Stochastic Nature of Physical Parameterizations in Ensemble Prediction: A Stochastic Convection Approach

JOÃO TEIXEIRA

UCAR Visiting Scientist Program, Naval Research Laboratory, Monterey, California, and NATO Undersea Research Centre, La Spezia, Italy

CAROLYN A. REYNOLDS

Naval Research Laboratory, Monterey, California

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ABSTRACT

In this paper it is argued that ensemble prediction systems can be devised in such a way that physical parameterizations of subgrid-scale motions are utilized in a stochastic manner, rather than in a deterministic way as is typically done. This can be achieved within the context of current physical parameterization schemes in weather and climate prediction models. Parameterizations are typically used to predict the evolution of grid-mean quantities because of unresolved subgrid-scale processes. However, parameterizations can also provide estimates of higher moments that could be used to constrain the random determination of the future state of a certain variable. The general equations used to estimate the variance of a generic variable are briefly discussed, and a simplified algorithm for a stochastic moist convection parameterization is proposed as a preliminary attempt. Results from the implementation of this stochastic convection scheme in the Navy Operational Global Atmospheric Prediction System (NOGAPS) ensemble are presented. It is shown that this method is able to generate substantial tropical perturbations that grow and "migrate" to the midlatitudes as forecast time progresses while moving from the small scales where the perturbations are forced to the larger synoptic scales. This stochastic convection method is able to produce substantial ensemble spread in the Tropics when compared with results from ensembles created from initial-condition perturbations. Although smaller, there is still a sizeable impact of the stochastic convection method in terms of ensemble spread in the extratropics. Preliminary simulations with initial-condition and stochastic convection perturbations together in the same ensemble system show a promising increase in ensemble spread and a decrease in the number of outliers in the Tropics.

1. Introduction

Traditionally, operational weather and climate prediction has been a purely deterministic exercise. Early on however, it has been realized that there is an inherent uncertainty in the initial state (i.e., the initial state of a certain variable should be seen as a probability distribution and not as a unique value) and that this uncertainty can severely affect the predictability of the system (e.g., Thompson 1957; Lorenz 1963; Leith 1974; Palmer 1995; Smith 2003; Kalnay 2003).

Ensemble forecasting is a practical and successful way of addressing the predictability problem associated with the uncertainty in initial conditions (e.g., Leith 1974). During the last 15 years or so, several operational weather prediction centers have addressed the issues associated with the uncertainty in the initial conditions by developing ensemble prediction systems (e.g., Toth and Kalnay 1993; Buizza and Palmer 1998).

Besides initial-condition error, weather and climate prediction models are also sensitive to errors associated with the model itself. In particular the uncertainty due to the parameterizations of subgrid-scale physical processes is known to play a crucial role in the predictability of a system (e.g., Palmer 2001). It has not been straightforward, however, to develop theoretically sound, and also practical, formulations for how to insert parameterization uncertainty into ensemble development. Also, a serious problem with operational en-

Corresponding author address: João Teixeira, Jet Propulsion Laboratory, 4800 Oak Grove Dr., MS 169-237, Pasadena, CA 91109.
E-mail: teixeira@jpl.nasa.gov

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semble methods is that the range of ensemble values for a particular field of interest often fails to include the verification value. It is believed that one of the reasons that this happens is that most ensemble systems do not handle model uncertainty in a realistic way (if at all). Some recent research has been developed in order to tackle this issue by trying to develop formulations that impose a stochastic term to the physical parameterizations (e.g., Buizza et al. 1999) or by using different parameterizations within ensemble prediction systems to mimic the parameterization error (e.g., Houtekamer et al. 1996).

It should be mentioned that in different contexts (other than ensemble prediction) there have been several recent studies on the development of stochastic parameterizations, in particular for moist convection (e.g., Lin and Neelin 2002, 2003; Majda and Khouider 2002) and for clouds/radiation (e.g., Barker 2002; Pincus et al. 2003; Evans and Wiscombe 2004; Larson et al. 2005). Also, the notion of probability distributions has been central for boundary layer and turbulence parameterization (e.g., Stull 1989; Garratt 1992; Golaz et al. 2002), and cloud parameterization methods based on probability density functions (PDFs) of moist conserved thermodynamic variables have been advocated and implemented in weather and climate prediction models (e.g., Sommeria and Deardorff 1977; Mellor 1977; Smith 1990; Cuijpers and Bechtold 1995; Bony and Emanuel 2001; Tompkins 2002; Teixeira and Hogan 2002; Chaboureau and Bechtold 2002).

The main point of the present paper is to argue that, in terms of physical parameterizations, ensemble prediction could be viewed as fundamentally different from deterministic prediction. Ensemble systems can be devised in such a way that for each ensemble member there is no a priori reason to assume that the physical parameterizations should be providing the evolution of the grid-mean value of a variable. Each ensemble member does not have to represent the evolution of the mean variables and could be providing a probable value of such a variable. These randomly selected values should be constrained by the PDFs that are implicitly associated with a particular physical parameterization. Although a complete knowledge of such distributions is impossible, approximations using only the mean and the variance can often be fairly straightforward to obtain, as is discussed in section 2. In this context, the approach that is being proposed differs from previous studies on stochastic parameterizations in the sense that the PDFs that are used to constrain the random determination of the future states of a variable are based on the parameterization schemes themselves.

As a summary, the main rationale behind the methodology being suggested is the following: ensemble prediction systems can be devised in which parameterizations are utilized in a stochastic manner, based on PDFs obtained from the parameterizations.

This paper is organized in the following manner: section 2 illustrates the methodology behind the approach; section 3 shows the results obtained with the simple approach proposed; and section 4 presents some conclusions.

2. Methodology

To describe in a simplified setting the way in which atmospheric models deal with subgrid-scale mixing, consider that any model equation can be written as $d\phi/dt = S$, where for simplicity it is assumed that S is a general source term. By using Reynolds decomposition, where a variable can be divided into its mean and perturbation components ($\phi = \overline{\phi} + \phi'$), averaging, neglecting the divergence of the subgrid horizontal fluxes (which is done for weather and climate prediction models), disregarding density for simplicity of presentation, and assuming (again for simplicity) that S is either linear or that the nonlinearities of S can be neglected, an equation for the mean value is obtained as

$$\frac{d\overline{\phi}}{dt} = -\frac{\partial}{\partial z}(\overline{w'\phi'}) + \overline{S}, \qquad (1)$$

where on the lhs is the Lagrangian tendency of the mean variable, which includes the fully resolved advection terms and on the rhs (first term) the divergence of the subgrid-scale vertical fluxes.

This predicted mean value is always implicitly associated with a PDF of the variable within the grid box, of which the mean is only the first moment. In generic terms, the problem of parameterization consists of estimating the PDF of a variable within a grid box, and in particular, of finding approximations to the subgrid-scale vertical flux, the covariance $\overline{w'\phi'}$.

