

Helioseismic constraints to the central solar temperature and neutrino fluxes

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Abstract

The central solar temperature T and its uncertainties are calculated in helioseismologically-constrained solar models. From the best fit to the convective radius, density at the convective radius and seismically determined helium abundance the central temperature is found to be $T = 1.58 \times 10^7 \text{K}$, in excellent agreement with Standard Solar Models. Conservatively, we estimate that the accuracy of this determination is $\Delta T/T = 1.4\%$, better than that in SSM. Neutrino fluxes are calculated. The lower limit to the boron neutrino flux, obtained with maximum reduction factors from all sources of uncertainties, is 2σ higher than the flux measured recently by SuperKamiokande.

Helioseismology allows us to look into the deep interior of the Sun, probably more efficiently than neutrinos (for reviews see [1–5]). The highly precise measurements of frequencies and the tremendous number of measured lines enable us to extract the values of sound speed and density inside the Sun with accuracy better than 1%. Recently it was demonstrated that a comparable accuracy can be obtained for the inner core of the Sun ($R/R_\odot < 0.1$) [6,7].

Helioseismic data are in agreement with recent Standard Solar Model (SSM) calculations, which use accurate equations of state, recent opacity tables and include helium and heavier elements diffusion [8,7,9–11], see also Ref. [12]. These SSMs yield central temperatures T_{SSM} which differ from each other by not more than 1%. However the uncertainties in input parameters, mainly the opacity κ and the heavy elements abundance Z/X , result in $(\Delta T/T)_{SSM} \approx 1 - 2\%$.

From helioseismological observations one cannot determine directly temperature of the solar interior, as one cannot determine the temperature of a gas from the knowledge of the sound speed unless the chemical composition is known. However, it is possible to obtain the

range of allowed values of the central temperature T , by selecting those solar models which are consistent with seismic data.¹

In this paper we shall address two related problems: accuracy in the determination of the central temperature in helioseismologically-constrained solar models and neutrino fluxes in these models.

Our calculations are not model-independent, but we shall use in principle a wider class of models in comparison with SSMS, which we call helioseismologically-constrained solar models (HCSM). These models are based on the same equilibrium and evolution equations as SSMS, but they differ in the choice of some input parameters. We generate this class of models by using the FRANEC code [10] for the SSMS and varying the input parameters. Each choice of the set of parameters gives some value of T .

We obtain the range of allowed values of T by selecting those solar models which are consistent with seismic data. More specifically, we shall determine the central temperature T_{HCSM} , as that of the model which gives the best fit to the seismic data and the uncertainties, ΔT_{HCSM} , corresponding to the range spanned by models consistent with these data.

As a method of calculation, we shall use the scaling approximation of the results of solar models evaluated with FRANEC. Namely we assume that every physical observable Q (e.g. temperature, photospheric helium abundance, radius of convective zone etc) depends on the input parameters P as

$$\frac{Q}{Q_{SSM}} = \left(\frac{P}{P_{SSM}} \right)^{\alpha_{Q,P}}, \quad (1)$$

where the subscript SSM is related to the (arbitrarily chosen) reference SSM. In this paper we use the new determination of the coefficients $\alpha_{Q,P}$ obtained with the latest version of FRANEC [10], which includes diffusion of all elements.

The precise value of the temperature is governed by four quantities P : the radiative opacity κ , the fraction of heavy elements Z/X , the astrophysical factor of the p+p reaction S_{11} , and the solar age t_{\odot} . If these four quantities are rescaled with respect to the values used in the SSM calculation by a multiplicative factor $p_i = P_i/P_{i,SSM}$ ($p_i=k, z, s$ and t respectively), the central temperature scales as:

$$T = T_{SSM} k^{0.14} z^{0.078} s^{-0.14} t^{0.084}. \quad (2)$$

We shall proceed now to determine the range of the scaling variables, p_i , allowed by seismic observations. We remark that the errors on S_{11} and age t_{\odot} are small (1% or less), so that they weakly affect T . On the other hand, the uncertainties on κ and Z/X are of the order of 10% and their influence on T is important. Furthermore, these uncertainties do not correspond to clear experimental or observational errors, rather they are determined by judicious comparison among published values. As an example, the uncertainty on κ can only be estimated from the comparison among recent theoretical calculations.

We shall thus concentrate on the two most important scaling variables, k and z .

¹Here and everywhere below T is the central temperature.

