On the parametrization of lateral dose profiles in proton radiation therapy

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Abstract
Hadrontherapy requires a good knowledge of the physical interactions of the particles when they cross the biological tissue: one of the aspects that determine the characterization of the beam is the study of the lateral profile. We study different parametrizations for the lateral dose profile of protons beam in water considering different energies at different depth. We compare six functions: we start from the well known Gaussian and Double Gaussian parametrizations and also analyse more recent parametrization obtained with Triple Gaussian and Double Gaussian Lorentz-Cauchy functions. Finally we propose alternative parametrizations based on the Gauss-Rutherford and Gauss-Levy functions. The goal is to improve the performances of the actual treatment planning used in proton beam therapy by suggesting alternative approaches to the Gaussian description typically employed.

1. Introduction

The propagation of a proton beam in water can be decomposed in a longitudinal and transverse profile. The longitudinal profile is well known as Bragg Peak, while laterally we have a beam spread.
The accurate study of lateral beam profile is important to delimit the dose to the cancer volume, saving as much as possible the surrounding healthy tissue.
The form of lateral beam profile comes from the combination of two processes: the Multiple Coulomb scattering and nuclear interactions. The multiple Coulomb scattering is described by Molière's theory [1] and takes into account only electromagnetic interaction. In addition on the tails of lateral profile, we have the effect of nuclear interactions, that produce nuclear fragmentation. The nuclear contribution for protons is between 5% to 15% and cannot be neglected [2].
Therefore, the shape of lateral beam profile is not Gaussian. Actually, the Treatment Planning System (TPS) parametrize the lateral profile with Gaussian or Double Gaussian.

2. Parametrizations

2.1 Gaussian

First of all, we try to fit the lateral dose profile with a single Gaussian, but the agreement is present in only first 2 decades of the deposited energy distribution. This parametrization does not take into account the tails of distribution.

$$f(y) = N \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{y^2}{2\sigma^2}\right] \tag{1}$$

This function, being centered at a zero mean value, has only 2 free parameters, the standard deviation σ and the normalization factor N.

2.2 Double Gaussian

An improvement of the previous model adds a second broader Gaussian to better describe the tails of dose distribution as proposed by R. Fruhwirth and M. Regler [3] and applied already in clinical environment.

$$f(y) = N\left\{(1-W)\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left[-\frac{y^2}{2\sigma_1^2}\right] + W\frac{1}{\sqrt{2\pi}\sigma_2}\exp\left[-\frac{y^2}{2\sigma_2^2}\right]\right\} \tag{2}$$

The idea behind this parametrization is to describe the core with a narrow Gaussian of width σ_1 whereas with the large Gaussian of width σ_2 the tails of lateral dose profile. This parametrization has 4 free parameters.

2.3 Triple Gaussian

Following this approach, we upgraded the previous model adding a further Gaussian of width σ_3 . This parametrization is better than double Gaussian but the number of free parameter increase to 6.

$$\begin{aligned} f(y) = N\Big\{&(1-W_1-W_2)\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left[-\frac{y^2}{2\sigma_1^2}\right] + W_1\frac{1}{\sqrt{2\pi}\sigma_2}\exp\left[-\frac{y^2}{2\sigma_2^2}\right] \\ &+ W_2\frac{1}{\sqrt{2\pi}\sigma_3}\exp\left[-\frac{y^2}{2\sigma_3^2}\right]\Big\} \end{aligned} \tag{3}$$

From the Gaussian approach, we select the triple Gaussian as a possible good parametrization and discard the single Gaussian.

2.4 Gauss-Rutherford

Taking into account that multiple scattering occurs at small angle and single scattering at wide angle, we considered a new model consists of a Gaussian core to describe the multiple scattering effect and a Rutherford-like hyperbole to represent single scattering.

$$f(y) = N\left\{(1-W)\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{y^2}{2\sigma^2}\right] + W\frac{2b^{3/2}}{\pi}\frac{1}{(y^2+b)^2}\right\} \tag{4}$$

This function depends only on 4 free parameters, the width σ of the Gaussian core, a relative weight W and a normalization factor N. The parameter b mathematically represents the horizontal shift of the hyperbolic function and physically allows to depict a smooth transition between small and wide angle distributions. Typically b gives information on the lateral position where the distribution loses its Gaussian profile and corresponds to about 1/600 of the peak value [4].
This parametrization, whose application to a TPS is innovative to our knowledge, is inspired by the Rutherford scattering experiment and seems physically well justified.

