A novel phase retrieval technique based on propagation diversity via a dielectric slab

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Abstract: This paper deals with a novel technique to determine the far field of an aperture starting from the knowledge of two near-field intensity data sets collected over the same measurement plane. The diversity between the two intensity data sets is achieved by ensuring different conditions of the near field propagation between the aperture and the measurement plane. In particular, one measurement is performed under free-space propagation condition while the second one is performed by exploiting a dielectric slab, with known properties, filling partly the space between the aperture and the measurement plane. A phase retrieval technique, that faces a non linear inverse problem, is solved by assuming as unknown the plane wave spectrum of the aperture field. The feasibility of the novel approach is presented also in comparison with the usual near field phase retrieval technique exploiting measurements of the near field intensity over two scanning planes.

OCIS codes: (100.5070) Phase retrieval; (100.3190) Inverse problems.

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1. Introduction

Phase retrieval problem is of significant interest in many areas of the theoretical and applied science such as: x-crystallography [1], electronic microscopy [2], astronomy [3], near field antenna diagnostics and inverse scattering [4, 5, 6] only to quote few examples.

The phase retrieval technique faces the nonlinear inverse problem of determining a complex function from the only amplitude information, and the solution is found as the global minimum of a functional whose unknowns best fit the phase-less data. Since the functional is non-quadratic with respect to the unknowns, it can exhibit local minima where the minimization algorithms can be trapped [7-9]. In this unfortunate case, the retrieved solution corresponds to a false solution that can be completely different to the true solution, so affecting the full reliability of the phase retrieval approach.

Two main classes of phase retrieval algorithms are available in literature and differ mainly in the choice of the objective functional to be minimized. A first class of phase retrieval algorithms is based on the minimization of the distance between the *amplitude* of the measured and the reconstructed near-field [2, 8, 10-11].

At variance, here, we adopt a cost function accounting the *square amplitude* of field as data of the problem. In this way, the phase retrieval problem is formulated as the inversion of a quadratic operator [7, 9, 12]. To deal with this simple nonlinearity allows to achieve good performances with respect to the local minima problem [9] and also to perform a thorough analysis of the effect of some parameters on the local minima problem [7, 9 12]. To this end, it has been already pointed out how the increase in the ratio between the number of independent data and the number of unknowns allows us to achieve a favorable effect on the local minima problem. This increases the amount of independent data that can be achieved by assuming "different" square amplitude sets. Diversity in data can be pursued in different way such as by: adopting the square amplitude of the aperture field and of the far field [12]; exploiting a priori information on the support of the unknown function [7, 9]; collecting square amplitude information on two scanning planes in near field [13, 14]; employing different receiving probes scanning the same measurement plane [4].

The aim of this paper is to present a new phase retrieval technique using only near-field intensities collected over a *single* scanning plane. The *diversity* between the two intensity data sets is achieved by ensuring different conditions of the near field propagation between the aperture and the measurement plane. In particular, a measurement is performed under free-space propagation and the other one by exploiting a dielectric slab with known properties (dielectric permittivity and thickness) that fills partly the space between the aperture and the measurement plane.

This technique is of particular applicative interest in antenna diagnostics at millimeter and THz frequencies [15] and /or when various factors make near-field phase measurements more and more difficult with increasing frequency: probe-positioning errors, temperature changes and undesired mechanical movement of the cables connecting the probes to the receiver, stability and accuracy of the transmitter and receiver [13, 15].

The paper is organized as follows. In Section 2, the far-field estimation problem from near-field intensity data is formulated for both the two scanning planes and the single plane/ dielectric slab implementations. In Section 3, the two planes implementation is discussed and

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numerical results are presented. In Section 4, the properties of the single plane /dielectric slab implementation are discussed and numerical results are presented with the aim to point out the feasibility of the proposed technique. Finally, conclusions follow.

2. The Formulation

This Section is concerned with the problem of determining the far-field radiated by a planar aperture starting from the only square amplitude of the radiated field in the near-zone, for the two formulations mentioned in the Introduction.

First we consider the case of the approach based on the measurement of the near field over two scanning planes [13, 14]. To this end, let us consider a planar aperture located within the *xy* plane and of extent *2a* and *2b* along the *x*- and *y*-axis respectively. The aperture field is assumed to have only the *y*-directed transverse component.

