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The Microcanonical Fermionic Average method for Asymptotically Free Theories: a test in the Schwinger Model. *

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We have applied the Microcanonical Fermionic Average method to QED_2, *i.e.* the Schwinger Model, to test its applicability to Asymptotically Free Theories. We present here the results of the simulations, compared to the continuum results. Since the M.F.A. method allows the study of the whole β, m_f plane at very small computer cost, we are able to verify the scaling of the chiral condensate at a high degree, and obtain the continuum result within 3 decimal places. We present also results for the plaquette energy.

1. INTRODUCTION

The Microcanonical Fermionic Average (M.F.A.) method for performing Lattice simulations with dynamical fermions [1] is ideally suited for discussing the phase structure of theories with phase transitions at finite couplings, and it has been applied so far in this context [2, 3].

The conventional wisdom, however, requires that physically interesting theories are Asymptotically Free like QCD. It is then interesting to test the applicability of the M.F.A. method to a theory without phase transitions at finite coupling.

In this paper we present an analysis of the Schwinger Model. We have chosen this model because it is the simplest model with fermions in which, like more physical theories, the continuum limit is approached at infinite lattice coupling, and moreover is confining and exactly solvable in the continuum at zero fermionic mass. We can therefore compare the results of our simulations with exact ones.

We have done so for the average plaquette, which has exactly been computed in the lattice, and for the chiral condensate in the non symmetric ($\theta = 0$) vacuum of the model.

The evaluation of the chiral condensate has been made easier by the fact that, in the M.F.A. approach, the main computer cost resides in the evaluation of an effective fermioninc action at fixed pure gauge energy, at $m_f = 0$. It is then essentially possible, at no extra cost, to move in the plane β, m_f to follow constant physics trajectories in approaching the correct continuum limit. This is easier in this model since here the Renormalization Group amounts to simple dimensional analysis.

The results of this simulation demonstrate the applicability of the M.F.A. method to Asymptotically Free theories, and are interesting by themselves since they are the best numerical results obtained in the Schwinger model so far.

2. THE METHOD

The M.F.A. method is fully described in [1], and in these proceedings in [4]. It is essentially based on the definition of an effective fermionic action, which is the microcanonical average (with respect to the pure gauge Energy) of the fermionic determinant

$$e^{-S^F_{eff}(m,n_f,E)} = \langle \det \Delta^{\frac{n_f}{2}} \rangle_E = \frac{\int [dU] \det \Delta^{\frac{n_f}{2}} \delta(S_{gauge}(U) - VE)}{N(E)} \quad (1)$$

where $N(E)$ is the density of states at energy E and the above expression is written for n_f flavours

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of staggered fermions. The effective action is computed for a number of values of E, on configurations separated by a large number of microcanonical sweeps, by finding all the eigenvalues of the fermionic determinant at $m_f = 0$, then interpolating to reconstruct the (derivatives of) the partition function.

3. THE SCHWINGER MODEL IN THE CONTINUUM

We report here only the features of the model in the continuum which are relevant for the present study. QED_2 is confining, superrenormalizable, and can exactly be solved at $m_f = 0$.

It can be shown that the partition function of the massless theory in the photonic sector is that of a theory of free massive vector bosons. In particular the Green functions of purely bosonic operators are the same in both theories. This fact has been used for obtaining the average plaquette in the lattice.

As for the chiral condensate, due to the rich structure of the vacua (labeled by a parameter θ) of the model, it is zero in the symmetric vacuum, which is selected in the theory at *exactly* $m_f = 0$. If however the chiral limit is obtained from $m_f \neq 0$, then the $\theta = 0$ vacuum is selected. In this vacuum the chiral condensate is (with one flavour)

$$\frac{1}{e}\langle\bar{\psi}\psi\rangle_c = \frac{e^{\gamma_e}}{2\pi\sqrt{\pi}} = 0.15995 \qquad (2)$$

while it diverges at zero flavour (*i.e.* the quenched limit) and is zero with two flavours.

