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Article *in* Physical Review C · April 2010

DOI: 10.1103/PhysRevC.81.044313

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PHYSICAL REVIEW C **81**, 044313 (2010)

Isospin-dependent pairing interaction from nuclear matter calculations

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(Received 16 February 2010; published 21 April 2010)

The isospin dependence of the effective pairing interaction is discussed on the basis of the Bardeen, Cooper, and Schrieffer theory of superfluid asymmetric nuclear matter. It is shown that the energy gap, calculated within the mean field approximation in the range from symmetric nuclear matter to pure neutron matter, is not linearly dependent on the symmetry parameter owing to the nonlinear structure of the gap equation. Moreover, the construction of a zero-range effective pairing interaction compatible with the neutron and proton gaps in homogeneous matter is investigated, along with some recent proposals of isospin dependence tested on the nuclear data table.

DOI: 10.1103/PhysRevC.81.044313 PACS number(s): 21.65.−f, 13.75.Cs, 21.30.Fe, 26.60.−c

I. INTRODUCTION

Recently density- and isospin-dependent pairing interactions have been proposed to explain the increasing number of nuclei far from the stability line [1–3]. The isospin dependence has been included, requiring the form factors to be separately dependent on neutron and proton densities. The main constraint imposed on these interactions was to be consistent with the gaps calculated for superfluid nuclear matter. This is made difficult by the need to introduce a cutoff in the energy (or momentum space), as such interactions have a zero range, whereas nuclear matter gaps are calculated with finite-range bare interactions. The key point is how to reduce the momentum-dependent interaction into a zero-range pairing interaction and how to consistently renormalize the single-particle (s.p.) spectrum. The weak coupling approximation (WCA) may be useful if it is applied with some care.

From investigation of the pairing in nuclear and neutron matter [4], it is expected that the isospin dependence of the effective interaction comes mainly from the medium polarization effects. Indeed, moving from pure neutron matter to symmetric nuclear matter, the screening of, say, the neutron pairing owing to neutron particle-hole excitations is converted into antiscreening by the coupling with proton particle-hole excitations. In nuclei these correlations could appear as surface vibrations and thus they could influence the pairing, which is supposed to be sizable just on the nuclear surface [5]. This idea is supported by microscopic calculations of pairing in the ^{120}Sn nucleus by the Milano group [6]. On the contrary, recent calculations reproducing the experimental gaps in nuclei with bare force [7] indicate that a compensation mechanism could occur in a full treatment of the medium polarization [4].

Before we consider medium polarization—simply on the basis of the pure Bardeen, Cooper, and Schrieffer (BCS) approach—the isospin effects that make neutron and proton gaps different from each other arise just from the isovector component of the nuclear mean field in asymmetric nuclear matter. As the full s.p. spectrum can be described in terms of effective mass in the gap equation, the neutron and proton gaps can probe the isovector component of the effective mass. This is still a controversial issue despite nucleon-nucleus collisions indicate that $m_n^* - m_p^* > 0$ [8]. Therefore, the isospin dependence of the effective mass is crucial for the study of pairing in asymmetric nuclear matter and the effective pairing interaction in nuclei.

In the present work we first discuss the neutron and proton $^1S_0$ pairing gaps in asymmetric nuclear matter calculated within the BCS theory in Sec. II. The reduction of the bare interaction into a zero-range pairing interaction, based on the WCA, is scrutinized starting from the exact renormalization of the higher-momentum components of the bare interaction in Sec. III. In Sec. IV the results are discussed in comparison with the neutron and proton gaps fixing the parameters of the effective pairing interactions used in recent fits of the nuclear mass data [1,3]. Concluding remarks are given in Sec. V.

II. PAIRING IN ASYMMETRIC NUCLEAR MATTER

In recent versions of the isospin-dependent effective pairing force, the isovector component is obtained from the interpolation between symmetric matter and pure neutron matter gaps. In the mean field approximation the potential turns out to be a linear function of the symmetry parameter $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$, but we do not expect this behavior for the pairing potential, because the gap equation is a highly nonlinear equation. This question demands the explicit calculation of the gap in the full range $0 \leqslant \beta \leqslant 1$. In this work we perform such a calculation only in the pure BCS approximation, leaving treatment of the medium polarization for a future investigation. In the pure BCS context the gap

