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Nonlinear Speed Control Scheme and Its Stability Analysis for SI Engines

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Abstract: For international combustion engines, due to the combustion cyclic nature, the intake-to-power stroke delay is inherent that causes additional difficulties in control design and validation phases. In this paper, a nonlinear speed control scheme is proposed based on the proportional feedback control method. From the consideration of improving the transient performance, a reference model is introduced to design the feedback controller. Then, the speed controller is formulated as a designed feedback control law connecting with a model-based feedforward compensation. The asymptotic convergence to the desired speed is guaranteed under the presented conditions of the feedback gains, which include the cases of using a speed-depended gain function and a constant gain, respectively. For the stability analysis of the proposed delayed control system, an initial method is presented via Lyapunov-Krasovskii functional stability theorem. Experimental results on the transition speed control are shown to demonstrate the control scheme.

Key Words: engine speed control, time-delay system, stability, Lyapunov-Krasovskii functional.

1. Introduction

Engine speed control is a classical issue in automotive control applications. The performance of engine speed has significant impacts on the vehicle design attributes such as comfort, emission, fuel economy, etc., especially during transitional operations [1],[2]. In the community of control engineering, this has led to many approaches to tackle the speed control problem, such as l_1 optimal control [3], sliding mode control [4], fuzzy control [1] and others referring to the references therein. However, one of the main characteristics of engine dynamics is that it involves the intake-to-power stroke delay, which was ignored in many works. As is well-known, the presence of time delay in the system may induce undesired behaviors included of oscillation and instability, therefore, this delay characteristic should be considered properly in investigating engine control problems [5].

Over the years, several speed control methods that take the intake-to-power stroke delay into account have been proposed [6]–[8], in which the controllers are constructed applying the design techniques for linear systems to the linearized engine model. On the other hand, the engine system, as an active benchmark example, has been used to assess the control design methods for time-delay systems [9],[10].

In this paper, we present a nonlinear feedback speed control scheme that consists of the speed error feedback with nonlinear proportional gain. The presented scheme focuses on dealing with transition speed control. A reference model, which generates a reference trajectory for any desired speed, is used to construct the feedback controller to ensure the tracking performance and a nominal model-based feedforward compensation is appended to achieve a rapid responsibility. Another contribution is to provide the stability analysis exactly based on an engine model involving the intake-to-power stroke delay. It is shown that the proposed control system is asymptotically stable at the desired speed if the nonlinear gain function satisfies the provided condition. Furthermore, the case that utilizes traditional proportional controller with constant feedback gain is discussed. In this case, a sufficient condition for the constant gain is given such that the control system is locally asymptotically stable, and an estimated domain of attraction which plays significant role from the view of practical applications is provided. The analysis results are obtained with Lyapunov-Krasovskii functional stability theorem for functional differential equations [11]. By using an engine test bench, the nominal model is identified and experimental validation results are carried out.

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2. Control Scheme and Model-Based Analysis

2.1 Control System Structure

The control scheme proposed in this paper is shown in Fig. 1, where engine speed $\omega[rad/s]$ denotes the angular velocity of crankshaft and u is the control input that is the normalized signal of throttle opening. The control input $u(t)$ is constructed by combining a feedforward compensation u^* with a proportional feedback controller.

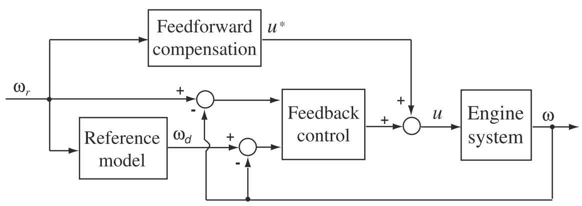


Fig. 1 Scheme of the speed control system.

The aim of this control system is to drive the engine speed ω to a given transition speed trajectory $\omega_d(t)[rad/s]$ asymptotically, which is generated by a reference model forced by the desired speed command $\omega_r[rad/s]$. To complete the proposal of the control scheme, we will show a detailed stability analysis for the control system. Precisely, we will present the conditions for the gain function and constant gain in the feedback control loop, respectively, that guarantee asymptotical stability. That is, the speed tracking error $e_\omega(t) \to 0$ as $t \to \infty$, $\forall e_\omega(0)$ where $e_\omega = \omega_r - \omega$.