4. THE MODEL IN THE LATTICE

For the present simulation we use n_f species of staggered fermions, coupled to *non compact* gauge fields. The results for the compact model will be presented elsewhere.

Since the continuum theory is equivalent to one of a free, massive vector boson, the average plaquette of the Schwinger model can be compared with that of the vector boson, which can be exactly computed:

$$\langle E\rangle_L = \frac{1}{2V}\sum_{p_\mu p_\nu}\frac{2-\cos p_\mu - \cos p_\nu}{2\beta\sum_\gamma(1-\cos p_\gamma)+\frac{1}{\pi}} \qquad (3)$$

where ($p_\mu = \frac{2\pi}{Na}k_\mu$).

The continuum limit of the theory is reached at $\beta \to \infty$. Since β is dimensionful, the limit must be reached keeping fixed the dimensionless ratio $\frac{m_c}{e_c} = \sqrt{\beta}m_f$. This ratio defines constant physics trajectories.

5. RESULTS AND DISCUSSION

We have performed simulations in lattices up to 100^2. We present here the results for the 64^2 lattice, where we have the best statistics (for a total of 70 Cray-equivalent hours). We will mainly discuss the 1−flavour case. The effective action has been computed at $m_f = 0$ for 20 values of the Energy (from 0.08 to 1.3). In the non compact, abelian models, the density of states can be computed analytically, since the underlying pure gauge theory is quadratic.

The average plaquette is obtained as

$$\langle E\rangle_L = \frac{\int dE\,E\,N(E)e^{-\beta V E}e^{-S^F_{eff}(E,m,n_f)}}{\mathcal{Z}} \qquad (4)$$

and can be directly computed at $m_f = 0$.

In Figure 1 we report the value of (2β times) the average plaquette energy (diamonds) compared with the exact result of eqn (3). As can be seen, apart from small β where we are far from the continuum limit, we have very good agreement. The straight line is the quenched value $2\beta\langle E\rangle = 1$. The chiral condensate

$$\langle\bar{\psi}\psi\rangle = -\frac{1}{V}\frac{\int dE e^{-S_{eff}}\frac{\partial}{\partial m}S^F_{eff}}{\int dE e^{-S_{eff}}} \qquad (5)$$

cannot be directly computed at $m_f = 0$, so it must be obtained as the limit $m_f \to 0$. To reach the correct continuum value, this limit has to be taken simultaneously with the $\beta \to \infty$ one, keeping the product $\sqrt{\beta}m_f$ fixed.

This can be easily done with this method, which does not require a separate simulation of the fermionic part for each pair of parameters (β, m).