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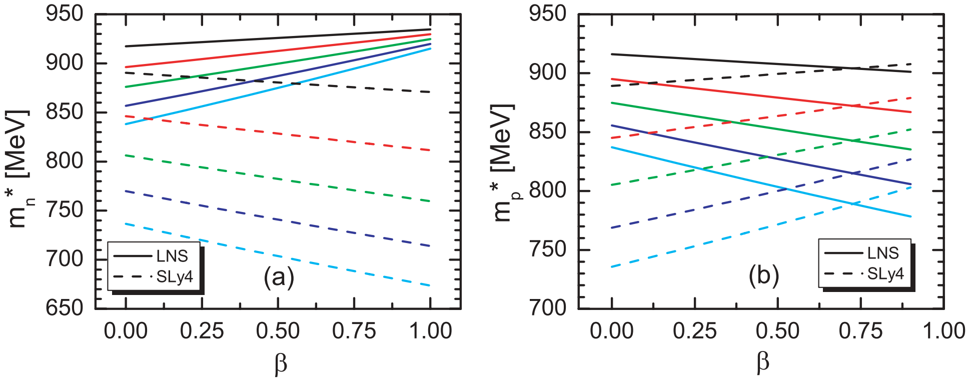

FIG. 1. (Color online) Neutron (a) and proton (b) effective masses in asymmetric nuclear matter for a set of densities. The density ranges from 0.02 to 0.1 fm^{-3}, with a step of 0.02 fm^{-3}, from top to bottom.

equation is written as [9,10]

$$\Delta_\tau(k) = -\sum_{k'} \frac{V(k,k')\Delta_\tau(k')}{2E_\tau(k')}, \tag{1}$$

$$N_\tau = \sum_k \left(1 - \frac{\epsilon_\tau(k) - \mu_\tau}{E_\tau(k)}\right), \tag{2}$$

where $E_\tau^2(k) = (\epsilon_\tau(k) - \mu_\tau)^2 + \Delta_\tau^2(k)$ and $\epsilon_\tau(k) = k^2/2m + U_\tau(k)$ is the s.p. energy. The two coupled equations give the energy gap and the chemical potential of the superfluid state. Neglecting other medium effects, the isospin dependence arises only from the nuclear potential U_τ.

The mean field can be split into two components that, in the Brueckner-Hartree-Fock (BHF) approximation [11], are given by the antisymmetrized matrix elements of the G matrix, as follows:

$$U_\tau(k) = U_{\tau,\tau} + U_{\tau,\tau'} = \sum_{k'} [\langle \vec{k}\sigma\tau, \vec{k}'\sigma'\tau|G|\vec{k}\sigma\tau, \vec{k}'\sigma'\tau\rangle_A + \langle \vec{k}\sigma\tau, \vec{k}'\sigma'\tau'|G|\vec{k}\sigma\tau, \vec{k}'\sigma'\tau'\rangle_A], \tag{3}$$

summed on $k' \equiv (\vec{k}', \sigma')$, that is, the momentum and spin of each nucleon inside the Fermi sphere. Moving from symmetric nuclear matter to pure neutron matter, U_{pp} bends to zero so that $U_p \to U_{pn}$, whereas $U_n \to U_{nn}$. Because the neutron and proton gaps are mainly sensitive to their effective masses, the isospin dependence of the nucleon pairing gaps turns out to be dependent just on the isospin splitting of neutron and proton effective masses in asymmetric nuclear matter. In this respect this investigation focused not only on the β dependence of the effective pairing interaction [1,3], but also on the consequences of whether, in asymmetric nuclear matter, the neutron effective mass is larger than the proton one, or vice versa. In the present calculations the so-called LNS potential was employed, as derived from a fit of the BHF calculations [12]. In the fit the full momentum dependence of the BHF potential is approximated by introducing the effective mass. But this is a good approximation because the dispersive effect of the momentum-dependent mean field in the gap equation is relevant only around the Fermi surface, where the effective mass is calculated as the slope of the mean field, whereas the momentum tail of the pairing interaction controls the integration cutoff (see, e.g., Fig. 9 in Ref. [13]). In Fig. 1 the LNS proton and neutron effective masses in asymmetric nuclear matter are plotted in comparison with the SLy4 predictions [14]. The two calculations exhibit opposite isospin slope, as is well known [8].

Solving the two gap equations, Eqs. (1) and (2), for the $^1S_0$ pairing in asymmetric nuclear matter, the pairing gaps were calculated as a function of the symmetry parameter. The numerical values are displayed in Fig. 2 for a set of nuclear matter densities.

