## "Telling Mathematics": Storytelling and Counting in Pre-Elementary School

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#### Abstract

This manuscript is the result of a didactical activity, "Telling mathematics", elaborated for pupils of Pre-Elementary School in Italy. The aim is to verify if storytelling is a methodology that allows developing approaches to the natural number in a significant way in children belonging to this age group and which mathematical skills are involved when certain processes are stimulated. The manuscript describes the phases in which the teaching experiment was articulated, framed in the Theory of Didactical Situations. In particular, I observe how, thanks to the storytelling, the acquisition of the notion of number in children is conveyed through different approaches to the natural number: cardinal, ordinal, recursive, geometric.

**Keywords:** *Pre-Elementary School; storytelling; counting; Theory of Didactical Situations; approaches to natural number.* 

### **1. INTRODUCTION**

Even today, many pre-elementary school teachers do not plan activities that children allow young to pursue mathematical concepts, relegating mathematics to a role of contour, almost occasional, in the various activities of everyday life in the classroom. For example, little importance is given to the conscious development of protomathematical aspects, leaving them to mere spontaneous development (Di Paola, n.a.). Probably this false belief can be traced back to the Piaget's (1972) thought, which for several years has influenced the ministerial programs and of mathematics teaching in Italy.

According to Piaget and his theory of cognitive development (Piaget, 1972; 2013), 2000; the child acquires arithmetic skills at an advanced stage of genetic development, following the appearance of logical abilities (concrete operational stage) that began at the age of six years. This inevitably led to the belief that the child, before then, could not have contact with the number, generating the unbelief that mathematics is a discipline to be addressed from primary school. Piaget has been denied by subsequent research (e.g. McGarrigle & Donaldson, 1974; Wynn, 1992) and it is now established that even very young

children are able to perform very simple arithmetic operations.

The Italian National Curriculum for the Pre-Elementary School (MIUR, 2012; 2018) identifies among the key skills for the mathematical learning, skill, understood as "the capacity to develop and apply mathematical thinking to solve series of problems in everyday а situations" [author's English translation] (MIUR, 2012, p. 21). Referring to these observations to the specificity of the Pre-Elementary School, and focusing on the skills to be developed in the child in this age group, in accordance with the National Curriculum, what the teacher should propose in classroom is "play, move, manipulate, look around, ask, describe, represent, imagine situations and events (...)" [author's English translation] (MIUR, 2012, p. 16). In this sense, the Pre-Elementary School is "the school of attention and intention, of the implicit curriculum (...)" [author's English translation] (MIUR, 2018, p. 8).

#### 2. LITERATURE REVIEW a. Approaches to natural number

There are different approaches to the natural number: cardinal, ordinal, recursive, geometric. These are different points of view regarding the acquisition of the notion of number (Di Paola, n.a.).

The cardinal approach determines the number of elements contained in a set (it answers the question "how many are there?"), not through a count, but through a comparison. Two sets A and B have the same cardinal number, when they can be placed in one-to-one correspondence (bijection). Thus, for example, the cardinal number "five" is determined by the correspondence of the elements with the number of fingers of a hand. This approach does not presuppose sorting operations. According to the ordinal approach, instead, children acquire the notion of number in the form of an ordered sequence and only later in the form of a quantity: the child number of objects determines the contained in a given set, not by comparison, but by counting "one, two, three" so that the last number pronounced is also the cardinal number. The ordinal number indicates the position of a certain element in the sequence (first, second...). The logical operations that allow a learning of the number according to the ordinal approach are: compare; relate; order. While in the cardinal aspect the number is seen as a quantity, in the ordinal aspect it is seen as an ordered sequence. For this reason, the spatio-temporal relations (front and before back, and after) are indispensable, since they determine the control of the order and favour the understanding of the subsequent and previous number. The understanding that the succession of natural numbers is characterized by the relation of order is not of immediate acquisition for children. In fact, children, especially in counting, perfectly capable of correctly are pronouncing the sequence of numbers and determining the cardinality of a whole; with difficulty, however, they fully understand the relationship between the numbers.

The recursive approach is based on the mathematical idea of succession and is characterised by its dynamic and constructive qualities and its interdisciplinary approach. In fact, recursive patterns can be found in language, music, geometry, and graphic representations. Among the first successions that initially enter into the mental organization of the child there are the temporal ones, and after the spatial ones to then arrive at the idea of recursiveness in numbers. From an early age, the child starts the "spontaneous count" and carries out some "recursive" play activities: jumping, bouncing the ball, drawing figures in a row. In this approach we understand how natural numbers are based on the idea of the subsequent number: each number has unit than the number one more immediately preceding it and one less unit than the subsequent number.

