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N. Paver

Riazuddin

and

M.D. Scadron

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N. Paver *
Dipartimento di Fisica Teorica, Università di Trieste, Trieste, Italy
and
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Trieste, Italy

Riazuddin ** and M.D. Scadron ***
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

We study SU(3) and SU(2) flavour symmetry violations in the vacuum from the viewpoint of nonperturbative quark mass generation and independently from charge symmetry-breaking considerations. The results for the ratios of quark condensates of different flavours are compatible with those of QCD sum rules. However, we find that very large SU(3) violating effects, suggested by some sum rule analyses, are barely accommodated in the present nonperturbative approach.

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** Permanent address: Department of Physics, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia.

*** Permanent address: Department of Physics, University of Arizona, Tucson, AZ 85721, USA.

1. For quite sometime since the notion of approximate chiral $SU(3)_f$ symmetry at the quark level was introduced [1], SU(3) invariance of the vacuum was taken as the natural starting assumption of most applications. This invariance, now expressed in terms of nonperturbative quark vacuum condensates, would read:

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle \quad . \tag{1}$$

In the real world, where the symmetry is broken, one would expect that (1) should hold to 10-30% accuracy, which is the range suggested e.g. by the observed departure from unity of the nonperturbative meson decay constant ratio $f_K/f_\pi \simeq 1.25$.

Recently, QCD sum rules have been applied to this question, with results which depend somewhat on the different sum rule versions, and on the phenomenological input used [2]. Taking that into account, the QCD sum rule results for the quark condensates ratios can be summarized as [3]:

$$\frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle} \simeq 1 - (1 \pm 0.3)\cdot 10^{-2} \tag{2a}$$

$$\frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \simeq 0.6 \pm 0.2 \quad . \tag{2b}$$

While (2a) is of the expected size for isospin violations, (2b) seems to suggest the possibility of SU(3) breaking effects in the vacuum appreciably larger than naïvely anticipated above.

In this paper we would like to consider this interesting problem from alternative conceptual perspectives, namely i) from the viewpoint of quark mass generation and ii) independently from charge symmetry breaking in nuclear forces. In doing so, we will also be able to write a relation between SU(2)- and SU(3)-violation in the vacuum, reminiscent of chiral perturbation theory. The results found from such combined analyses are, within the quoted errors, not incompatible with (2a) and (2b). Concerning the latter, however, they point to the smaller values of SU(3) violation, so that large effects, of the order of 50% or so, cannot be accommodated in this framework in any natural way.

2. We start by specifically considering the mechanism of nonperturbative quark mass generation via quark vacuum condensates, generalized away from the chiral limit. In this limit, with the "current" quark masses $(m_{u,d,s})_{curr} = 0$, the graph in Fig.1 is gauge invariant at the pole position, and leads to the self-consistent expression [4]:

$$m_{dyn} = \left[4\pi\, \alpha_s(M^2)\, |\langle \bar{q}q \rangle_M|/3 \right]^{1/3} \approx 320 \text{ MeV} \quad . \tag{3}$$

This expression truncates after leading order in the operator-product expansion of the quark propagator, and no higher-dimension condensates appear to shift the pole position [4]. The numerical quark mass in (3) follows from the values [5,6] $\alpha_s(1\ \text{GeV}) \approx 0.5$ and $|\langle \bar{q}q \rangle_{1\,\text{GeV}}| \approx (250\ \text{MeV})^3$ respectively, and the $M_N/3$ mass scale of (3) is, of course, expected.

Actually (3) can be derived alternatively [7] from the quark condensate graph of Fig.2, which in a sense is the "inverse" of Fig.1 as $x \to y$. In particular the loop integral for the chiral-limiting condensate of Fig.2 is

$$\langle \bar{q}q \rangle_M = \frac{4N_c i}{(2\pi)^4} \int_\Lambda^M \frac{d^4p\; m_{dyn}(p^2)}{p^2 - m^2_{dyn}(p^2)} \quad . \tag{4}$$

