Available online at www.sciencedirect.com
ScienceDirect
Nuclear Instruments and Methods in Physics Research A 577 (2007) 745–750
NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A
www.elsevier.com/locate/nima

Experimental requirements for X-ray compact free electron lasers with a laser wiggler

R. Bonifacio[a,b], N. Piovella[a,c,*], M.M. Cola[a,c], L. Volpe[a,c]

[a] INFN-Sezione di Milano, Via Celoria 16, 20133 Milano, Italy
[b] Centro Brasileiro de Pesquisas Fisicas, Rio de Janeiro, Brazil
[c] Dipartimento di Fisica, Università degli Studi di Milano,via Celoria 16, 20133 Milano, Italy

Received 8 March 2007; received in revised form 23 March 2007; accepted 24 March 2007
Available online 28 March 2007

Abstract

In this paper we specify the experimental parameters required to operate a Free Electron Laser with a laser wiggler in the Angstrom region. Both the quantum and the classical regimes are discussed. The quantum regime of SASE can be reached with more realistic parameters than the classical one. The fundamental feature of the quantum SASE is the extremely narrow single-line radiation spectrum, whose line width can be four orders of magnitude smaller than the bandwidth of the classical spiky SASE spectrum.

PACS: 41.60.Cr; 42.50.Fx

Keywords: FEL; Laser wiggler; X-ray source; Quantum recoil

1. Introduction

It has been shown that the quantum effects in a Free Electron Laser (FEL) are ruled by the quantum FEL parameter $\bar{\rho} = \rho(mc\gamma/\hbar k)$ [1], where ρ is the classical FEL parameter [2]. The classical analysis is valid only for $\bar{\rho} \gg 1$, whereas for $\bar{\rho} < 1$ the quantum effects dominate [3,4].

In particular, in the quantum Self Amplified Spontaneous Emission (SASE) mode operation, the quantum purification of the radiation spectrum has been predicted [3,4], i.e. the broad and chaotic spectrum of the classical SASE [5–8] shrinks to a very narrow spectrum when $\bar{\rho} \ll 1$. More specifically, in this work we show that the spectrum is a single line whose width is $\Delta\omega/\omega = \lambda_r/L_b$ instead of 2ρ as in classical SASE. The ratio between the two line-widths is the total number of spikes in the classical regime, $L_b/(2\pi L_c)$ where L_b is the bunch length and L_c is the cooperation length [9]. This means that the quantum line width is the same as the width of the single spike of the classical regime. In the X-ray region this implies a difference of four orders of magnitude. Hence, the quantum regime, contrary to the classical one, exhibits a full temporal coherence.

It has been suggested that a quantum SASE FEL could be built using a laser wiggler [10,11] in a Compton backscattered configuration, instead of the static wiggler used in the planned classical SASE experiments [12–14]. In a laser wiggler configuration, a low-energy electron beam back scatters the photons of a counter-propagating high power laser, with a frequency up-shifted by a factor $4\gamma^2$. The use of a laser wiggler has been discussed in the past [15,16] in a classical theory.

In the following, we propose a way to calculate the experimental parameters for an X-ray FEL with a laser wiggler both in the classical and in the quantum regime. The analysis shows that the quantum regime appears, in general, more feasible than the classical regime for the state-of-art of the electron beam and laser technologies. We outline again that only in the quantum SASE regime the FEL is a temporally coherent X-ray source, whereas in the classical SASE regime the FEL radiation spectrum is composed by many random spikes with little temporal

*Corresponding author. Dipartimento di Fisica, Università degli Studi di Milano, via Celoria 16, 20133 Milano, Italy. Tel.: +390250317266; fax: +390250317675.
E-mail address: nicola.piovella@mi.infn.it (N. Piovella).

0168-9002/$ - see front matter
doi:10.1016/j.nima.2007.03.024

coherence. Furthermore, the quantum FEL with a laser wiggler would be two or three orders of magnitude smaller in size (possibly table-top) and cost. The results of the present parametric study are rather encouraging with respect to a future realization of a quantum SASE X-ray FEL source.