We want to emphasize that this not a determination of the above-mentioned parameters directly from seismic data, since solar models are involved in this evaluation. Rather we intend to determine those solar models which are consistent with helioseismology. Let us also remark that we are restricting to *uniform*, although generous, variations of opacity.

As seismic “observables” we choose three independent physical quantities determined most accurately by seismic observations (see Ref. [7]), namely the photospheric helium abundance, Y_{ph} , the depth of convective envelope, R_b , and the density, ρ_b , at the bottom of convective envelope (see the data in Table 1). A fourth seismic “observable”, the sound speed at the convective radius is traditionally considered, e.g. [5]. We have not included it in our list since, as shown in Ref. [4], it is not an independent one.

By numerical experiments, using FRANEC, we found the following dependence on z and k (for dependence on the other variables see Table I)

$$R_b = R_{b,SSM} z^{-0.046} k^{-0.0084} \quad (3a)$$

$$\rho_b = \rho_{b,SSM} z^{0.47} k^{0.095} \quad (3b)$$

$$Y_{ph} = Y_{ph,SSM} z^{0.31} k^{0.61} \quad (3c)$$

We remark that ρ_b and R_b have almost equal restriction power. In fact the error in R_b is much smaller than that in ρ_b (see Table I), but the dependence on the scaling parameters is much stronger for ρ_b , see Eq.(3b).

We also remark that T is mainly determined by Y_{ph} . One sees from Eqs. (2) and (3c) that to a good approximation the dependence of T and Y_{ph} on z and k is just through $\eta = z \cdot k^2$, so that one can express T as a function of Y_{ph} :

$$T_{HCSM} = T_{SSM} (Y_{ph,\odot}/Y_{ph,SSM})^{0.2} \quad , \quad (4)$$

Let us determine now the values of z and k which give the best fit to all three quantities $Q_{\odot,i}$, i.e. Y_{ph} , R_b and ρ_b . For this we minimize the function

$$\chi^2(z, k) = \sum_i \left(\frac{Q_{\odot,i} - Q_i(z, k)}{\Delta Q_{\odot,i}} \right)^2 \quad . \quad (5)$$

The corresponding value of the central temperature $T(z, k)$ gives our best estimate T_{HCSM} .

By starting with BP95 model (the “best model with metal and helium diffusion” of Ref. [9]) with a central temperature $T_{SSM} = 1.584 \times 10^7 K$, one arrives at $T_{HCSM} = 1.587 \times 10^7 K$, slightly higher but fully consistent with the former value within the uncertainties of SSMs. In fact, this shows that BP95 is in good agreement with helioseismic data. We repeated the same procedure starting from a few different SSM calculations, on which enough information is available to us: FR97 is the “best” model with He and heavier elements diffusion of Ref. [10]; FR96 is another variant of the same model with the Livermore opacities tables calculated for just 12 elements [7]; JCD is the “model S” of Ref. [11].

We remark that BP95 and FR96 adopt the same 12 elements composition [13], whereas FR97 uses the newest Livermore opacities calculated for 19 elements [14]. The values found are all within the range $T_{HCSM} = (1.573 - 1.587) \times 10^7 K$, slightly tighter than that of SSM predictions, $T_{SSM} = (1.567 - 1.584) \times 10^7 K$, see also Fig. 1.

By averaging over all available SSM as starting models we obtain the best estimated seismic temperature

$$T_{HCSM} = 1.58 \times 10^7 \text{ K} . \quad (6)$$

The range of acceptable temperatures is determined by those values of z and k such that *each* $Q(z, k)$ is in agreement with helioseismology within the estimated uncertainty. By using BP95 as a reference SSM, the allowed domain of z and k is shown in Fig. 2, where we also show the limiting temperatures. The resulting uncertainty is $\pm 1\%$.

A conservative estimate of $(\Delta T/T)_{HCSM}$ is obtained by adding the spread due to starting with different SSMs. The result is shown together with $(\Delta T/T)_{SSM}$ estimated with the same procedure, assuming a 10% uncertainty in κ and Z/X :

$$(\Delta T/T)_{HCSM} = \pm 1.4\%, \quad (\Delta T/T)_{SSM} = \pm 2.7\% \quad (7)$$

We remark that Eq. (7) for both uncertainties is derived using a similar and conservative approach, of *adding up linearly all error sources*.