D'Amore and Aglì (1995) argue that the number, according to the geometric approach, expresses a measure, or rather the quantities of units of measure that serve to measure a certain size. The expressions of measurement are already part of the daily experience of children: the terms connected with estimation and measurement (near, far, long, short...) are already part of their vocabulary. Children spontaneously begin to use occasional and personal measuring instruments (a step, a stick...) and express the measurement with a number accompanying it to an arbitrary unit of measurement. The opportunities offered by everyday life and game are many and

must be well exploited by the school. The skills that need to be strengthened and expanded are: recognising and applying orderly relations; making comparisons of quantities and sizes.

These different approaches are part of the different representations of numbers, so it is essential that the teacher does not privilege and/or neglect one to the disadvantage of another. The experience of "Tellina didactic Mathematics", through the activities it proposes in the phases in which it is carried out, as we will see below, considers the different approaches listed above, taking advantage of the benefits of storytelling.

## b. Storytelling and Mathematics

It may seem unusual to talk about storytelling in the context of teaching and learning mathematics, since generally the storytelling is linked to leisure time or other disciplines far from mathematics, considering the latter rather as arid and difficult. However, men have always told stories: even before the invention of writing, storytelling was the only means able to spread values and customs. Using storytelling in disciplines other than literature can give the learner several benefits such as: promoting listening skills, verbal communication and group discussion; reducing stress related to school activities in support of emotional and social development; promoting the development of logical skills (Aiex, 1988; Mello, 2001).

The storytelling of stories to explain mathematical content is a way to try to introduce mathematics to children in a new way: less difficult, more creative, fun and motivating, able to involve children emotionally, in order to make more understandable and more real concepts that would otherwise be abstract and difficult to understand at certain ages (Longo, 2008). This process is possible thanks to the storytelling of mathematical content, which approach the experiences of children to make them feel involved, motivated and driven by curiosity to discover the world of mathematics, contrasting the typical anxiety in front of numbers, recorded in most schoolchildren (Zan, 2000; Di Martino, 2009).

In this manuscript I will describe an activity called "Telling Mathematics",

which comes from the Master's thesis in Primary Education Sciences conducted by Cinzia Ciulla, of which I was a supervisor, at the Faculty of Classical, Linguistics and Educational Studies of the University of Enna KORE. This activity was first experimented during Cinzia's thesis work. Later it was expanded by me to be tested again by another pre-elementary teacher, Laura.

The story that forms the background to the different phases of the activity is "The great invention of Bubal" (Figure 1) by Anna Cerasoli (2012). It tells the story of a prehistoric girl, named Bubal, who is left to take care of her father's flock of sheep, who has gone away for a hunt.



Figure 1. Cover of the book "The great invention of Bubal" by A. Cerasoli (2012)

Bubal still does not know the numbers, how to count, and what can be useful to count. Yet, in order to avoid losing her sheep, she thinks up a way to summarize their quantity with a few marks drawn on the rock, as I will describe below. The mathematical concepts that the story holds and is able to convey are the oneto-one correspondence; the different representations of the number; the numbering system. These, from a didactic point of view, are connected to the development of the sense of number and to the approach to counting.

The aim of this manuscript is to answer the following research question: *is storytelling a methodology that allows approaches to natural numbers to be developed significantly in pre-elementary schoolchildren?* 

In the following, we will focus on the experiments conducted by Cinzia and Laura with children from two sections of the pre-elementary school (5 years old). The phases in which the experiments were articulated will be described. The experiments are both framed in the Theory of Didactical Situations.

### **3. THEORETICAL FRAMEWORK**

*mathematical situation* is the Α modelling of the conditions under which human beings produce, communicate and learn knowledge that is recognised mathematical. as Α mathematical situation consists of set а of relationships, established explicitly or implicitly, between the teacher, the pupil who has to live the situation to learn, and the *milieu*, i.e. everything on which the pupil can act and that in return gives him feedback. Brousseau (1986) in his "Theory of Didactic Situations" (TDS), identifies three types of situations: the non-didactical situation, the didactical situation and the a-didactical situation. I will briefly describe the first two, then I will focus instead on the third, which is the situation that has been considered to prepare the experimentation.