The interaction chosen for this calculation is the Argonne AV_{18} [15], which is also used to calculate the BHF potential [11], and the mean field was approximated with the LNS effective mass. In symmetric nuclear matter $\Delta_n = \Delta_p$ because the nuclear interaction is charge independent (we neglect the Coulomb force). In the case of neutron gaps [Fig. 2(a)], the β-slope is positive for $\rho \leqslant 0.02$ fm^{-3} and negative for $\rho > 0.02$ fm^{-3}, as one may expect from the fact that $\rho = 0.02$ fm^{-3} is the density corresponding to peak value of the gap. In the case of proton gaps [Fig. 2(b)], the β-slope is negative for $\rho \leqslant 0.02$ fm^{-3} and it should be positive for $\rho > 0.02$ fm^{-3}. But in the latter range the expected trend is in competion with the mean field effect, which becomes more and more attractive for increasing β.

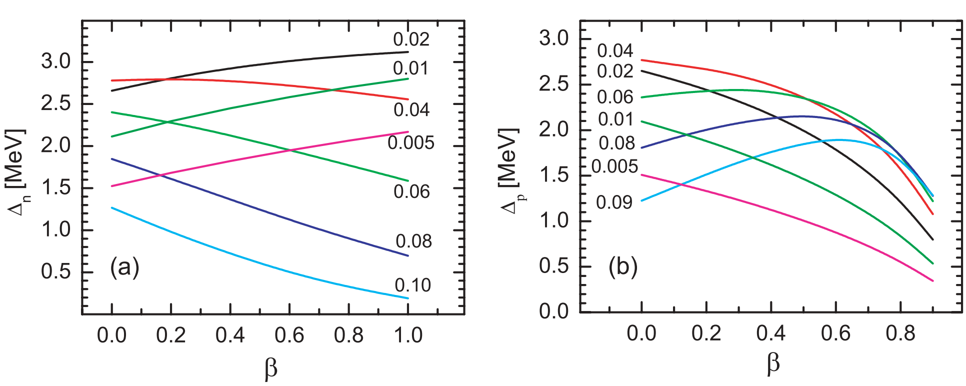

FIG. 2. (Color online) Neutron (a) and proton (b) gaps in asymmetric nuclear matter. Numbers on the curves are the corresponding densities (fm^{-3}).

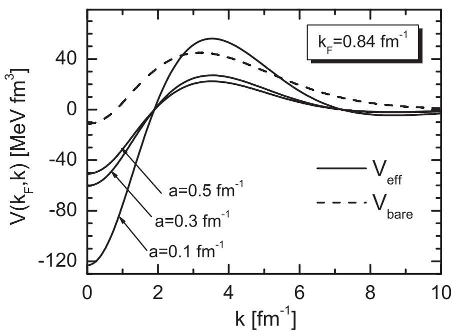

FIG. 3. Renormalized neutron-neutron pairing interaction in neutron matter for different values of the cutoff $a = |k_F - k|$.

III. RENORMALIZATION OF THE BARE INTERACTION

A convenient way to solve the gap equation is to use a renormalized pairing interaction that embodies the lower- and higher-momentum components so that the integration of the gap equation is restricted to the region around k_F [16]. This procedure leads to splitting the BCS gap equation into two coupled equations:

$$\Delta(k) = -\sum_{|k'-k_F|<\Delta k} \frac{\tilde{V}(k,k')\Delta(k')}{2E(k')}, \tag{4}$$

$$\tilde{V}(k,k') = V(k,k') - \sum_{|k''-k_F|>\Delta k} \frac{V(k,k'')\tilde{V}(k'',k')}{2E(k'')}. \tag{5}$$

In principle, the cutoff in the momentum integration is absolutely arbitrary. The effective interaction must change with the cutoff for the gap to be a unique solution of the gap equation. This procedure is valid regardless of the complexity of the pairing interaction $V(k,k')$ and the s.p. energy $\epsilon(k)$ [16]. The comparison between the effective interaction $\tilde{V}$ and the bare interaction as a function of the cutoff is illustrated in Fig. 3. $\tilde{V}$ is calculated with the exact value of the gap. The effect of renormalization is to reduce the higher-momentum repulsive components of the interaction and to enhance the lower-momentum attractive ones until $\tilde{V}$ becomes singular. This behavior can be understood with a toy model, based on a rank 1 separable interaction $V(k,k') = V_0 g(k)g(k')$. In this case $\Delta(k) = \Delta_0 g(k)$ and the gap equation takes the form

$$V_0^{-1} = -\sum_k \frac{g(k)^2}{2E(k)}, \tag{6}$$

from which the gap strength Δ_0 in the energy denominator is calculated. For any choice of the cutoff the effective interaction is given by

$$\tilde{V}_0^{-1} = V_0^{-1}\left(1 + V_0 \sum_{|k-k_F|>\Delta k} \frac{g(k)^2}{2E(k)}\right). \tag{7}$$