The stability analysis is performed based on the so-called mean-value engine model, which ignores the characteristics of individual cylinders and captures the average features of engine physics, and is widely used for engine control. Under the ideal air-fuel ratio and spark timing, the model for speed control, which includes the dynamics of air intake and crank rotation, is described as follows [12] (see Table 1 for the symbol nomenclature of physical parameters)

$$\dot{p}_m = \frac{RT_m}{V_m}(\dot{m}_i - \dot{m}_o) \tag{1}$$

$$J\dot{\omega} = \tau_e - \tau_f - \tau_l \tag{2}$$

$$\dot{m}_i = s_0(1 - \cos\phi) \cdot \frac{p_a}{\sqrt{RT_a}} \sqrt{\psi\left(\frac{p_m}{p_a}\right)} \tag{3}$$

$$\dot{m}_o = \frac{\rho_a V_c \eta}{4\pi p_a} p_m \omega \tag{4}$$

where $\dot{m}_i[kg/s]$ represents the air mass flow rate through the throttle, $\dot{m}_o[kg/s]$ represents the air mass flow rate into the cylinders, $\tau_e[Nm]$, $\tau_f[Nm]$ and $\tau_l[Nm]$ denote the engine torque, the frication torque and the external load torque, respectively. $\psi(\cdot)$ is defined by

$$\psi(s) = \begin{cases} s^{\frac{2}{\kappa}} \left[\frac{2\kappa}{\kappa-1}(1-s)\right]^{\frac{\kappa-1}{\kappa}}, & s \geq \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} \\ \kappa\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}, & \text{others} \end{cases}$$

The mean-value expression exploited to model the engine torque is as follows

$$\tau_e = c \cdot \frac{\dot{m}_o(t - t_d)}{\omega(t - t_d)} \tag{5}$$

where $c[Nm/kg/rad]$ denotes the maximum torque capacity and $t_d[s]$ denotes the intake-to-power stroke delay, which is determined by engine speed. The friction torque is modeled simply as [13]

$$\tau_f = D\omega + D_0 \tag{6}$$

where D and D_0 are constants.

Table 1 Notations of physical parameters.

symbol	meaning
κ	specific heat ratio ($[-]$)
R	gas constant ($[J/(kg \cdot K)]$)
p_m	intake manifold pressure ($[Pa]$)
T_m	intake manifold temperature ($[K]$)
V_m	intake manifold volume ($[m^3]$)
V_c	volume of six cylinders ($[m^3]$)
J	crank inertia moment ($[kg \cdot m^2]$)
ϕ	throttle opening ($[rad]$)
s_0	throttle area ($[m^2]$)
p_a	air pressure ($[Pa]$)
T_a	air temperature ($[K]$)
ρ_a	air density ($[kg/m^3]$)
η	volumetric efficient ($[-]$)

The engine model (1)~(6) can be rewritten as follows

$$\begin{cases} \dot{\omega}(t) = a_1 p_m(t - t_d) - \bar{D}\omega(t) - \bar{D}_0 - \bar{\tau}_l \\ \dot{p}_m(t) = u(t) - a_2 p_m(t)\omega(t) \end{cases} \tag{7}$$

where the parameters are defined by

$$a_1 = \frac{c\rho_a V_c \eta}{4J\pi p_a}, \quad \bar{D} = \frac{D}{J}, \quad \bar{D}_0 = \frac{D_0}{J}, \quad \bar{\tau}_l = \frac{\tau_l}{J}$$
$$a_2 = \frac{R_m T_m}{V_m} \cdot \frac{\rho_a V_c \eta}{4\pi p_a},$$

and control input $u(t)$ is defined by

$$u(t) := \frac{RT_m}{V_m}\dot{m}_i(\phi, p_m)$$

which can be realized by the throttle opening ϕ.

For desired speed ω_r, we denote the equilibrium of system (7) as $(\omega_r\ p_m^*)$ which satisfies the following conditions

$$p_m^* = \frac{\bar{D}\omega_r + \bar{D}_0 + \bar{\tau}_l}{a_1} \tag{8}$$

$$u^* = a_2 \omega_r p_m^* \tag{9}$$

It can be seen from (8) ~ (9) that the value p_m^* varies with the changes of load torque τ_l, in other words, the control action u^* depends on the desired speed and external loads (see Fig. 2 for an image illustration).

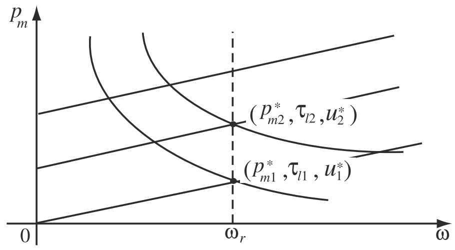


Fig. 2 Image for the equilibrium of model (7).

Furthermore, it is clear from the physics that the parameters a_1, a_2 and $\bar{D}$ are positive numbers. Taking these physics into account and assuming that the model parameters and load torque are known and measurable, the following control laws are proposed for each block in the control scheme.

First, the reference model is introduced as

$$\dot{\omega}_d(t) = \sigma(\omega_r - \omega_d(t)) \tag{10}$$

with a positive number σ, and the feedback control law u is designed as

$$u(t) = u^* - a_2 p_m^* e_\omega(t) + k_p(\omega)(\omega_d(t) - \omega(t)) \tag{11}$$

where $k_p(\omega)$ is the feedback gain function. In the following, two cases of the feedback gain $k_p(\omega)$, a nonlinear function and constant, will be addressed for ensuring the stability of the tracking dynamics.