The *y*-component of the radiated field over the plane at $z = z_1$ can be expressed as

$$E(x,y,z_1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{E}(u,v)\exp[-j(ux+vy)]\exp(-jwz_1)dudv = T_1(\hat{E}) \quad (1)$$

where $u = \beta\sin\theta\cos\varphi$, $v = \beta\sin\theta\sin\varphi$, $w = \sqrt{\beta^2 - u^2 - v^2}$, $\beta = 2\pi/\lambda$ and λ is the wavelength. $\hat{E}(u,v)$ is the plane wave spectrum (PWS) of the aperture field and the time dependence $\exp(j\omega t)$ has been assumed and omitted.

According to eq. (1), the problem of determining the phase of $E(x,y,z_1)$ from the knowledge of its square amplitude is equivalent to the one of reconstructing the PWS $\hat{E}(u,v)$ from the knowledge of the square amplitude of the field $E(x,y,z_1)$. Differently from the conventional technique based on the phase and amplitude measurement of the near field over a single plane, for phaseless near field techniques a second set of data is required. The two-planes implementation in [13, 14] assumes as data the square amplitude distribution of the near-field over a second scanning surface located at distance $z = z_2$ (in this paper we assume $z_1 > z_2$). The near-field over the second scanning surface is related to the near field over the first surface through the following linear integral relation:

$$E(x,y,z_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{E}(u,v)\exp[-j(ux+vy)]\exp(-jwz_2)dudv = \quad (2)$$
$$T_2(\hat{E}) = T_1(\exp[-jw(z_2 - z_1)]\hat{E}) = T_1(\exp[\ jw(z_1 - z_2)]\hat{E})$$

As it has been already shown in [13, 14], when the spacing between the two planes increases, the role of the term $\exp(-jw(z_2 - z_1))$ becomes more and more relevant to ensure diversity between the two data sets so that the information content increases.

The same effect can be also achieved by a second approach that exploits measurements on the single measurement plane in the near-zone at $z = z_1$. In particular, it exploits as a first measurement the same of eq. (1). Differently, the second measurement is achieved by partially filling the space between the aperture plane and the measurement plane located at $z = z_1$ with a dielectric slab of relative dielectric permittivity ε_r and thickness *d*. By neglecting depolarization effects, the second near field measurement is given as

$$E_d(x,y,z_1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tau(u,v)\hat{E}(u,v)\exp[-j(ux+vy)]\exp[-j(w_d - w)d]\exp(-jwz_1)dudv \quad (3)$$
$$= \hat{T}_2(\hat{E}) = T_1(\exp[-j(w_d - w)d]\tau\hat{E})$$

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where $\tau(u,v)=[u^2T_o(u,v)+v^2T_p(u,v)]/(u^2+v^2)$ accounts for the transmission of the PWS through the slab (apart from the $\exp(-jw_d d)$ term), $T_p(u,v), T_0(u,v)$ are the transmission coefficients in the perpendicular and horizontal polarization with respect to the incidence plane of each plane wave, respectively. It can be verified easily that in many circumstances the phase function of $\tau(u,v)$ is slowly varying so that in (3) it can be now appreciated how the term $\exp[(-j(w_d-w)d]$, where $w_d=\sqrt{\beta^2\varepsilon_r-u^2-v^2}$, has the same role as the defocusing exponential factor $\exp[-jw(z_2-z_1)]$ in (2).

For both implementations, the problem can be formulated in an unified way as the reconstruction of the PWS from knowledge of the square amplitudes

$$(M_1^2, M_2^2)=(|L_1\hat{E}|^2, |L_2\hat{E}|^2) \tag{4}$$

being the linear operators $L_1=T_1$ and $L_2=T_2$, $L_2=\hat{T}_2$ for the two-planes and the single plane/dielectric slab case, respectively. Thus the phase-retrieval problem at hand can be cast as the solution of the equation (4), which is searched for as the *global minimum* of the functional

$$\Phi(\hat{E})=\|\,|L_1\hat{E}|^2-\tilde{M}_1^2\,\|^2+\|\,|L_2\hat{E}|^2-\tilde{M}_2^2\,\|^2 \tag{5}$$

In (5), $\|\cdot\|^2$ is the usual quadratic norm in the data space, $\tilde{M}_1^2$ and $\tilde{M}_2^2$ are the measurement errors and noise affected versions of the actual square amplitudes M_1^2 and M_2^2, respectively.