For $\Delta k \to 0$ the right-hand side of Eq. (7) tends to zero according to Eq. (6) and thus the effective interaction becomes singular.

It is desirable to reduce the cutoff as much as possible to apply the WCA, but for $\Delta k \to 0$ the effective interaction increases rapidly, owing to the resonant behavior of the correlation function. On the contrary, for large Δk the WCA cannot be applied. To avoid strong variations of the effective interaction with respect to the cutoff, a good guide is that the size of the window is of the same order as the aforementioned resonance, namely, $\Delta k \sim \sqrt{\Delta}$.

One of the main advantages of the renormalized interaction is that, for small enough values of Δk, WCA can be adopted to solve the gap equation; that is,

$$\frac{1}{\tilde{V}_F} = -\sum_{|k'-k_F|<\Delta k} \frac{1}{2\sqrt{[\epsilon(k')-\mu]^2 + \Delta^2}}, \tag{8}$$

where $\tilde{V}_F \equiv \tilde{V}(k_F,k_F)$ and $\Delta \equiv \Delta(k_F)$. A further advantage is that one can replace the momentum dependence of the mean field with the effective mass, so that $E(k) = \sqrt{[(k^2-k_F^2)/2m^*]^2 + \Delta^2}$.

As already mentioned, the isospin dependence of the energy gap is caused by the isospin splitting of the effective mass. Even for low asymmetry the neutron and proton gaps are expected to vary considerably, being exponentially dependent on the respective effective mass. This can easily be seen in the WCA limit. In the framework of the WCA one can introduce the energy cutoff ϵ_c [17], and for $\epsilon_c \ll \Delta$ one easily gets

$$\frac{2\epsilon_c}{\Delta_\tau} = e^{m_\tau^*/mN_0\tilde{V}_F} - 1, \tag{9}$$

where N_0 and $\tilde{V}_F$ are the level density of the free proton fraction and the effective interaction at the Fermi energy, respectively.

Because zero-range effective interactions are often used to calculate the pairing gap in nuclei, the introduction of a cutoff k_{cut}, that is, an upper limit in momentum space, is needed to solve the BCS equations. Nuclear matter calculations can give

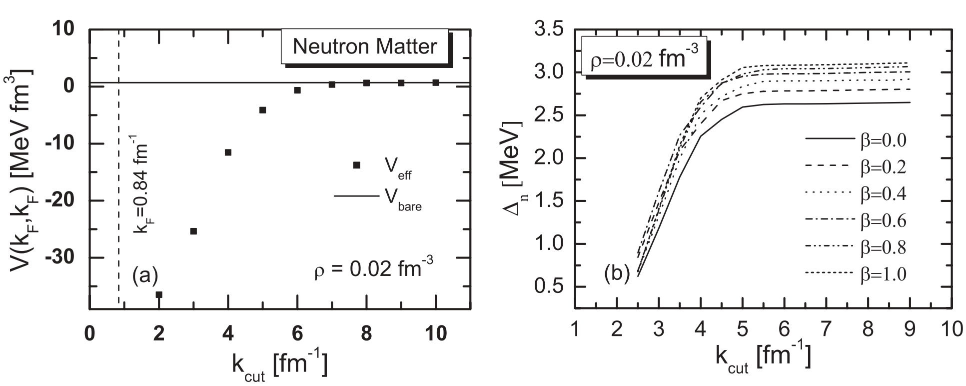

FIG. 4. Sensitivity of the pairing effective interaction (a) and the energy gap (b) to the cutoff chosen.