The non-didactical situation does not require that there is specific knowledge at stake. The explicit intention to teach is missing, there is no learning as a goal: the milieu is not aimed at learning. In the didactical situation, instead, there is an explicit intention to teach. Pupil and teacher are aware of their mutual role (the first knows that he/she is learning, the second knows that he/she is teaching) and of the evolution of the situation. The teacher immediately declares the cognitive goal she wants to reach, often also declares what she expects the students to do. The teacher structures the milieu in an appropriate way: she stimulates with concrete suggestions to do activities, to solve problems, to carry out deliveries. The pupil knows that at that moment notions that are part of the school's knowledge are being outlined and devolved.

The *a-didactical situation*, among the three, is the ideal one to favour the construction of knowledge. Here the protagonists are the students and the object of knowledge, not the teacher. The situation suggests needs and the pupils respond to them. There are no didactic duties, so what is done is not linked to pressure from the teacher, who in this case does not intervene, but only plays the role of 'director'. In an adidactical situation, students participate in something that is not explicitly cognitive; only the teacher is aware of the purpose of the activity but does not declare it. The pupil makes attempts (alone or in groups), verifies if these fail or are ineffective; if she has to make several attempts, interacting with the elements of the milieu, the pupil modifies her system of knowledge because of the adaptations she assumes in using various strategies. Carrying out that certain mathematical activity that she has decided to undertake is not a proposal that comes from the teacher, but a need motivated by the activity itself. If such an activity, pertinent to mathematics, is not solved with the first attempt and provokes in the student, or among the students, a discussion to agree on other modalities, then there is the production of knowledge, explicitly but not by the requested teacher, not institutionalized. Brousseau (1986)argues that the pupil builds knowledge only if she is personally interested in the resolution of what has been proposed to her through the situation.

The a-didactical situation is divided into six phases, which we will briefly describe (D'Amore & Sbaragli, 2011): (i) Devolution: the teacher transfers to the pupil the responsibility of solving a activity; problem, а cognitive (ii) Implication: the pupil accepts the teacher's offer and is involved in the proposed activity, i.e. she accepts the responsibility of personally dealing with the resolution of the problem/activity without the teacher's proposed, continuous quidance; (iii) Construction of student, private knowledge: each interacting with the milieu, creates her own individual internal knowledge, which must then be translated and reorganized when it is communicated to others; (iv) Validation: the student accepts the

invitation from the teacher-director to verbalize her privately constructed knowledge (e.g. the solution to the problem) and to defend it by arguing it, putting herself in an explicitly communicative situation in order to explain her idea to her classmates; (v) Socialisation: personal knowledge built and validated by individual students is discussed, presented, agreed with others, that is, it becomes part of the common heritage and shared by the class; (vi) Institutionalisation of *knowledge*: explicit act that the teacher makes to allow knowledge built by the students and socially shared in the class to be officially recognized. The knowledge at stake is validated by the teacher and then transformed into official knowledge by the students.

### 4. METHODOLOGY

This work is based on qualitative empirical research which draws its conclusions from direct or indirect observation of the facts. Two experiments have been carried out. The first was carried out by Cinzia, in a section of the pre-elementary school of the Verga plexus of the Vincenzo Guarnaccia Comprehensive Institute (Pietraperzia, Enna), in November 2017, with 18 children of 5 years old, for a total of 35 hours. Before the experimentation, careful observation of the context of the chosen section carried was out. Information was found regarding the spaces of the section, the choice of materials and the way in which they would be made available to the children.

The observation lasted four weeks and took place in October 2017, shortly after the start of the school year, for a total of 20 hours. Thanks to this phase, it was possible to create a relationship with the children, useful for the subsequent phase of implementation of the project.

Compared natural to number experimentation approaches, Cinzia's proposes activities for the development of cardinal and recursive approaches. The section's teacher took part in the experimentation, who only acted as an observer/helper.

The second experimentation was carried out by Laura, a teacher of the pre-elementary school of the Pergusa plexus of the Istituto Comprensivo F.P. Neglia - N. Savarese (Enna), in November 2018, with 16 children of 5 years old, for a total of 50 hours. Together with Laura, I reviewed all the activity and I added some steps (the 4 and 5 of Table 1) to develop also the geometric and ordinal approaches. Laura's section colleague, Alessandra, also participated in the experimentation, and she only acted as an observer/helper.

### a. Experimentation phases

Table 1 shows the phases of the experimentations, specifying the used methodologies and materials.