Here, we make the further hypothesis that the aperture radiates all its power within a limited angular region of the visible domain. In this way, $\hat{E}(u,v)\approx 0$ outside a circular domain Ω of radius ρ comprising all the directions $u^2+v^2\le\rho^2$ contained within the *visible domain*, namely $\rho<\beta$. Due to the Fourier transform relationship between the PWS $\hat{E}(u,v)$ and the aperture field, that is in turn is defined on the rectangular domain of extent $2a\times 2b$, the $\hat{E}(u,v)$ PWS function is amenable of a representation through a Shannon sampling series [4, 13, 14]

$$\hat{E}(u,v)=\sum_{n=-N}^{N}\sum_{m=-M}^{M}\hat{E}_{nn}\,\mathrm{sinc}(au-n\pi)\,\mathrm{sinc}(bv-m\pi) \tag{6}$$

where $\hat{E}_{nm}=\hat{E}(n\pi/a, m\pi/b)$ and $\mathrm{sinc}(x)=\sin(x)/x$.

The indices N and M in (6) are chosen in order to satisfy the relation $(N\pi/a)^2+(M\pi/b)^2\le\rho^2$; this relation means that the spectrum samples searched for as actual unknowns are the ones belonging to the a priori known circular domain Ω of radius ρ where the spectrum is assumed to be significantly different from zero. The square amplitude of the near field are represented through their samples at the uniform step of a quarter of a wavelength [4, 13, 14], because of the band-limited properties of the near field functions over a plane.

As far as the minimization of the functional in (5) is concerned, an iterative procedure based on the Pollak-Ribiere method [16] is applied. The evaluation of the updating direction requires the computation of the gradient of the cost function with respect to the real and imaginary parts of the PWS samples $\hat{E}_{nn}$. This operation, as well as the other ones of the iterative procedure, are performed in an efficient way thanks to the FFT technique [4, 13].

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The convergence properties of the minimization scheme are improved by the accurate and fast evaluation of the optimal step, ensuring the maximum decrease of the cost function along the updating direction, by solving in an exact way a third-degree algebraic equation [7, 9, 12].

3. The two planes implementation

As shown in the above Section, the solution approach for both the implementations is cast as the global minimization of the functional (5).

For both the implementations, the cost function in eq. (5) arises as a non quadratic function with respect to the unknown parameters $\hat{E}_{nn}$ and thus the local minima problem arises. In fact, the large number of unknown parameters to be searched for such a problem makes it feasible only the adoption of deterministic minimization schemes in order to achieve the global minimum of the functional in (5). However, these schemes are able only to achieve a minimum closer to the starting point of the procedure and can be trapped into a local minimum that can be completely different from the global one.

As shown in [7, 9, 12], the local minima problem for a quadratic operator has been thoroughly investigated; in particular, it has been pointed out how the increase in the ratio between the number of independent data and the dimension of the unknowns space has a favourable effect on the local minima problem.

It is to be pointed out that, within operators (1) and (2), diversity between the sets of intensity data on the two scanning planes is ensured by the $\exp[jw(z_1 - z_2)]$ term that accounts for the back-propagation from the plane at z_1 to the plane at z_2. Since this term has unit modulus in the visible domain, the possibility to achieve the diversity in near field data is related to the variability of the phase function in $\exp[jw(z_1 - z_2)]$. In particular, as long as the phase term is increasingly varying in the *(u,v)* plane, the more and more different the two sets of the near–field intensity data are. This allows to establish the positive role of the increase in the distance z_1- z_2 since this arises a more pronounced variation of the phase of the $\exp(jw(z_1 - z_2))$ term. Figure 1 depicts the behaviour of the phase term of $\exp\left[j(w(u,v) - w(0,0))(z_1 - z_2)\right]$ at $v=0$: as expected the phase is increasingly varying as the distance $(z_1 - z_2)$ increases.