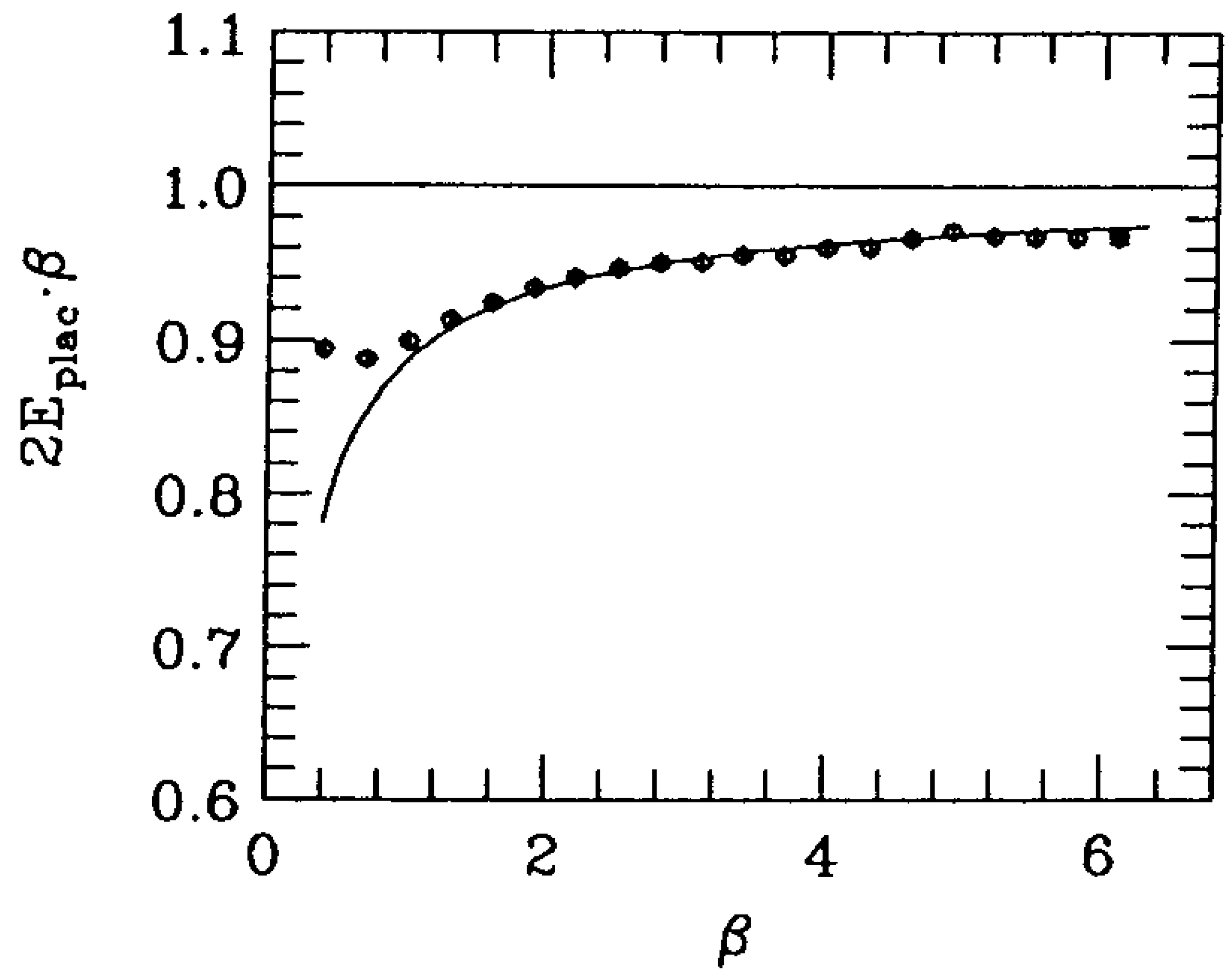

Figure 1. Average plaquette $64^2, m_f = 0, n_f = 1$.

In Figure 2 we report the value of the chiral condensate for two values of the ratio $\frac{m_c}{e_c}$. As it can be seen, scaling sets up already near $\beta \sim 1$. This procedure has been repeated for 12 values of