Should one add errors in quadrature (so that e.g. $\Delta Y_{ph}/Y_{ph} = 1.4\%$, see [7]), it gives

$$(\Delta T/T)_{HCSM} = \pm 0.5\%, \quad (\Delta T/T)_{SSM} = \pm 1.7\% \quad (8)$$

Fig. 1 summarizes what we have found so far: the central temperatures in HCSMs agree very well with ones in SSMs, though the uncertainties in the former models are smaller.

Finally, we shall discuss a question which a concerned reader certainly must ask: why do we not use the sound speed profile as a constraining condition? Indeed, in the region $0.2 < R/R_\odot < 0.6$ the accuracy on the isothermal sound speed squared $U = \mathcal{P}/\rho$, where \mathcal{P} is pressure, is better than 0.5%. Why this tremendous accuracy, taken as a function of distance, does not give the strongest constraints?

Let us look to this problem quantitatively. At each fractional distance $x = R/R_\odot$ we parametrize U as:

$$U(x) = U_{SSM}(x) z^{\alpha_z(x)} k^{\alpha_k(x)} s^{\alpha_s(x)} \quad (9)$$

The calculated coefficients, $\alpha_p(x) \equiv d \ln U / d \ln p$, are plotted in Fig. 3. For each parameter p the excluded value of Δp is given by

$$\Delta p \geq \frac{\Delta U}{U \alpha_p} . \quad (10)$$

In case of Z/X , for example, the maximum α_z is 0.05 (see Fig.3) and for $\Delta U/U \approx 0.005$ we can exclude variations in Z/X of order of 10% or more, which does not improve our knowledge.

As noted in Ref. [15], a change in $\kappa(\rho, T)$ by a multiplicative constant factor can largely be compensated by change in the composition, the sound speed profile remaining approximately the same in the intermediate region. Actually the opacity coefficients are an order of magnitude smaller than the others, so that a 10% uniform variation of κ , which affects T to the 1.5% level [see Eq. (2)], cannot be excluded by studying $U(x)$.

Let us come over to *neutrino fluxes*, Φ_i ($i=pp, Be, B$). Their dependence on the central temperature T is parametrized as:

$$\Phi_i = \Phi_{i,SSM} \left(\frac{T}{T_{SSM}} \right)^{\beta_i}. \quad (11)$$

From numerical experiments with FRANEC, we found: $\beta_{pp} = -0.92$, $\beta_{Be} = 7.9$, $\beta_B = 18$.

We have determined neutrino fluxes by starting with different SSMs, renormalizing their predictions to the same temperature $T_{HCSM} = 1.58 \cdot 10^7$ K and to the same (updated) nuclear cross sections. The resulting fluxes and signals, all very close to each other, have been averaged to determine the HCSM predictions shown in Table II, where the uncertainties corresponding to $(\Delta T/T)_{HCSM} = \pm 1.4\%$ are also indicated.

In order to discuss nuclear physics uncertainties, we shortly review the present status for three most important cross-sections. The numbers quoted below refer to 3σ errors.

a) ${}^3\text{He}+{}^3\text{He} \rightarrow {}^4\text{He} + 2p$. This cross-section was measured recently in LUNA experiment [16] at energy corresponding to the solar Gamow peak (~ 20 KeV). The uncertainty in astrophysical factor is $\Delta S_{33}/S_{33} = \pm 6\%$.

b) ${}^3\text{He}+{}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$. The uncertainty is $\Delta S_{34}/S_{34} = \pm 12\%$.

c) $p+{}^7\text{Be} \rightarrow {}^8\text{B} + \gamma$. S_{17} is determined from direct measurements with an error of about $\pm 30\%$ [17–19]. The indirect measurement [20] gives S_{17} consistent with the lower limit of direct measurements, although the accuracy of this method is unclear [21–23].

The errors in fluxes due to cross-sections are calculated using the relations between fluxes and cross-sections [24,25], $\Phi_{Be} \propto S_{34} S_{33}^{-1/2}$ and $\Phi_B \propto S_{34} S_{33}^{-1/2} S_{17}$. We remark that the (3σ) uncertainty on Φ_{Be} due to that on S_{34} is slightly larger than that corresponding to temperature. The 30% uncertainty on Φ_B due to that of S_{17} exceeds that due to the solar temperature.

One concludes, see Table II, that (3σ) nuclear physics uncertainties are at least as important as the (generously estimated 1.4%) temperature uncertainty.

In the end we shall discuss the problem of boron neutrinos.

