

## Full QCD in external chromomagnetic field

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We investigate the deconfining phase transition in full QCD with two flavors of staggered fermions in presence of a constant abelian chromomagnetic field. We find that the deconfinement temperature decreases and eventually goes to zero by increasing the strength of the chromomagnetic field. Moreover our results suggest that the chiral transition coincides with the deconfinement transition and therefore even the chiral critical temperature depends on the applied chromomagnetic field. We also find that the chiral condensate increases with the strength of the chromomagnetic field.

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## 1. Introduction

In a previous study [1] on the vacuum dynamics of non abelian gauge theories we found that the deconfinement temperature depends on the strength of an external abelian chromomagnetic field. In particular we found that the deconfinement temperature decreases when the strength of the applied field is increased and eventually goes to zero. This effect corresponds to the reversible Meissner effect in the case of ordinary superconductors therefore we refer to it as "vacuum color Meissner effect". This effect could shed light on confinement/deconfinement dynamics, therefore in our opinion it is important to test if this effect continues to hold even when switching on fermionic degrees of freedom.

The aim of the present work is to understand if the deconfinement temperature depends on the strength of an external abelian chromomagnetic field even in the case of full QCD.

To this purpose we performed numerical simulations for finite temperature  $N_f = 2$  QCD in an external abelian chromomagnetic field. Simulations have been done using APEmille crate in Bari and the recently installed computer facilities at INFN apeNEXT Computing Center in Rome.

## 2. The method

To investigate the QCD dynamics in presence of a external background field at finite temperature we use the following lattice thermal partition functional [2–4]

$$\begin{aligned} \mathcal{Z}_T [\vec{A}^{\text{ext}}] &= \int_{U_k(L_t, \vec{x})=U_k(0, \vec{x})=U_k^{\text{ext}}(\vec{x})} \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-(S_W + S_F)} \\ &= \int_{U_k(L_t, \vec{x})=U_k(0, \vec{x})=U_k^{\text{ext}}(\vec{x})} \mathcal{D}U e^{-S_W} \det M, \end{aligned} \quad (2.1)$$

where  $S_W$  is the Wilson action,  $S_F$  is the fermion action, and  $M$  is the fermionic matrix. The spatial links are constrained to values corresponding to the external background field, whereas the fermionic fields are not constrained. The relevant quantity is the free energy functional defined as

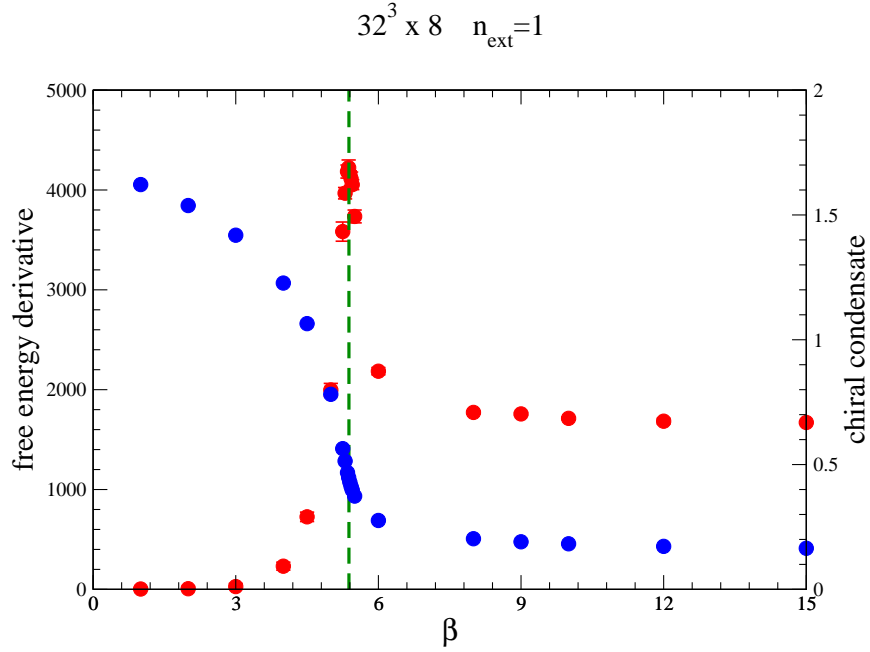
$$\mathcal{F}[\vec{A}^{\text{ext}}] = -\frac{1}{L_t} \ln \left\{ \frac{\mathcal{Z}_T[\vec{A}^{\text{ext}}]}{\mathcal{Z}_T[0]} \right\}. \quad (2.2)$$