2. Magnetic versus laser wiggler

The quantum FEL (QFEL) parameter is given by [1]

$$\bar{\rho} = \rho \frac{mc\gamma}{\hbar k} = \gamma\rho \frac{\lambda_r}{\lambda_c}, \tag{1}$$

where λ_r and $\lambda_c = h/mc \approx 0.024\,\text{Å}$ are, respectively, the radiation and the Compton wavelengths, γ is the resonance energy in units mc^2 given by

$$\gamma = \sqrt{\frac{\lambda_w(1+a_0^2)}{2\lambda_r}}, \tag{2}$$

λ_w is the period of the magnetic wiggler and a_0 is the wiggler parameter. Using Eqs. (1) and (2), the QFEL condition $\bar{\rho} \leqslant 1$ becomes

$$\rho \leqslant \frac{\sqrt{2}\lambda_c}{\sqrt{\lambda_r\lambda_w(1+a_0^2)}}, \tag{3}$$

so that, to reach the high-gain regime, a number N_w of wiggler periods of the order of $1/\rho$ is required, i.e., a wiggler length L_w given by

$$L_w = N_w\lambda_w \approx \frac{\lambda_w}{\rho} \geqslant \frac{\sqrt{\lambda_r\lambda_w^3(1+a_0^2)}}{\sqrt{\lambda_c}}. \tag{4}$$

If a laser wiggler with wavelength λ_L is used, the substitution

$$\lambda_w \to \frac{\lambda_L}{2} \tag{5}$$

must be done everywhere in Eqs. (2)–(4). For instance, the resonance condition (2) becomes

$$\gamma = \sqrt{\frac{\lambda_L(1+a_0^2)}{4\lambda_r}}. \tag{6}$$

To lase at $\lambda_r = 1\,\text{Å}$, Eqs. (2)–(4) for a magnetic wiggler with $\lambda_w \approx 1$ cm yield $E = 3.5$ GeV, $\rho \leqslant 3.4\times10^{-6}$ and $L_w \geqslant 3$ km, whereas for a laser wiggler with $\lambda_L \approx \mu$m, they yield $E = 25$ MeV, $\rho \leqslant 5\times10^{-4}$ and $L_w \geqslant 2$ mm (for simplicity we have assumed $a_0 \ll 1$). The previous considerations clearly show that a QFEL with a magnetic wiggler is unpractical, whereas with a laser wiggler it can be a table-top apparatus for a sufficiently high quality beam, as it will be discussed in the following.

3. Scaling laws for a laser wiggler

In the following we will discuss the scaling laws and possible experimental parameters for an FEL with a laser wiggler both in the classical and quantum regimes.

Using (1) and (6) the quantum FEL parameter $\bar{\rho}$ is related to the classical FEL parameter ρ by

$$\rho = \bar{\rho}\frac{\lambda_c}{\gamma\lambda_r} = \bar{\rho}\frac{2\lambda_c}{\sqrt{\lambda_r\lambda_L(1+a_0^2)}}. \tag{7}$$

In Eq. (7) ρ is given by

$$\rho = \frac{1}{2\gamma}\left(\frac{I}{I_A}\right)^{1/3}\left(\frac{k_e\lambda_L a_0}{4\pi\sigma}\right)^{2/3}, \tag{8}$$

where $I_A \approx 17$ kA is the Alfven current. Assuming as current density the peak current I divided by the effective surface $2\pi\sigma^2$ for a transversally Gaussian or $\pi\sigma^2$ for a flat top shape of the current, the parameter k_e is 1 or $\sqrt{2}$, respectively. Here σ is the rms electron beam radius. Eq. (8) is a generalization of the usual expression (see for instance Ref. [17]) to a laser wiggler.

From Eqs. (7) and (8) with some algebra we obtain

$$I(A) = 3\times10^2\frac{\bar{\rho}^3\sigma^2}{k_e^2\lambda_r^3\lambda_L^2a_0^2}, \tag{9}$$

where the units are λ_r (Å), λ_L (μm) and σ (μm). For a given σ, the electron current is proportional to $\bar{\rho}^3$, so that, going from the quantum to the classical regime, if $\bar{\rho}$ increases for instance by a factor 10, the current increases by a factor 10^3. This is one of the reasons why the use of a laser wiggler may be much more convenient in the quantum regime ($\bar{\rho} < 1$) than in the classical one ($\bar{\rho} \gg 1$).