In atmospheric models, turbulence and dry convection in the boundary layer are typically parameterized using the eddy-diffusivity (ED) approach, where the flux is approximated by the vertical gradient of the mean variable times a diffusivity coefficient k that is a function of the turbulent state of the atmosphere [see Louis et al. (1982) or Troen and Mahrt (1986) for methods that are typically used operationally in global weather prediction models]. Currently, for moist convection the subgrid-scale vertical flux is often parameterized using a mass-flux (MF) approach, where a grid box is decomposed into regions of upward motion (updrafts), and quiescent or downward motion. In the MF parameterization, assuming that the area of convective updrafts is small relative to the grid size, the flux can be approximated by the mass flux (a function of the intensity of convection) times the difference between the updraft and the mean value of a variable (e.g., Arakawa 1969; Tiedtke 1989).

Although in climate and weather prediction models the mean value of a variable is often the only moment of the PDF that is predicted, it is possible by using Reynolds decomposition and averaging, to determine an equation for the variance of a generic variable $\sigma_\phi^2 = \overline{\phi'\phi'}$ (e.g., Stull 1989). In simplified form, this variance equation can be written as

$$\frac{\partial}{\partial t}\sigma_\phi^2 = -2\overline{w'\phi'}\frac{\partial\overline{\phi}}{\partial z} - \frac{\partial}{\partial z}(\overline{w'\phi'\phi'}) - \frac{\sigma_\phi^2}{\tau_\phi}, \qquad (2)$$

where τ_ϕ is a dissipation time scale. To fully solve this prognostic equation, a parameterization of the third moment has to be developed, which serves as a simple illustration of the turbulence closure problem. Often, however, in order to simplify the problem and in the interest of computational efficiency a steady-state version of the variance prognostic equation is used where the third-order transport term is neglected

$$\sigma_\phi^2 = -2\tau_\phi\overline{w'\phi'}\frac{\partial\overline{\phi}}{\partial z}. \qquad (3)$$

With this simplification, it is only necessary to solve a simple algebraic equation in order to determine the variance, instead of a partial differential equation. The problem of estimating the flux can be dealt with by using either the ED or the MF closures that were discussed above, or some combination of both (e.g., Siebesma and Teixeira 2000; Soares et al. 2004). To obtain Eq. (2) in such a general and simplified form, a number of assumptions and simplifications are made, which include neglecting large-scale advection, density, and any nonlinear source/sink terms of each of the variables, and assuming that dissipation can be represented as the ratio between the variance and a dissipation time scale.

Note that although the first two rhs terms of Eq. (2) follow directly from the nonlinear transport terms of the original flow equations, the dissipation term is a simple parameterization of a term that is relatively unknown for moist convection. The dissipation time scale is associated with the physical processes that control the dissipation of variance of a particular variable and a general physical theory to determine the time scale for each of the variables is currently not available. In some situations the time scale is replaced by the ratio between a length scale and a velocity scale. This is often done for the turbulent kinetic energy equation in boundary layer convection (e.g., Stull 1989). For deep convection, Chaboureau and Bechtold (2002) use this approach to estimate the variance of the thermodynamic variables in their PDF-based convection cloud parameterization. The dissipation time scale has also been approximated as a constant in convection studies (e.g., Randall and Pan 1993; Lenderink and Siebesma 2000) in which analogies with the typical cloud lifetime are made. Also, care must be taken when trying to extrapolate results from boundary layer studies to deep convection where the dissipation term may have a different behavior (e.g., Khairoutdinov and Randall 2002).

The discussion above shows that there are ways of determining the variance of a certain variable due to subgrid-scale motions that can be used in the context of each individual turbulence or convection parameterization, as long as it is straightforward to estimate the subgrid fluxes explicitly.

A possible methodology to utilize physical parameterizations in a stochastic manner in the context of ensemble prediction, based on the knowledge of the variance provided by the parameterization, is as follows. Assume that the value of a generic variable after being updated by a certain physical parameterization (moist convection in the present study) can be written as

$$\phi_{\rm conv}^{\rm stoch} = \overline{\phi}_{\rm conv} + \varepsilon, \qquad (4)$$

where $\overline{\phi}_{\rm conv}$ is the mean value of the variable after convection, $\phi_{\rm conv}^{\rm stoch}$ is the stochastic value of the same variable after convection, and ε is a normally distributed stochastic variable with mean $\mu(\varepsilon) = 0$ and standard deviation $\sigma(\varepsilon) = \sigma_{\phi,\rm conv}$, where $\sigma_{\phi,\rm conv}$ is the standard deviation of the variable due to moist convective processes.

It should be noted that the assumption of a normal distribution for moist convection processes is a simplification and skewness can be significant in PDFs associated with moist convection. Studies using cloud-resolving models (CRMs) and large-eddy simulation (LES) models have shown substantial departures from the mean in shallow and deep moist convection and the important role of skewness (e.g., Xu and Randall 1996; Klein et al. 2005; Khairoutdinov and Randall 2006; Siebesma et al. 2003).

After discretizing the first term on the rhs of Eq. (4), the following equation is obtained

$$\phi_{\rm conv}^{\rm stoch} = \overline{\phi} + \Delta t\left(\frac{\Delta\overline{\phi}}{\Delta t}\right)_{\rm conv} + \varepsilon, \qquad (5)$$

where $\overline{\phi}$ is the mean value before the moist convection parameterization.

In general, the standard deviation could be determined using versions of Eqs. (2) or (3) for the moist convection parameterization. In the present paper however, a simpler approach will be followed in order to perform a preliminary study of the impact of this general methodology on an ensemble prediction system.

Shutts and Palmer (2004) use CRM simulations to show histograms of deep convection tendencies (temperature tendencies in particular). In a first approximation, the distributions implied by these histograms resemble normal PDFs. As a way to achieve similar results, it is assumed that the standard deviation of a generic variable due to moist convection is proportional to its tendency.

Assuming that the standard deviation is proportional to the tendency and rearranging leads to

$$\frac{\phi_{\mathrm{conv}}^{\mathrm{stoch}} - \overline{\phi}}{\Delta t} = (1 + \eta\beta)\left(\frac{\Delta\overline{\phi}}{\Delta t}\right)_{\mathrm{conv}}, \tag{6}$$

where β is a constant of proportionality and η is a normally distributed stochastic variable with mean $\mu(\eta) = 0$ and standard deviation $\sigma(\eta) = 1$. Note that β could be estimated from CRM studies such as the ones performed by Shutts and Palmer (2004). For example, Fig. 10a in Shutts and Palmer (2004) suggests a value of β between 1 and 2 for the temperature tendency.

Taking $\Delta t \to 0$ and assuming that we can write

$$\lim_{\Delta t \to 0} \frac{\phi_{\mathrm{conv}}^{\mathrm{stoch}} - \overline{\phi}}{\Delta t} = \left(\frac{\partial\phi}{\partial t}\right)_{\mathrm{conv}}^{\mathrm{stoch}}$$

leads to

$$\left(\frac{\partial\phi}{\partial t}\right)_{\mathrm{conv}}^{\mathrm{stoch}} = (1 + \eta\beta)\left(\frac{\partial\overline{\phi}}{\partial t}\right)_{\mathrm{conv}}. \tag{7}$$

There are similarities between this method and the one proposed in Buizza et al. (1999). In a certain sense our methodology can be interpreted as an attempt to provide the theoretical framework from which methods of the type of Buizza et al. (1999) could be derived. A major advantage of this is that with a well-founded formalism it is possible to devise subsequent developments (improvements) of such methods (e.g., utilize more realistic distributions; include more complex variance equations) in a relatively straightforward manner.