Before going to the detail of system analysis, we give a short comment on the structure of control law (11). Noticing that $\omega_d - \omega = e_\omega - e_r$ where $e_r = \omega_r - \omega_d$, control law (11) can be rewritten as

$$u(t) = u^* + \left(k_p(\omega) - a_2 p_m^*\right)e_\omega(t) - k_p(\omega)e_r(t)$$

where u^* is the constant determined by (9). Moreover, as we can see below, the error system of (7) under the control of (10) and (11) can be represented with the state variables (e_ω, e_p, e_r) where $e_p = p_m^* - p_m$. Therefore, the presented controller is a partial state feedback of e_r and e_ω with nonlinear gains. It should be noted that a natural idea of output feedback with feedforward is to choose the control law as

$$u = u^* + k(\omega)(\omega_d - \omega)$$

however, with this control law, we cannot obtain the theoretical results on the convergence of the error system.

2.2 Stability Analysis

Substituting (8) ~ (11) into (7), we get the following error dynamics

$$\begin{cases} \dot{e}_\omega = -\bar{D}e_\omega + a_1 e_p(t - t_d) \\ \dot{e}_p = -k_p(\omega)e_\omega - a_2\omega e_p + k_p(\omega)e_r \\ \dot{e}_r = -\sigma e_r \end{cases} \tag{12}$$

Denote $x_t = [e_{\omega t}\ e_{pt}\ e_{rt}]^T$. The notation x_t represents the function in the space $C_r := \{x \mid x : [0, r] \to R^3\}$, where $r > 0$ is a constant, and $x(t - t_d) = x_t(t_d)$, $t_d \in [0, r]$ denotes the value of function x_t at t_d.

To analyze the stability of system (12), we choose a candidate of Lyapunov-Krasovskii functional as follows

$$V(x_t) = \frac{\gamma_1}{2}e_\omega^2 + \frac{1}{2}e_p^2 + \frac{\gamma_2}{2}e_r^2 + \frac{1}{2}\int_{t-t_d}^{t} e_p^2(s)ds \tag{13}$$

where γ_1 and γ_2 are given by

$$\gamma_1 = \frac{\bar{D} + \sqrt{\bar{D}^2 - 2a_1^2\varepsilon}}{a_1^2}, \quad \gamma_2 = \frac{\varepsilon}{\sigma} \tag{14}$$

with a given ε satisfying

$$0 < \varepsilon < \frac{\bar{D}^2}{2a_1^2} \tag{15}$$

For the sake of simplicity, we treat the delay time t_d in the model (5) as constant, i.e. it takes the nominal value at the desired speed ω_r, $t_d = \pi/\omega_r$. Then, it is clear that for the candidate $V(x_t)$ given by (13), the derivative $\dot{V}(x_t)$ on the trajectory of system (12) can be calculated as

$$\dot{V}(x_t) = \frac{\partial V(x_t)}{\partial x_t(0)}\dot{x} + \frac{1}{2}(e_p^2 - e_{pt}^2) \tag{16}$$

where $\dot{V}$ is defined as the right-hand upper derivative, i.e.

$$\dot{V}(x_t) = \limsup_{h\to 0^+} \frac{V(x_{t+h}) - V(x_t)}{h}$$

In addition, it is noticed that $2a_2\omega > 1$ under the allowable operation condition of the engine.

Proposition 1. *For any given ω_r and $\sigma > 0$, if the feedback gain $k_p(\omega)$ is given by*

$$k_p(\omega) = \rho(t)\sqrt{\varepsilon(2a_2\omega - 1)} \tag{17}$$

with a given function $|\rho(t)| < 1$, $\forall t \geq 0$, then the time derivative of Lyapunov-Krasovskii functional (13) along the trajectory of system (12) satisfies

$$\dot{V}(x_t) \leq -\lambda\|x\|^2 \tag{18}$$

for a sufficiently small $\lambda > 0$. In other words, for any initial condition $x_0(t_d) \in C_r$, x_t asymptotically converges to zero as $t \to \infty$.

Proof: Calculating $\dot{V}(x_t)$ obtains

$$\begin{aligned} \dot{V}(x_t) &= -\gamma_1\bar{D}e_\omega^2 + \gamma_1 a_1 e_{pt}e_\omega - k_p(\cdot)e_\omega e_p - a_2\omega e_p^2 \\ &\quad + k_p(\cdot)e_p e_r - \gamma_2\sigma e_r^2 + \frac{1}{2}e_p^2 - \frac{1}{2}e_{pt}^2 \\ &= -\gamma_1\bar{D}e_\omega^2 + \frac{\gamma_1^2 a_1^2}{2}e_\omega^2 - \left(\frac{a_1\gamma_1}{\sqrt{2}}e_\omega - \frac{1}{\sqrt{2}}e_{pt}\right)^2 \\ &\quad + \frac{1}{2}e_{pt}^2 - k_p(\cdot)e_\omega e_p - a_2\omega e_p^2 + k_p(\cdot)e_p e_r \\ &\quad - 2\gamma_2\sigma e_r^2 + \frac{1}{2}e_p^2 - \frac{1}{2}e_{pt}^2 \\ &\leq -\frac{1}{2}\left[2\gamma_1\bar{D} - a_1^2\gamma_1^2\right]e_\omega^2 - \gamma_2\sigma e_r^2 - k_p(\cdot)e_\omega e_p \\ &\quad - \frac{1}{2}\left[2a_2\omega - 1\right]e_p^2 + k_p(\cdot)e_p e_r \end{aligned}$$

