Intra-channel nonlinearity in differentially phase-modulated transmission

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Abstract: The mechanisms responsible for nonlinear impairments in single-channel phase modulated system employing differential detection are investigated. The role of dispersion precompensation is discussed. It is shown that precompensation may be designed as to minimize the in-phase components of the fluctuations thus reducing nonlinear impairments. In differential-phase-shift-keying the effect of precompensation is stronger than in differential-quadrature-phase-shift-keying. The results of an analytic theory are compared with split-step based computer simulations using realistic system parameters.

OCIS codes: (060.2330) Fiber optics communications; (060.4370) Nonlinear optics, fibers.

References and links

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1. Introduction

In the pioneering era of optical communication it was unconceivable that optical transmission will ever go beyond simple intensity modulation. At the dawn of the new millennium, rapid progresses in the field of integrated optics made systems based on differential phase modulation more and more practical, and over performing their intensity modulated counterparts [1].

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On one side, the signal to noise ratio of a differential-binary-phase-shift-keying (DBPSK) system is potentially 3 dB larger than an intensity modulated system with the same average power [2]. On the other, differential quadrature phase shift keying (DQPSK) has a larger spectral efficiency than conventional on-off keying intensity modulation because two bits per symbol are transmitted. The effect of the nonlinear impairments on these schemes is still an open issue over many aspects. The purpose of this paper is to discuss the role of phase and amplitude nonlinear perturbations in DBPSK and DQPSK transmission schemes, and show that an optimized dispersion precompensation may significantly improve the transmission performance. We will first discuss the physical mechanisms behind nonlinear fluctuations, and give suggestions for their mitigations. We will then compare the results of a recently developed theory [3] with the outcome of computer simulations of both DBPSK and DQPSK systems. We will neglect polarization effects all throughout this paper.

2. Analysis

Let us assume that a sequence of Gaussian pulses of equal amplitude A and pulse-width τ, equally spaced by the symbol time T_s, is injected in a fiber with dispersion β'' and nonlinear coefficient γ. Let the input field be $u_0(0,t) = \sum_k a_k v_0(t - T_k)$ with $v_0(t) = A\exp[-t^2/(2\tau^2)]$, where $a_k = \exp(-i\varphi_k)$ are unit amplitude complex parameters specifying the message, and $T_k = kT_s$ are the center timing of the pulses. The pulse-width τ is related to the 3dB pulse-width by $\tau_{\mathrm{FWHM}} = 2\sqrt{\log 2}\,\tau$. Let us assume first that dispersion compensation is applied only at the input and output of the system, leaving the analysis of the more general case of lumped in-line dispersion compensations to a discussion at the end of this section. The fiber input, where precompensation is applied, is at $z = 0$, and the fiber output, where dispersion postcompensation is applied is at $z = L$. We assume that the sum of pre and postcompensation is the total dispersion of the line. It was shown in ref. [3] using a first order perturbation theory, that is retaining in the analysis the only terms that depend linearly on γ, that the nonlinear perturbation ΔI of the photocurrent I originating from the overlapping of the pulse centered at $T_0 = 0$ with the pulse centered at $T_1 = T_s$ in the delay interferometer is, after differential detection, $\Delta I = [e\eta/(\hbar\omega_0)]\mathrm{Re}[\exp(-i\varphi_d)(\Delta I_1 + \Delta I_0^*)]$. Here, e is the electron charge, η is the detector efficiency, $\hbar\omega_0$ is the photon energy at the signal center frequency. A system employing DBPSK uses for detection a single delay interferometer with $\varphi_d = 0$, whereas a system employing DQPSK uses two delay interferometers with unbalancing $\varphi_d = \pm\pi/4$. The complex quantities ΔI_1 and ΔI_0 are defined as $\Delta I_1 = \sum_{j,l} a_1^* a_j a_{j+l}^* a_l J_{l,j}$ and $\Delta I_0 = \sum_{j',l'} a_0^* a_{j'} a_{j'+l'-1}^* a_{l'} J_{l'-1,j'-1}$, where, assuming A as real and normalized such that A^2 is the peak power of the signal,

$$J_{j,l} = i\gamma\sqrt{2\pi^3}A^4\tau^3 \int_{-z^*}^{L-z^*} f(z+z^*)G(T_j,T_l;z)\,\mathrm{d}z, \tag{1}$$

and

$$G(T_j,T_l;z) = \frac{1}{2\pi\tau^2\sqrt{(z/z_d)^2+1}} \exp\left\{-\frac{T_j^2+T_l^2-2i(z/z_d)T_jT_l}{2\tau^2[(z/z_d)^2+1]}\right\}. \tag{2}$$