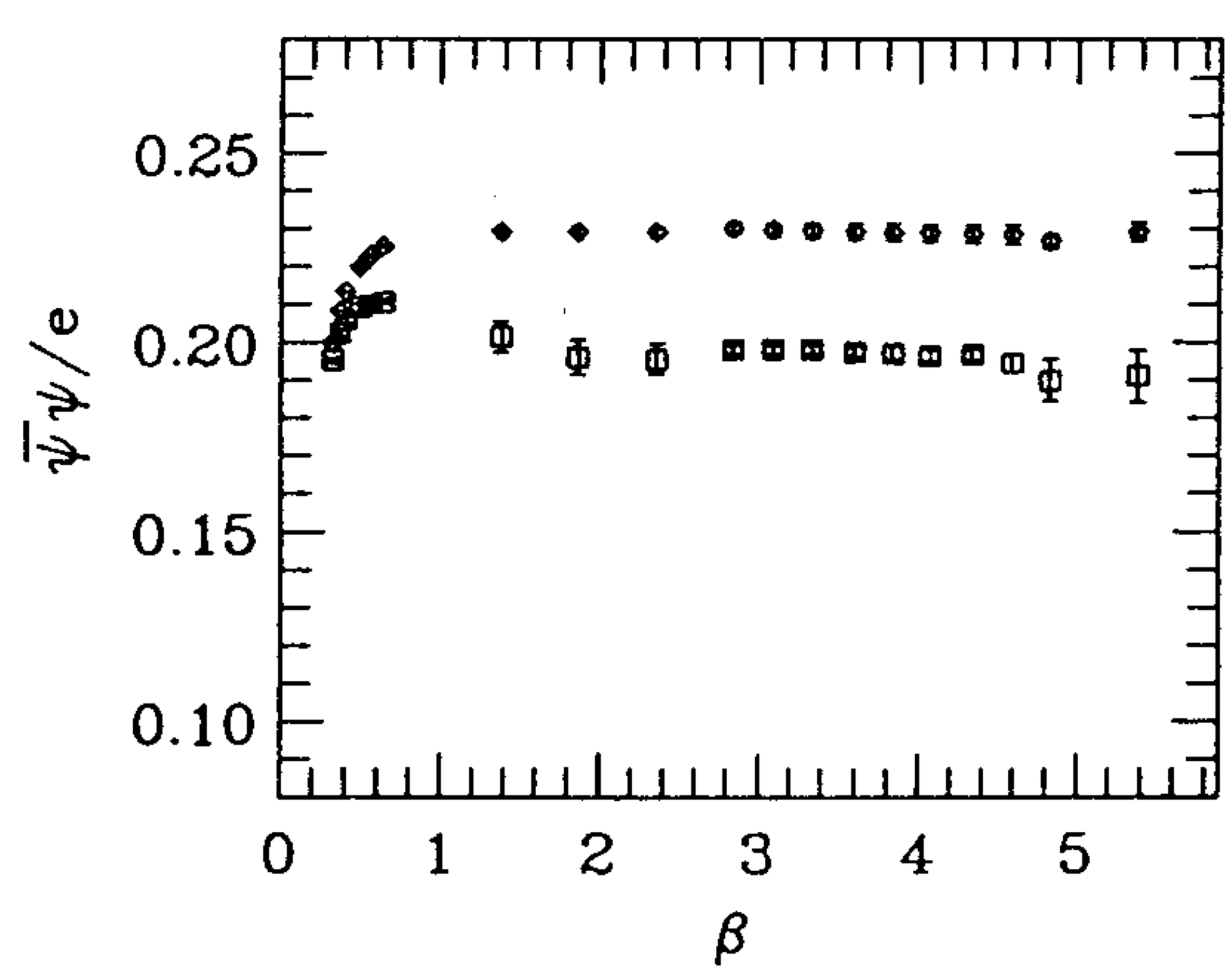

Figure 2. Chiral condensate, 64^2, $\frac{m}{e} = 0.04, 0.08$

the ratio, and the values of the chiral condensate in the scaling window so obtained have been reported in Figure 3. The behaviour of the condensate is very clear towards the continuum value, indicated in the figure as a circle. By fitting the last points with a straight line, we obtain

$$\langle\bar{\psi}\psi\rangle = 0.160 \pm 0.002 \quad (6)$$

in perfect agreement with the theoretical value.