However, there are also some fundamental differences that should be highlighted: (i) the current methodology is based on using the present formalism of the convection parameterization and on a reinterpretation of how to utilize a parameterization in the context of an ensemble prediction system; (ii) the stochastic component affects only the moist convection parameterization and not the entirety of the physics tendencies; (iii) it is assumed that the PDF is well approximated by a normal distribution with a variance given by the convection parameterization; and (iv) in the current method no horizontal or temporal correlations are applied (see below).

The vertical correlation of the perturbations is achieved in a simple manner by drawing only a single random number per column at each time step. The correlation between the different variables is also achieved in this simple manner, by using the same random number for all the variables. Note that Eq. (7) with such a simple assumption for the vertical correlations naturally leads to similar conservation properties for both the deterministic and stochastic values.

At this stage, it was decided to neglect the horizontal and temporal correlations for this study. There are some important reasons for this. The more practical reason is to maintain the simplicity of the approach in the sense of not introducing extra unknowns. A more theoretical reason pertains to the question of what part of the subgrid variability is this method trying to represent. Ignoring horizontal correlations is akin to assuming that the perturbations that are being represented by the stochastic convection method are much smaller than the grid size. In fact, in the rationale behind what was presented in this section on how to relate the stochastic noise to a PDF that is produced by the parameterization itself, it is implicitly assumed that this PDF is representing the physical subgrid variability in a statistically significant manner, with no sampling issues related to the scale of the physical variability versus the grid size. This implies that the physical subgrid variability that is being considered by the stochastic convection method is actually occurring at scales much smaller than the grid size. As such, it can be argued that the stochastic method presented in this paper should be utilized without horizontal correlations.

On the other hand, it is also unclear how to relate current parameterization schemes with the parameters that control the temporal and spatial (horizontal) correlations. In fact, the problem that is being studied in the present paper (how to relate stochastic terms with the PDF given by the parameterization schemes) is quite different from the one associated with the determination of spatial (horizontal) correlations. This latter one is connected to the scale (as compared with the horizontal grid size) of physical processes that are somewhere between being fully resolved and fully parameterized. In this context, there has been some recent promising work using, for example, concepts associated with cellular automaton (e.g., Palmer 2001; Shutts and Palmer 2004).

It should also be noted that in weather prediction models, convection is often associated with systems that are larger than the typical grid scale (i.e., gridpoint storms are uncommon). This means that if the stochastic method is associated, as it is suggested in this work, with estimates of the variance that are directly produced by the convection scheme, then there are implicit spatial and temporal correlations present in the values of the variance that are associated with the mesoscale convective systems generated by the model. In this sense, the present stochastic method already possesses a certain degree of spatial and temporal correlations. Note, however, that moist convection parameterizations can suffer from problems associated with a discontinuous response to forcing (i.e., "on–off" behavior). This is not necessarily the case for the convection parameterization used in this study (Emanuel and Zivkovic-Rothman 1999; Peng et al. 2004), probably because it uses a prognostic equation for the cloud-base mass flux.

It should be noted that in the present study this stochastic method is used for the wind and temperature variables, but is not directly used for the specific humidity. The determination of the stochastic value for the specific humidity is based on the assumption that relative humidity should be kept constant during this process and as such is indirectly based on the stochastic temperature value. Although, in reality, variability is to be expected in the relative humidity field, this was done in order to avoid additional interactions with the cloud and condensation parameterizations. A problem with this approach is that the good conservative properties of Eq. (7) (with the simple vertical correlation) do not apply to humidity. It is unclear at this stage how serious a problem this may be and it is not our intention to argue that this is necessarily the only way to proceed. Sensitivity studies to this assumption (see below) identify it as an important component.

3. Results

In this section results are shown that use the stochastic approach discussed above (with $\beta = 1$) for the moist convection parameterization within the Navy Operational Global Atmospheric Prediction System (NOGAPS). Sensitivity studies to different values of β are also discussed at the end of the section. The ensemble system is used in the following configuration: 32-member ensembles are run for 10 days at a T119L30 resolution, once a day for the month of May 2005. In some experiments, there are no initial-condition perturbations and the ensemble spread is based entirely on the stochastic convection forcing. In other experiments, initial perturbations are introduced based on randomly sampling the analysis error variance estimate produced by the Naval Research Laboratory (NRL) Atmospheric Variational Data Assimilation System (NAVDAS; Daley and Barker 2001) in a method similar to that described in Errico and Baumhefner (1987).

NOGAPS (Hogan and Rosmond 1991) is a global spectral model in the horizontal and energy conserving finite difference (hybrid-sigma coordinate) in the vertical. The model uses vorticity and divergence, virtual potential temperature, specific humidity, and terrain pressure as the dynamic variables, with a semi-implicit treatment of gravity wave propagation. The physical parameterizations include boundary layer turbulence (Louis et al. 1982), shallow and deep moist convection (Emanuel and Zivkovic-Rothman 1999; Peng et al. 2004), convective and stratiform clouds (Teixeira and Hogan 2002), and solar and longwave radiation (Harshvardhan et al. 1987). NOGAPS is the global weather prediction model of the U.S. Navy, and drives several applications such as the Coupled Ocean Atmosphere Mesoscale Prediction System (COAMPS; Hodur 1997; Hodur and Doyle 1998) and the Navy aerosol prediction model.

Global maps are shown in Fig. 1 for the ensemble spread of geopotential height at 500 hPa for six forecast periods (from 24 to 144 h) averaged for the month of May 2005. In these experiments, there are no initial-condition perturbations. Two aspects are particularly noticeable in this evolution. The ensemble spread at 24 h grows substantially over time reaching values at 144 h that are over one order of magnitude larger than at 24 h. As expected, because of the typical geographical distribution of deep moist convection, the perturbations at 24 h occur mostly in the Tropics and subtropics, but as time evolves the largest values of ensemble spread migrate toward the higher latitudes. Note, however, that migration must be understood in this case in a general sense, as this may not necessarily represent direct forcing of the midlatitudes by the Tropics, but may just reflect the different perturbation growth rates and saturation levels in the midlatitudes and in the Tropics. This assumption could be tested by utilizing the stochastic convection method only between certain latitudes.

Similar global maps but for the wind at 250 hPa and temperature at 850 hPa are shown in Figs. 2 and 3. A similar behavior as the one discussed for Fig. 1 is present in these variables, as well as in the wind at 850 hPa (not shown). The initial spread (at hour 24) grows to be quite large by 144 h and there is a migration of the highest values of ensemble spread from the Tropics to the midlatitudes. Within this context, however, the ensemble spread at late (early) forecast times is more (less) uniformly distributed for the wind field than for the temperature field.