Here $f(z)$ is the signal power variation caused by linear loss and gain, i.e. the ratio of the signal power at a generic position z to the input power at position $z = 0$, and $z_d = \tau^2/\beta''$ is the dispersion length, positive or negative depending upon the sign of β''. The sum should be extended to all pulses of the stream significantly overlapping with the pulses centered at $T_0 = 0$ and $T_1 = T_s$. The parameter z^* is the length of transmission fiber whose dispersion is precompensated, more precisely z^* is defined as the coordinate of the point of the link where the total accumulated dispersion, added by dispersion precompensation and by the linear propagation up to z^*, is zero.

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By a different and more transparent rearrangement of the terms of ref. [3] we obtain a simpler expression for the variance of the nonlinear fluctuations

$$\langle \Delta I_{\mathrm{mod}}^2 \rangle = \frac{e\eta}{\hbar\omega_0} \left(C_{\mathrm{mod,FWM}} + C_{\mathrm{mod,corr}} \right), \tag{3}$$

where the subscript "mod" identifies the modulation format, DBPSK or DQPSK. In Eq. (3), the first term accounts for the contributions of the intra-channel four-wave mixing (FWM), and the second for the correlation of the FWM fluctuations [4]. For DBPSK we have

$$C_{\mathrm{DBPSK,FWM}} = \sum_{j,l} f_{j,l} \left[|J_{l,j}|^2 + \mathrm{Re}\left(J_{l,j}^2 \right) \right], \tag{4}$$

$$C_{\mathrm{DBPSK,corr}} = \sum_{j} \mathrm{Re}\left[q_j \left(J_{1,j} J_{-1,j}^* + J_{1,j} J_{-1,j} \right) + s_j \left(|J_{j,1-j}|^2 + J_{j,1-j}^2 \right) \right], \tag{5}$$

whereas for DQPSK

$$C_{\mathrm{DQPSK,FWM}} = \sum_{j,l} f_{j,l} |J_{l,j}|^2, \quad C_{\mathrm{DQPSK,corr}} = \sum_{j} \mathrm{Re}\left(q_j J_{1,j} J_{-1,j} \right). \tag{6}$$

We defined the functions $f_{j,l} = 2$ always, except $f_{j,j} = 1$ and $f_{j,0} = f_{0,l} = f_{0,0} = 0$; $q_j = 4$ except $q_1 = q_{-1} = 2$ and $q_0 = 0$; and finally $s_j = 2$ except $s_0 = s_1 = 0$.

Let us define the Q factor at the receiver as $Q = \langle I \rangle / \sqrt{\langle \Delta I^2 \rangle}$. The average signal at detection is $\langle I \rangle = e\eta/(\hbar\omega_0)\mathrm{Re}[\exp(-i\varphi_d)E_0]$, where $E_0 = \sqrt{\pi} A^2 \tau = P_{\mathrm{av}} T_s$ is the pulse energy and P_{av} the average launched power. Using $\varphi_d = 0$ for DBPSK and $\varphi_d = \pm\pi/4$ for DQPSK, we obtain $\langle I_{\mathrm{DBPSK}} \rangle^2 = e\eta P_{\mathrm{av}}^2 T_s^2/(\hbar\omega_0)$ for DBPSK, and $\langle I_{\mathrm{DQPSK}} \rangle^2 = e\eta P_{\mathrm{av}}^2 T_s^2/(2\hbar\omega_0)$ for DQPSK. Putting all these results together, we obtain for the Q factor at the receiver the expressions $Q_{\mathrm{DBPSK}} = P_{\mathrm{av}} T_s / \sqrt{\langle \Delta I_{\mathrm{DBPSK}}^2 \rangle}$ for DBPSK, and $Q_{\mathrm{DQPSK}} = P_{\mathrm{av}} T_s / \sqrt{2 \langle \Delta I_{\mathrm{DQPSK}}^2 \rangle}$, for DQPSK, where the variance of the nonlinear fluctuations is given by Eq. (3). For a given pulse-width, we have $\langle I \rangle \propto P_{\mathrm{av}}$ and $\sqrt{\langle \Delta I^2 \rangle} \propto |J_{j,l}| \propto A^4 \propto P_s^2$, so that the nonlinear contribution to the Q factor is inversely proportional to P_{av}.