The HCSM boron neutrino flux (see Table II) is 18σ higher than than the combined Kamiokande [26] and SuperKamiokande [27] flux, $\Phi_K = (2.58 \pm 0.19) \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}$). However, the uncertainties in the predicted flux are of great importance for this comparison.

During the last several years an idea of reconciling the predicted boron neutrino flux with that measured by Kamiokande was widely discussed [28–31]. This discussion was inspired by the low value of S_{17} measured in indirect experiments [20] and by a possible decrease of the central temperature T due to collective plasma effects (through opacity [32]) and due to abundance of heavy elements Z . These effects, when correlated, can result in the agreement between predicted and measured B-neutrino flux.

Since in our calculations the opacity and heavy elements abundance are constrained by seismic observations, the status of this problem has changed. Indeed, diminishing T_{HCSM} by 1.4% and S_{17} by 30% one obtains from the boron-neutrino flux in HCSM the minimum flux $3.24 \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}$, which is 3.5σ higher than the combined Kamiokande and SuperKamiokande flux.

If one accepts a more reasonable 1% reduction of T_{HCSM} , but takes other errors in a correlated way, all diminishing the B-neutrino flux (namely S_{17} by 30% smaller, S_{34} by 12%

smaller and S_{34} by 6% higher) we obtain $\Phi_B = 2.97 \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}$, i.e. 2.1σ higher than SuperKamiokande flux. Only in case of largest possible correlated errors the HCSM flux can be reconciled with the SuperKamiokande data.

Therefore, we have now three solar-neutrino problems: (i) the controversy of Homestake and SuperKamiokande results, (ii) the Beryllium neutrino problem, as controversy of gallium and SuperKamiokande (or Homestake) experiments, and (iii) Boron neutrino problem, as it is described above.

In conclusion, the helioseismologically constrained solar models (HCSM) give the central solar temperature in excellent agreement with SSMs, and with smaller uncertainties. The boron neutrino problem exists in this class of models.

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REFERENCES

- [1] D. O. Gough, J. W. Leibacher, P. H. Scherrer and J. Toore, *Science* 272 (1996) 1281.
- [2] W. Unno, Y. Osaki, H. Ando, H. Saio and H. Shibahashi, "Non radial oscillations of stars", University of Tokio Press, Tokio 1989.
- [3] D. O. Gough and J. Toomre, *Ann. Rev. Astron. Astroph.* 29 (1991) 627.
- [4] J. Christensen-Dalsgaard, D. O. Gough and M. J. Thompson *Ap. J.* 378 (1991) 413.
- [5] J. Christensen-Dalsgaard, *Nucl. Phys. B (Proc. Suppl.)* 48 (1996) 325.
- [6] W.J.Dziembowski, *Bull. Astr. Soc. India*, 24 (1996) 133.
- [7] W. J. Dziembowsky, S. Degl'Innocenti, G. Fiorentini and B. Ricci, *Astr. Phys.* (1997) to appear.
- [8] J. N. Bahcall and M. H. Pinsonneault, S. Basu and J. Christensen-Dalsgaard, *Phys. Rev. Lett.* 78 (1997) 171; J. N. Bahcall, S. Basu and P. Kumar, *ApJ. Lett.* (1997) to appear.
- [9] J. N. Bahcall and M. H. Pinsonneault, *Rev. Mod. Phys* 67 (1995) 781.
- [10] S. Degl' Innocenti, F. Ciacio and B. Ricci, *Astr. Astroph. Suppl. Ser.* 123 (1997) 1.
- [11] J. Christensen-Dalsgaard et al. , *Science* 272 (1996) 1286.
- [12] O.Richard, S.Vauclair, C.Charbonnel and W.J.Dziembowski, *Astr. Astroph.* 312 (1996) 1000.
- [13] C. A. Iglesias and F. J. Rogers, *Ap. J. Suppl.* 79 (1992) 507; C. A. Iglesias and F. J. Rogers, and B. G. Wilson *Ap. J.* 397 (1992) 717.
- [14] C. A. Iglesias and F. J. Rogers, *Ap. J.* 464 (1996) 943.
- [15] S. C. Tripathy, S. Basu and J. Christensen-Dalsgaard, *astro-ph/9703170* (1997).
- [16] C. Arpesella et al., LUNA collaboration, *Phys. Lett. B* (1997) to appear.
- [17] C. W. Johnson, E. Kolbe, S. E. Koonin and K. Langanke, *Ap. J.* 392 (1992) 320.
- [18] R. W. Kavanagh, T. A. Tombrello, J. M. Mosher and D. R. Goosman, *Bull. Am. Phys. Soc.* 14 (1969) 1209.
- [19] B. W. Filippone, A. J. Elwyn, C. N. Davids and D. D. Koetke, *Phys. Rev. C* 28 (1983) 2222.
- [20] T. Motobayashi et al., *Phys. Rev. Lett.* 20 (1994) 2680; *J. Phys. Soc. Jpn.* 65 (1996) 25.
- [21] K. Langanke and T. D Shoppa, *Phys. Rev. C* 49 (1994) 1771;
errata Phys. Rev. C 51 (1995) 2844.
- [22] M. Gai and C. A. Bertulani, *Phys. Rev. C* 52 (1995) 1706.
- [23] K. Langanke and T. D. Shoppa, *Phys. Rev. C* 52 (1995) 1709.
- [24] V. Castellani, S. Degl'Innocenti and G. Fiorentini, *Astr. Astr.* 271 (1993) 601.
- [25] V. Castellani, S. Degl'Innocenti, G. Fiorentini, M. Lissia and B. Ricci, *Phys. Rep* 281 (1997) 309.
- [26] Kamiokande coll., *Phys. Rev. Lett.* 77 (1996) 1683
- [27] Y. Totsuka, preprint 1997, based on the talk at TEXAS Symposium, Chicago, December 1996.
- [28] X.Shi, D.N.Schramm and D.S.Deaborn, *Phys. Rev. D* 50 (1994) 2414.
- [29] A.Dar and G.Shaviv, Preprint Technion Ph-94-5 (1994)
- [30] V.Berezinsky, *Comm. Nucl. Part. Phys.* 21 (1994) 249
- [31] P.I.Krastev and A.Yu.Smirnov, *Phys. Lett. B* 338 (1994) 282,
V.Berezinsky, G.Fiorentini and M.Lissia, *Phys. Lett. B* 341 (1994) 38.