Note that such an investigation performed along the cut-line $v=0$ has a more general validity, since, in virtue of the dependence of w from the variable u^2+v^2 , this behaviour is similar to the one along all the cut-lines passing through the origin of the *(u,v)* plane.

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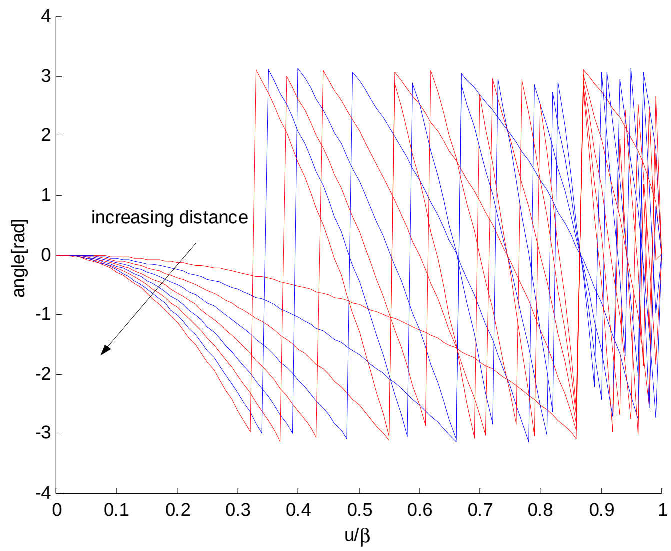

Fig. 1. Phase variation of $\exp[j(w(u,v)-w(0,0))(z_1-z_2)]$ for distances (z_1-z_2) ranging from 1λ to 9λ with a step of 1 λ .

Let us turn now to consider a first numerical result for the two-planes implementation. We refer also in the following to the same source with the aperture field exhibiting a beam squint, given by

$$E_a(x,y)=\cos(\pi x/(2a))\cos(\pi y/(2b))\exp(-j\beta x\sin(5\pi/180)-j0.03y^2) \tag{7}$$

where *2a=2b=14*λ. The square amplitude of the near field is measured over two planes at $z_1=10\lambda$ and $z_2=6\lambda$ at 128x128 measurement points equally spaced by λ/4.

A completely random set of the PWS samples, the same for all the numerical test cases shown in this paper, is considered as starting point of the minimization procedure. For every presented case, we adopt the same solution strategy in the minimization. In particular, in the early stages of the minimization procedure, only the significant samples of the PWS $\hat{E}(u,v)$ are considered [4, 7]. This makes it possible to deal with with an enlarged ratio between the amount of independent data and the number of unknowns so that a favourable effect on the local minima is achieved [4, 7]. Once the significant samples of $\hat{E}(u,v)$ are reliably estimated, the solution is improved by gradually increasing the number of unknowns until all the samples falling within the domain Ω with radius $\rho=0.8\beta$ are considered. Finally, the result is improved by adopting the weighted formulation [7, 9, 12],

$$\Psi(\hat{E})=\sum_{i,j}\frac{(|L_1\hat{E}|^2_{ij}-\tilde{M}^2_{1ij})^2}{\tilde{M}^2_{1ij}}+\frac{(|L_2\hat{E}|^2_{ij}-\tilde{M}^2_{2ij})^2}{\tilde{M}^2_{2ij}} \tag{8}$$

where $|L_1\hat{E}|^2_{ij},\tilde{M}^2_{1ij}$, $|L_2\hat{E}|^2_{ij},\tilde{M}^2_{2ij}$ are the samples of the theoretical and measured intensity data over the first and second scanning planes, respectively.

The adoption of this solution strategy allows us to achieve the global minimum starting from the previously mentioned random starting point. This can be inferred by the comparison

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between the modulus and the phase of the measured and retrieved near field over the cut lines x=0 and y=0 of the plane at $z_2 = 6\lambda$ (see Fig. 2).