The relation between a_0 and the laser power P is, in agreement with [16],

$$P(\text{TW}) = \left(\frac{Ra_0}{2.4k_L\lambda_L}\right)^2, \tag{10}$$

where R is the minimum radius of the laser, P is in TW and k_L is, as above, 1 or $\sqrt{2}$ for a Gaussian (with beam section $2\pi R^2$) or for a flat top (with beam section πR^2) transverse profile of the laser, respectively. From Eqs. (9) and (10), we obtain the important relation between the electron current and the laser power:

$$I(\text{A}) \approx 50\bar{\rho}^3\frac{R^2\sigma^2}{k_e^2k_L^2\lambda_L^4\lambda_r^3}\frac{1}{P(\text{TW})}, \tag{11}$$

with λ_r(Å), λ_L(μm), R(μm) and σ(μm). Using Ref. [10], the gain length and the cooperation length for the field can be written in the form

$$L_g = \frac{\lambda_L}{8\pi\rho}\sqrt{\frac{1+\bar{\rho}}{\bar{\rho}}}, \quad L_c = \frac{\lambda_r}{4\pi\rho}\sqrt{\frac{1+\bar{\rho}}{\bar{\rho}}}, \tag{12}$$

where the factor $\sqrt{1+\bar{\rho}}$ in the numerator has been added by hand to obtain the classical expression when $\bar{\rho} \gg 1$ and the quantum expression when $\bar{\rho} \ll 1$. Note that L_g has a factor 8π instead of 4π in the denominator, since a laser wiggler is assumed (see Eq. (5)).

We impose that the interaction length, $L_{\rm int}$, the laser Rayleigh range, $Z_{\rm L}$, and the gain length for the field, $L_{\rm g}$, should satisfy the following relations:

$$L_{\rm int} = c\tau_{\rm L} \approx 2Z_{\rm L} = a_1 L_{\rm g}, \quad a_1 \geqslant 1, \tag{13}$$

where a_1 is the number of $L_{\rm g}$ in the interaction region $2Z_{\rm L}$, $\tau_{\rm L}$ is the laser pulse duration and

$$Z_{\rm L} = \frac{4\pi R^2}{\lambda_{\rm L}} \tag{14}$$

is the laser Rayleigh range. Hence, the total energy of the laser pulse is given by

$$U = P\tau_{\rm L} = a_1 P \frac{L_{\rm g}}{c}. \tag{15}$$

From Eqs. (13) and (14), we obtain the following self-consistent value for the rms laser radius at the focus:

$$R = \sqrt{\frac{a_1 L_{\rm g} \lambda_{\rm L}}{8\pi}}. \tag{16}$$

Note that the gain length for the power is half of the gain length for the field, given by Eq. (12). For instance, for $a_1 = 5$ the interaction length is ten times the gain length for the power.

Concerning the requirement on the beam emittance, a very important geometrical matching condition is the following:

$$\beta^* \equiv \frac{\gamma\sigma^2}{\varepsilon_{\rm n}} \geqslant Z_{\rm L}, \tag{17}$$

where $\varepsilon_{\rm n}$ is the normalized beam emittance. Eq. (17) imposes that the electron beam should be contained in the laser beam, provided $\sigma \leqslant R$, and the electron beam should not diverge appreciably in a laser Rayleigh range $Z_{\rm L}$. From Eqs. (14) and (17), it follows:

$$\varepsilon_{\rm n} \leqslant \varepsilon_{\rm n}^{\rm (hom)} \equiv \frac{\gamma\lambda_{\rm L}}{4\pi}\left(\frac{\sigma}{R}\right)^2. \tag{18}$$

This is the correct condition on the emittance to be satisfied in a laser wiggler, which becomes quite restrictive when $\sigma \ll R$.

As discussed in Ref. [11], the emittance criterium for the FEL radiation, $\varepsilon_{\rm n} < \gamma\lambda_{\rm r}/4\pi$, does not apply in a laser wiggler, since it would imply $\beta^* > Z_{\rm r}$ where $Z_{\rm r} = 4\pi\sigma^2/\lambda_{\rm r}$ is the Rayleigh range of the FEL radiation (where, for simplicity, we have assumed an equal radius for the radiation and the electron beams), so that the emitted radiation would get outside of the electron beam, making impossible the amplification process. To avoid this, we should reverse the criterion, i.e. $\beta^* < Z_{\rm r}$, so that $\varepsilon_{\rm n} > \gamma\lambda_{\rm r}/(4\pi)$.

