Quantum properties of optical field in photonic band gap structures

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Abstract: A theoretical analysis of the quantum behaviour of radiation field's propagation in photonic band gaps structures is performed. In these initial calculations we consider linear inhomogeneous and nondispersive media.

OCIS codes: (270.0270) Quantum optics ; (160.1190) anisotropic optical materials.

References and links

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1. Introduction

Photonic band gap (PBG) structures have been extensively studied during these last years [1] , due to the possibility of handling light . The propagation can occur in 1D,2D, or 3D periodical structures and gives rise to gaps in the transmission as for electron energy in crystals. In what follows we consider a 1D PBG .

The propagation in inhomogeneous materials is described by the following equation: $[\nabla^2+k^2\varepsilon(z,\omega)]f(z,\omega)=0$, it does not have closed solution in the general form and for the general case [1]. Of course this problem remains even when we work in a quantum domain. In the absence of absorption and dispersion, in the quantum domain, we have the operator equation

$$\left[\frac{\partial^2}{\partial z^2}+\frac{\omega^2}{c^2}\varepsilon(z)\right]\hat{A}(z,\omega)=0 \qquad (1)$$

In the previous equation we refer to an angular frequency of the vector potential operator; the time-independent potential operator given by

$$\hat{A}(z)=\int_0^\infty \hat{A}(z,\omega)e^{-i\omega t}d\omega \qquad (2)$$

and the electric field operator is:

$$\hat{E}(z)=\int_0^\infty i\omega\hat{A}(z,\omega)e^{-i\omega t}d\omega \qquad (3)$$

These operators satisfy the well-known canonical commutation relation [2]:

#35162 - $15.00 US Received September 06, 2001; Revised October 19, 2001

$$\left[\hat{A}(z),\hat{E}(z')\right]=\frac{i\hbar}{V}\delta(z-z') \tag{4}$$

in which V is normalization constant and it is linked to the quantization volumes.

2. PBG structure.

We are interested to the study of Eq.(1) when ε(z) is function of the propagation variable z , for example in the particular case of 1 D photonic band gap (PBG) structures. Consider the dielectric permittivity function given by:

$$\varepsilon(z)=\sum_{j=1}^{M-1} rect_{B_j}(z-\frac{z_{i+1}-z_i}{2})\varepsilon_j \tag{5}$$

where $B_j=|z_{i+1}-z_i|$ and $z_1=-\infty$ $z_N=+\infty$. j is an index related to the number of interfaces, and M M is an index which defines the space: i.e. M=1 is the semi-space on the left side of the structure. No absorption and dispersion are considered.

Let us consider the simplest case in which M=4. In this situation we have:

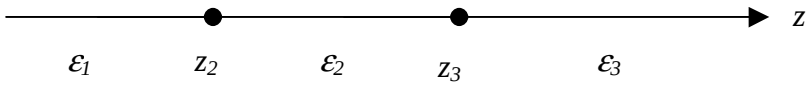

Fig. 1 In this figure the three homogeneous regions are represented, in which the permittivity $\varepsilon(z)$ is subdivided.

In homogeneous regions, i.e. where ε=Cost., the $\hat{A}$–potential operator is of the form :

$$\hat{A}(z,\omega)=C_\omega \hat{a}e^{ikz}+H.c. \tag{6}$$

where C_ω is a normalization constant and $\hat{a}$ is independent of z. We observe that $\hat{a}\neq\hat{a}^+$ instead $\hat{A}=\hat{A}^+$. Operators $\hat{a}$ and $\hat{a}^+$ satisfy the well know boson commutation relations:

$$\left[\hat{a}_{j+},\hat{a}_{j-}\right]=0 \quad ; \quad \left[\hat{a}_{j+}(x,\omega),\hat{a}^+_{i+}(x',\omega')\right]=\delta_{j,i}\delta(x-x')\delta(\omega-\omega') \tag{7}$$

where we have considered two distinct fields, in region 1:

$$\text{for } z<z_2 \Rightarrow \hat{A}(z,\omega)=C_\omega\left[\hat{a}_{1+}e^{ikz}+\hat{a}_{1-}e^{-ikz}\right]+H.c. \tag{8}$$

forward (1+) and backwards (1–) propagating fields, respectively. Subscripts j and I are related to modes of the radiation field . The same considerations apply to the other external region 3:

$$\text{for } z>z_3 \Rightarrow \hat{A}(z,\omega)=C_\omega\left[\hat{a}_{3+}e^{ikz}+\hat{a}_{3-}e^{-ikz}\right]+H.c. \tag{9}$$