Taking the ultraviolet cut-off in the Euclidean region, $M \sim 1$ GeV, where m_{dyn} falls off rapidly as [8] $m_{dyn}(p^2) \sim p^{-2}(\ln p^2)^{-1+d}$ with $d = 12/(33-2N_f)$, one may ignore the small mass term in the denominator of (4) and evaluate the integral in closed form:

$$\langle \bar{q}q \rangle_M = -\frac{N_c}{4\pi^2 d}\left(\ln M^2/\Lambda^2\right)^{1-d} M^2 m_{dyn}(M^2)\left(\ln p^2/\Lambda^2\right)^d \Big|_{\Lambda^2}^{M^2} \quad . \tag{5}$$

Since the contribution at the lower end-point vanishes, we recover the desired relation (3) upon using the asymptotic freedom relation

$$\alpha_s(p^2) = \pi d / \ln\left(p^2/\Lambda^2\right) \tag{6}$$

provided we also invoke in (5)

$$m_{dyn}(M^2) = m^3_{dyn}/M^2 \quad . \tag{7}$$

Although (3) is a renormalization-group-invariant expression, it is derived from Fig.1 for $M > 1$ GeV in the Euclidean region, while Fig.2 and (4) are actually dominated by the "inverse", nonperturbative (infrared) region, and Eq.(7) in practice signifies the value of M where these two regions meet. Inspired by the σ-model, we would suggest that here $M \approx m_\sigma$, the scalar-isoscalar $\bar{q}q$ σ mass. In fact Fig.1 can be topologically converted to the quark or tadpole graph of Fig.3, and then m_σ can be self-consistently found to obey the "gap equation" [9]

$$m_\sigma / m_{dyn} \approx \sqrt{\pi / \alpha_s(m_\sigma^2)} \approx 2 \tag{8}$$

at $m_\sigma \sim 650$ MeV. The numerical value of 2 in (8) follows from $\alpha_s((0.65\ \text{GeV})^2) \approx 0.74$ as implied by (6) and $\alpha_s(1\ \text{GeV}) \approx 0.5$ (for $\Lambda = 250$ MeV). Below $p^2 \approx m_\sigma^2$ lattice calculations [10] suggest that α_s should freeze out. In fact theoretical considerations [11] also show that $m_{dyn}(p^2)$ freezes out below $p^2 = m^2_{dyn}$.

Away from the chiral limit, we continue to apply Fig.2, with flavour-dependent quark masses, but with Λ remaining 250 MeV and still $M \simeq 2m_{dyn}$ as in the cut-offs of the chiral-limiting integral (4). For the strange vs. nonstrange flavours we consider the difference $\langle \bar{s}s \rangle - \langle \bar{u}u \rangle$ in place of $\langle \bar{q}q \rangle$ in (4). In the resulting integral we invoke the flavour independence of $m_{dyn}(p^2)$ in $m(p^2) = m_{dyn}(p^2) + m(p^2)_{curr}$ to replace the total quark mass difference by the quark current mass difference:

$$m_s(p^2) - m_u(p^2) = (m_s - m_u)_{curr} \quad . \tag{9}$$

In this way we obtain the following expression, where the logarithmic dependence of m_{curr} has been neglected, as we are below freeze-out for current masses in any case,

$$\langle \bar{s}s \rangle_M - \langle \bar{u}u \rangle_M \approx \frac{12i(m_s - m_u)_{curr}}{(2\pi)^4} \int_\Lambda^{M=2m_{dyn}} \frac{d^4p\,(p^2 + m_s(p^2)\, m_u(p^2))}{(p^2 - m_s^2(p^2))(p^2 - m_u^2(p^2))} \quad . \tag{10}$$

Since the RHS of (10) has the explicit (linear) symmetry breaking factor $(m_s - u_u)_{curr}$, we may estimate as a first approximation the remaining integral in the chiral limit, and replace there $m_{u,s}(p^2)$ by $m_{dyn}(p^2)$ as given by (7) and (3). In this approximation the final result would be:

$$\langle \bar{s}s \rangle - \langle \bar{u}u \rangle \approx -\frac{3(m_s - m_u)_{curr}}{4\pi^2}\,(-\,3.44\,\Lambda^2) \quad . \tag{11}$$