Furthermore, defining

$$a_2 = \frac{R}{\sigma} \geqslant 1, \tag{19}$$

from Eq. (18) it follows

$$\varepsilon^{\rm (hom)} \equiv \frac{\gamma\lambda_{\rm L}}{4\pi a_2^2}. \tag{20}$$

Up to now, all we have written is valid either in the classical or in the quantum regime. In both the cases, the condition on the energy spread is

$$\frac{\Delta\gamma}{\gamma} < \Gamma, \tag{21}$$

where Γ is the FEL line width. In Ref. [3] we have estimated that the line width in the quantum regime is

$$\Gamma \approx \rho\sqrt{\bar{\rho}} \quad \text{if } \bar{\rho} < 1, \tag{22}$$

whereas in the classical regime it is the well known expression [2]

$$\Gamma \approx \rho \quad \text{if } \bar{\rho} \gg 1. \tag{23}$$

Emittance is one of the causes of the energy spread increasing. In fact, since the resonant wavelength depends on the divergence angle θ according to

$$\lambda_{\rm r} = \frac{\lambda_{\rm L}(1 + a_0^2 + \gamma^2\theta^2)}{4\gamma^2}, \quad \text{with} \quad 0 \leqslant \theta \leqslant \frac{\varepsilon_{\rm r}}{\sigma} \tag{24}$$

we have

$$\frac{\Delta\lambda}{\lambda_{\rm r}} \approx \frac{2\Delta\gamma}{\gamma} \approx \frac{\varepsilon_{\rm n}^2}{\sigma^2(1 + a_0^2)} \leqslant 2\Gamma. \tag{25}$$

Hence, we obtain the following ‘inhomogeneous’ condition for emittance:

$$\varepsilon_{\rm n} \leqslant \varepsilon_{\rm n}^{\rm (in\,hom)} \equiv \sigma\sqrt{2\Gamma(1 + a_0^2)}. \tag{26}$$

Eq. (26), using Eq. (12), can be combined with the geometrical condition $\varepsilon_{\rm n} > \gamma\lambda_{\rm r}/(4\pi)$ to give [11,18]:

$$\frac{\gamma\lambda_{\rm r}}{4\pi} < \varepsilon_{\rm n} \leqslant \frac{\gamma\lambda_{\rm r}}{2\pi}\sqrt{\frac{Z_{\rm r}}{L_{\rm g}}}. \tag{27}$$

We remark that the inequality (18) must be strictly satisfied, otherwise the FEL action is destroyed. The inequality (26) arises from an inhomogeneous broadening of the resonance, which reduce the gain, since only the electrons whose θ is small enough will participate to the emission process [19]. There are many different effects which may contribute to the broadening of the resonance, such as the fluctuations of the wiggler parameter a_0 and of the laser wavelength $\lambda_{\rm L}$ or the variation of a_0 due to the laser section expansion away from the waist (this can be seen as a kind of wiggler tapering or anti-tapering, as discussed in Ref. [15]). All these effects will be investigated in a future paper with a full 3D analysis. In particular, the fluctuations of the wiggler parameter a_0 are negligible if

$$\frac{\Delta a_0}{a_0} \leqslant \frac{1 + a_0^2}{a_0^2}\Gamma, \tag{28}$$

where Eqs. (6) and (21) have been used.

The peak power of the FEL radiation is given by [2]

$$P_r = P_{beam}(\rho|A|^2) = (I/e)\hbar\omega\bar{\rho}|A|^2, \tag{29}$$

where A is the dimensionless field amplitude in the 'universal scaling' and $P_{beam} = (I/e)mc^2\gamma$ is the beam power. At saturation in the classical regime $|A|^2 \approx 1$ [2] whereas in the quantum regime $|A|^2 \approx 1/\bar{\rho}$ [20], so that, in a unique form, we can write

$$|A|^2 \approx 1 + \frac{1}{\bar{\rho}} \tag{30}$$

and Eq. (29) with (30) yield

$$P_r \approx (I/e)\hbar\omega(1+\bar{\rho}). \tag{31}$$

As a consequence the number of emitted photons is

$$N_{ph} = \frac{Q}{e}(1+\bar{\rho}), \tag{32}$$

where Q is the beam charge. Eq. (32) shows that the average number of the emitted photons per electron at saturation is $1+\bar{\rho}$. Note that the power in the quantum regime is larger than the value predicted by the classical theory. The meaning of Eqs. (31) and (32) is that in the classical regime each electron emits in average $\bar{\rho}$ photons, whereas in the quantum regime it emits exactly a single photon. As a consequence, an FEL operating in the quantum regime will produce less photons than in the classical regime, but with a monochromatic spectrum, as it will be discussed in the next section.