In the following calculations, we neglect the H.c. specifications. The operators $\hat{a}_j$ and $\hat{a}_j{}^+$ inside the two external regions (j=1,3) are linked through a linear transformation:

$$\begin{aligned}\hat{a}_{3+}&=U_{11}\,\hat{a}_{1+}+U_{12}\,\hat{a}_{1-}\\ \hat{a}_{3-}&=U_{21}\,\hat{a}_{1+}+U_{22}\,\hat{a}_{1-}\end{aligned} \tag{10}$$

Coefficients of this transformation are determined starting from boundary conditions [3]. We omit tedious calculations for the specific case of interest [2]; but it's interesting to point out the symmetries of this transformation: $U_{22}{}^*=U_{11}=F$; $U_{21}{}^*=U_{12}=G$ and det $\underline{\underline{U}}$ =1. Matrix $\underline{\underline{U}}$ could be easily linked to the Transmission matrix $\underline{\underline{T}}$:

$$\begin{bmatrix}a_{1-}\\ a_{3+}\end{bmatrix}=\underline{\underline{T}}\begin{bmatrix}a_{1+}\\ a_{3-}\end{bmatrix}\Rightarrow$$

#35162 - $15.00 US Received September 06, 2001; Revised October 19, 2001

$$\Rightarrow \underline{\underline{T}} = \frac{1}{F^*}\begin{pmatrix} -G^* & 1 \\ 1 & G \end{pmatrix} \tag{11}$$

The coefficients *F* and *G* will depend on the particular shape of region between 1 and 2 half spaces. Using definition (5), for the permittivity coefficient, the *F* and *G* coefficients are

$$F = \left[cos(k' B_2) + i \frac{k^2 - k'^2}{2kk'} sin(k' B_2) \right] \cdot e^{-ikB_2} \tag{12}$$
$$G = i \frac{k'^2 - k^2}{2kk'} sin(k' B_2)$$

where $B_2=|z_3-z_2|$, $k = \frac{\omega}{c}$, $k' = \frac{\omega}{c}\sqrt{\varepsilon_2} = \frac{\omega}{c} n_2 = k \cdot n_2$.

We can also consider a more complicated structure, in which we have ε_2 constituted of N periodic regions (a real PBG structure), as reported in fig. 2.

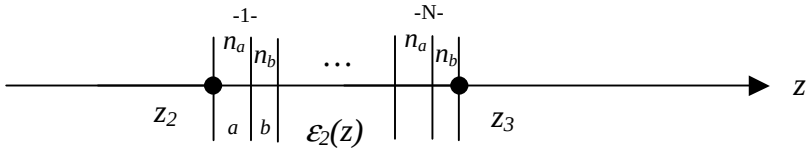

Fig. 2 Extension of the previous calculations to a real PBG structure. Now ε_2 is z-dependent, and it consists of N regions in which we have n=constant.

In this case, in the further hypothesis of considering quarter-wave stacks (optical path in every region it's equal to each other and it's a quarter of wave length), we have for the F and G coefficients [4]

$$F = 1/\left(\frac{4n_a n_b (n_a + n_b)^2}{\left(e^{-i\pi\tilde{\omega}} - \frac{(n_a - n_b)^2}{(n_a + n_b)^2} \right) \Xi_N(\beta) - 4n_a n_b (n_a + n_b)^2 \Xi_{N-1}(\beta)} \right)^* \tag{13}$$

$$G = \left(\frac{(n_a - n_b)}{4n_a n_b (n_a + n_b)^3} \cdot \left(e^{i\pi\tilde{\omega}} - 1 \right) \Xi_N(\beta) \right)^* \tag{14}$$

where $\Xi_N(\beta)=Sin(N\beta)/Sin(\beta)$ is the modified Chebyshev function, β is the Bloch phase and in this particular case it's defined as

$$Cos(\beta) = \frac{Cos(\pi\tilde{\omega}) - \frac{(n_a - n_b)^2}{(n_a + n_b)^2}}{4n_a n_b (n_a + n_b)^2} \tag{15}$$

and $\tilde{\omega}$ is normalized to midgap frequency ($\equiv\omega/\omega_0$).