Let us now consider the case where at $N-1$ lumped locations equally spaced along the line, which we assume coincident with the amplifier locations, one Nth of the total line dispersion is compensated. Compensation at the transmitter and receiver takes care of the residual one Nth of the total dispersion such that the full line dispersion is compensated. Such a line may always be represented as the concatenation of N identical sections of length L made of precompensation, line propagation and postcompensation with the sum of pre and postcompensation of each section equal to the total dispersion of the section. In this case, $\langle \Delta I^2 \rangle$ is N^2 times the photocurrent fluctuations for a single section, because the fluctuations, in the linear regime, are additive. The total system length is now $L_{\mathrm{tot}} = NL$.

3. Discussion

The physical understanding is facilitated by the analysis of the simplest case in which the loss is locally compensated by a distributed gain [5], so that $f(z) = 1$. This case is fully equivalent to a transmission through an ideal lossless fiber, so that in the following we will refer to it as such. In this case, the imaginary part of $G(T_j, T_l; z)$ is an antisymmetric function of z so that, if we chose z^* such that the integration interval in Eq. (1) is itself symmetric, the real part of $J_{j,l}$, proportional to the integral of the antisymmetric imaginary part of $G(T_j, T_l; z)$ over a symmetric interval, becomes zero, and $J_{j,l}$ purely imaginary. The value of z^* that makes the integration interval symmetric is half of the span length $z^* = L/2$, corresponding to a symmetric profile where in each fiber sections between dispersion compensating stations half of the dispersion is

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compensated at the input, half at the output of the section. Let us now analyze separately the effect of a symmetric compensation profile on DBPSK and DQPSK system performance.

In DBPSK, we have $\varphi_d = 0$ and $a_j = \pm\exp(-i\varphi)$, where φ is an arbitrary bit-independent phase added by the link, so that the products $a_0^* a_{j'} a_{j'+l'-1}^* a_{l'}$ and $a_1^* a_j a_{j+l}^* a_l$ may assume in the expression of ΔI only the values ± 1. With symmetric compensation, $J_{j,l}$ is purely imaginary, so that both ΔI_1 and ΔI_0 are purely imaginary hence $\Delta I = 0$. Physically, the absence of nonlinear fluctuations with a symmetric dispersion profile reflects the fact that in this case the nonlinear fluctuations are in quadrature with the field. With symmetric compensation, therefore, the photocurrent is not affected by the nonlinear fluctuations in the lossless case, within our first order perturbation theory.

The analysis of DQPSK shows a different scenario. The possible values of a_j are now four, namely $\pm\exp(-i\varphi)$ and $\pm i\exp(-i\varphi)$, and the products $a_0^* a_{j'} a_{j'+l'-1}^* a_{l'}$ and $a_1^* a_j a_{j+l}^* a_l$ may assume in the expression of ΔI the four values ± 1 and $\pm i$, with equal probability for random codes with equally probable symbols. Therefore, ΔI_1 and ΔI_0 are with equal probability purely real or purely imaginary. This implies that in the symmetric case the nonlinear fluctuations of a DQPSK pulse are either pure amplitude noise or pure phase noise. The effects of the fluctuations on the receiver are however identical for pure in quadrature or in phase fluctuations. This is because if we represent the electric field of the four symbols at the center of the four quadrants of a complex plane, the photo-currents detected after the two interferometers, whose arms are unbalanced by $\varphi_d = \pm\pi/4$, measure the projection of the field on the real and imaginary axis and are therefore sensitive to both amplitude and phase noise of the field. The reduction of one of the two components of the nonlinear fluctuations is however still beneficial because the total fluctuations become smaller, although they do not become zero in the lossless case as they do in DBPSK.