		Table 1. Experimentation phases		
Mathematical content	Methodologies	Materials		
- First counting ideas	- Cirlce-time	- Book (Figure 1)		
- One-to-one	- Learning by discovery			
correspondence	- Stimulus questions			
- Counting	- Role Playing			
	- Circle time			
<ul> <li>Cardinal approach</li> </ul>	- Learning by doing	- Cardboard		
- Recursive approach	- Practical-manipulative	- Cotton		
	activities	- Vinyl glue		
	- Circle time	<ul> <li>Marker pens</li> </ul>		
		- Two circles		
- Geometric approach	- Circle-time	<ul> <li>Building blocks</li> </ul>		
	- Team play	- Bag		
- Ordinal approach	- Team play	- Building blocks		
	<ul> <li>First counting ideas</li> <li>One-to-one correspondence</li> <li>Counting</li> <li>Cardinal approach</li> <li>Recursive approach</li> <li>Geometric approach</li> </ul>	<ul> <li>First counting ideas</li> <li>One-to-one</li> <li>Correspondence</li> <li>Counting</li> <li>Counting</li> <li>Cardinal approach</li> <li>Recursive approach</li> <li>Geometric approach</li> <li>Geometric approach</li> <li>Circle time</li> <li>Circle time</li> <li>Learning by discovery</li> <li>Stimulus questions</li> <li>Role Playing</li> <li>Circle time</li> <li>Learning by doing</li> <li>Practical-manipulative activities</li> <li>Circle time</li> </ul>		

Table 1 Experimentation phases

Since the experimentations were carried out in a similar way up to Phase 3 (Table 1) in both sections, I will report only the data of Cinzia's experimentation (which are also those discussed in the degree thesis). While for Phases 4 and 5

(Table 1), I report the data related to Laura's experimentation.

### **b.** Data collection tools

In Cinzia's experimentation, a video camera and a logbook were used to collect the data. Thanks to the video camera, Cinzia was able to dedicate herself to the observation of the children during the execution phase, without having to interrupt herself to note down the strategies implemented by them. The logbook was drawn up daily and this allowed the free collection of information on the children's strategies of action and their difficulties.

In Laura's experimentation, an audio recorder and a logbook were used to collect the data. The audio recorder allowed to analyse, at a later stage, discussions between the teacher and the children. Here, too, the logbook was drawn up on a daily basis, taking into account the children's action strategies and/or difficulties.

### 5. EXPERIMENTATION: DATA ANALYSIS AND RESULTS

## a. Phase 1: The encounter with the story

The data below refer to the experimentation conducted by Cinzia.

Initially, the children are left free to view and explore the book independently (there was a copy for the entire classroom), through the methodology of circle-time. The children are given the sole opportunity to dwell initially only on the outer cover of the book (Figure 1) and to pass the book between them. It was not made clear that the content of the book would be used for subsequent educational activities, in order to recreate an a-didactical situation. The *devolution* phase of TDS begins when the teacher transfers to the children the responsibility for the resolution of cognitive activity. Cinzia asks them to imagine what story is told in the book. The children elaborate and express different hypotheses. Then, she asks the children to explore the pages of the book independently, looking in particular at its figures. The intention was to stimulate learning by the discovery (a methodology that differs from the typical frontal lesson) since the child assumes the role of active "discoverer" of knowledge. The figures of a book, in general, have in fact the ability to develop a child's reading skills even if she is not able to read, thanks to their evocative and concrete power (Samuels, 1970). This autonomous exploration has continued to contribute the to elaboration by children of different hypotheses about the content of the book, as shown by some examples below:

# "The book tells the story of so many sheep".

"The book tells the story of a little girl who counts the sheep before she sleeps".

This phase coincides with that of the *implication* of the TDS, in which the child accepted the teacher's offer and is involved in the activity proposed without her guidance. At this point, Cinzia moves on to the actual reading of the story. The reading does not happen all at once, but proceeds through successive pauses, in each of which she dwells on the piece of history that she has just finished reading,

to guide the children in the construction of knowledge.

The first break from reading occurs in the part of the story where Bubal loses one of the sheep after having taken them out to graze the grass. Here Cinzia puts in place a further *devolution*, asking the children if they can find a way to help the prehistoric girl not to lose more. With a rapid and almost implicit implication, the children immediately move on to the next phase of the TDS, the construction of private knowledge, in which they create their own individual internal knowledge and, thanks to their imagination, express their ideas about what is required, thus moving on to the *validation* phase. Here are some examples:

"Maybe if she gives a name to the sheep then she can call them back one by one and make them come back. My father does this with my dogs";

"She could make them go out one by one",

"Instead of letting them out one by one, why doesn't she let them out more?";

"She could count all the sheep".