4. Radiation line width

The maximum induced energy spread in an FEL is

$$\frac{\sigma(\gamma)}{\gamma} \approx \rho, \tag{33}$$

which can be written, using the definition (1) of $\bar{\rho}$, in terms of momentum spread

$$\frac{\sigma(p_z)}{\hbar k} = \frac{mc\sigma(\gamma)}{\hbar k} \approx \bar{\rho}, \tag{34}$$

where $p_z = mc(\gamma-\gamma_r)$ and γ_r is the resonant electron energy in mc^2 units. The QFEL parameter $\bar{\rho}$ can be interpreted as the ratio between the classical momentum spread and the photon recoil $\hbar k$, so that quantum effects become important when $\bar{\rho} \leqslant 1$, since in this case the discreteness of momentum exchange is relevant. This allows to explain in a simple way the origin of the broad and spiky classical spectrum and the reduction to a single line in the quantum regime. The radiation emission is due to the transition between adjacent recoil momentum states ($p_z^{(n)} = n\hbar k$), which are equally spaced by the photon momentum $\hbar k$. The emitted frequencies in the transitions $n \to n-1$ are also equally spaced, since they are proportional to the difference between the corresponding kinetics energies. In the classical regime ($\bar{\rho} \gg 1$) many momentum states become occupied (see Eq. (34)), and the multiple transitions between the different momentum states lead to a multi-frequency spectrum with equally spaced lines and an envelope width equal to $\Delta\omega/\omega \approx 2\sigma(\gamma)/\gamma \approx 2\rho$. The several transitions $n \to n-1$ occur randomly under the gain curve and this leads to the multiple-line chaotic spectrum observed in the classical SASE. Note also that, since the radiation is emitted in a time L_b/c (where $L_b = c\tau_b = cQ/I$ is the beam length), each line has a Fourier transformed line width $\Delta\omega/\omega \approx \lambda_r/L_b$. Hence, the number of spikes in the classical regime is [5]

$$N_S = \frac{2\rho L_b}{\lambda_r} = \frac{L_b}{2\pi L_c}, \tag{35}$$

where $L_c = \lambda_r/4\pi\rho$ is the classical cooperation length. Conversely, in the quantum regime $\bar{\rho} < 1$, $\sigma(p_z)$ cannot be larger than the photon recoil $\hbar k$ and a single transition occurs, whose line width is

$$\left(\frac{\Delta\omega}{\omega}\right)_{QFEL} \approx \frac{\lambda_r}{L_b}. \tag{36}$$

Hence, the ratio between the quantum line width and the classical line width 2ρ is given by Eq. (35). For instance, for a beam with $Q = 1\,nC$ and $\tau_b = 1$ ps in the Angstrom region the QFEL line width (see Eq. (36)) is of the order of 10^{-7}, whereas the line width of the envelope of the classical SASE spectrum is of the order of 10^{-3}. Hence QFEL naturally produces transform-limited radiation, which would be useful for ultra-high resolution studies of processes.

5. Parameters for the classical and the quantum regimes

In order to discuss some specific example for the classical and the quantum regimes, we take as independent the following six parameters: $\bar{\rho}$, λ_r(Å), λ_L(μm), a_0 (the wiggler parameter), a_1 (the number of amplitude gain lengths in the interaction region $2Z_L$) and a_2 (the ratio between the laser and the radiation rms beam radius at the focal point). Furthermore, we assume a given beam charge $Q = 1$ nC. All the other parameters are deduced self-consistently as follows. From Eq. (6) and (7) we deduce γ and ρ, and from Eq. (12) we deduce L_g and L_c. Then, from Eqs. (16) and (19) we deduce R and σ. Introducing these values in Eqs. (9), (10) and (15) we calculate the current I, the laser power P and the laser duration τ_L. From Eqs. (20) and (26), using Eqs. (22) or (23) for Γ, we deduce the homogeneous and inhomogeneous emittance limit values. Finally, from Eq. (32) we calculate the total number of emitted photons. We remark that the values of R and σ cannot be arbitrarily chosen, but must be deduced consistently by the independent parameters of the system (see also the scaling laws given in Appendix A). A different but less accurate approach to the classical case with similar parameters has been recently published [21].[1]

[1] Ref. [21] contains some incorrect and misleading results, which can generate confusion in the scientific community: (a) As it is well known, the spectrum of the SASE radiation is affected by random spikes when the electron beam contains many cooperation lengths [5]. Up to now, this spiking behavior has been confirmed by all the numerical simulations [12–14] and experiments [6–8]. For the case discussed in Ref. [21] we