2.5 Gauss-Levy

An alternative parametrization with a similar behavior on the tails is given by adding to the Gaussian core a Levy function.

$$f(y) = N\left\{(1-W)\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{y^2}{2\sigma^2}\right] + W\frac{1}{\mathscr{I}}\frac{\exp\left[-\frac{y+c}{2}\right]}{(y+c)^{3/2}}\right\} \tag{5}$$

where the Levy function normalization integral $\mathscr{I}$ does not exist in analytic form and must be computed numerically and stored in a look-up table:

$$\mathscr{I} = \int_{-\infty}^{\infty} \frac{\exp\left[-\frac{y+c}{2}\right]}{(y+c)^{3/2}} dy$$

This function depends only on 4 free parameters as above: in addition to σ, W and N we introduce the parameter c as a scale parameter that describes the dispersion of the tails in analogy with σ in the core. Also this function is original when applied to TPS, although less accurate than the Gauss-Rutherford.

2.6 Double Gaussian and Lorentz Cauchy

Finally, we also analysed a recent parametrization proposed by Soukup et al. [5] and by Li et al. [6], where a second Gaussian function with a large standard deviation and a modified Cauchy-Lorentz distribution function are added to the Gaussian profile of the core.
These functions are used to describe the long-range scatters caused by nuclear interaction and large-angle Coulomb scattering that are not accounted for using the primary Gaussian components, but requires 6 parameters.

$$\begin{aligned} f(y) = N\Big\{&(1-W_1-W_2)\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left[-\frac{y^2}{2\sigma_1^2}\right] + W_1\frac{1}{\sqrt{2\pi}\sigma_2}\exp\left[-\frac{y^2}{2\sigma_2^2}\right] \\ &+ W_2\frac{1}{\mathscr{I}}\frac{1-A\exp\left[-\frac{y^2}{2b^2\sigma_2^2}\right]}{\pi b\left(\frac{y^2}{b^2}+1\right)}\Big\} \end{aligned} \tag{6}$$

where:

$$\begin{aligned} A &= \frac{2\sigma_2^2}{2\sigma_2^2+1} \\ \mathscr{I} &= \int_{-\infty}^{\infty} \frac{1-A\exp\left[-\frac{x^2}{2b^2\sigma_2^2}\right]}{\pi b\left(\frac{x^2}{b^2}+1\right)} dx \end{aligned}$$

Obviously other parametrizations are possible and are currently being proposed (up to 25 free parameters [7]), but for the purpose of this study we decided to limit the number of parameters in view of a possible use in a future customizable TPS.

3. Simulations & Data

The functions reported in the previous sections were compared with experimental data measured at CNAO and with Monte Carlo simulations obtained with FLUKA. We have implemented a C++ program based on ROOT and MINUIT to fit lateral dose distribution obtains by FLUKA simulation with six different functions.
The simulation setup implemented in FLUKA is very simple. The geometry consists in a parallelepiped of water that represents the water phantom, into a parallelepiped of air. The position of isocenter and the geometry reflects the CNAO facilities (Fig. 1). The Physic setting used is HADRONTHE and the source is the CNAO phase space. A phase space distribution is a file containing the parameters for a large set of particles: in particolar the energy, the position and the directional cosine. In these simulation, we have scored the energy deposition in water in a mesh of this dimension:

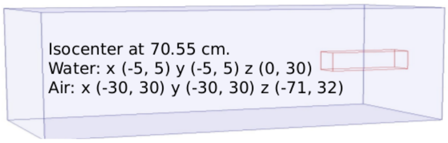

Fig. 1: The geometry implemented in FLUKA simulation.

- 1bin in x: (-0.1, 0.1)*cm*
- 400 bins in y: (-5, 5)*cm*
- 2000 bins in z: (0, 20)*cm*

We have made 10 run to 10^7 particles each and we consider the Fluka errors.

This simulation are compared with CNAO data. The measurements are the transversal dose profile of the proton pencil beam and were acquired using a cylindrical PinPoint ionization chamber in a remotely-controlled 3D motorized water phantom.