We can evaluate by numerical simulations the derivative of the free energy functional with respect to the gauge coupling

$$F'(\beta) = \frac{\partial \mathcal{F}(\beta)}{\partial \beta} = V \left[ \langle U_{\mu\nu} \rangle_{\vec{A}^{\text{ext}}=0} - \langle U_{\mu\nu} \rangle_{\vec{A}^{\text{ext}} \neq 0} \right] \equiv Vf(\beta), \quad (2.3)$$

where the subscripts on the averages indicate the value of the external field. The generic plaquette  $U_{\mu\nu}(x) = U_\mu(x)U_\nu(x + \hat{\mu})U_\mu^\dagger(x + \hat{\nu})U_\nu^\dagger(x)$  contributes to the sum in Eq. (2.3) if the link  $U_\mu(x)$  is a "dynamical" one, i.e. it is not constrained in the functional integration eq. (2.1). Observing that  $f[\vec{A}^{\text{ext}}] = 0$  at  $\beta = 0$ , we may eventually obtain  $f[\vec{A}^{\text{ext}}]$  from  $f'[\vec{A}^{\text{ext}}]$  by numerical integration:

$$f[\vec{A}^{\text{ext}}] = \int_0^\beta f'[\vec{A}^{\text{ext}}] d\beta'. \quad (2.4)$$



**Figure 1:** The derivative of the free energy (blue circles) and the chiral condensate (red circles) vs.  $\beta$  on a  $32^3 \times 8$  lattice and bare quark mass  $am_q = 0.075$  in correspondence of external field strength  $n_{\text{ext}} = 1$ .

### 3. Numerical results

We consider an external abelian chromomagnetic field directed along spatial direction 3 and color direction 3. On the lattice this corresponds to constrain the spatial links to

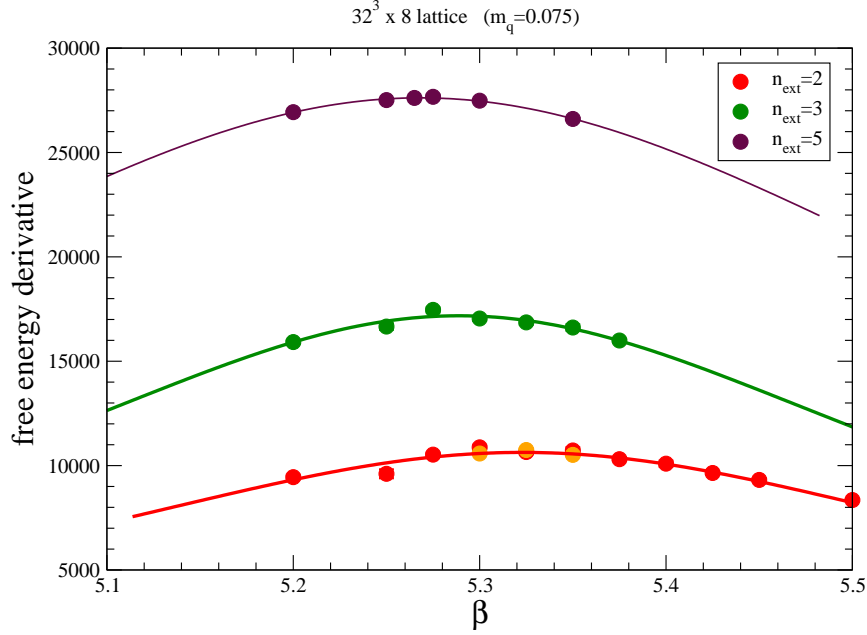
$$\begin{aligned}
 U_1^{\text{ext}}(\vec{x}) &= U_3^{\text{ext}}(\vec{x}) = \mathbf{1}, \\
 U_2^{\text{ext}}(\vec{x}) &= \begin{bmatrix} \exp(i\frac{gHx_1}{2}) & 0 & 0 \\ 0 & \exp(-i\frac{gHx_1}{2}) & 0 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned} \tag{3.1}$$

where due to toroidal topology of the lattice the field strength gets quantized

$$a^2 \frac{gH}{2} = \frac{2\pi}{L_1} n_{\text{ext}}, n_{\text{ext}} \text{ integer.} \tag{3.2}$$

In Fig. 1 we display the derivative of the free energy and the chiral condensate at fixed external field strength. The peak in the derivative of the free energy correlates to the drop of the chiral condensate.