Table 1

QFEL parameter	$\bar{\rho}$	0.2	5
$(2Z_L)/L_g$	a_1	5	5
R/σ	a_2	1.5	1.5
Laser wave length	λ_L (μm)	1	1
Radiation length	λ_r (Å)	1.5	1.5
Wiggler parameter	a_0	0.3	0.8
FEL parameter	ρ	7.5×10^{-5}	1.5×10^{-3}
Gain length	L_g (mn)	1.2	0.03
Cooperation length	L_c (nm)	356	3.5
Laser Rayleigh range	Z_L (mm)	3	0.07
Laser radius	R (μm)	15.4	2.4
Laser power	P_L (TW)	1.85	0.32
Laser duration	τ_L (ps)	19.7	0.47
Interaction length	L_{int} (mm)	6	0.14
E-beam energy	γ	42.6	52.3
E-beam radius	σ (μm)	10.3	1.6
Bunch length	L_b (μm)	720	13.6
Peak current	I (kA)	0.42	22
Emittance hom limit	$\varepsilon_n^{(hom)}$ (mm mrad)	1.5	1.85
Emittance inhom limit	$\varepsilon_n^{(in)}$ (mm mrad)	0.09	0.11
Gain band width	Γ	3.4×10^{-5}	1.5×10^{-3}
FEL line width	$\Delta\omega/\omega$	2.1×10^{-7}	3.1×10^{-3}
Number of spikes	N_s	1	278
FEL power	P_r (MW)	3.45	912
Photons' number	N_{ph}	6.2×10^{9}	3.1×10^{10}
Peak brilliance	B^a	10^{28}	1.6×10^{26}

[a]photons/(s mm^2)mrad^2 0.1% BW).

In Table 1 we give two numerical examples for a quantum case ($\bar{\rho}=0.2$) and a classical case ($\bar{\rho}=5$), considering a laser wiggler and an electron beam both with transverse and longitudinal flat-top profiles ($k_L=k_e=\sqrt{2}$), $a_1=5$, $a_2=1.5$ and $\lambda_L=1\,\mu$m. In the quantum case we have chosen a wiggler parameter $a_0=0.3$, whereas in the classical case we assumed $a_0=0.8$, as in ref. [21]. Notice that the self-consistent value of the peak current in the classical case results irrealistic (about 20 kA) and the correspondent beam radius (1.6 μm) is very small. Conversely, in the quantum regime the peak current and the beam radius have feasible values. On the base of these considerations, we may state that it should be easier to operate an FEL with a laser wiggler in the quantum regime rather than in the classical one. This explains also why, up to now, FELs with a laser wiggler have not been realized experimentally in the classical regime. Finally, we observe that for high current and low energy beams the space charge effect (neglected in the present analysis) could become relevant [22]. This effect, together with all the other non ideal conditions, will be investigated in a full 3D quantum FEL model, including also the microscopic space charge term [23].

6. Conclusions

On the basis of the analysis and of the examples of experimental parameters considered above, we conclude that the quantum regime of an FEL with a laser wiggler can be a convenient X-ray source, since the emitted radiation has the important property of high temporal coherence with no spiking, whereas in the classical regime many random spikes are expected. This is the fundamental difference between the quantum and the classical regimes. Furthermore, the full line width of the spectrum in the classical regime can be three or four order of magnitude larger than the single spectral line width obtained in the quantum regime. As a consequence, in the quantum regime the brilliance can be largely enhanced with respect to the classical one [13,14]. In conclusion, the quantum regime not only appears easier to be reached experimentally than the classical regime, but its properties definitely allows to claim that QFEL is a full temporal, compact, coherent X-ray source, whose dimension and cost could be three orders of magnitude less than the large and expensive present projects of classical SASE-FELs [13,14], which will produce temporally incoherent X-ray light. Although beams with emittance below the required inhomogeneous limit, Eq. (26), are presently far from being produced, the big advantages of QFEL motivate a large effort for its realization. In any case, even if the inhomogeneous limitation is not met, the FEL action would occur anyway but at a power reduction, since only the electrons within the gain line width participate to the lasing process. However, this reduction is compensated by the high degree of coherence of the emitted radiation. On the contrary,