Clearly, Eq.(11) depends numerically on one's preferred values of quark mass differences. Choosing $(m_s - m_u)_{curr} \simeq 250$ MeV *) as found from QCD sum rules [12] or from scaling integrals [13], we obtain from (11):

$$1 - \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \equiv \epsilon_s \approx 0.26, \quad \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} \equiv 1 - \epsilon_s \approx 0.74 \quad . \tag{12}$$

While being close to the limit allowed by (2b), the value in Eq.(12) is certainly compatible with the observed SU(3) breaking effects observed in meson and baryon phenomenology. Moreover, as the chosen value of $(m_s - m_u)_{curr}$ is on the "large" side, and the numerical value of the integral in (10) is found to decrease if we take different choices of quark masses (with $m_s > m_u$), Eq.(12) should ultimately represent an upper limit for ϵ_s. Also, we emphasize that the quadratic sensitivity on the upper end-point in the integral in (10) should be somehow minimized by the condition (7), which forces the flavour-neutral freeze-out at $M \approx 2m_{dyn}$.

Turning next to SU(2) breaking in the quark condensates, the analogues of (10) and (11) will be:

$$\langle \bar{d}d \rangle_M - \langle \bar{u}u \rangle_M \approx \frac{12i(m_d - m_u)_{curr}}{(2\pi)^4} \int_{\Lambda}^{M=2m_{dyn}} \frac{d^4p\,(p^2 + m_d(p^2)m_u(p^2))}{(p^2 - m_d^2(p^2))(p^2 - m_u^2(p^2))} \tag{13}$$

and

$$\langle \bar{d}d \rangle - \langle \bar{u}u \rangle \approx - \frac{3(m_d - m_u)_{curr}}{4\pi^2} \cdot (-3.44\,\Lambda^2) \quad . \tag{14}$$

Taking $(m_d - m_u)_{curr} \approx 7$ MeV [3, 12] we should obtain from (14):

$$1 - \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle} \equiv \epsilon_d \approx 0.007 \quad , \quad \frac{\langle \bar{d}d \rangle}{\langle \bar{u}u \rangle} \equiv 1 - \epsilon_d \approx 0.993 \quad . \tag{15}$$

It is satisfying that Eq.(15) is compatible with (2a), which gives ϵ_d 0.010 ± 0.003.

Another way to obtain the above results, which separates out the upper-end-point-dependent terms, is to combine (10) and (13) to

*) It might be amusing to note that, with our choice of parameters, $(m_s - m_u)_{curr} \sim (\langle \bar{u}u \rangle)^{1/3} \sim \Lambda$.

$$\epsilon_d \equiv \frac{(m_d - m_u)_{curr}}{(m_s - m_u)_{curr}} \left[\epsilon_s + \frac{(m_s - m_u)_{curr}(m_s - m_d)_{curr}}{\langle \bar{u}u \rangle} \cdot \frac{12i}{(2\pi)^4} \cdot \int_{\Lambda}^{M} d^4p \frac{[(m_s + m_u + m_d)p^2 + m_s m_u m_d]}{(p^2 - m_u^2)(p^2 - m_d^2)(p^2 - m_s^2)} \right] . \tag{16}$$

In this form, (16) is similar to the chiral perturbation theory result [14]

$$\epsilon_d = \frac{(m_d - m_u)_{curr}}{(m_s - m_u)_{curr}} \left[\epsilon_s + \frac{1}{16\pi^2 f_\pi^2} \left(m_K^2 - m_\pi^2 - m_\pi^2 \ln \frac{m_K^2}{m_\pi^2} \right) \right] , \tag{17}$$

since the integral in (16) has also a logarithmic form. Furthermore, numerically evaluating this integral as $i\pi^2(1.92\,\Lambda)$, both (16) and (17) have approximately the same numerical structure, respectively given by

$$\epsilon_d \approx 0.028 \cdot (\epsilon_s + 0.15) \; , \quad \epsilon_d \approx 0.023 \cdot (\epsilon_s + 0.13) \quad . \tag{18}$$

Of course the overall multiplicative scales in (18) are due, in the former case, to our input $(m_d - m_u) \approx 7$ MeV and $(m_s - m_u) \approx 250$ MeV vs. those of Ref.[14] $(m_d - m_u) \approx 2.8$ MeV and $(m_s - m_u) \approx 120$ MeV. However, all that matters in (16) and (17) is the ratio of these estimates, and the closeness of the coefficients 0.028 and 0.023 in (18) indicates the stability of relating ϵ_d to ϵ_s in this manner.