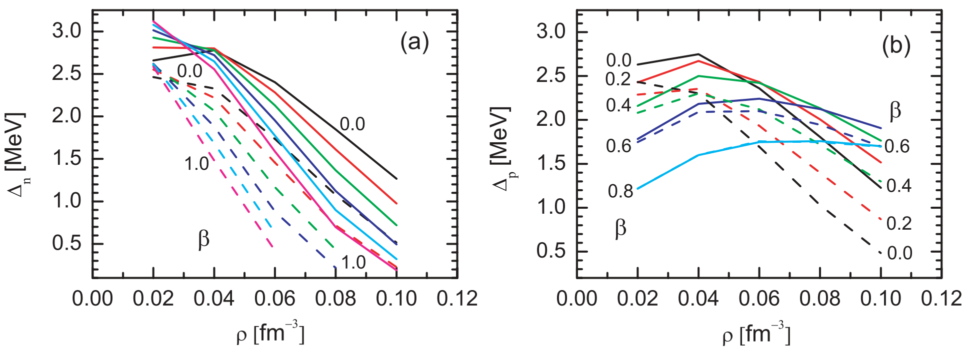

FIG. 5. (Color online) Neutron (a) and proton (b) gaps in the $^1S_0$ channel from the BCS equations with Argonne AV_{18}. Predictions using the LNS mean field (solid lines) are depicted in comparison with those using the SLy4 mean field (dashed lines). For neutron gaps the asymmetry parameter ranges from 0 to 1, with a step of 0.2.

an indication of how to choose such a cutoff. As a function of k_{cut} the gap is plotted in Fig. 4(b). For a large enough cutoff the gap saturates to the exact value; how large it is depends on the interaction used. With Argonne AV_{18} its value is $k_{\text{cut}} \approx 5\,\text{fm}^{-1}$. The corresponding effective interaction, as given by Eq. (5) with $0 < k < k_{\text{cut}}$, is reported in Fig. 4(a). This value of k_{cut} can be interpreted as the range of nonlocality of the chosen potential for pairing correlations. Its large value casts doubt on zero-range pairing interactions unless the previously discussed procedure to get the effective interaction is permitted.

IV. EFFECTIVE PAIRING INTERACTION

The increasing interest in nuclei far from the stability line demands special attention to isospin effects in pair correlations, and in fact, density-dependent zero-range pairing interactions have recently been generalized to exhibit an isospin dependence, disentangling the dependence of the proton density from that of neutrons [1,3]. The most straightforward way to do that consists in a linear interpolation of the interaction between the symmetric nuclear system and the pure neutron matter, as proposed in Ref. [1]; namely,

$$v(\rho, \beta) = 1 - \eta_s(\rho/\rho_0)^{\alpha_s}(1-\beta) - \eta_n(\rho/\rho_0)^{\alpha_n}\beta, \qquad (10)$$

where the parameters are adjusted to reproduce the exact values of the gaps in nuclear matter. In Ref. [1] the fit was performed solving the gap equation, Eq. (4), with the above interaction and introducing a cutoff to remove the high-momentum divergence. The cutoff was tuned so as to best fit the nuclear matter gaps in the two limits $\beta = 0$ and $\beta = 1$ taken from the calculation reported in Ref. [4]. The resulting gaps were compared with the predictions in Ref. [4]. They do not reproduce the exact values of the unscreened pairing at higher densities and ill reproduce the values including screening (see Fig. 2 in Ref. [1]). Unfortunately, they used a SLy4 s.p. energy whose isospin dependence is in strong disagreement with the LNS one, as mentioned. Calculation of the gaps in asymmetric nuclear matter with the two s.p. energies, depicted in Fig. 5, was performed to emphasize how the different isospin properties of the two s.p. spectra can affect the gaps at various densities. However the results, as also shown in Fig. 2, do not support any linear interpolation for the proton gap.

A different approach was proposed by Goriely, Chamel, and Pearson (GCP) [3], which does not assume any explicit form for the density dependence of the effective interaction, but the latter is built up directly from the neutron and proton gaps of nuclear matter. The idea is to employ the gap equation, in the form of Eq. (4), to determine the effective interaction, adopting the exact gaps of nuclear matter as inputs. In this way the density and isospin dependence of the effective interaction are automatically ensured. The delicate point of this approach is that the cutoff must be chosen small enough, so as to neglect the momentum dependence of the interaction. In Ref. [3] this interaction was successfully applied to reproduce a huge number of nuclear data. The neutron and proton gaps adopted in calculating the pairing interaction were taken from a symmetric nuclear matter and pure neutron matter calculation, including medium polarization effects [4]. The gaps in asymmetric nuclear matter were obtained from the interpolation between symmetric nuclear matter and pure neutron matter gaps. The interpolation ansatz is

$$\Delta_q(\rho_n, \rho_p) = \Delta_{\text{SM}}(\rho)(1 - |\beta|) \pm \Delta_{\text{NM}}(\rho_q)\beta\frac{\rho_q}{\rho}. \qquad (11)$$