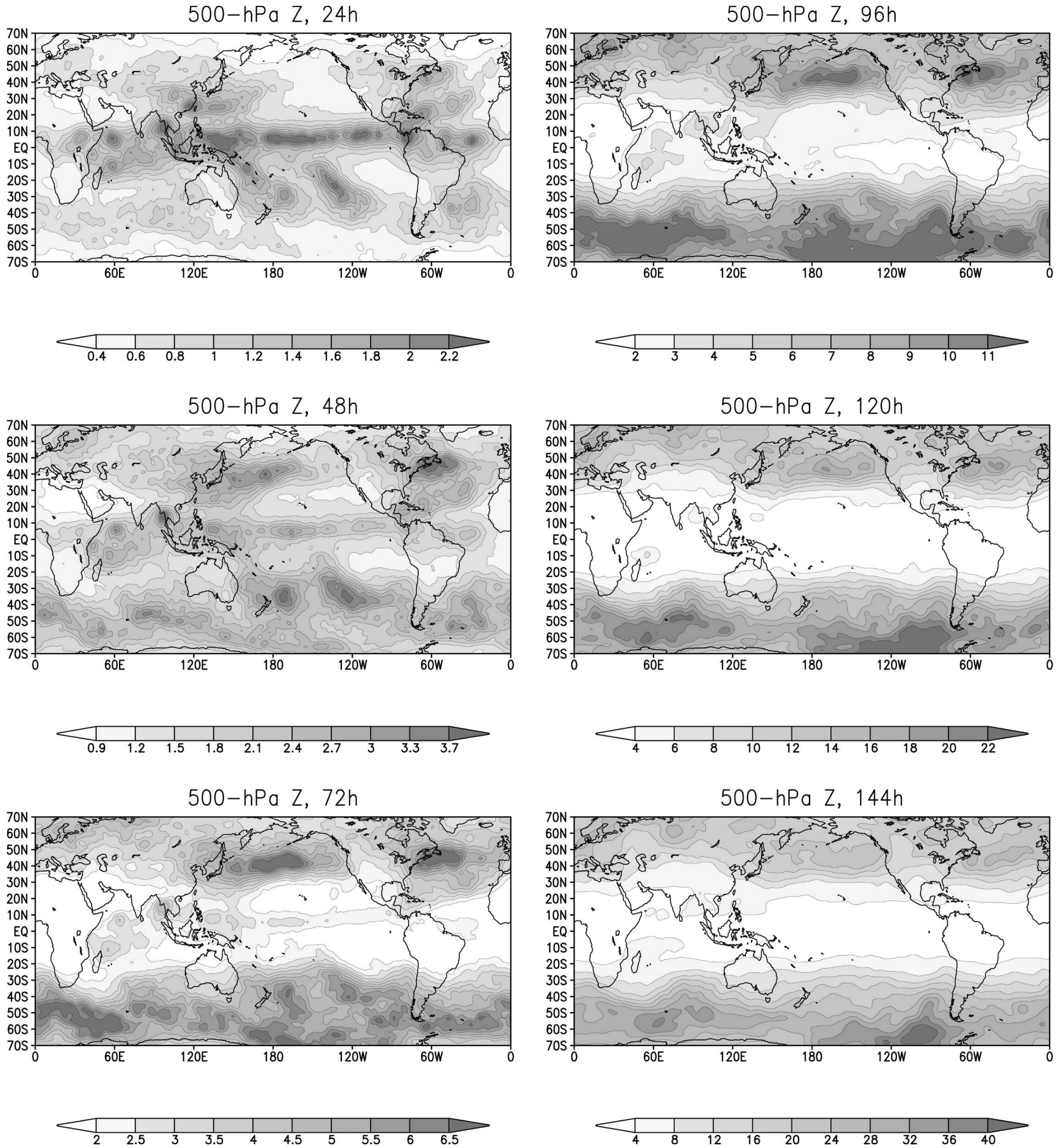

FIG. 1. Global maps of the ensemble spread of geopotential height (m) at 500 hPa for six forecast periods (from 24 to 144 h).

To analyze the migration of the largest values in ensemble spread from the tropical regions to the higher latitudes, Fig. 4 shows the total energy difference between an ensemble member with stochastic convection and the control simulation with no stochastic convection, as a function of total wavenumber (out to T119) for the Tropics (20°S–20°N) and the Northern Hemisphere (NH) midlatitudes (30°–70°N) for several forecast time ranges (from 12 to 240 h), averaged for daily forecasts from May 2005. For the tropical case, it appears that the total energy is starting to saturate after a few days on all but the largest scales. In contrast, for the midlatitudes, saturation of the small scales occurs later, while a distinct peak at the synoptic scales develops and

