonium states J/ψ and η_c , respectively, and branching fractions of $\sim 0.2\%$ for the decays into the P -wave charmonium states. Considering the fact that there will be a yield of up to 10^{10} B_c mesons per year at the Tevatron and LHC the semileptonic decays of the B_c mesons into charmonium states studied in this paper offers a fascinating area of future research.

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APPENDIX: CONVENTION FOR DIRAC γ -MATRICES AND THE ANTISYMMETRIC TENSOR IN MINKOWSKI SPACE

We use the conventions of Bjorken-Drell. Thus we define the metric tensor and the totally antisymmetric ε -tensor in Minkowski space by $g^{\mu\nu} = g_{\mu\nu} = \text{diag}(+, -, -, -)$ and $\varepsilon_{0123} = -\varepsilon^{0123} = 1$. For the partial and full contractions of a pair of ε -tensors one finds

$$\begin{aligned}
\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\varepsilon^{\nu_1\nu_2\nu_3\mu_4} &= -g_{\mu_1}^{\nu_1}g_{\mu_2}^{\nu_2}g_{\mu_3}^{\nu_3} - g_{\mu_1}^{\nu_2}g_{\mu_2}^{\nu_3}g_{\mu_3}^{\nu_1} - g_{\mu_1}^{\nu_3}g_{\mu_2}^{\nu_1}g_{\mu_3}^{\nu_2} \\
&\quad + g_{\mu_1}^{\nu_1}g_{\mu_2}^{\nu_3}g_{\mu_3}^{\nu_2} + g_{\mu_1}^{\nu_2}g_{\mu_2}^{\nu_1}g_{\mu_3}^{\nu_3} + g_{\mu_1}^{\nu_3}g_{\mu_2}^{\nu_2}g_{\mu_3}^{\nu_1} \\
\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\varepsilon^{\nu_1\nu_2\nu_3\mu_4} &= -2(g_{\mu_1}^{\nu_1}g_{\mu_2}^{\nu_2} - g_{\mu_1}^{\nu_2}g_{\mu_2}^{\nu_1}) \\
\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\varepsilon^{\nu_1\mu_2\mu_3\mu_4} &= -6g_{\mu_1}^{\nu_1}
\end{aligned}$$

TABLE III: Semileptonic decay rates in units of 10^{-15} GeV. We use $|V_{cb}| = 0.04$.

	This model	[6, 21]	[7]	[8]	[9]	[12]	[23]
$B_c \rightarrow \eta_c e \nu$	10.7	14.2	11.1	8.6	11 ± 1	2.1 (6.9)	5.9
$B_c \rightarrow \eta_c \tau \nu$	3.52			3.3 ± 0.9			
$B_c \rightarrow J/\psi e \nu$	28.2	34.4	30.2	17.2	28 ± 5	21.6 (48.3)	17.7
$B_c \rightarrow J/\psi \tau \nu$	7.82			7 ± 2			
$B_c \rightarrow \chi_{c0} e \nu$	2.52	1.686					
$B_c \rightarrow \chi_{c0} \tau \nu$	0.26	0.249					
$B_c \rightarrow \chi_{c1} e \nu$	1.40	2.206					
$B_c \rightarrow \chi_{c1} \tau \nu$	0.17	0.346					
$B_c \rightarrow h_c e \nu$	4.42	2.509					
$B_c \rightarrow h_c \tau \nu$	0.38	0.356					
$B_c \rightarrow \chi_{c2} e \nu$	2.92	2.732					
$B_c \rightarrow \chi_{c2} \tau \nu$	0.20	0.422					
$B_c \rightarrow \psi(3836) e \nu$	0.13						
$B_c \rightarrow \psi(3836) \tau \nu$	0.0031						

$$\varepsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} = -24$$

We employ the following definition of the γ^5 -matrix

$$\gamma^5 = \gamma_5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \frac{i}{24} \varepsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix},$$

$$\text{Tr}(\gamma_5 \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4}) = 4 i \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4}.$$

The leptons with negative charge ($l = e^-, \mu^-, \tau^-$) are referred to as “leptons” whereas the positively charged leptons $\bar{l} = e^+, \mu^+, \tau^+$ are referred to as “antileptons”.

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TABLE IV: Partial helicity rates in units of 10^{-15} GeV.