4. Results

Results of the best fits for the simulated beam profiles are shown in Fig.2-3 for two energies at different depth in water.

The grey line represents 0.1% of the central axis dose, assumed as the lowest level of clinical relevance.

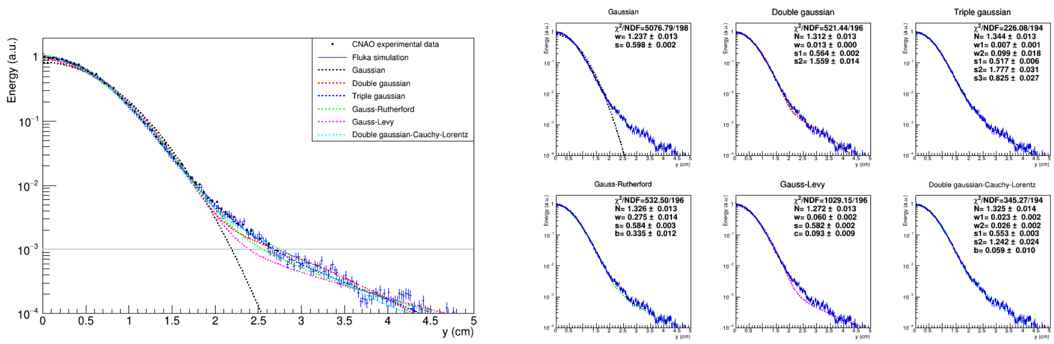

Fig. 2: Left: Lateral half-profile of deposited energy for the CNAO beamline measured and simulated from a phase space beam of 117.75 MeV protons in 2.5 cm water depth. Data are normalized to the central axis. Right: different parametrizations fitted to the same simulated data.

The constant *N* is an additional normalization that varies with energy and depth and in a TPS, where the lateral and longitudinal profiles are factorized, it is typically described by the Depth Dose Deposition (DDD) profile. The values of N are used as a cross-check of the goodness of fit: for each E and z, N is similar among all six functions and this indicates that the overall profile is normalized correctly.

4.1 Accuracy

For the accuracy, we have analysed the trend of the reduced χ^2 as a function of the depth z for six different measurements as well seen in Fig.4. The χ^2 has acceptable values for each function apart for the single Gaussian (reported in the inset because of the different scale) at all depths analysed. The fit quality depends on the water depth and all function show an acceptable value: clearly the triple

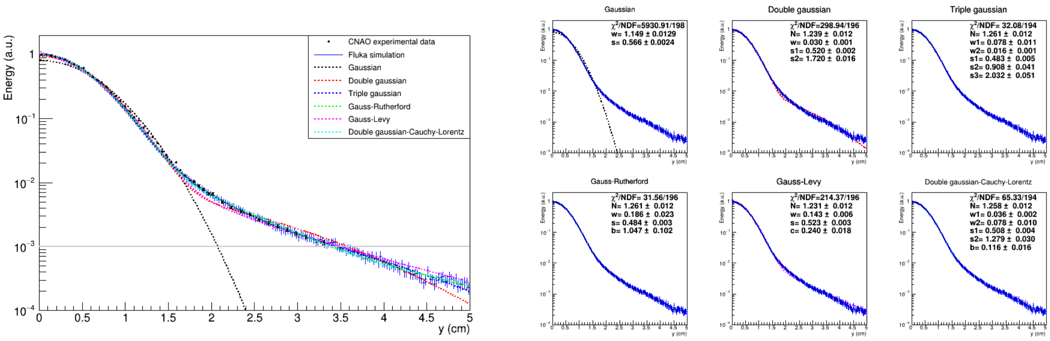

Fig. 3: As in Fig.2 for a phase space beam of 154.25 MeV protons in 10 cm water depth.

Gaussian and the double Gaussian Cauchy- Lorentz give better results, but the double Gaussian and the Gauss-Rutherford are both comparable.