#35162 - $15.00 US Received September 06, 2001; Revised October 19, 2001

3. Correlation Functions.

We can write the potential operator $\hat{A}$, in the following form [5]:

$$\hat{A}_j(z,\omega) = \hat{A}_j^{(+)}(z,\omega) + \hat{A}_j^{(-)}(z,\omega) \tag{16}$$

where the positive part of $\hat{A}$ is defined as

$$\hat{A}_j^{(+)}(z,\omega) = C_\omega a_{j+} e^{ikz} \tag{17}$$

It is immediately evident that $\hat{A}^{(-)}(z,\omega) = \hat{A}^{(+)}(z,-\omega)^\dagger$ and obviously $\hat{A}_j(z,\omega) = \hat{A}_j(z,\omega)^\dagger$. If we use identical assumptions, used for the potential operator, for the electric field operator, we find the output photon-number density, i.e. the correlation function $\langle \hat{E}_3^{(+)} \hat{E}_3^{(-)} \rangle$, is directly linked to the following quantity

$$\left\langle \hat{a}_{3+}^+ \hat{a}_{3+} \right\rangle = \left\langle \left(F^* \hat{a}_{1+}^+ + G^* \hat{a}_{1-}^+\right)\left(F\hat{a}_{1+} + G\hat{a}_{1-}\right)\right\rangle =$$
$$= |F|^2 \left\langle \hat{a}_{1+}^+ \hat{a}_{1+} \right\rangle + \left(|F|^2 - 1\right)\left\langle \hat{a}_{1-}^+ \hat{a}_{1-} \right\rangle \tag{18}$$

Using the following photon-number densities (number of photons per unit frequency) functions for the fields in the two regions (the 1st and the 3rd one) [6]:

$$N_{1out}(\omega) = \left\langle \hat{a}_{1-}^+ \hat{a}_{1-} \right\rangle; \quad N_{1in}(\omega) = \left\langle \hat{a}_{1+}^+ \hat{a}_{1+} \right\rangle$$

$$N_{3out}(\omega) = \left\langle \hat{a}_{3+}^+ \hat{a}_{3+} \right\rangle \quad N_{3in}(\omega) = \left\langle \hat{a}_{3-}^+ \hat{a}_{3-} \right\rangle \tag{19}$$

if we consider $N_1(\omega)=N_{1out}(\omega)/N_{1in}(\omega)$, and $N_3(\omega)=N_{3out}(\omega)/N_{1in}(\omega)$, in the further hypothesis of irradiating the dielectrics from one side (the input field *3*– is in the vacuum state):

$$N_{1in}(\omega) \geq 0, \quad N_{3in}(\omega) = 0 \tag{20}$$

and using the transmission matrix formalism (see Eq.(16)), we have:

$$N_{1out}(\omega) = \left\langle \hat{a}_{1-}^+ \hat{a}_{1-} \right\rangle = \left\langle \left(t_{11}^* \hat{a}_{1+}^+ + t_{12}^* \hat{a}_{3-}^+\right)\left(t_{11}\hat{a}_{1+} + t_{12}\hat{a}_{3-}\right)\right\rangle =$$

$$= |t_{11}|^2 \, N_{1in}(\omega) \tag{21}$$

where $|t_{11}|^2=|G/F|^2$. Similar calculations, performed for to the $N_{3out}(\omega)$ field, give the following results:

$$N_{3out}(\omega) = |t_{21}|^2 \, N_{1in}(\omega) \tag{22}$$

where $|t_{21}|^2=|1/F|^2$.