Table 1. Numerical parameters

Quantity	Symbol	Value	Units
Fiber dispersion	β''	-20.4	ps^2/km
Nonlinear coefficient	γ	1.3	$W^{-1}km^{-1}$
Pulse-width (FWHM)	τ_{FWHM}	5	ps
Symbol time (inverse of baud rate)	T_s	25	ps
Number of spans	N	7	
Span length	L	100	km

4. Results

To illustrate these results, we report in Figs. 1 and 2 the Q factors for DBPSK and DQPSK systems with the parameters listed in Tab. 1 vs. the precompensated fiber dispersion z^*. We assume full dispersion compensation, the sum of precompensation and postcompensation, after each of the $N = 7$ amplifier spans. For instance, $z^* = 0$ corresponds to full span dispersion compensation at each amplifier station and at the receiver; $z^* = L$ to full span dispersion compensation at the line input and at each amplifier station; $z^* = L/2$ to half span dispersion compensation at the line input, full span dispersion compensation at each amplifier station, and half span dispersion compensation at the receiver. Notice that, although the baud rate 40Gbaud/s is the same in both cases, the bit rate of the DQPSK system (two bit/symbol) is 80Gbit/s whereas that of the DBPSK (one bit/symbol) is 40Gbit/s. Lines are a plot of the theory. Circles are the results of a split-step based simulation program, using at receiver a third order Butterworth optical filter of 1.5 nm bandwidth, and an electrical filter of approximately 30 GHz bandwidth compliant

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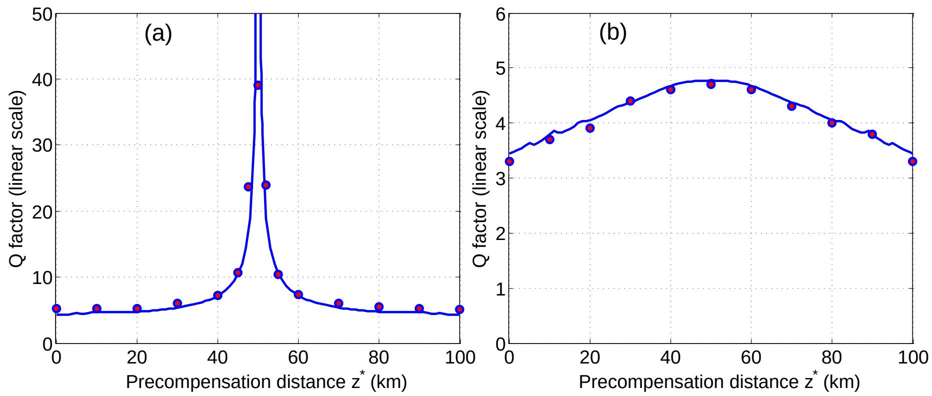

Fig. 1. Q factor in linear scale vs. the precompensation distance z^* in the lossless case, with 3 dBm average input power, and the parameters listed in Table 1. a) Solid line and circles, theory and simulations for DBPSK; b) Solid line and circles, theory and simulations for DQPSK.

with the G.957 ITU-T Recommendation, with a bandwidth 0.8 the baud rate. We used in the simulations De Brujin sequences of 2^9 symbols for DBPSK and of 4^4 symbols, each encoding two bits, for DQPSK. The number of pulses was sufficient in our case where full compensation is performed every $L = 100$ km, and the number of overlapping pulses, approximately $(L/z_d)(\tau_{\mathrm{FWHM}}/T_s)$, is about 45. The results are represented with circles in figs. 1 and 2.

To substantiate the previous qualitative considerations, we plot in Fig. 1 the Q factors vs. the precompensated fiber dispersion z^* for a lossless fiber system with the parameters listed in table 1. The input average power was $P_{\mathrm{av}} = 3$ dBm. Fig. 1a refers to DBPSK, the solid line is the plot of the theory for DBPSK, circles are the results of the computer simulations. The theoretical curves have been obtained by truncating the sums in Eq. (4), (5) and (6) up to $|j|, |l| \leq 200$. This means that we include the interaction with the 200 pulses preceding and the 200 following any given pulse. We have verified that increasing this number does not change the first few digits of the results. The benefits of the cancellation of the in-phase components of the nonlinear fluctuations with symmetric compensation show up clearly [6]. For symmetric compensation, our first order perturbation theory predicts infinite Q, that is zero nonlinear fluctuations, whereas the numerical solution of the complete equations show a large, but finite, Q, about one order of magnitude larger than the Q in the absence of precompensation. Fig. 1b refers to DQPSK, the solid line being again the result of the theory, the circles of the simulations. It shows up clearly that in the lossless case, although the benefits of a symmetric compensation are still measurable, the impact of a symmetric compensation is significantly smaller than in DBPSK [7].