In view of (14), it yields

$$\dot{V}(x_t) \leq -x^T Q(\omega)x$$

where

$$Q(\omega) = \begin{bmatrix} \varepsilon & \frac{1}{2}k_p(\omega) & 0 \\ \frac{1}{2}k_p(\omega) & a_2\omega - \frac{1}{2} & -\frac{1}{2}k_p(\omega) \\ 0 & -\frac{1}{2}k_p(\omega) & \varepsilon \end{bmatrix}$$

Taking the condition (17) into account, it is easy to verify that a sufficiently small $\lambda > 0$ can be found such that

$$x^T Q(\omega)x \geq \lambda\|x\|^2$$

where $\|\cdot\|$ denotes the Euclidean norm. Namely, condition (18) holds. Moreover, it is clear that for the Lyapunov-Krasovskii functional (13), there exist continuous nondecreasing functions $\mu_i(s)(> 0, s > 0)$ and $\mu_i(0) = 0$ $(i = 1, 2)$ such that

$$\mu_1(\|x\|) \leq V(x_t) \leq \mu_2(\|x_t\|_c) \tag{19}$$

Hence, the asymptotic stability of system (12) at the origin follows by the Lyapunov-Krasovskii functional stability theorem 2.1 in [11]. ■

As mentioned above, the delay time t_d is determined by engine speed as π/ω exactly. If we take this time-variability into account, the time derivative of $V(x_t)$ will depend on dt_d/dt. However, as we can see below if t_d does not vary much quickly, then stability of the error system can be also guaranteed by the same Lyapunov-Krasovskii functional with a slightly modified coefficients.

In fact, since in Proposition 1 ε satisfies condition (15) and $|\rho| < 1$, there exists a $0 < \delta < 1$ such that

$$\frac{2a_1^2\rho^2\varepsilon}{\bar{D}^2} = 1 - \delta \tag{20}$$

Let $M < \delta$. There exists a constant $\gamma_1' > 0$ such that

$$\bar{D}\gamma_1' - \frac{a_1^2}{2(1 - M)}\gamma_1'^2 > \rho^2\varepsilon \tag{21}$$

With γ_1', define a constant γ_2' as

$$\gamma_2' = \frac{\bar{D}\gamma_1'}{\sigma} - \frac{a_1^2\gamma_1'^2}{2\sigma(1-M)} \tag{22}$$

and it is clear that $\gamma_2' > 0$. We now choose a candidate of Lyapunov-Krasovskii functional as

$$V(x_t) = \frac{\gamma_1'}{2}e_\omega^2 + \frac{1}{2}e_p^2 + \frac{\gamma_2'}{2}e_r^2 + \frac{1}{2}\int_{t-t_d}^{t} e_p^2(s)ds \tag{23}$$

which along the trajectory of the error system (12) with feedback gain $k_p(\omega)$ given by Proposition 1, we have

$$\begin{aligned}
\dot{V}(x_t) &= -\gamma_1'\bar{D}e_\omega^2 + \gamma_1' a_1 e_{pt}e_\omega - k_p(\cdot)e_\omega e_p - a_2\omega e_p^2 \\
&\quad + k_p(\cdot)e_p e_r - \gamma_2'\sigma e_r^2 + \frac{1}{2}e_p^2 - \frac{1}{2}\left(1 - \frac{dt_d}{dt}\right)e_{pt}^2 \\
&\leq -\frac{1}{2}\left[2\gamma_1'\bar{D} - \frac{a_1^2}{1-M}\gamma_1'^2\right]e_\omega^2 - \gamma_2'\sigma e_r^2 - k_p(\cdot)e_\omega e_p \\
&\quad - \frac{1}{2}\left[2a_2\omega - 1\right]e_p^2 + k_p(\cdot)e_p e_r - \frac{1}{2}\left(M - \frac{dt_d}{dt}\right)e_{pt}^2 \\
&= -x^T Q'(\omega)x - \frac{1}{2}\left(M - \frac{dt_d}{dt}\right)e_{pt}^2
\end{aligned}$$

where

$$Q'(\omega) = \begin{bmatrix} \gamma_1'\bar{D} - \dfrac{a_1^2\gamma_1'^2}{2(1-M)} & \dfrac{1}{2}k_p(\omega) & 0 \\ \dfrac{1}{2}k_p(\omega) & a_2\omega - \dfrac{1}{2} & -\dfrac{1}{2}k_p(\omega) \\ 0 & -\dfrac{1}{2}k_p(\omega) & \gamma_1'\bar{D} - \dfrac{a_1^2\gamma_1'^2}{2(1-M)} \end{bmatrix}$$

and it is easy to obtain that in view of (21) and (22), there exists a sufficiently small λ such that

$$x^T Q'(\omega)x \geq \lambda\|x\| \tag{24}$$

Obviously, in the case of $dt_d/dt \leq 0$, the following inequality holds definitely,

$$\dot{V}(x_t) \leq -\lambda\|x\| \tag{25}$$

on the other hand, it also holds if $dt_d/dt > 0$ but $dt_d/dt \leq M$.