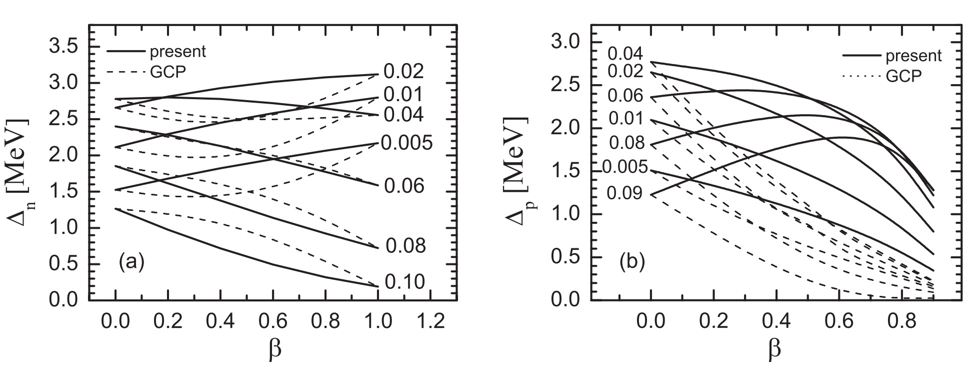

FIG. 6. $^1S_0$ neutron (a) and proton (b) gaps. Comparison between the calculations reported in Fig. 2 and the calculations based on the GCP interpolation method (see text).

For small positive asymmetry values this equation is just a linear interpolation between the gap Δ_{SM} in symmetric nuclear matter and the gap Δ_{NM} in pure neutron matter. This behavior does not take into account the isospin deepening of the mean field. The comparison, reported in Fig. 6, between the exact BCS gaps in Fig. 2 and the gaps calculated from Eq. (11) clearly shows that the GCP interpolation formula is not valid. Nevertheless, the GCP model is reasonable because it does not require any interpolation formula once the gaps in asymmetric nuclear matter are made available.

V. CONCLUSIONS

In this paper the isospin dependence of neutron and proton pairing gaps has been discussed within the pure BCS pairing theory. A realistic interaction, that is, Argonne AV_{18}, was adopted as the pairing interaction in the gap equation, and the Skyrme-LNS potential was used for the mean field in the gap equation. The isospin dependence, even in the asymmetry range typical of nuclei ($0 \leqslant \beta \leqslant 0.2$), turns out to be quite strong, for the combined effects of density and effective mass dependence. In particular, the latter should allow us to distinguish the isospin splitting of the neutron and proton effective masses, because the gaps are exponential functions of the effective mass. The exact pairing gaps are compared to the recent predictions in Refs. [1] and [3]; it is shown that the isospin dependence is far from being a linear function of isospin.

The rigorous construction of the renormalized pairing interaction in nuclear matter has been discussed, along with the role played by the cutoff. This investigation could be useful to fix a cutoff suitable for reliable application of the WCA. Imposing a cutoff without suitable renormalization of the interaction could result in arbitrary values of the gap.

As pointed out in Sec. I, a large isospin effect is expected to come from the induced interaction. In fact, in a comparison of neutron pairing in nuclear matter versus neutron matter, it was shown that this effect is caused by the interplay between neutron and proton particle-hole collective excitations [4]. The calculations will be extended to the superfluidity of asymmetric nuclear matter. The main question is how much the antiscreening effect owing to proton ph excitations will be compensated by the gap quenching owing to self-energy corrections [4]. This compensation can also be explained by comparing the many-body treatment of the pairing with recent Monte Carlo calculations [18].

ACKNOWLEDGMENTS

Discussions with Dr. M. Baldo are gratefully acknowledged. This study was performed in the framework of the Agreement CAS-INFN (2008-2010): research project between the ITP (Beijing), IMP (Lanzhou) and LNS (Catania). It was supported in part by the Project of Knowledge Innovation Program (PKIP) of Chinese Academy of Sciences, Grant No. KJCX2.YW.W10 and the National Natural Science Foundation of China under Grant No. 10875150 and 10875157.

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