The deconfinement phase transition is located by looking at the peak of the derivative of the free energy. In pure gauge theories we found [1] that the peak position depends on the strength of the external chromomagnetic field. Indeed by varying the external field strength we found even in the case of full QCD that the phase transition shifts by varying the field strength. In Fig. 2 the free energy derivative vs.  $\beta$  in correspondence of three values of the field strength is displayed. And one can see that the peak of the free energy derivative shifts towards lower values of  $\beta$  by increasing the field strength. It is also interesting to notice that (see Fig. 3) the value of the chiral condensate depends on the strength of the applied chromomagnetic field.



**Figure 2:** The peak position in the derivative of the free energy in correspondence of three values of the external field strength.

#### 4. The critical field strength

Now we want to repeat the analysis of Ref. [1] in order to ascertain if (i) the deconfinement temperature goes to zero as increasing the chromomagnetic field strength (ii) there exists a critical field strength beyond which the system is deconfined even at zero temperature.

The deconfinement temperature can be derived using the values of the critical coupling obtained by locating the peak of the derivative of the free energy in correspondence of each value of the external field strength.

In our previous study [1] in pure gauge SU(3) we used the string tension as a physical scale (see Fig. 4).

$$\frac{T_c}{\sqrt{\sigma(\beta_c)}} = \frac{1}{L_t \sqrt{\sigma(\beta_c)}}. \quad (4.1)$$

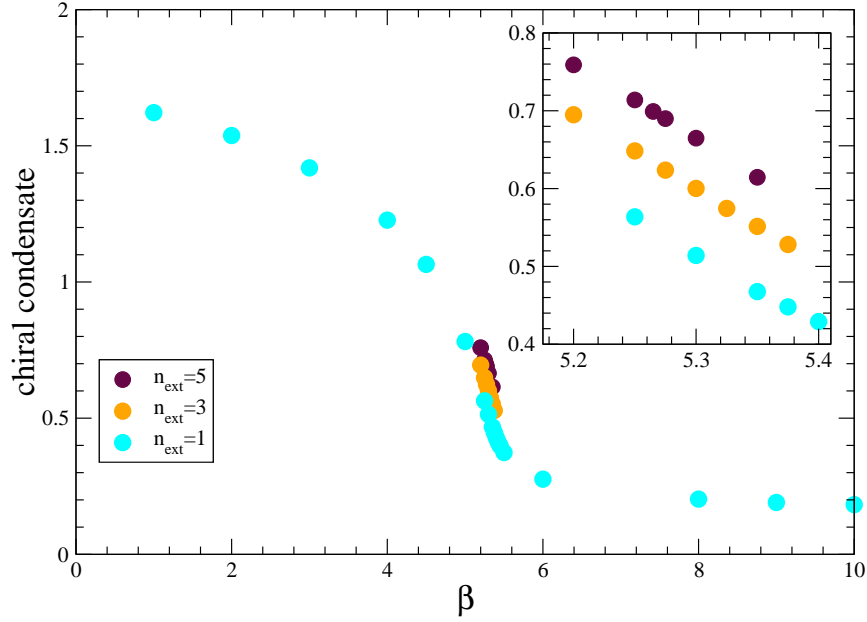
Moreover, using eq. (3.2), the field strength is

$$\frac{\sqrt{gH}}{\sqrt{\sigma(\beta_c)}} = \sqrt{\frac{4\pi n_{\text{ext}}}{L_x \sigma(\beta_c)}}. \quad (4.2)$$