From the above discussion, we have the following result which is guaranteed by the Lyapunov-Krasovskii stability theorem.

Corollary 1. Consider system (7), (10) with controller (11) given by Proposition 1. The error system (12) is asymptotically stable at the origin if the delay time t_d satisfies

$$\frac{dt_d}{dt} \leq M \tag{26}$$

where $M < \delta$.

For the proposed control scheme, Proposition 1 shows that the speed tracking control can be achieved by using the nonlinear feedback gain in the proportional controller. Now, it is natural to consider a constant feedback gain k_p which can also guarantee the control objective. The following conclusion is obtained with the same Lyapunov-Krasovskii functional candidate (13).

Proposition 2. *For any given ω_r and $\sigma > 0$, the derivative of (13) along the trajectory of system (12) satisfies*

$$\dot{V}(x_t) \leq -\lambda\|x\|^2 \tag{27}$$

over the domain $\mathcal{D} = \{x_t(t_d) \in C_r \mid |e_\omega| \leq \zeta\}$, where $\lambda > 0$ is a sufficiently small number and ζ is a constant satisfying

$$0 < \zeta < \omega_r - \frac{1}{2a_2}$$

with respect to ω_r, if the constant feedback gain k_p satisfies

$$|k_p| < \sqrt{\varepsilon[2a_2(\omega_r - \zeta) - 1]} \tag{28}$$

i.e. the closed-loop system (12) is locally asymptotically stable at the origin. Furthermore, the set

$$\Omega = \left\{ x_t \in C_r \;\middle|\; \|e_{\omega t}\|_c^2 + \frac{1+t_d}{\gamma_1}\|e_{pt}\|_c^2 + \frac{\gamma_2}{\gamma_1}\|e_{rt}\|_c^2 \leq \zeta^2 \right\} \tag{29}$$

is an estimated attraction domain of the solution $x_t = 0$ of system (12), i.e. $x_t \to 0$ as $t \to \infty$, $\forall x_0(t_d) \in \Omega$.

Proof: Similar to the proof of Proposition 1, the time derivative of $V(x_t)$ along he trajectory of system (12) satisfies

$$\dot{V}(x_t) \leq -x^T Q' x$$

where

$$Q' = \begin{bmatrix} \varepsilon & \dfrac{1}{2}k_p & 0 \\ \dfrac{1}{2}k_p & a_2(\omega_r - \zeta) - \dfrac{1}{2} & -\dfrac{1}{2}k_p \\ 0 & -\dfrac{1}{2}k_p & \varepsilon \end{bmatrix}$$

Taking (28) into account, we obtain that Q' is positive definite which implies that there exists a sufficiently small $\lambda > 0$ such that (27) holds $\forall x_t \in \mathcal{D}$. Therefore, by associating with condition (19), the local asymptotic stability of system (12) at the origin follows by the Lyapunov-Krasovskii functional stability theorem 2.1 in [11].

On the other hand, we have that

$$V(x_t) \leq \mu_2(\|x_t\|_c), \quad \forall x_t \in \Omega \tag{30}$$

where

$$\mu_2(\cdot) = \frac{\|e_{\omega t}\|_c^2}{\zeta^2} + \frac{(1+t_d)\|e_{pt}\|_c^2}{\gamma_1\zeta^2} + \frac{\gamma_2\|e_{rt}\|_c^2}{\gamma_1\zeta^2}$$

Moreover, it is clear that the set Ω defined by (29) is bounded and

$$\Omega \subset \mathcal{D} \tag{31}$$

then, it follows from the condition (27) that

$$\dot{V}(x_t) \leq -\lambda[\mu_2^{-1}(V(x_t))]^2 \tag{32}$$

According to Lemma 4.4 in [14], this implies that for any initial condition $x_0(t_d) \in \Omega$

$$V(x_t) \to 0 \text{ as } t \to \infty \;\Rightarrow\; x_t \to 0 \text{ as } t \to \infty$$

This concludes the proof. ■

Remark 1. It should be noted that Proposition 1 provides a sufficient condition for choosing the gain function such that the tracking system is globally asymptotically stable in the practical sense. Observing the proof of the proposition, it can be seen that the free coefficient ρ affects the damping rate λ of the Lyapunov functional. A larger λ is recommended if a quick response is required.