Examples of photon number densities ratio are presented in figures 3 and 4 for one single layer. Figure 3 represents $N_1(\omega)$ as a function of the frequency and as a function of the thickness B_2 of the layer. Figure 4 represents $N_3(\omega)$ as a function of the frequency and as a function of the thickness B_2 of the layer.

#35162 - $15.00 US Received September 06, 2001; Revised October 19, 2001

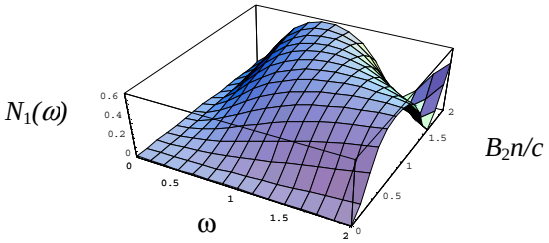

Fig. **3**. Photon number densities ratio of the reflected outgoing field over the incoming field, $N_1(\omega)=N_{1out}(\omega)/N_{1in}(\omega)$, as a function of frequency and dielectric thickness. ω is of the order of 10^{14} s^{-1} and B_2 is of the order of 10^{-6} m. In this simulation N=1 (one layer).

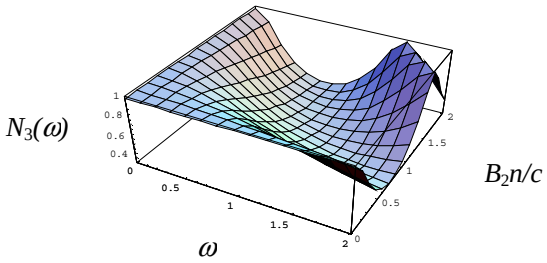

Fig. **4** Photon number densities ratio of the transmitted outgoing field over the incoming field, $N_3(\omega)=N_{3out}(\omega)/N_{1in}(\omega)$, as a function of frequency and dielectric thickness. ω is of order 10^{14} s^{-1} and B_2 is of order 10^{-6} m. In this simulation N=1 (one layer).

In Figure 5 the photon number densities of the reflected outgoing field over the incoming field, $N_1(\omega/\omega_0)$, is plotted as a function of the normalized frequency $\omega/\omega_0=\omega B_2 n_2/c$. In green the plot of the photon number densities of the transmitted outgoing field over the incoming field, $N_3(\omega/\omega_0)$, as a function of the normalized frequency $\omega/\omega_0=\omega B_2 n_2/c$ is presented , B_2 is the dielectric thickness and n_2 is the refractive index (≈3 in our example). In this simulation N=1 (single layer).

#35162 - $15.00 US Received September 06, 2001; Revised October 19, 2001

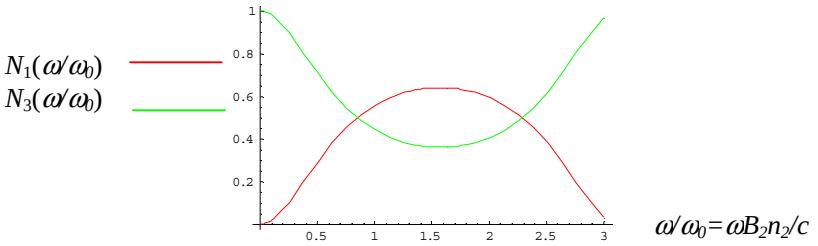

Fig. **5** In red: Photon number densities of the reflected outgoing field over the incoming field, $N_1(\omega/\omega_0)$, as a function of the normalized frequency $\omega/\omega_0=\omega B_2 n_2/c$. In green: Photon number densities of the transmitted outgoing field over the incoming field, $N_3(\omega/\omega_0)$, as a function of the normalized frequency $\omega/\omega_0=\omega B_2 n_2/c$. B_2 is the dielectric thickness and n_2 is the refractive index (≈3 in our example). In this simulation N=1 (single layer).