The lattice data can be reproduced by a linear fit

$$\frac{T_c}{\sqrt{\sigma}} = \alpha \frac{\sqrt{gH}}{\sqrt{\sigma}} + \frac{T_c(0)}{\sqrt{\sigma}}, \quad (4.3)$$

with  $T_c(0)/\sqrt{\sigma} = 0.643(15)$ . Where our determination for  $T_c(0)/\sqrt{\sigma}$  is consistent with the determinations  $T_c/\sqrt{\sigma} = 0.640(15)$  obtained in the literature without external field [5]. Using eq. (4.3)



**Figure 3:** The chiral condensate vs.  $\beta$  in correspondence of different values of the external field strength. The region near the chiral phase transition is displayed in the inset .

the critical field can now be expressed in units of the string tension

$$\frac{\sqrt{gH_c}}{\sqrt{\sigma}} = 2.63 \pm 0.15. \quad (4.4)$$

Assuming  $\sqrt{\sigma} = 420$  MeV, eq. (4.4) gives for the critical field

$$\sqrt{gH_c} = (1.104 \pm 0.063) \text{ GeV} \quad (4.5)$$

corresponding to  $gH_c = 6.26(2) \times 10^{19}$  Gauss.

In the present work we perform a preliminary analysis of the lattice data obtained in full QCD with  $N_f = 2$  using the  $\Lambda$  scale introduced in [6, 7]

$$\Lambda = \frac{1}{a} f(g^2) (1 + c_2 \hat{a}(g)^2 + c_4 \hat{a}(g)^4) \quad (4.6)$$

with

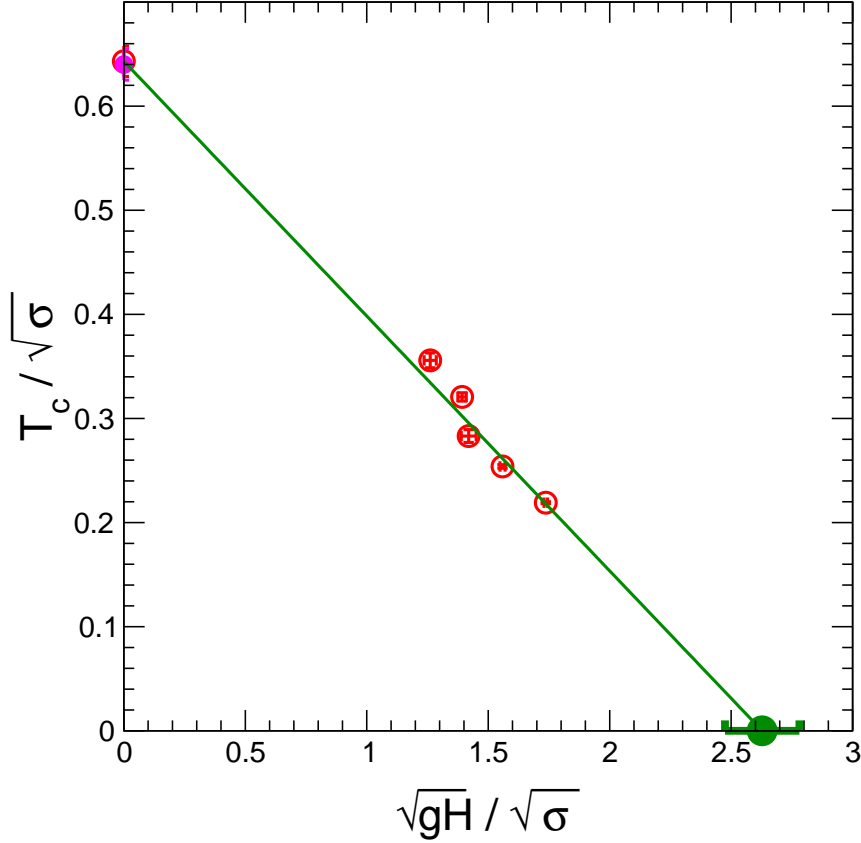
$$\hat{a}(g)^2 \equiv \frac{f(g^2)}{f(g^2 = 1)} \quad (4.7)$$

and  $f(g^2)$  the 2-loop  $\beta$ -function with  $N_f = 2$ .