Remark 2. In the meantime, Proposition 2 presents a range for constant feedback gain that guarantees local convergence, and an estimated attraction domain with the feedback gain is given. As shown in (29), to enlarge the provided attraction domain, larger ζ should be chosen which according to condition (28) will cause limitations on the selection of feedback gain k_p consequently. Moreover, it can be noted that in this case, the value of ζ also affects the damping coefficient of Lyapunov functional $V(x_t)$, specifically, the term of $-1/2[2a_2(\omega_r - \zeta)]e_p^2$ in $\dot{V}$. Therefore, a tread-off between a large attraction domain and the response performance should be considered to choose k_p according to condition (28).

Remark 3. It is of interest to notice that condition (26) provides sufficient margin to cover the real dt_d/dt $(= -\pi\dot{\omega}/\omega^2)$ in practical situation. For example, if $\delta = 0.5$ and let $M = 0.499$, according to (26) the acceleration of engine speed should be more than $-0.5\omega^2/\pi$. In other words, if the engine runs in $1000\ rpm \sim 3000\ rpm$, the acceleration should be large than $-277\ rpm/s^2$. This means that the feedback gain (17) is applicable for engine operations with time-varying intake-to-power stroke delay.

3. Experimental Results

The engine used for verification experiments is a 2GR-FSE (3.5L-V6) internal combustion engine supported by Toyota Motor Corporation as shown in Fig. 3. Figure 4 shows a sketch of the engine test bench set up in the laboratory. It includes the software module ECU that serves for engine control in practical vehicles. The control software of the test bench has been modified such that the engine can accept the control commands of designers, which is realized by the dSPACE instrument. Control algorithms that are built in Matlab/Simulink by the designers are downloaded to dSPACE, then are delivered to ECU to control engine through a standard CAN bus. Several sensors are equipped to measure physical variables of engine (e.g. manifold pressure, etc.). A dynamometer that acts the load of vehicles is connected to the engine. During the experiments, the control commands expect for throttle opening are all from the initial controllers of ECU, and the control results of corresponding indexes (spark timing and air-fuel ratio) satisfy the requirements of ideal conditions.

It is clear that to conduct validation experiments, the nominal parameter values of model (7) are needed to obtain the control law (11). The model identification is performed first. By changing the given throttle opening command $\hat{\phi}$, we obtain experimental data of engine speed, intake manifold pressure and throttle opening from ECU as shown in Fig. 5. With these off-line data, recursive least square algorithm is applied to the dynamic model of air intake and crank rotation (7), respectively. Considering that $u(t) = RT_m/Vm \cdot \dot{m}_i = a_u(1 - \cos\phi)$, the identified nominal values at $\omega \simeq 1500\ rpm$ are $a_u = 9.51 \times 10^6$, $a_1 = 5.8\times10^{-3}$, $\bar{D} = 3.5\times10^{-3}$, $\bar{D}_0 = 111.924$, $a_2 = 4.91\times10^{-3}$. The fitting effects of the model output to the experimental date are illustrated by Fig. 5, where $\hat{p}_m$ and $\hat{\omega}$ are the output of model (7). It should be noted that the identification result is applicable to the proposed speed control scheme under the corresponding operation mode. Practically, engine model with constant parameters is effective for a certain working range with respect to a nominal engine speed.

Based on the identification result, experiments are conducted to validate the proposed control scheme shown in Fig. 1. Under an attached external load torque $\tau_l = 15[Nm]$ provided by the dynamometer, the control performance is assessed by applying a series of step tracking commands of $\pm 200\ rpm$, respectively, at operating speed $\omega = 1500\ rpm$. Throttle opening is the control input command computed from $u(t)$ by $\phi = \arccos(1 - u/a_u) \cdot 180/\pi$. For providing a proper response time of the transient speed, the time constant of reference model (10) is set as $\sigma = 1.5$. According to condition (15), the advisable ε belongs to (0, 0.0421). For comparison, experiments are also conducted with the proposed control law that is with the

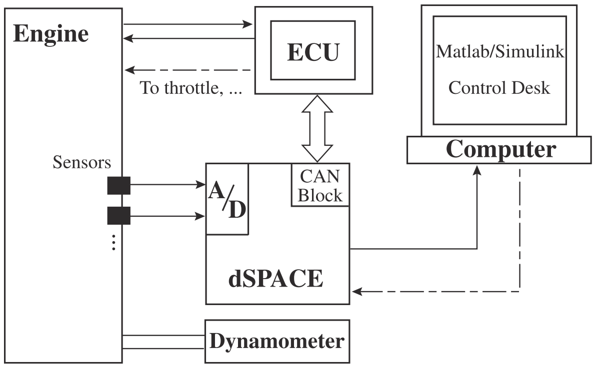


Fig. 4 Sketch of engine test bench.

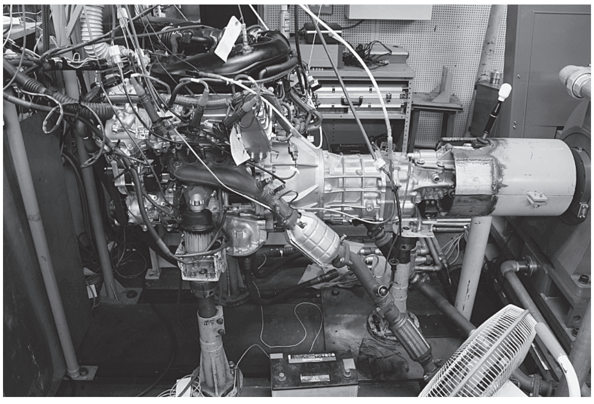


Fig. 3 Engine test bench.