(*footnote continued*)
should expect about 500 spikes. However, no evidence of these spikes is reported in the paper and no plausible reason is given for their disappearance. (b) The expression of ρ and L_g are incorrect, since they are just the same of those for a static wiggler without the necessary substitution $\lambda_w=\lambda_L/2$. Hence, L_g is missing a factor 2, whereas ρ is smaller by a factor $2^{2/3}\sim1.6$. As a consequence, $\bar{\rho}$ should be decreased by a factor 1.6 and their "classical" regime (obtained for $\bar{\rho}=2$) describes actually a quantum case with $\bar{\rho}\sim1$, so that the classical model cannot be used. (c) Referring to Ref. [3], the authors incorrectly state: "a recent *numerical calculation* has shown that quantum effects appears appreciable if $\bar{\rho}\geqslant0.4$". A part of the misprint ($\bar{\rho}\leqslant0.4$), in Ref. [3] we have demonstrated *analytically* that the classical limit occurs for $\bar{\rho}\gg1$ (showing that, for $\bar{\rho}\to\infty$, a finite difference term in the equation for the Wigner function becomes a continuous derivative, originating a classical Vlasov equation). The value 0.4 appears in a subsequent paper [4], where we demonstrated analytically that the quantum spectrum, made up by discrete lines, becomes continuous like as the classical spectrum when $\bar{\rho}\geqslant0.4$, since in this limit the line width becomes larger than the lines separation. However, the spectrum is still far to be classical. Hence, the classical theory, strictly speaking, can be used only when $\bar{\rho}\gg1$. (d) The parameters used for simulations in Ref. [21] are: $a_0=0.8$, $R=50\,\mu$m, $\sigma=10\,\mu$m, $\lambda_L=0.8\,\mu$m, $\lambda_r=3.64\,$Å, $\tau_{int}=5\,$ps, $\gamma=30$, $\rho=4.3\times10^{-4}$, $\bar{\rho}=2$, $I=1.5\,$kA and 0.3 mm mrad $<\varepsilon_n<0.88$ mm mrad. The laser power and the energy are not given. Moreover, only the inhomogeneous emittance limitation of Eqs. (26) and (27) is considered, whereas the more fundamental homogenous emittance limitation of Eq. (18) is ignored. Using their parameters in Eqs. (9), (10) and (15), we obtain $I=6\,$kA, $P=200\,$TW, $U=10^3\,$J. These values appear rather difficult to be obtained. (e) Discussing the inhomogeneous emittance criterium, the authors neglect a_0^2 in the denominator in their Eq. (26) (which is the analogue of our Eq. (26)), and this would be correct only if $a_0^2\ll1$, whereas it is not for $a_0\approx0.8$.

if the homogeneous limit, Eq. (18), is not met, the FEL action is suppressed.

Acknowledgments

We thank the Editor, Dr. William Barletta, whose many comments and suggestions have improved the quality of this work.

Appendix A. Scaling laws

Here we give the formulae, obtained using the chain of equations discussed in Section 5, for the numerical value of the main experimental parameters, expressed as a function of the independent parameters $\bar{\rho}$, $\lambda_r(\text{Å})$, $\lambda_L(\mu\text{m})$, a_0 (the wiggler parameter), a_1 (the number of amplitude gain lengths in the interaction region $2Z_L$), a_2 (the ratio between the laser and the radiation rms beam radius at the focal point) and $k_e(k_L) = 1$ or $= \sqrt{2}$ for a gaussian or a flat top electron (laser) beam transverse profile:

$$\gamma \approx 50\sqrt{\frac{\lambda_L}{\lambda_r}(1+a_0^2)}, \tag{A.1}$$

$$\rho \approx 4.8 \times 10^{-4}\frac{\bar{\rho}}{\sqrt{\lambda_r\lambda_L(1+a_0^2)}}, \tag{A.2}$$

$$L_g(\text{mm}) \approx 8.3 \times 10^{-2}\sqrt{\lambda_r\lambda_L^3(1+a_0^2)\frac{1+\bar{\rho}}{\bar{\rho}^3}}, \tag{A.3}$$

$$\sigma(\mu\text{m}) = \frac{R(\mu\text{m})}{a_2} \approx 1.8\frac{\sqrt{a_1}}{a_2}\left[\lambda_r\lambda_L^5(1+a_0^2)\frac{1+\bar{\rho}}{\bar{\rho}^3}\right]^{1/4}, \tag{A.4}$$

$$P(\text{TW}) \approx 0.57\frac{a_1 a_0^2}{k_L^2}\sqrt{\lambda_r\lambda_L(1+a_0^2)\frac{1+\bar{\rho}}{\bar{\rho}^3}}, \tag{A.5}$$

$$\tau(\text{ps}) \approx 0.27a_1\sqrt{\lambda_r\lambda_L^3(1+a_0^2)\frac{1+\bar{\rho}}{\bar{\rho}^3}}, \tag{A.6}$$

$$I(\text{A}) \approx 989\frac{a_1}{a_2^2 a_0^2 k_e^2}\sqrt{\frac{\lambda_L}{\lambda_r^5}(1+a_0^2)\bar{\rho}^3(1+\bar{\rho})}, \tag{A.7}$$

$$\varepsilon_n^{(\text{hom})}(\text{mm mrad}) \approx \frac{4}{a_2^2}\sqrt{\frac{\lambda_L^3}{\lambda_r}(1+a_0^2)}, \tag{A.8}$$

$$\varepsilon_n^{(\text{inhom})}(\text{mm mrad}) \approx 0.06\frac{\lambda_L}{a_2}\sqrt{a_1(1+a_0^2)}. \tag{A.9}$$