The data for  $T_c/\Lambda$  versus the external field strength  $(gH)^{1/2}/\Lambda$  are reported in Fig. 5. In Fig. 5 we display also the pure gauge data analysed using the same scale  $\Lambda$  given in Eq. (4.6). Data reported in Fig. (5) shows that the deconfinement temperature goes to zero, both in quenched and unquenched case, in correspondence of a given critical field strength.

In particular, assuming a linear dependence on  $\sqrt{gH}$  as in the quenched case

$$\frac{T_c(gH)}{T_c(0)} = 1 - \frac{\sqrt{gH}}{\sqrt{gH_c}} \quad (4.8)$$



**Figure 4:** SU(3) pure gauge. The critical temperature  $T_c$  estimated on a  $64^3 \times 8$  lattice in units of the string tension, eq. (4.1), versus the square root of the field strength  $\sqrt{gH}$  in units of the string tension, eq. (4.2). Solid line is the linear fit eq. (4.3). In correspondence of zero vertical axis: open circle is  $T_c/\sqrt{\sigma}$  at zero external field; full circle is the determination of  $T_c/\sqrt{\sigma}$  obtained in the literature [5] The green full point is the critical field in units of the string tension, eq. (4.4).

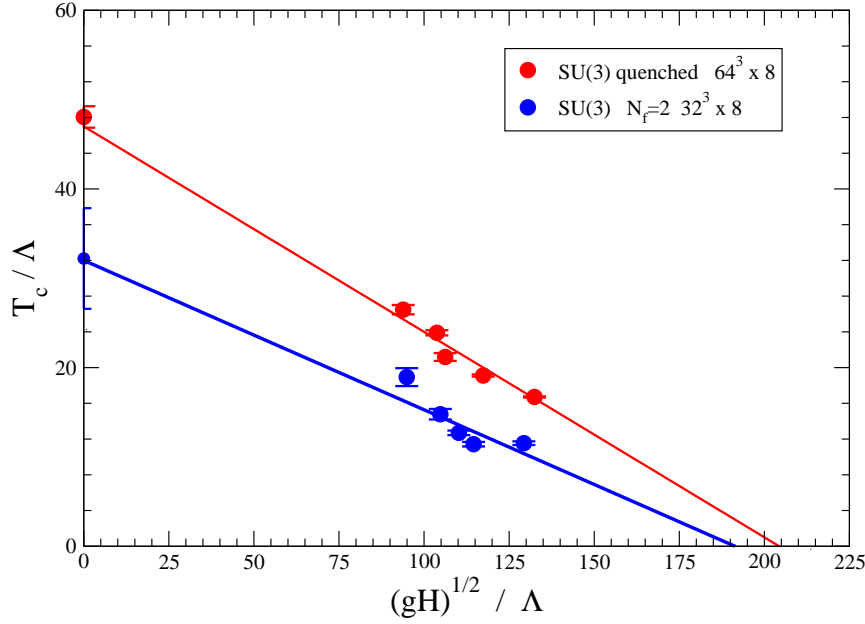
and using the value  $\Lambda/\sqrt{\sigma} = 0.01338$  (Eq. 4.3) of Ref. [7]) we found that

$$\sqrt{gH_c}(N_f = 2) \sim \sqrt{gH_c}(\text{quenched}) \simeq 1.1\text{GeV}. \quad (4.9)$$

The value for the critical field strength in the present analysis for the quenched case is well consistent with the value obtained in Ref. [1] using the scale of the string tension and reported here in Eq. (4.5).

## 5. Conclusions

We studied full QCD with  $N_f = 2$  flavors of staggered fermions in presence of a constant chromomagnetic field. We found that, as in the quenched case (see ref. [1]), the critical temperature for the deconfinement phase transition depends on the strength of the applied field and eventually goes



**Figure 5:**  $T_c/\Lambda$  versus the strength of the applied chromomagnetic field in unit of the scale  $\Lambda$  (Eq. 4.6).

to zero. The value of the critical field strength is  $\simeq 1$  GeV. Moreover numerical results suggest that the chiral critical temperature is consistent with the deconfinement temperature and both depend on the strength of the external chromomagnetic field.

Finally our numerical results shows that the value of the chiral condensate increases with the strength of the external chromomagnetic field. This is an intriguing effect that deserves further investigations.

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