3. As further support to our flavour-dependent quark-loop estimates (12), (15) and (18) for ϵ_s and ϵ_d, we may also turn to charge symmetry-breaking considerations.

The flavour-dependent effects on the pion-nucleon coupling constant can be estimated with the aid of QCD three-point function sum rules [15,16]. It was recently shown that in this approach the SU(2) x SU(2) symmetry breaking of $g_{\pi^0 NN}$ can be related to the condensate breaking by [17]

$$\frac{g_{\pi^0 pp}}{g_{\pi^0 nn}} = \left(1 + \frac{2}{3}\epsilon_d\right) \frac{m_n}{m_p} \quad , \tag{19}$$

where the 2/3 factor is due to the flavour-difference of the proton relative to the neutron.

On the other hand, we may independently estimate chiral symmetry breaking corrections to $g_{\pi^0 NN}$ via the $\langle\pi^0|H_{Tad}(\lambda_3)|\eta_8\rangle$ coupling of the η_8 pole as shown in Fig.4. In order that chiral symmetry breaking corrections are consistently fed into $g_{\pi^0 NN}$ (thus into $\langle\bar{u}u\rangle$ and $\langle\bar{d}d\rangle$ by virtue of (19)), only the SU(3)-octet η_8 must be considered, rather than the singlet η_0, because only the former is a Nambu-Goldstone boson along with the π^0. To evaluate $\langle\pi^0|H_{Tad}(\lambda_3)|\eta_8\rangle$ independently of $(m_d - m_u)_{curr}$, we follow the approach [18] of writing $H_{em} = H_{JJ} + H_{Tad}(\lambda_3)$ and using

$$\langle\pi^0|H_{em}|\eta_8\rangle = \frac{1}{\sqrt{3}}\left[\left(m^2_{K^+} - m^2_{K^0}\right) - \left(m^2_{\pi^+} - m^2_{\pi^0}\right)\right] \approx -0.00304\ \text{GeV}^2 \qquad (20)$$

$$\langle\pi^0|H_{JJ}|\eta_8\rangle = \left(1/2\sqrt{3}\right)\left(m^2_{\pi^+} - m^2_{\pi^0}\right) \gtrsim 0.00036\ \text{GeV}^2 \quad . \qquad (21)$$

Eq.(20) is a standard SU(3) result, and the meaning of (21) is that, while the K^+ and π^+ have the same charge and self-mass, the π^0 has no electromagnetic self-mass [19]. Subtracting (21) from (20) gives the chiral SU(2) symmetry breaking result

$$\langle\pi^0|H_{Tad}(\lambda_3)|\eta_8\rangle = \frac{1}{\sqrt{3}}\left[\left(m^2_{K^+} - m^2_{K^0}\right) - \frac{3}{2}\left(m^2_{\pi^+} - m^2_{\pi^0}\right)\right] \approx -0.0034\ \text{GeV}^2 \quad . \qquad (22)$$

One then finds [20]

$$\frac{g_{\pi^0 pp}}{g_{\pi^0 nn}} = 1 + \frac{2\langle\pi^0|H_{Tad}(\lambda_3)|\eta_8\rangle}{m^2_{\pi^0} - m^2_{\eta_8}}\ \frac{g_{\eta_8 pp}}{g_{\pi^0 pp}} \quad , \qquad (23a)$$

$$\frac{g_{\eta_8 pp}}{g_{\pi^0 pp}} = (3f - d)/(f + d)\sqrt{3} \approx 0.25 \pm 0.09 \quad , \qquad (23b)$$

for d/f varying between the SU(6) value of 1.5 and the phenomenological estimate ≈ 2. With $m^2_{\eta_8} \simeq 17 m^2_\pi$ from the Gell-Mann-Okubo mass formula, (22), (23) and (19) lead to the estimate

$$\frac{g_{\pi^0 pp}}{g_{\pi^0 nn}} - 1 \approx 0.0056 \pm 0.0020, \quad \epsilon_d \approx 0.0063 \pm 0.0030 \quad . \qquad (25)$$

It is interesting to note that, within the quoted error, the result for ϵ_d in Eq.(25) is consistent with our previous quark-loop determination (15), with the QCD sum rule (2a) and with the observed difference of nucleon scattering lengths $a_{pp} - a_{nn}$ [17,20,21].