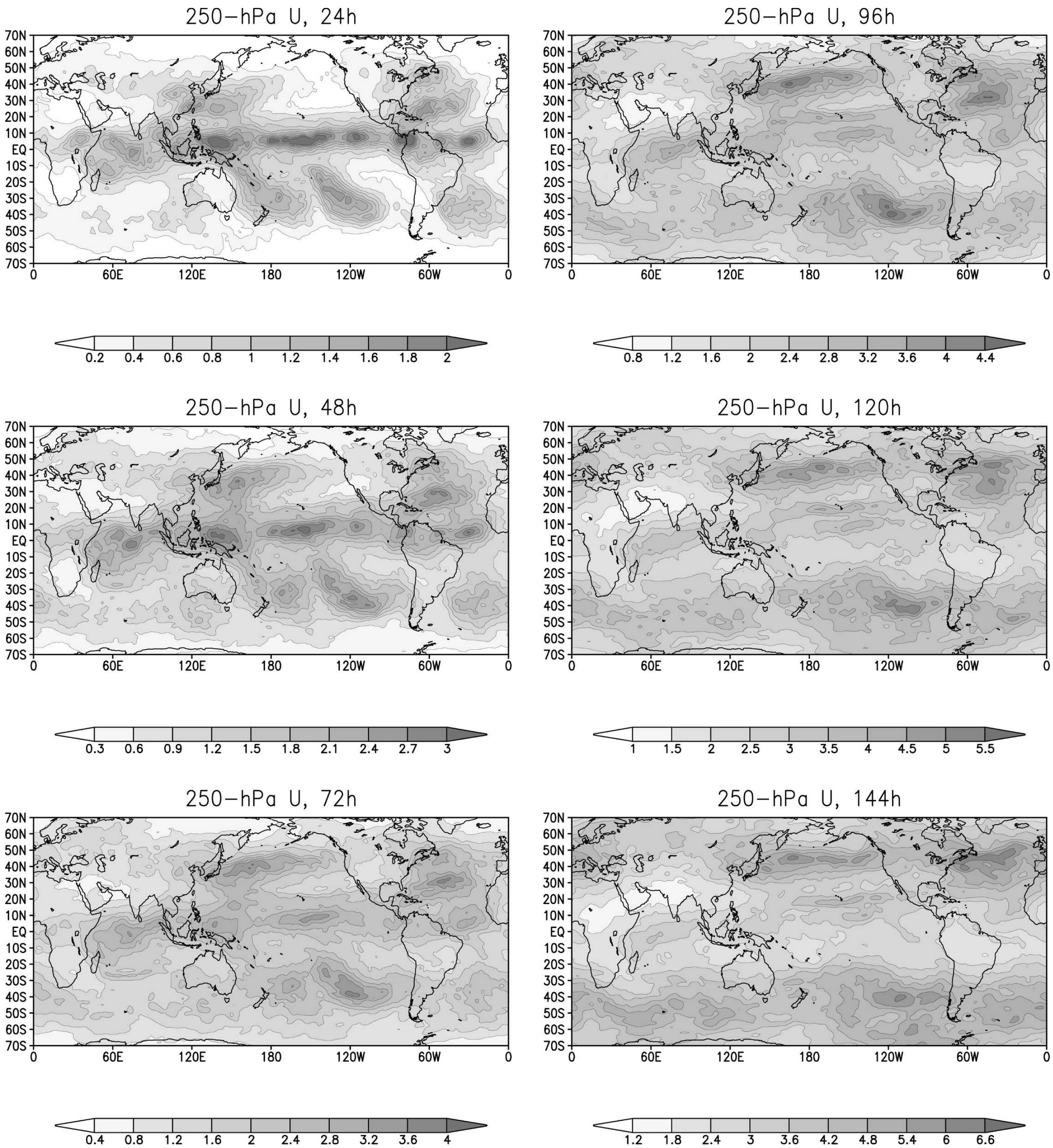

FIG. 2. As in Fig. 1, but for the wind speed (m s^{-1}) at 250 hPa.

continues to grow up to 10 days.[1] The reader should be reminded that the stochastic convection perturbations that are being introduced are uncorrelated in space and time, and as such represent perturbations that are only occurring at small spatial and temporal scales and only when the moist convection parameterization is being triggered. These plots illustrate the significant transfer of perturbation energy to larger scales as the forecasts proceed, especially in the extratropics.

[1] There is some aliasing of perturbations onto the largest scales (lowest wavenumbers) because of the local projection operator that sets the perturbation field to zero outside the region of interest.

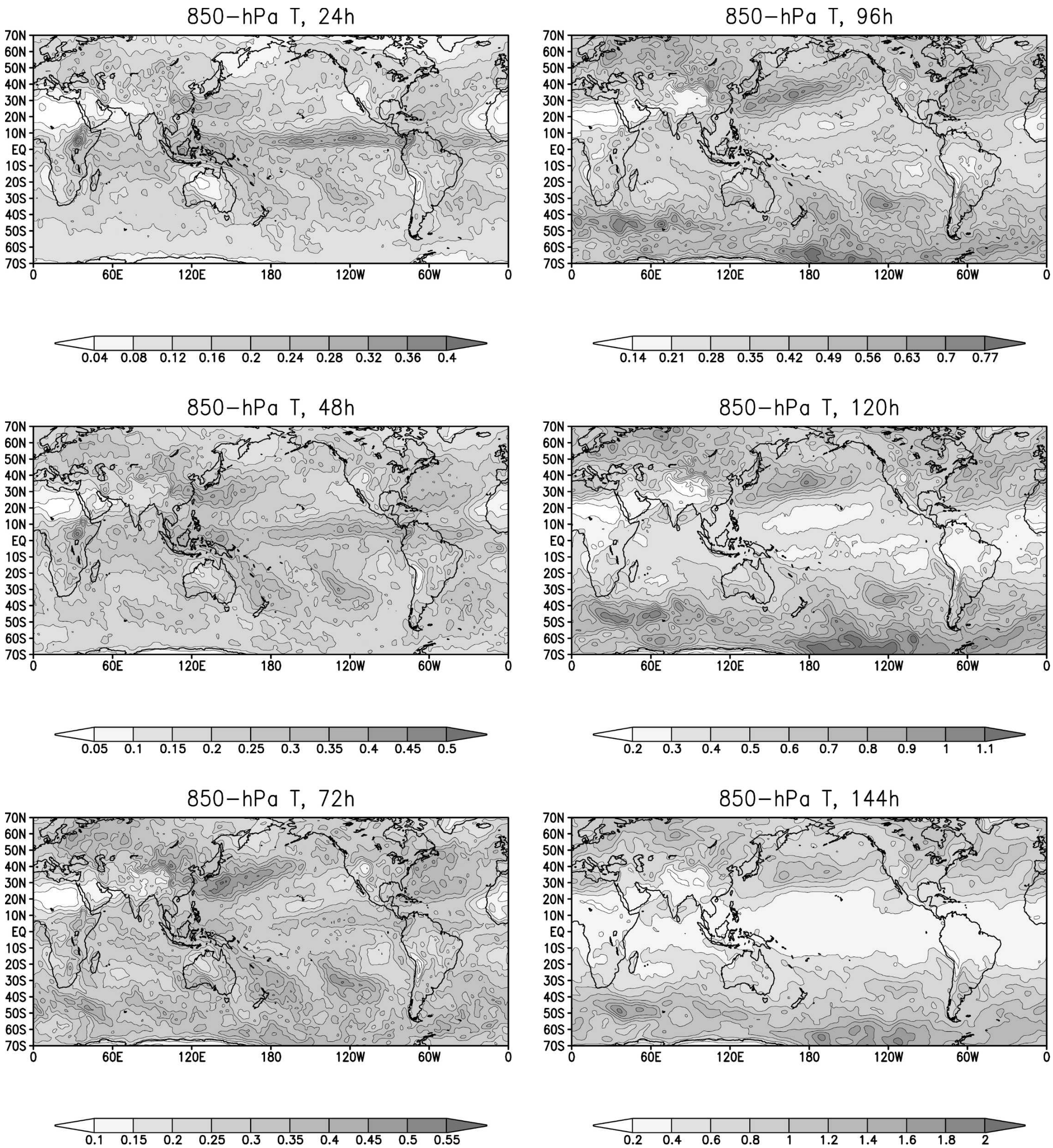

FIG. 3. As in Fig. 1, but for the temperature (K) at 850 hPa.

To further investigate the differences in terms of time evolution of the energy for the Tropics and the midlatitudes, Fig. 5 shows the evolution of the differences (between one ensemble member and the control simulation with no stochastic convection) in total, potential, and kinetic (rotational and divergent) energies (TE, PE, RKE, and DKE, respectively) for the entire globe, the Tropics, and the NH midlatitudes, averaged for the daily forecasts from May 2005. The potential and kinetic energy are computed as in Rosmond (1997). Also shown are curves with a linear behavior proportional to time *t*. The total energy growth appears linear for the first 2 or 3 days. This initial linear growth is in agreement with recent studies (although in different con-