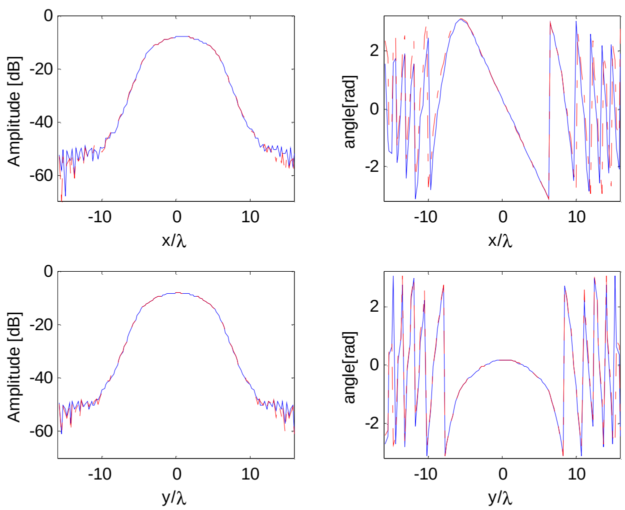

Fig. 2. Comparison between the ideal near field (blue line) and the retrieved near field (dashed red line) at the cut lines y=0 and x=0 of the plane at $z_2 = 6\lambda$.

4. The single plane/dielectric slab implementation

Let us turn now to present the second formulation based on the exploitation of the dielectric slab, of relative dielectric permittivity ε_r and thickness *d*, when the two measurements of the intensity of the near field are performed over the same scanning plane at $z = z_1$.

Now, the propagation through the dielectric slab in the second measurement (see eq. (3)) provides a velocity of the plane waves of the spectrum smaller that in free-space. This effect roughly corresponds to consider the second intensity data set obtained as the near-field propagation occurred in free space when the second measurement is performed at a scanning plane at quota $z<z_1$. This can be also understood by the considerations below, when we first neglect the effect of the transmission coefficient $\tau(u,v)$.

By observing the operator in eq. (3), the presence of the dielectric slab involves the propagation term $\exp[-j(w_d - w)d]$ playing the same role as the above $\exp[jw(z_1 - z_2)]$ term for the two-planes implementation. This consideration drives the choice of the relative dielectric permittivity and the thickness of the so that the phase of the $\exp[jw(z_1 - z_2)]$ and $\exp[-j(w_d - w)d]$ terms behave in a very similar way. Once the value of the thickness of the slab $d = 6\lambda$ is assumed, the similarity between the two phase terms is achieved by choosing the relative dielectric permittivity ε_r=6 (see Fig. 3).

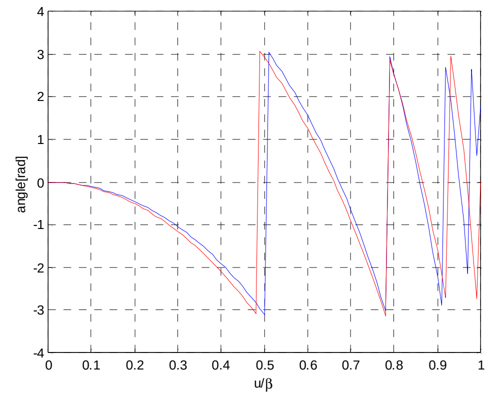

Fig. 3 Comparison between the phase of $exp\{j[w(u,v)-w(0,0)](z_1-z_2)\}$ for $z_1=10\lambda, z_2=6\lambda$ (blue line) and $exp\{j[w_d(u,v)-w(u,v)-w_d(0.0)+w(0,0)]d\}$ for $\varepsilon_r=6, d=6\lambda$ (red line).

By performing the minimization from the same starting point as the above Section, with $L_2(\hat{E})=T_1(\exp[-j(w_d-w)d]\hat{E})$, ε_r =6 and d=6λ the global minimum is achieved. For sake of brevity, we do not report any figure of reconstruction relative to this test case.

When, we now take into account the transmission coefficient $\tau(u,v)$ in the operator $\hat{T}_2$ and, for the minimization, we follow the same two-step minimization strategy as the above Section with a progressive increase in the number of the searched unknowns followed by the exploitation of the weighted formulation in (8). This strategy allows us to achieve the actual solution as depicted by Figs. 4 and 5, where an excellent comparison is shown between the actual and the retrieved PWS along the cut lines at u and v constant passing through the point of maximum PWS modulus.

Finally, we have verified the good stability of the solution approach by considering noisy data by superposing an additional 10% uniformly distributed noise on the intensity data. The reconstruction results are shown in Figs. 6 and 7 that are analogous of Figs. 4, 5 of the noise-free case.