4. Briefly summarizing our preceding analysis, we have attempted to study the role of flavour symmetry breaking in quark vacuum condensates, starting from a purely nonperturbative viewpoint, as an alternative to the QCD sum rule method.

The first important point to mention is that the results thus found for ϵ_d and ϵ_s, Eqs. (12), (15) and (17), confirm the rule that larger mass quarks should have correspondingly smaller (in absolute value) vacuum condensates. This is in accord with the findings from QCD sum rules (2a,b) and also with general expectation [22].

The values of the isospin breaking parameter ϵ_d reported in (15) and (25) are compatible with (2a) and with the observed charge symmetry violation in nucleon-nucleon scattering. Their size certainly complies with the accuracy expected a priori for SU(2).

Regarding SU(3), the value of $1 - \epsilon_s$ quoted in (12), at the upper edge of the range suggested by (2b), points to 20%-30% violation effects, as guessed e.g. from f_K/f_π, and conforms with the notion of approximate first order SU(3) breaking found elsewhere in nature. By changing the values of quark masses in (10) we could not obtain the 50% or larger symmetry breaking effects still allowed by QCD sum rules (2b), unless we substantially increase the integration end-point, thereby voiding the prescription following Eqs.(3) to (8) on which our whole approach is conceptually based. The reason for this only partial agreement between (12) and (2b) could lie in the phenomenological input. In fact sum rules estimates of ϵ_s rely on the saturation of spectral integrals by $(K - \pi)$ phase shifts [23] or by an equivalent, effective 0^{++} κ-resonance with $M_\kappa \simeq 1.35$ GeV. The latter is not the lowest lying κ-resonance, with $M_\kappa \simeq 800$ GeV, expected from current algebra or from the σ-model, which has not shown up in the data yet. Were such a lower M_κ used in QCD sum rules, then presumably smaller values of ϵ_s than (2b) would be obtained, bringing the two approaches to a complete agreement. In fact such an accord exists for the ϵ_d channel, where the δ is present, and saturates the QCD sum rules. Clearly all this takes us back to the problem of the experimental determination of scalar mesons.

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FIGURE CAPTIONS

Fig.1 Dynamical quark mass generation due to the quark condensate $\langle \bar{q}q \rangle$.

Fig.2 Quark condensate generation due to the running dynamical quark mass $m_{dyn}(p^2)$.

Fig.3 Quark σ tadpole generation of m_{dyn} due to $\langle \bar{q}q \rangle$

Fig.4 Charge symmetry breaking of $\pi^0 pp$ and $\pi^0 nn$ coupling constants due to $\langle \pi^0 | H_{tad} | \eta_8 \rangle$.

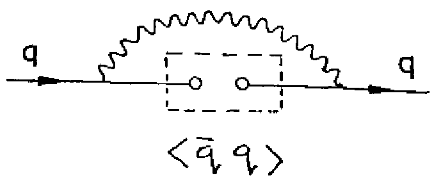

Fig.1

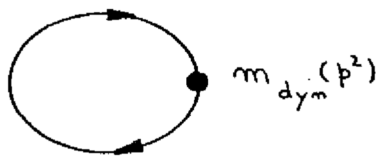

Fig.2

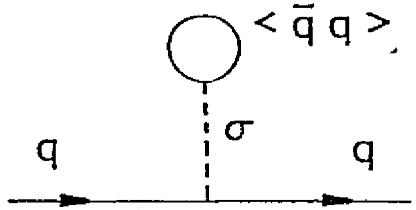

Fig.3

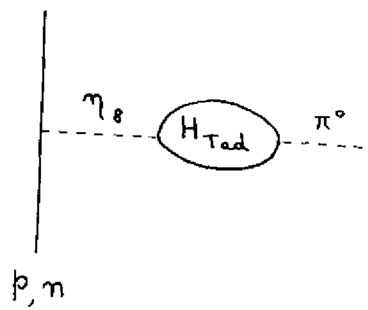

Fig.4