So spontaneously begins the first idea of counting the sheep, which is supported by the continuation of the reading of the story, in which you observe the application of one-to-one correspondence. In fact, Bubal began to let out of the fence first a sheep for each finger of a hand, then as many sheep as all the fingers of a hand.



Figure 2. V symbol to indicate the open hand. Taken from A. Cerasoli (2012)

Continuing with the reading of the story, the children are told that little



Figure 3. Institutionalisation of knowledge

Bubal traces a mark on the wall similar to her open hand (Figure 2).

The story guides the children to understand the need to indicate numbers in synthetic form. Bubal needs to keep track of the amount of sheep to come out each time, so using the V symbol, similar to her open hand, allows her to indicate the number 5 in a more compact way.

Children *socialize* this knowledge that they have built and validated individually by listening to the story. They start to stretch out a hand and observe its five fingers, simulating Bubal's hand. Cinzia joins them in this socialization of knowledge (Figure 3).

For the children, it is clear that Bubal takes the sheep out in groups of five and keeps track of them by marking on the stone how many groups of five he takes out each time. Before continuing with the reading of the story, Cinzia asks the children some stimulating questions, so as to encourage further elaboration of hypotheses:

"How can little Bubal save time for playing, since there are so many sheep and in this way [making them go out in groups of five] it would take a whole day?".

Here are some answers from the children:

"She lets out the thinnest ones who are hungry and tomorrow she lets out the biggest ones";

"It's better if she lets out more than one hand";

"*True! She can use the other hand too*";



Figure 4: X symbol to indicate two hands. Taken from A. Cerasoli (2012)

"Hands together are more than one hand and she hurries first so she can play".



Figure 5: I symbol to indicate a finger. Taken from: A. Cerasoli (2012)

After this further moment of *socialization*, Cinzia *institutionalizes* the knowledge that emerged, that is, she

makes the knowledge built by the children and socially shared by the group, be reconstructed and officially recognized as knowledge. Specifically, Cinzia points out to the children that while using one hand Bubal could only count groups of 5, using two hands Bubal can count groups of 10. These can be counted with individual fingers. Figure 4 and Figure 5 are clearly identified and understood by the children. In fact, we can observe how, with two open hands, one below the other, the symbol X originates, which allows us to represent the quantity of 10 units; while the single finger is represented by a rod "I", which indicates the single unit. With this set of symbols, Bubal can keep track of all the quantities she wants.

# b. Phase 2: Dramatization of the story

Phase 2 began by asking the children to recall the story of Bubal, telling the sequences that were most important to them. Subsequently, the story was dramatized, using the methodology of role-playing, in a playful atmosphere and, once again, framed in an a-didactical situation, where the pupil acts in the foreground with the dramatization and the object of knowledge (the concept of number and counting). Cinzia's aim was to start the phase of *devolution*, leading pupils to become personally the *implicated*, through dramatization, in the activity of building knowledge. The children carried out the dramatization of the whole story told, impersonating in turn little Bubal and the various sheep. Each "Bubal child" had to count the number of children in the classroom, that is, the number of children in the role of sheep. The fence was represented by the arrangement of the children in circletime. During the counting phase, it was noticed that: some children, if they realized they had made a mistake, go back and start counting again; others, even if they realized they had made a mistake, went on. In general, all the children felt the necessity to touch their classmates to count them (Figure 6).



Figure 6: Counting your classmates



Figure 7: Sheep creation

#### c. Phase 3: Sheep creation

different approaches The to developing the concept of number and counting have been proposed through the methodology of learning by doing, which aims not only at memorizing concepts, but also at understanding them through doing. Through a practicalmanipulative activity, with reference to the history of little Bubal, the children created sheep on cards (Figure 7). Each child had his own card, in which he was helped by the teacher to write his name with a marker. The card showed a silhouette previously made by Cinzia, on which cotton was glued with glue (the use of cotton is justified by the sensory purpose with the softness and the reference to the color of the real sheep). The sheep thus created were then used in subsequent moments, as we shall see,

and also every morning by the children to take the daily presences-absences, for duration entire of the the experimentation. In fact, thanks to the fact that each card bore the name of the child who had made it, it was possible to the one-to-one encourage correspondence between a child and his own card sheep. In order to pursue the cardinal approach to number, that is, the approach that answers the question "How many are there?", the following adidactical situation has been prepared. The milieu was so articulated: each child was asked to take the sheep he had made and to arrange himself in circletime. In the centre of the circle thus formed, two coloured circles are placed with the function of a quantity container (Figure 8).