We observe that, in the classical regime, for large values of the QFEL parameter $\bar{\rho}$, the electron beam section (see Eq. (A.4)) decreases as $1/\bar{\rho}$, and the peak current I in Eq. (A.7) increases as $\bar{\rho}^2$, so that the current density increases as $\bar{\rho}^3$. Furthermore, if a_0 increases the laser power increases, but the peak current strongly decreases and the emittance limitations become more relaxed.

References

[1] R. Bonifacio, F. Casagrande, Nucl. Instr. and Meth. A 237 (1985) 168.
[2] R. Bonifacio, C. Pellegrini, L.M. Narducci, Opt. Commun. 50 (1984) 373.
[3] R. Bonifacio, N. Piovella, G.R.M. Robb, Nucl. Instr. and Meth. A 543 (2005) 645.
[4] R. Bonifacio, N. Piovella, G.R.M. Robb, A. Schiavi, Phys. Rev. ST Accel. Beams 9 (2006) 090701.
[5] R. Bonifacio, L. De Salvo, P. Pierini, N. Piovella, C. Pellegrini, Phys. Rev. Lett. 73 (1994) 70.
[6] M.J. Hogan, et al., Phys. Rev. Lett. 81 (1998) 4867.
[7] V. Ayvaziyan, et al., Phys. Rev. Lett. 88 (2002) 104802.
[8] A. Murokh, et al., Nucl. Instr. and Meth. A 507 (2003) 417.
[9] R. Bonifacio, F. Casagrande, Nucl. Instr. and Meth. A 239 (1985) 36.
[10] R. Bonifacio, Nucl. Instr. and Meth. A 546 (2005) 634.
[11] R. Bonifacio, M. Ferrario, G.R.M. Robb, N. Piovella, A. Schiavi, L. Serafini, Proceedings of the 27th International Free Electron Laser Conference, 2005, p. 71.
[12] A VUV Free Electron Laser at the TESLA Test Facility at DESY. Conceptual Design Report, DESY, TESLA-FEL 95-03, 1995.
[13] The LCLS Design Study Group: LCLS Design Study Report, SLAC-R521, Stanford, 1998, ⟨http://www-ssrl.slac.stanford.edu/lcls/CDR⟩.
[14] R. Brinkmann et al., TESLA XFEL: First stage of the X-ray laser laboratory-Technical Design Report, DESY 2002-167, 2002, ⟨http://xfel.desy.de⟩.
[15] J. Gea-Banacloche, G.T. Moore, R.R. Schlicher, M.O. Scully, H. Walther, IEEE J. Quantum Electron. 23 (1987) 1558.
[16] J.C. Gallardo, R.C. Fernow, R. Palmer, C. Pellegrini, IEEE J. Quantum Electron. 24 (1988) 1557; see Eq. (4) with $k_L = 1$ for a gaussian shape.
[17] M. Xie, Proceedings of the Particle Accelerator Conference, Dallas, 1995, p. 183.
[18] R. Bonifacio, L. de Salvo Souza, B.W.J. McNeil, Opt. Commun. 93 (1992) 179.
[19] N. Piovella, R. Bonifacio, Nucl. Instr. and Meth. A 560 (2006) 240.
[20] R. Bonifacio, N. Piovella, G.R.M. Robb, M.M. Cola, Opt. Commun. 252 (2005) 381.
[21] A. Bacci, M. Ferrario, C. Maroli, V. Petrillo, L. Serafini, Phys. Rev. ST Accel. Beams 9 (2006) 60704.
[22] Z. Huang, D. Dowell, P. Emma, C. Limborg-Deprey, G. Stupakov, J. Wu, Proceedings of the 2005 Particle Accelerator Conference, Knoxville, Tennessee, 2005, p. 3570.
[23] J.B. Murphy, C. Pellegrini, R. Bonifacio, Opt. Commun. 53 (1985) 197.