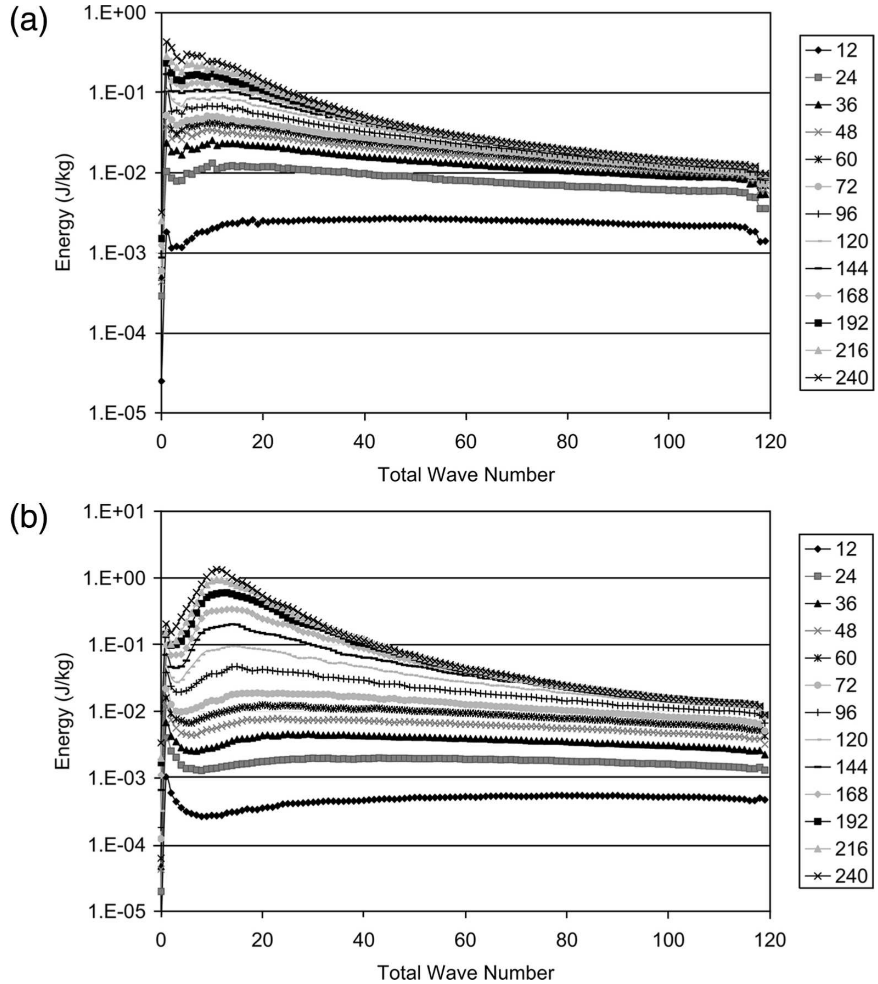

FIG. 4. Total energy difference (J kg^{-1}) between an ensemble member with stochastic convection and the control simulation with no stochastic convection, as a function of total wavenumber (out to T119) for several forecast ranges (from 12 to 240 h) for (a) the Tropics (20°S–20°N) and (b) the NH midlatitudes (30°–70°N), averaged for the month of May 2005.

texts) of model error growth from Vannitsem and Toth (2002), Nicolis (2003), and Teixeira et al. (2007). During the early stages (after 12 h) the perturbation energy is at similar levels in the RKE PE fields. However, the growth in the RKE component during the first 48 h is significantly faster than the growth in the PE component. This eventually leads to values of RKE that are almost one order of magnitude larger than the values of PE in the Tropics (about one-half of an order of magnitude in the NH midlatitudes) after 10 days. During the early stages (at 12 h) the perturbation energy is larger in the Tropics than in the midlatitudes, but it is clear that the growth rate is much less in the Tropics (with a quasi-linear behavior) than in the midlatitudes. This leads to values of TE, RKE, and PE that are much larger in the midlatitudes, than in the Tropics, after 10 days.

One of the more practical motivations for this study, as well as for some of the recent studies on stochastic physics in general, is (as mentioned in the introduction) the general lack of spread in current ensemble prediction systems. In particular, in the Tropics the problem seems to be worse than elsewhere in the globe. To investigate the impact of the approach proposed here on this issue, a comparison is performed between an ensemble that uses only the stochastic convection method and an ensemble that uses only initial-condition perturbations as described in the beginning of this section.

Figure 6 shows the ensemble spread for the mean sea level pressure (MSLP), the temperature at 850 hPa, and the wind at 250 hPa for the Tropics and the extratropics, averaged for the month of May 2005. In the Tropics, Fig. 6a, the simulations with initial-condition (IC) perturbations show an initial decay or slow growth of the

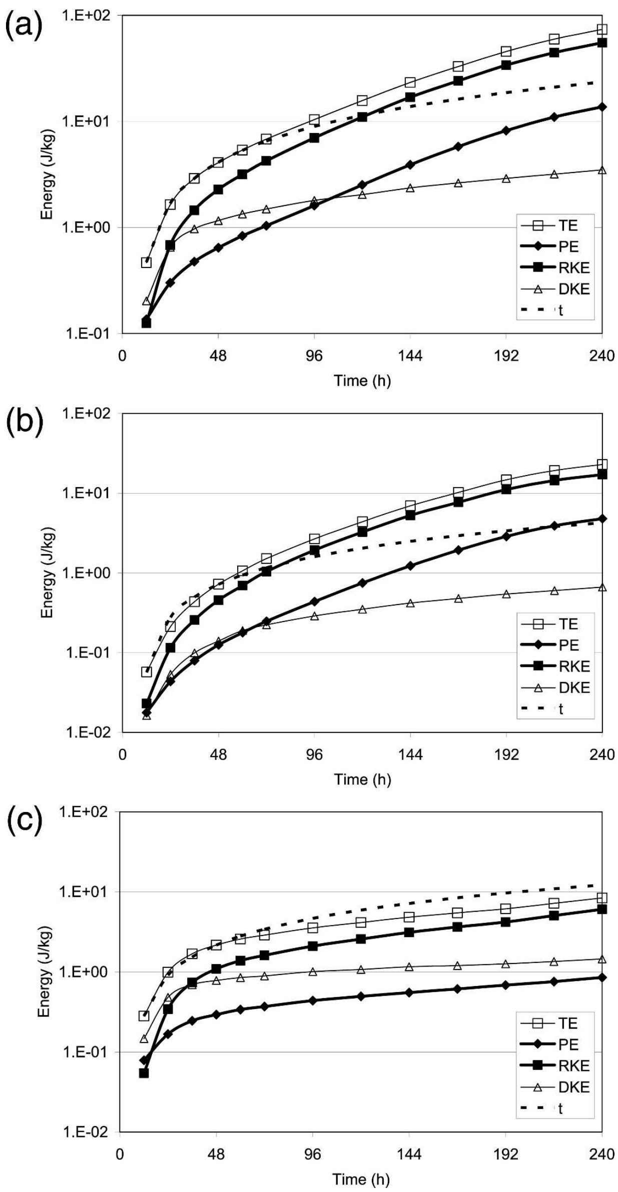

FIG. 5. Evolution of the differences (J kg^{-1}), between one ensemble member and the control simulation with no stochastic convection in total, potential, kinetic rotational, and kinetic divergent energies for (a) the entire globe, (b) the NH midlatitudes, and (c) the Tropics, averaged for the month of May 2005.