In general, all functions show a worse description of the true profile in correspondence of the Bragg

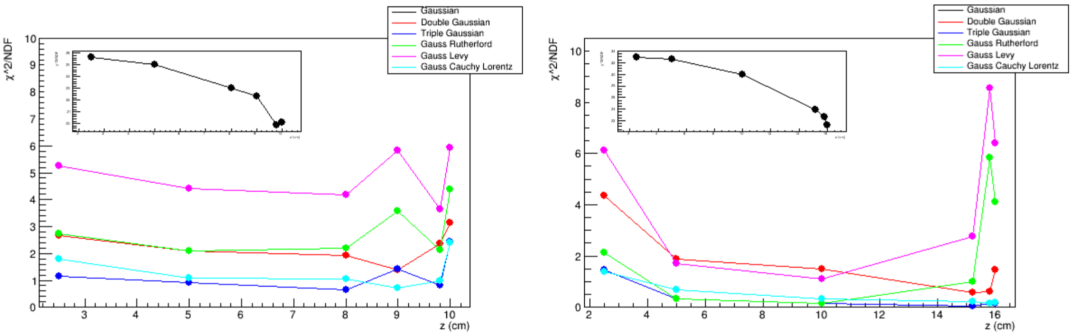

Fig. 4: Trend of reduced χ^2 as a function of water thickness for energy 117.75 MeV (left) and 154.25 MeV (right) beam energies.

peak depth, where inelastic processes, that are not easy to parametrize by simple functions with few parameters, dominate.

Qualitatively the six functions can be classified in 3 groups: the Gaussian and Gauss-Levy functions that show an average value greater than 3 (bad fit), the double Gaussian and Gauss-Rutherford function that are on average between 2 and 3 (good fit) and the triple Gaussian and the double Gaussian Cauchy-Lorentz that are close to one (best fit).

Moreover, we have performed a Kolmogorov-Smirnov test between the FLUKA distribution and the six functions to assess the goodness of fit: all the functions pass the test but have different maximum distances that we report in Tab.1 and that we use as a criterion to evaluate the accuracy of the parametrization.

The use of the maximum distance D in this test is complementary to the use of the standard p-value. In case of binned data D is a parameter which, although not having a universal statistical meaning, allows to estimate the different fits quality in a relative manner. In addition this method is suggested as an alternative to the χ^2 method for histograms with low statistics [8].

Looking at the values of D the Gauss-Rutherford parametrization shows an accuracy comparable or better than the one of the double Gaussian, making it an interesting option.

Table 1: Maximum distance D of Kolmogorov-Smirnov test for two depths at energies 117.75 MeV and 154.25 MeV.

Function	$E = 117.75$ MeV		$E = 154.25$ MeV	
	2.5 cm	8 cm	10 cm	15.2 cm
Gaussian	0.0471	0.0517	0.0493	0.0512
Double Gauss	0.0247	0.0229	0.0216	0.0155
Triple Gauss	0.0025	0.0013	0.0029	0.0009
Gauss-Rutherford	0.0168	0.0178	0.0024	0.0119
Gauss-Levy	0.0216	0.0216	0.0142	0.0149
Double Gauss Lorentz Cauchy	0.0111	0.0073	0.0047	0.0051

Table 2: Computational time relative to double Gaussian τ for energies 117.75 MeV and 154.25 MeV.

Function	number of parameters	τ 117.75 MeV	τ 154.25 MeV
Gaussian	2	0.74	0.76
Double Gaussian	4	1.00	1.00
Triple Gaussian	6	1.19	1.16
Gauss-Rutherford	4	1.11	1.10
Gauss-Levy	4	0.87	0.91
Double Gauss-Lorentz Cauchy	6	1.76	1.73

4.2 Time Calculation

With this program we have also estimated the calculation time τ for all the functions relative to the double Gaussian function which is available in the commercial TPS used at CNAO. The results on the computation time for all functions are shown in Tab.2.

Most of the computation time in a TPS is determined by the dose calculation in a longitudinal profile and not by the lateral parametrization. Therefore the impact of these times is not crucial and we report them for completeness as a possible optimization to shorten the calculation without a loss of accuracy.

5. Conclusions

In this study we have analysed different parametrizations currently available to describe the lateral dose profile in proton therapy. In addition to functions already published, we propose in particular a new parametrization: Gauss-Rutherford.
The Gauss-Rutherford function is a good compromise to evaluate the lateral energy deposition of real beam shapes: indeed, with only 4 free parameters, it ensures a good accuracy, but also a fast calculation time. Moreover, this parametrization is firmly justified by a physical explanation.

This study was recently published in Physica Medica [9].

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