Figure 8. In circle-time with your sheep

Children are asked to take turns placing their sheep inside one of the circles (Figure 9). The *devolution* begins with the request to collect the desired number of sheep from the circle in which they were placed and put them inside the other, and then answer the question: "How many are they?". Then the children are invited to count the sheep moved into the second circle, choosing their own strategy. The activity is carried out in turn by each child, starting the phase of *implication*. The counting ability had already been resumed and experienced in Phase 2 with the dramatization. Here the children recall their *private knowledge* and proceed to *validate* it with different strategies.



Figure 9: Arrange the sheep in a circle

All the children in the section did the same activity, demonstrating different levels of ability. In the socialisation phase, the children noticed that the last number pronounced corresponded to the quantity contained in the circle. The teacher then institutionalised this knowledge. In the following days, to consolidate the acquisition of this concept, each child, after the routine welcome of each morning, had to put his sheep inside one of the two circles. In the end, it was necessary to count how many children were present. In mid-November, the difficulty was increased by putting the sheep of the children present in one circle and those of the

For example, some children have chosen to count the sheep by passing them from the circle to the floor; others have chosen to pass them on to the experimenter (Figure 10); still, others have chosen to count them by pointing them out with the finger inside the circle itself.



Figure 10: Counting of the sheep in the circle

children absent in the second circle and asking for both quantities to be counted.

After the first weeks of experimentation, the children had begun to enjoy playing the nursery rhyme of numbers. Counting for counting, that is, the oral enunciation of numbers one after the other in order without reference to objects or actions, or the recitation of the nursery rhyme of numbers, involves for the child not only the experience of recurrence (repetition and proceeding in orderly manner), but also the an acquisition of the laws of linguistic production of the verbal sequence. This is an important procedure because it allows the child to proceed continuously in the recitation of numbers and also to gradually experience the feeling of the unlimitedness of the procedure (Liverta Sempio, 1997).

However, we have had the opportunity to experience the recursive approach also benefiting from an observation advanced by a child. After a few days that the sheep count was repeated, by positioning them within one of the two circles, a child proposed to operate the count by moving the sheep from the full circle to the empty circle saving:

"I put the sheep in this empty circle so that from empty it passes to full every time I put more sheep in it".

From this observation, we have taken the cue to say that, by making the sheep jump, to pass from one circle to another, it is possible to consider the "plus one" in the count. Moreover, it was specified that the initially empty circle represented the concept of "zero".

### d. Phase 4: The highest tower

The data below refer to the experimentation conducted by Laura. The children of Laura's section have carried out the same phases described in Cinzia's experimentation, with similar results. Laura carried out Phases 4 and 5 of the experimentation in the school gym.

The children were arranged in circletime, taking their seats on the ground. After having the children tell Bubal's story again, saying one piece each, Laura added a new piece to the story<sup>1</sup>. She explained to the children that some of Bubal's father's friends had moved near her cave because the pastures there are greener and more luxuriant. Since Bubal was so good at looking after her father's flock, these friends asked her if she could look after their flock for the time when they, like her father, were leaving to go hunting. However, now the sheep have become so many. In order to be able to control them all, Bubal would need to look at them from above, perhaps from a tower. A phase of *devolution* begins, asking the children the following question: "Would you help Bubal to build a tower?". The children accept to be *implicated* in this proposal very willingly: it almost seems that an affective bond has been created between the children and the protagonist of the story.

"Yes, of course we are helping Bubal";

## "How beautiful it is, Bubal needs us".

Laura, helped by her colleague Alessandra, divides the section into two teams, A and B, of which they will be respectively the team leaders. She involves the children in a game called "Highest Tower Wins". She explains that the tower will have to be made with coloured building blocks that the Bubal's sheep had fun hiding in the gym. On the floor adjacent to the sides of the gym, in fact, Laura had arranged, disorderly, 20 building blocks per side. Team A is assigned the task of controlling the right wall, team B controls the left wall. We

<sup>&</sup>lt;sup>1</sup> The additional piece of the story was invented by me, it is not contained in the book.

start after the "Go" of the experimenter and stop at her "Stop". All the building blocks had to be collected and gradually brought to the team leader who would collect them in a bag.