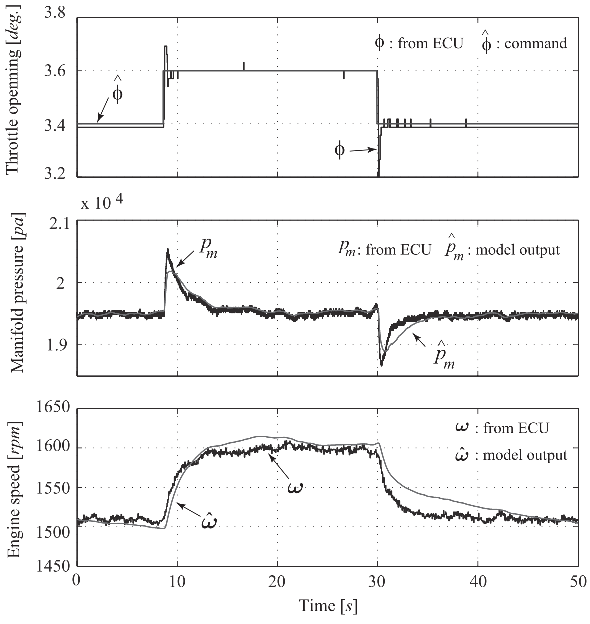


Fig. 5 Identification result of model (7).

feedback of speed error $\omega_r - \omega$ instead of $\omega_d - \omega$.

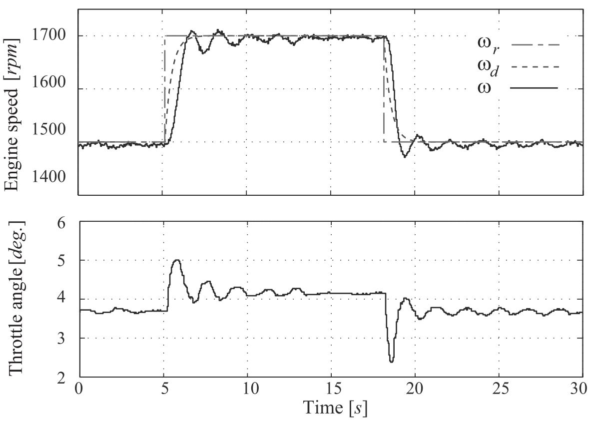
In the experiments carried out under the feedback control with gain function (17), the turning parameters are set as $\varepsilon = 0.04$ and $\rho = 0.65$, which are selected from compromising consideration between quickness and oscillation of speed responses. First experiment is to follow an accelerating-decelerating operation mode, and the result is shown in Fig. 6(a). Then, experiment is conducted with twice accelerating operations. The response curve shown in Fig. 6(b) indicates that the presented speed control algorithm can work effectively under different operation range. Finally, the test is carried out with the control law using a constant feedback gain. In the experiment, ω_r is set at 1500 rpm and 1700 rpm, respectively. Consider a constant $\zeta = 300$ rpm. Solving the condition (28) of Proposition 2 with $\varepsilon = 0.04$, the constant gain k_p can be chosen as $|k_p| < 0.091$ and $|k_p| < 0.128$ with respect to each ω_r, respectively. Furthermore, for a given $\varepsilon = 0.04$, the attraction domain presented in Proposition 2 can be obtained, for example, as $w_r = 1500$ rpm, substituting (14) into (29) obtains

$$\Omega = \Big\{ x_t \in C_r \mid \|e_{\omega t}\|_c^2 + 1.7 \times 10^{-2} \|e_{pt}\|_c^2 + 5.0 \times 10^{-4} \|e_{rt}\|_c^2 \le 986.96 \Big\}$$

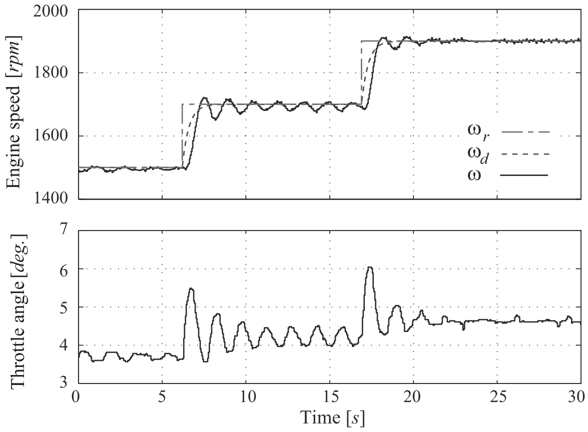
By using feedback gain $k_p = 0.085$, experimental result in Fig. 7 shows similar performance of speed response to the case with the nonlinear feedback gain in Fig. 6(a).