Let us now analyze the impact of precompensation on the more practical configuration in which lumped amplifiers replace distributed amplification. The fiber loss is 0.25 dB/km, and the average input power is 15dBm. Figure 2a refers to DBPSK whereas Fig. 2b to DQPSK. To sort out the effect of the nonlinear impairments only, no amplified spontaneous emission was added in the simulations at the amplifier locations. As expected, the amount of precompensation that gives the highest Q is smaller than in the lossless case, because the efficiency of the nonlinear effects in the final part of each span is negligible, so that the effective span length is smaller than the actual length. With loss, the in-phase components of the nonlinear fluctuations are never zero hence, at optimum precompensation, the theory does not predict for DBPSK an

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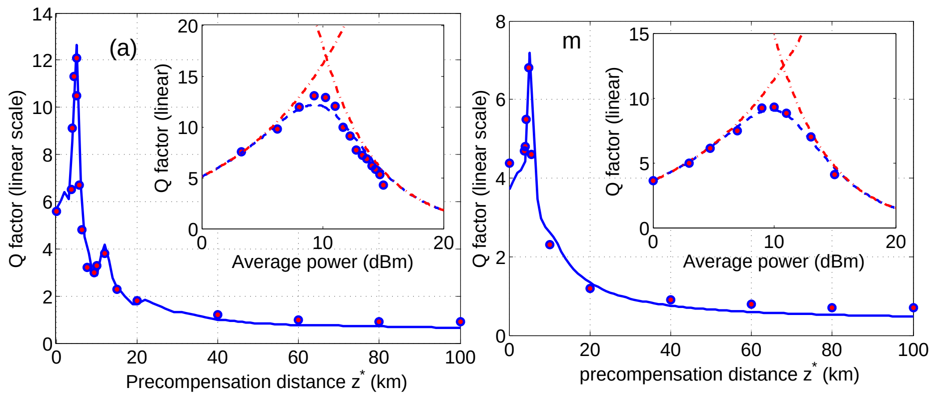

Fig. 2. Q factor in linear scale vs. the precompensation distance z^* with lumped amplification, 0.25 dB/km of fiber loss, 15 dBm average input power, and the parameters listed in Table 1. a) Solid line and circles, theory and simulations for DBPSK; b) Solid line and circles, theory and simulations for DQPSK. Insets: Q vs. input power when ASE noise is included for zero precompensation (see text).

infinite Q. With loss the benefit of precompensation, still larger for DBPSK than for DQPSK, becomes comparable in the two transmission formats.

Our analysis neglects amplified spontaneous emission (ASE) noise and nonlinear noise coupling. To check whether the results are still meaningful, we performed simulations in the lossy case including noise, assuming amplifiers having 6 dB noise figure (3 dB above the quantum limit), and a wavelength of 1.55 μm. The Q factor for DBPSK vs. the input power P_{av} are reported in the insets of Fig. 2, for zero dispersion precompensation. Dots are the results of the simulations, dot-dashed red lines the Q caused by the ASE noise alone, $Q^2 \propto P_{\mathrm{av}}$ and nonlinear intrachannel effects alone, $Q^2 \propto 1/P_{\mathrm{av}}^2$. Dashed blue lines are the total Q factor, assuming the variance of the noise as the sum of the variances caused by the ASE noise and intrachannel effects alone, i.e. neglecting their interaction (i.e. the nonlinear noise coupling, $Q^2 \propto 1/P_{\mathrm{av}}$). It is apparent that nonlinear noise coupling, if present, does not modify the qualitative and even quantitative behavior of the two curves.

5. Conclusions

To conclude, we discussed the physical mechanisms responsible for the nonlinear fluctuations in differentially phase modulated transmission. We showed that the nonlinear fluctuations can be mitigated by a careful choice of the precompensation of the in-line dispersion [6]. Precompensation impacts the performance of DBPSK, insensitive to first order to the in-quadrature fluctuations of the field, more than it does DQPSK, which is sensitive to the in quadrature as well as the in-phase components of the field fluctuations. In DBPSK, the impact of nonlinear fluctuations on the received photocurrent may be reduced ideally to zero in the lossless case.

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