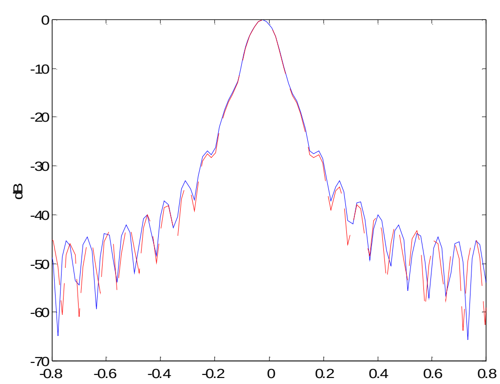

Fig. 4. Comparison between the ideal PWS (blue line) and the retrieved PWS (red line) at the cut line at constant v and passing through the point where the modulus of the PWS attains its maximum. Noise-free data.

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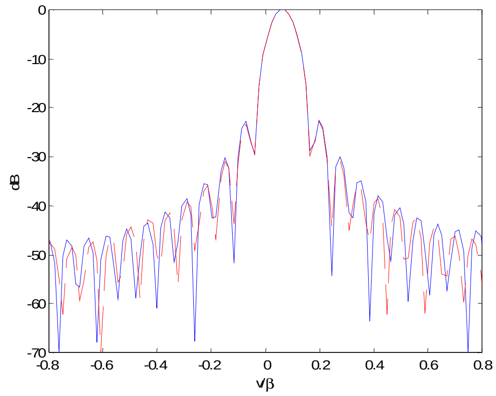

Fig. 5. Comparison between the ideal PWS (blue line) and the retrieved PWS (red line) at the cut line at constant u and passing through the point where the modulus of the PWS attains its maximum. Noise-free data.

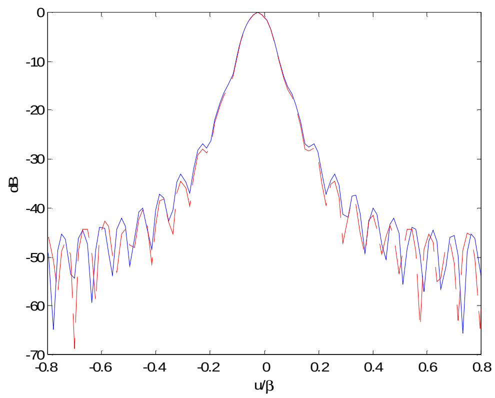

Fig. 6. Comparison between the ideal PWS (blue line) and the retrieved PWS (red line) at the cut line at constant v and passing through the point where the modulus of the PWS attains its maximum. Noisy data.

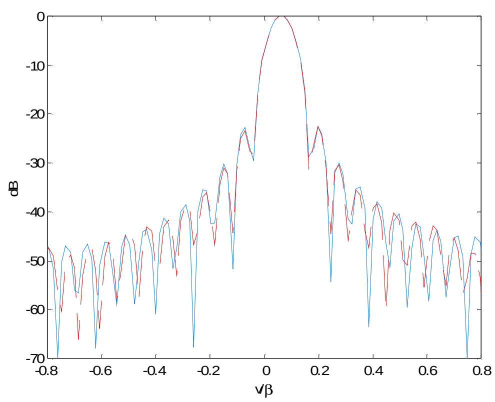

Fig. 7. Comparison between the ideal PWS (blue line) and the retrieved PWS (red line) at the cut line at constant u and passing through the point where the modulus of the PWS attains its maximum. Noisy data.

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5. Conclusions

In this paper we have presented first results concerning with a novel technique for antenna diagnostics from near field phaseless measurement. The technique exploits measurement on a single plane and the diversity between the two data sets (necessary to avoid the local minima problem) is ensured by *different conditions* under which the propagation of the near field from the aperture plane and the measurement plane occurs. In particular, one condition is concerned with the free-space propagation, while the other one is concerned with a propagation where the space between the aperture and measurement planes is partly filled with a dielectric slab of known properties.

Further topics deserving investigation concern with: the study of the effects of the inaccuracy in the knowledge of the dielectric slab properties on the reconstruction results; the combined effect of the two probes approach [4] with the present technique; the experimental validation of the technique; the possibility to exploit intensity measurements in Fresnel zone [17].

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