[32] V.N.Tsyтович et al, *Astrop. Phys.* 5 (1996) 197.

TABLES

TABLE I. Seismically determined quantities, Q , and the exponents $\alpha_{Q,P}$ of the scaling approximation given by Eq.(1).

Q	Q_{\odot}	$\alpha_{Q,k}$	$\alpha_{Q,z}$	$\alpha_{Q,s}$	$\alpha_{Q,t}$
Y_{ph}	0.249 ($1 \pm 4.2\%$)	0.61	0.31	0.14	0.20
R_b/R_{\odot}	0.711 ($1 \pm 0.4\%$)	-0.0084	-0.046	-0.058	-0.080
ρ_b [gr/cm ³]	0.192 ($1 \pm 3.7\%$)	0.095	0.472	0.86	0.85

TABLE II. Predictions for neutrino fluxes and signals in the Cl and Ga detectors from SSM and HCSM. Uncertainties corresponding to $(\Delta T/T)_{HCSM} = \pm 1.4\%$ are shown (first error) together with those from nuclear cross sections (second error).

		SSM				HCSM
		BP95	FR97	FR96	JCD	
Φ_{Be}	[10 ⁹ /cm ² /s]	5.15	4.49	4.58	4.94	4.81±0.53 ± 0.59
Φ_B	[10 ⁶ /cm ² /s]	6.62	5.16	5.28	5.87	5.96±1.49 ± 1.93
Cl	[SNU]	9.3	7.3	7.5	8.2	8.4±1.9 ± 2.2
Ga	[SNU]	137	128	129	132	133±11 ± 8

FIGURES

FIG. 1. For a few recent SSM calculations we present : a) the predicted central solar temperatures T_{SSM} (diamonds) with the conservative uncertainty (thin bars); b) the values T_{HCSM} derived by best fit of helioseismic observables at the convective radius (circles) with the uncertainties (bars) calculated in the same way as for SSM's. Labels indicating solar models are defined in the text.

FIG. 2. In the (z,k) plane we show the position of the best-fit HCSM (diamond) and of the models consistent with helioseismic data (crosses) obtained by using BP95 as a starting model. The curves labelled by ρ_b , Y_{ph} and R_b show the helioseismic constraints due to these quantities. Curves labelled by the numbers are isothermperature curves in kz plane. Numbers are values of T/T_{SSM} .

FIG. 3. Logarithmic derivates of the isothermal sound speed squared U with respect to S_{11} (dot-dashed line), opacity (dashed line) and Z/X (full line). Note that the opacity coefficient have been multiplied by a factor 10.