Figures 8 and 9 show the response curves under the control law that does not involve reference trajectory ω_d. The results are obtained with same command ω_r and feedback gain parameters ρ, ε and constant k_p in the above experiments for the results Figs. 6(a) and 7, respectively.

It can be observed from experimental results Figs. 6 and 7 that engine speed converges to the desired speed value with tolerant transient performance under the presented nonlinear control law, and both overshoot and adjusting time are reduced by comparing Figs. 6(a) and 7 with the curves in Figs. 8 and 9, respectively. In other words, the introduced reference trajectory can improve the transient performance to some extent for the considered set-point speed tracking control problem. On the other hand, it can be seen from the response result shown in Fig. 6(a) and Fig. 7 ~ Fig. 9 that deceleration response shows larger overshoot than during acceleration. This gives an intuitively suggestion that using small turning parameters in the



(a)



(b)

Fig. 6 Control result with nonlinear $k_p(\omega)$.

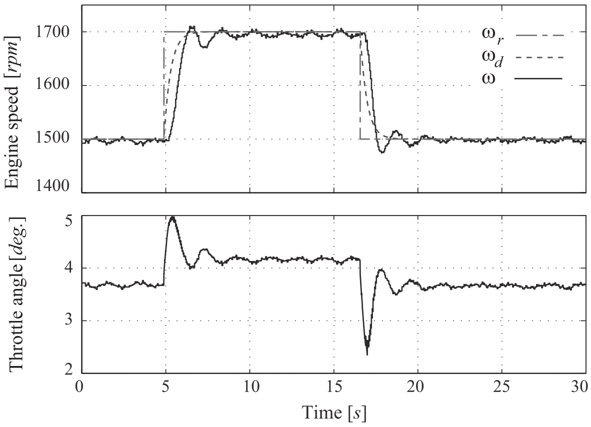


Fig. 7 Control result with constant k_p.

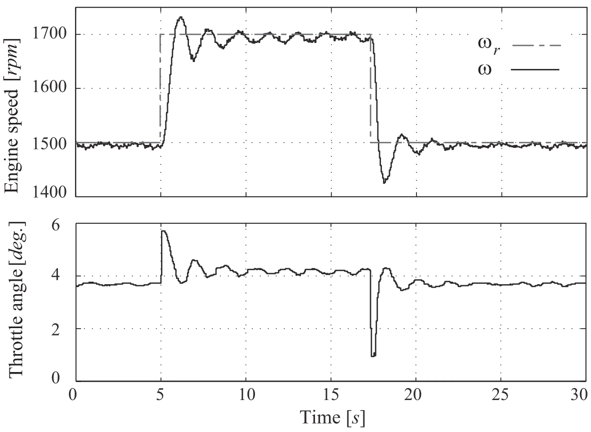


Fig. 8 Control result with nonlinear $k_p(\omega)$ but without ω_d.

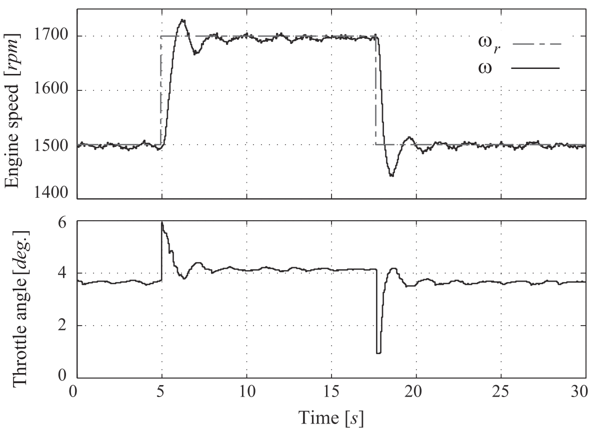


Fig. 9 Control result with constant k_p but without ω_d.

feedback gain during deceleration.

4. Concluding Remarks

A nonlinear speed control scheme was discussed for SI engines. By applying the Lyapunov-Krasovskii stability theory, a precise stability analysis of the error system was provided with explicit consideration of the intake-to-power stroke delay, which is an essential physics of combustion engines. Moreover, it was also shown that under the proposed control scheme, the stability is guaranteed even though the delay time is varied according to the engine speed. The situation of a more simple case of the controller with constant feedback gain was addressed. The effectiveness of the proposed control scheme was demonstrated by experimental results conducted on a practical engine test bench.

Finally, it should be noted that the proposed design and analysis are based on the engine model. Hence, the performance will depend on the precision of the model. For example, the load torque in the model is assumed to be known, in practical applications, which will be provided by the vehicle management level, or the load torque estimation might be a significant research issue from the view of control theory. Another important issue for practical applications is to improve the robustness on the model parameter uncertainty. In fact, the parameter is dependent on the operation modes. As was presented in section 3, the experimental validation was performed around certain operating range. From the view of control theory, the approaches such as gain scheduling, parameter adaption might be feasible ways to enlarge the effective range in application. The authors should like to keep these issues in the next stage.

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