At Laura's "Stop", precise deliveries were given to each team. To build the tower, you had to take one building block at a time from the bag; all the members of the team together, aloud, had to count as you pulled a building block out of the bag and gradually stack the building blocks. The team that finished first had to say "Twenty" (the last number that would have finished the count that corresponds to the cardinal of the buildings used) and the team that had built the highest tower would win.

Team B managed to finish first. Laura arranged the two teams on the ground, one in front of the other, asking Alessandra to keep the two towers apart for a moment. So she asked the children: "*In your opinion, did team B build the tallest tower?*". Two children from team A responded.

"Yes, because they screamed "Twenty";

"They were faster than us".

It is as if these children had expressed *validation* of their *private knowledge*: each team had 20 pieces. If team B finished earlier, it means that they had used all the pieces, while team A remained behind.

At this point Laura asks Alessandra to put the two towers next to each other, in the middle, between the two teams. And she asks again: "*Did Team B build the highest tower?*". "*Yes, yes. You can see that it is higher*".

Laura insists: "And if I couldn't see them, how could I say that team B tower is higher than team A one?".

The children think about it a bit, then one of them says:

C (child): "You have to count the bricks".

L (Laura): "Why do I have to count the bricks?".

C: "Because so you know ... you know that if the tower of B has more bricks than that [indicates the team A tower] ... that [the team A tower] does not win, because it is lower".

L: "So help me count the bricks of A and B?".

All children, together with Laura, almost wanting to *socialize* the knowledge built and validated by their classmates, count the individual building blocks that make up the team A tower: "18' and then the one of team B: "20'.

Laura asks if we can now be sure that the tower of team B is exactly the highest and a little girl answers:

*Yes, because 18 is less than 20'*. Laura *institutionalized* this knowledge: *That's right, 18 bricks are less than 20 bricks. 18 is smaller than 20'*.

At first, the children operated by making comparisons between sizes: "highest tower/lowest tower". Subsequently, with the counting of the single building blocks, they made comparisons of quantities, identifying in the single building block the unit of measure to establish the height of the tower. By establishing that the highest tower is the one with the highest number of building blocks together, they are able to recognize and correctly apply order relationships. The two towers have certainly benefited from their mutual come up beside, which has provided a concrete visual representation of what larger and smaller numbers mean.

## e. Phase 5: Colors of the tower

Again in the gym, there is the last game for the children. They are always divided by team and each one is given a pile of 15 building blocks: 5 blue, 5 red, and 5 yellow. A speed challenge is launched again. Now the tower for Bubal must be built by stacking the building blocks in a precise order: the first piece must be blue, the second red, the third yellow, and so again until all the pieces are exhausted. Children have no trouble stacking the constructions according to the instructions. Team A ends first, with a clear difference from team B.

L (Laura): "*Team A, who explains to me how you got to finish first?*"

C (child): "*Chiara said blue, red, yellow and we did that when Chiara said the color, we gave it to Giorgio*".

The children spontaneously made cooperative learning. They designated Giorgio as the child who had to stack the pieces, Chiara as the little girl who had to remember the orderly sequence of colours and the others had to take what Chiara said from the pile.

Laura also asked team B how they had proceeded.

C (child): "Because of Elisa we lost";

L (Laura): "*Why? What did Elisa do?* [Elisa in the meantime cried and was consoled by Alessandra]".

C: "She wanted to make bunches of color and then took a brick from the blues, one from the reds and one from the yellows and stuck them one under the other. But when we finished the bunches, the others had already won".

L: "Elisa's idea seems to me to be a good one because she divided the building blocks by colour [which had been given to the children in a disorderly manner in a single bunch], she first put the bunch of blues, second the bunch of reds and third the bunch of yellows. So you can build the tower right away. Do we all want to try Elisa's method?".

Without any more competition, the children have challenged themselves again and again, in turn, in the building of the tower, with the "method of Elisa". Someone also said while stacking:

"For first, for second, for third".

This game on the one hand still insists on the recursive approach, because they come across a recursive composition: alternating blue-red-yellow building blocks. On the other hand, children are invited to take into account the position of that certain building block during stacking. They dominated the control of the order, fostering the understanding of successor and predecessor, with non-trivial strategies for 5-year-old children. They then carried out operations of comparison, relationship and order of buildings,

putting into practice the ordinal approach.