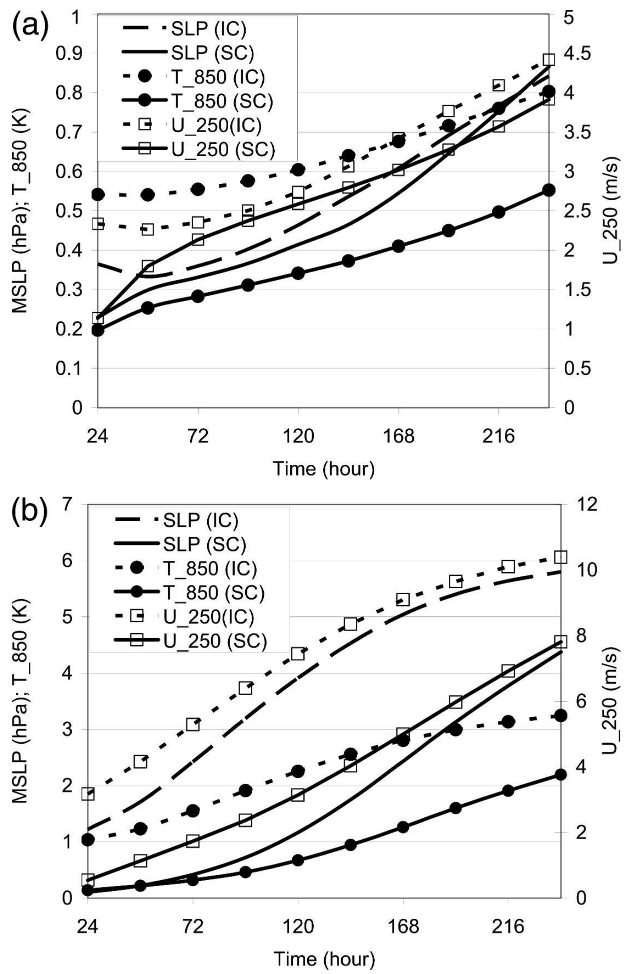

FIG. 6. Evolution of the ensemble spread of the mean sea level pressure (MSLP, hPa), the temperature at 850 hPa (T_850, K) and the wind speed at 250 hPa (U_250, m s^{-1}), averaged for the month of May 2005 for (a) the Tropics and (b) the extratropics. Two different ensembles are analyzed: one that uses only initial-condition perturbations and one that uses only stochastic convection perturbations.

ensemble spread for the first 3 forecast days and only after this does the IC spread start to increase.[2] On the other hand, the simulations with the stochastic convection (SC) approach show a continuous increase of the ensemble spread, with the SC spread reaching values comparable to the IC spread after about 72 h for both the MSLP and the wind at 250 hPa.

In the extratropics (Fig. 6b), although there is, as expected, a smaller ensemble spread for the SC method relative to the IC method, this ensemble spread is still substantial. For the IC ensemble, the spread is rapidly increasing in the beginning but starts leveling off after about 7 forecast days. The SC ensemble spread, although always less than that of the IC simulations, is still far from saturation even after 10 forecast days.

In addition, simulations were performed to examine the sensitivity of the results to the formulation of the stochastic convection perturbations. Because of computational constraints, these additional experiments were only run for the first week of May 2005, rather than the full month. Note that the limited experiment size may

[2] This characteristic of initial decay is not common to all types of IC perturbations.

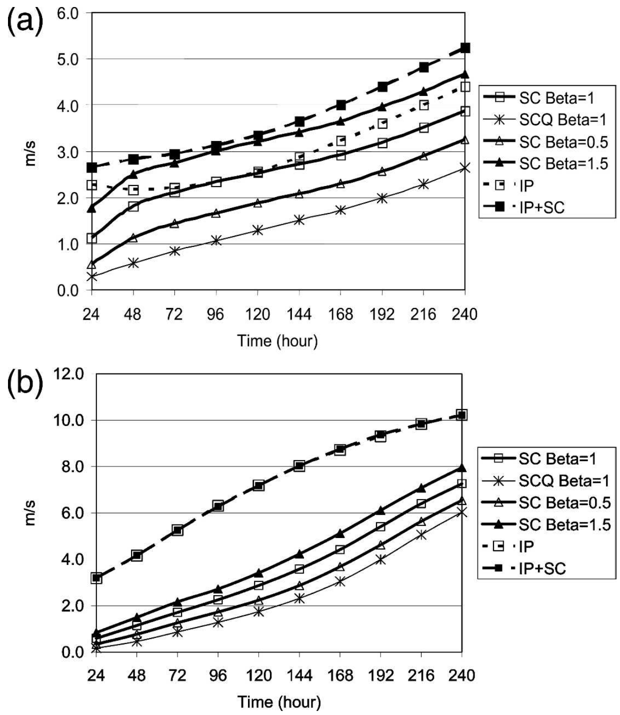

FIG. 7. Evolution of the ensemble spread of the 250-hPa wind speed (m s^{-1}), averaged for the first week of May 2005 for (a) the Tropics and (b) the NH extratropics. Several different ensembles are analyzed: SC perturbations only, with different values of β; stochastic convection without the constant relative humidity assumption (labeled SCQ); initial-condition perturbations only; and initial-condition perturbations plus stochastic convection.

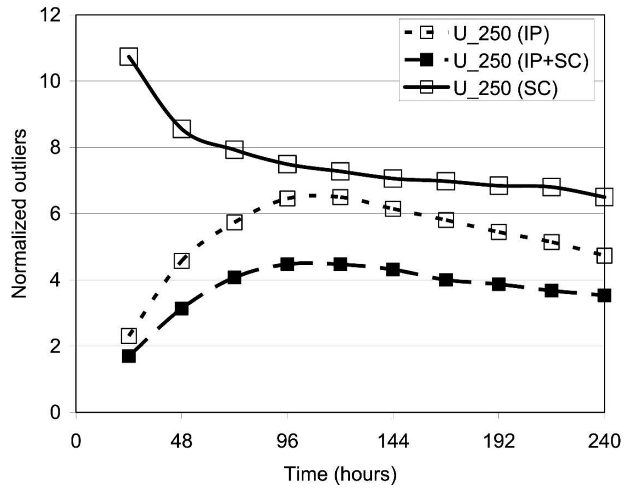

FIG. 8. Evolution of the number of outliers (normalized by the expected number of outliers for a flat distribution) of 250-hPa wind speed averaged for the first week of May 2005 in the Tropics for SC, IP, and IP + SC.

have an impact on the interpretation of the results in terms of ensemble performance.

Figure 7 shows the ensemble spread for the 250-hPa wind speed for the Tropics and NH extratropics. Ensembles were produced with the same SC formulation previously described, but in one case $\beta = 0.5$ and in another case $\beta = 1.5$. One can see that ensemble spread is increased (decreased) when β is increased (decreased). For example, at 24 h, in the Tropics, the spread is roughly proportional to β: the spread for $\beta = 1.5$ is 1.57 times greater than the spread for $\beta = 1$, and the spread for $\beta = 1$ is 2.02 times greater than the spread for $\beta = 0.5$. However, as integration time increases, these ratios decrease, eventually toward 1.

Also included in Fig. 7 are results for a different stochastic formulation, one in which the relative humidity does not remain constant, but rather the specific humidity is perturbed in a similar manner as the temperature and wind fields (with $\beta = 1$) denoted as SCQ in Fig. 7. The ensemble spread in this case is smaller than in the initial formulation, indicating that the perturbations introduced to the moisture in order to keep relative humidity constant result in larger spread than the direct random perturbations to the specific humidity. It should be noted however that there is the possibility that β could be increased, allowing for a similar impact in terms of ensemble spread even with direct perturbations to the specific humidity field.