Mode	U	\tilde{U}	L	\tilde{L}	P	\tilde{S}	$\tilde{S}\tilde{L}$
$B_c \rightarrow \eta_c e \nu$	0	0	10.7	$0.34 \cdot 10^{-5}$	0	$0.11 \cdot 10^{-4}$	$0.34 \cdot 10^{-5}$
$B_c \rightarrow \eta_c \tau \nu$	0	0	1.18	0.27	0	2.06	0.42
$B_c \rightarrow J/\psi e \nu$	14.0	$3.8 \cdot 10^{-7}$	14.2	$0.31 \cdot 10^{-5}$	8.00	$0.88 \cdot 10^{-5}$	$0.30 \cdot 10^{-5}$
$B_c \rightarrow J/\psi \tau \nu$	3.59	0.73	2.35	0.50	1.74	0.63	0.31
$B_c \rightarrow \chi_{c0} e \nu$	0	0	2.52	$0.11 \cdot 10^{-5}$	0	$0.32 \cdot 10^{-5}$	$0.11 \cdot 10^{-5}$
$B_c \rightarrow \chi_{c0} \tau \nu$	0	0	0.13	0.037	0	0.089	0.033
$B_c \rightarrow \chi_{c1} e \nu$	1.08	$0.51 \cdot 10^{-7}$	0.33	$0.11 \cdot 10^{-6}$	-0.35	$0.32 \cdot 10^{-6}$	$0.11 \cdot 10^{-6}$
$B_c \rightarrow \chi_{c1} \tau \nu$	0.11	0.029	0.024	0.0064	-0.066	$0.42 \cdot 10^{-2}$	$0.29 \cdot 10^{-2}$
$B_c \rightarrow h_c e \nu$	0.33	$0.13 \cdot 10^{-7}$	4.10	$0.24 \cdot 10^{-5}$	0.21	$0.72 \cdot 10^{-5}$	$0.24 \cdot 10^{-5}$
$B_c \rightarrow h_c \tau \nu$	0.045	0.012	0.13	0.037	0.023	0.15	0.044
$B_c \rightarrow \chi_{c2} e \nu$	0.98	$0.53 \cdot 10^{-7}$	1.93	$0.10 \cdot 10^{-5}$	0.62	$0.30 \cdot 10^{-5}$	$0.99 \cdot 10^{-6}$
$B_c \rightarrow \chi_{c2} \tau \nu$	0.073	0.021	0.066	0.019	0.036	0.025	0.012
$B_c \rightarrow \psi(3836) e \nu$	0.075	$0.06 \cdot 10^{-7}$	0.052	$0.36 \cdot 10^{-7}$	-0.036	$0.10 \cdot 10^{-6}$	$0.35 \cdot 10^{-7}$
$B_c \rightarrow \psi(3836) \tau \nu$	$0.16 \cdot 10^{-2}$	$0.53 \cdot 10^{-3}$	$0.66 \cdot 10^{-3}$	$0.22 \cdot 10^{-3}$	$-0.13 \cdot 10^{-2}$	$0.16 \cdot 10^{-3}$	$0.11 \cdot 10^{-3}$

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TABLE V: Forward-backward asymmetry A_{FB} and the asymmetry parameter α^* .

Mode	$A_{FB}(l^-)$	$A_{FB}(l^+)$	α^*
$B_c \rightarrow \eta_c e \nu$	$9.64 \cdot 10^{-6}$	$9.64 \cdot 10^{-6}$	-
$B_c \rightarrow \eta_c \tau \nu$	0.36	0.36	-
$B_c \rightarrow J/\psi e \nu$	0.21	-0.21	-0.34
$B_c \rightarrow J/\psi \tau \nu$	0.29	-0.05	-0.24
$B_c \rightarrow \chi_{c0} e \nu$	$1.29 \cdot 10^{-6}$	$1.29 \cdot 10^{-6}$	-
$B_c \rightarrow \chi_{c0} \tau \nu$	0.38	0.38	-
$B_c \rightarrow \chi_{c1} e \nu$	-0.19	0.19	-
$B_c \rightarrow \chi_{c1} \tau \nu$	-0.24	0.34	-
$B_c \rightarrow h_c e \nu$	0.036	-0.036	-
$B_c \rightarrow h_c \tau \nu$	0.39	0.30	-
$B_c \rightarrow \chi_{c2} e \nu$	0.16	-0.16	-
$B_c \rightarrow \chi_{c2} \tau \nu$	0.32	0.05	-
$B_c \rightarrow \psi(3836) e \nu$	-0.21	0.21	-0.17
$B_c \rightarrow \psi(3836) \tau \nu$	-0.21	0.41	0.006