### 6. DISCUSSION AND CONCLUSION

The results obtained, considering the small number of children, are not statistically significant. However, they allow us to answer the research question. In fact, they highlight how storytelling is a methodology that allows specific mathematical content to be conveved in pre-elementary school children. All the phases of the experimentation conducted always revolve around the reference of Bubal and her sheep. I will, therefore, examine the salient points of the phases and the mathematical content they addressed. In the encounter with the story (Phase 1), the children know Bubal and immediately come into contact with a problem that worries the little protagonist: the loss of the sheep. Immersed in the story, the children spontaneously come up with a mathematical solution: suggesting to Bubal to count the sheep. This idea is gradually reinforced with the application of one-to-one correspondence, using as a starting set the fingers of one hand to form groups of 5 sheep, and then the fingers of two hands to form groups of 10 sheep. With the dramatization of the story (Phase 2), the children take turns in Bubal and count the other classmates who play the role of sheep. Not everyone can count perfectly and they all need to point and/or touch their classmates in order to associate a number with them. They slowly begin to practice the concept of counting. This aspect is reinforced

with the creation of the sheep (Phase 3), in which each child creates his or her own sheep on paper and writes his or on her name it. Α one-to-one correspondence between each child and his paper sheep takes place. The paper sheep is used in various counting activities that make it possible to introduce two approaches to the number: the cardinal and the recursive. The first is pursued using two sets represented by two circles, in which a certain number of paper sheep are placed by the children. The children are invited to count the quantities of sheep present in the circles and understand that the last number pronounced corresponds to the cardinal number, i.e. the quantity of sheep contained in the circles. In the following days, this concept will be consolidated by replacing the sets represented by the circles with the sets represented by the children present and absent from school. The children acquire with ease the rhyme of numbers and gradually experience the feeling of the unlimitedness of the procedure. The repetition of the numbers in succession allows the children to experience the recursive approach. This is also presented with the use of circles: one can start from the empty circle representing zero and make the sheep jump from the full circle to the empty circle. The circle gradually fills up because you add one sheep to the previous quantity. The geometric approach and the ordinal approach are with the tower pursued stratagem (Phases 4 and 5). The narrative background is inspired by Bubal's story,

which is expanded by the author of the manuscript, with the aim of continuing the activity in a context now familiar to children. In fact, it is said that the number of sheep that little Bubal has to look after has increased considerably, so she needs a tower to look after all her sheep. The children are divided into teams and compete in the construction of the highest tower, using a number of building blocks. By placing the towers next to each other, the children visually understand that the tallest tower is the one made up of the largest number of building blocks. The building blocks are thus perceived as a unit of measurement to determine the height of the tower. Finally, the construction of the tower with specific colors that alternate neatly allows for an orderly approach. Children relate to space-time relationships (first comes blue, then red, after yellow) and maintain control of the order, thus also internalizing the meaning of previous and next numbers.

The children follow all these didactic proposals with lightness, on the one hand, because they are linked to the story and its protagonist; on the other hand, because each activity has been designed as an a-didactical situation, often with playful components. It is therefore considered desirable to set up mathematical activities at the preelementary school creating links with other areas, such as narrative, practicalmanipulative and motor one. In this way, children would be given the opportunity to develop their skills in a richer way and to live deep learning experiences, with a positive and creative attitude.

### 7. RECOMMENDATIONS

The research that has been carried out on the effectiveness of storytelling as a methodology to convey mathematical to pre-elementary school content children has shown that storytelling can start from simple stories that are of interest to children. The protagonist of the story is a little girl, just like the audience to whom the story is addressed. The children feel involved and identify themselves with the character, sharing her needs, on an empathic level. This allows affecting their learning. The story is read, then dramatized. This improves memory over time. In addition, for two months the activities that are carried out all focus on the story and its characters, Bubal and her sheep. This makes it possible to construct shared mathematical meanings, based on memories that belong to the heritage of the class. The activities related to history different are of various types: methodologies are used (circle-time, learning by doing, practical-manipulative activities, ...) and different materials buildina (book, cardboards, cotton, blocks, ...). In particular, children's ideas are given wide space. Everyone is free to discuss. Some link the activities to concrete experiences made before. For example, it is a child who suggests to Bubal to give a name to the sheep because his daddy does so to call his dogs. The lesson continues using some of the children's observations: for

example, they say that Bubal could count her sheep so as not to lose them. Every attempt, every strategy of the children is listened to and sometimes enhanced. For example, the strategy adopted by Elisa to build the tower using the coloured blocks in a certain order had first been rejected by her team, who considered her guilty of the match lost. Thanks to the intervention of the teacher, all the children have the opportunity to better understand that Elisa's strategy is still valid and they all try to use it. The

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lesson, which wants to pursue specific mathematical aims, is built together with the children.

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