In Figure 6 the photon number densities of the reflected outgoing field over the incoming field, $N_1(\omega/\omega_0)$, is plotted as a function of the normalized frequency ω/ω_0. In green the plot of the photon number densities of the transmitted outgoing field over the incoming field, $N_3(\omega/\omega_0)$, as a function of the normalized frequency ω/ω_0 is presented , B_2 is the dielectric thickness and n_2 is the refractive index (≈3 in our example). In this simulation a quarter-wave stack has been considered, and $\omega_0=2\pi c/\lambda_0$. The refractive index of each layer is $n_a=1$, $n_b=2$, and the number of cells (see fig. 2) is N=3 (multi-layer material: PBG structure). We can observe as the photon number density follows the classical transmission spectrum of the layered structure [4].

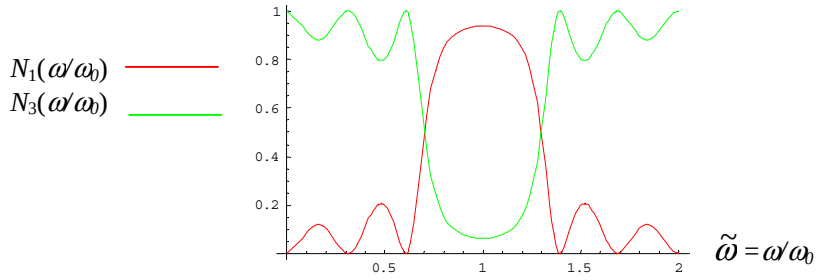

Fig. **6** In red: Photon number densities of the reflected outgoing field and incoming field, $N_1(\omega/\omega_n)$, as a function of the normalized frequency $\tilde{\omega}=\omega/\omega_0$. In green: Photon number densities of the transmitted outgoing field and incoming field, $N_3(\omega/\omega_n)$, as a function of the normalized frequency $\tilde{\omega}$. In this quarter-wave stack, $\omega_0=2\pi c/\lambda_0$. In this simulation $n_a=1$, $n_b=2$ and the number of cells (see fig. 2) is N=3 (multi-layer material: PBG structure).

4. States symmetries.

Starting from U (or T) matrix, we obtain in the general case, the following property:

$$\hat{n}_{3+}+\hat{n}_{1-}=\hat{n}_{1+}+\hat{n}_{3-} \qquad (23)$$

that is the conservation energy relation for the total optical system, involving the 4 fields. The Hilbert Space describing the system, has the following base vector (Fock state):

#35162 - $15.00 US
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$$|\varphi\rangle = |n, m, s, p\rangle \tag{24}$$

where labels in the ket are the photon numbers of modes 1–,1+,3–,3+, respectively. The set $\{|\varphi\rangle\}$ is an ortho-normal set. If we try to calculate the average on this (general) state, of equation 23, we obtain:

$$p + n = m + s \tag{25}$$

where *n,m,s,p* are positive integer numbers. If we are in a state in which we have *N* photons (in total), the general expression (24) gives the following state:

$$|\varphi_N\rangle = |n, m, s, N - (n + m + s)\rangle \tag{26}$$

where $n+m+s \leq N$. The number of such ortho-normal states is $(6+11N+6N^2+N^3)/6$. If we consider the further condition (25), the number of ortho-normal states will became $(2+N)^2/4$ and the general *N* photon state is:

$$|\varphi_N\rangle = \left|n, m, \frac{N}{2} - m, \frac{N}{2} - n\right\rangle, \quad n, m \in \left[0, \frac{N}{2}\right] \cap \aleph \tag{27}$$

where the numbers *n,m* and *N* are positive integers. Replacing *N* with 2*N*', and renaming *N*' by *N*, equation (27) becomes:

$$|\varphi_{2N}\rangle = |n, m, N - m, N - n\rangle, \quad n, m \in [0, N] \cap \aleph \tag{28}$$

This is the structure of the 2*N* photon field in the system.

6. Conclusion.

These calculations show the study of the quantum correlation behaviour of the optical field in the space regions external to the PBG structures. The transfer matrix formalism has been applied to follow the output behaviour of photon number density of a quantum field exiting from the PBG structure. The knowledge of these properties will be of interest for implementation of quantum computing and quantum gates. These calculations are only the starting analysis of the interesting quantum properties of the inhomogeneous media. We'll extend the study of the state's symmetries and the correlation functions from linear to nonlinear cases.

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