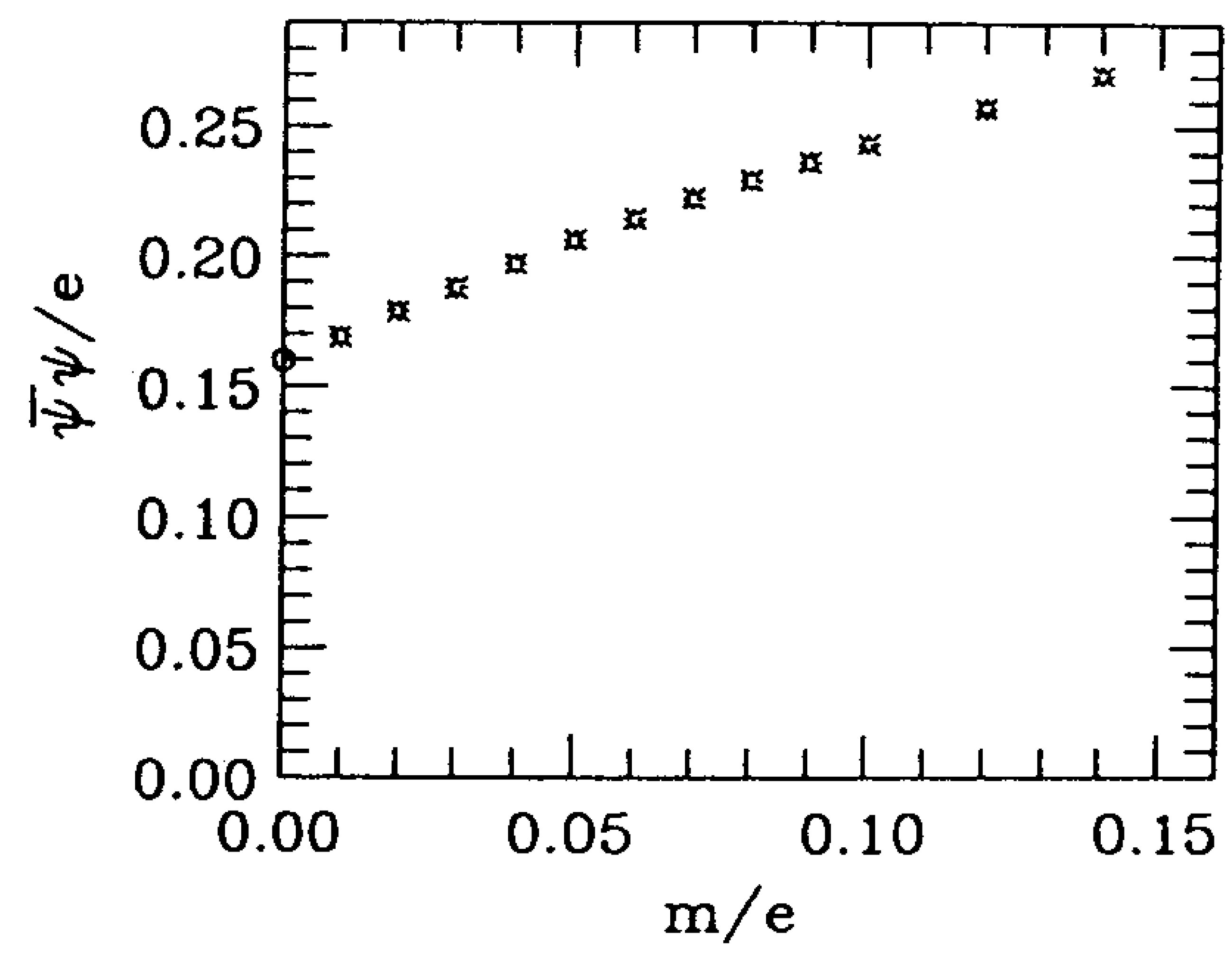

Figure 3. Chiral condensate vs $\frac{m}{e}, n_f = 1$, errors are smaller than symbols

We have also analyzed the zero and two flavours cases. At zero flavours there is really no scaling region, with the chiral condensate increasing at large β, indicating that it diverges as expected. On the contrary in the two flavour case, the behaviour of the chiral condensate at finite mass indicates a vanishing value in the chiral limit, again in agreement with expectations.

In conclusion, we find no reason why the M.F.A. method could not be succesfully applied in realistic, Asymptotically Free theories.

As for the numerical results in the Schwinger model, we believe that the potentialities of the method have been fully exploited; in particular, the fact that the mass dependence of the Dirac operator has become trivial in this approach, allows us to move easily in the parameter space.

All the above simulations have been performed on the LNF Transputer Network .

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