Preliminary tests with ensembles that contain both initial-condition perturbations and stochastic convection (IP + SC) were also performed. One can see in Fig. 7 that the addition of stochastic convection perturbations to an ensemble based on initial-condition perturbations, has a positive impact on the spread in the Tropics (in the sense of increasing the spread), but a minimal impact in the NH extratropics. Note that there is a small increase in spread in the Southern Hemisphere (SH) extratropics (results not shown). The addition of stochastic convection perturbations to the initial perturbations results in the increase of ensemble spread in the Tropics by between 15% and 30%, depending on forecast lead time, in the 250-hPa wind.

As shown in Fig. 8, the addition of stochastic convection to the initial-condition perturbations substantially decreases the number of "outliers" in the Tropics (i.e., the number of times the value of the verifying field lies outside the range of ensemble values). Figure 8 shows that the number of outliers (normalized by the expected number of outliers for a flat distribution) in 250-hPa wind speed in the Tropics decreases by up to about 30%, depending on lead time.

The root-mean-square error (RMSE) of the ensemble mean 250-hPa zonal wind is shown in Fig. 9. In the extratropics (Fig. 9a), the addition of stochastic convection to the initial-condition perturbations has a negligible impact on the ensemble mean RMSE. In the

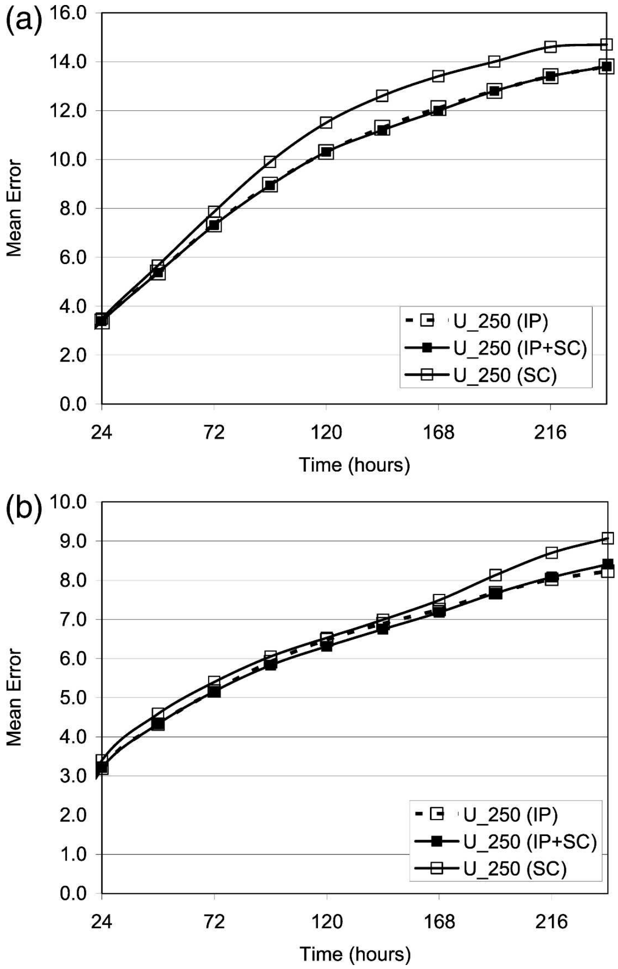

FIG. 9. Evolution of the RMSE of the ensemble mean 250-hPa zonal wind speed (m s^{-1}), averaged for the first week of May 2005 for SC, IP, and IP + SC for (a) the NH extratropics and (b) the Tropics.

Tropics (Fig. 9b), however, the addition of stochastic convection to the initial-condition perturbations has a visible impact on the error. There is a slight improvement at forecast days 4–6, and a slight degradation at day 10. These small changes, however, could be due to sampling errors associated with the limited number of forecasts included in these statistics.

It is clear that further work is necessary to examine the impact of using stochastic convection in addition to initial-condition perturbations, but these preliminary results encourage further study.

4. Conclusions

In this paper it is argued that parameterization methods in ensemble prediction do not have to be viewed as deterministic in the sense of providing a deterministic tendency of a grid-mean variable. Instead, parameterizations in ensemble prediction could be used as stochastic schemes that estimate a probable value of a certain variable that is constrained by distributions calculated within the context of each individual parameterization. A general methodology to incorporate a stochastic component into a physical parameterization of subgrid-scale mixing is presented, where in particular well-known turbulence closure equations to calculate the variance of a distribution are discussed.

A simplified approach for a stochastic convection parameterization is tested in the context of the NOGAPS ensemble prediction system. In this particular approach it is assumed that the standard deviation of the PDF of each variable is proportional to the tendency of the same variable produced by the moist convection parameterization.

Results with this simplified version show that the stochastic convection perturbations alone result in significant ensemble spread of the 500-hPa geopotential height, the 250- and 850-hPa winds, and the 850-hPa temperature. At the early stages the largest values of ensemble spread occur in the Tropics and subtropics, but then "migrate" toward the higher latitudes where they are concentrated after about six forecast days. The average evolution of differences between a control forecast and a stochastic convection ensemble member, for the total energy wavenumber spectra, illustrates the reason for this apparent migration. In contrast to the quasi-saturation exhibited in the tropical regions, perturbation growth in the midlatitudes continues through the forecast, and begins to accumulate in the synoptic and near-synoptic scales.

The evolution of similar differences in total, potential, and kinetic (rotational and divergent) energies shows an initial linear behavior proportional to time t. It also shows that while initially (after 12 h) the rotational kinetic energy and the potential energy components have similar values, after 10 forecast days the rotational kinetic energy component dominates the perturbation energy. This is true for both the Tropics and the NH midlatitudes.

A comparison for MSLP, 850-hPa temperature, and 250-hPa wind between simulations with an ensemble that uses only stochastic convection and an ensemble that uses only initial-condition perturbations, illustrates well the potential of the stochastic convection approach. In the tropical regions, the stochastic convection ensemble produces a significant increase in spread to values comparable to the initial-condition ensemble after about 3 forecast days. Although less prominent than in the Tropics the impact of the stochastic convection in the extratropics is still relatively large.

Sensitivity studies show that ensemble spread increases with the value of β and that perturbing the specific humidity directly (instead of keeping the relative humidity constant) has less impact (for the same β) in terms of ensemble spread. Preliminary tests with an ensemble composed of initial-condition perturbations together with stochastic convection are encouraging in the sense of showing a substantial increase in ensemble spread and decrease in the number of outliers in the Tropics.

In this paper an attempt is made to formalize and argue for a stochastic nature of parameterizations in ensemble prediction systems. A simplified version of the general approach proposed here is tested in the NOGAPS ensemble system. This initial simplified attempt should be viewed as a feasibility study. In the near future more sophisticated versions will be tested based on the variance equation [Eq. (2)]. Further work is also needed to refine the simple vertical correlation assumed in the method tested here. Because of the nature of moist convection (e.g., skewness is often important) it is also plausible that disregarding higher moments of the PDF (as is being currently done) may well be an unrealistic simplification. As such, the extension of the present stochastic approach to more complex distributions should also be an important focus of future research.

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