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STOCHASTICITY AND PERSISTENCE OF SOLAR CORONAL MASS EJECTIONS

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ABSTRACT

The study of the statistical properties of coronal mass ejections (CMEs) reveals that their properties depend on the period of solar activity. In particular, when investigating the origin of the waiting time distribution between CMEs, a significant departure from a Poisson process during periods of high solar activity has been found, thus suggesting the existence of at least two physical processes underlying the origin of CMEs. One acts continuously, perhaps related to randomly occurring magnetic reconfigurations of the solar corona at large scales. The other plays a role only during the solar maximum, probably due to the photospheric emergence of magnetic flux as a statistically persistent mechanism, which generates long correlation times among CME events strong enough not to be destroyed by the former random process.

Key words: methods: statistical – Sun: activity – Sun: corona – Sun: coronal mass ejections (CMEs) – Sun: evolution

Online-only material: color figures

1. INTRODUCTION

The distribution of the time interval between two successive coronal mass ejections (CMEs) provides information regarding whether CMEs occur as independent events, which may cast some light on the nature of the processes underlying their origin and on the understanding of their relationship with solar flares.

The waiting time distribution (WTD) of CMEs has recently been the object of investigation. Using the catalog of CME events from the Large Angle Spectrometric Coronagraph (LASCO; Brueckner et al. 1995) onboard the *Solar and Heliospheric Observatory* (*SOHO*; Domingo et al. 1995) from 1996–2001, Yeh et al. (2002) and Wheatland (2003) have independently claimed that the distribution follows a power-law behavior, $P(\Delta t) \sim \Delta t^{-\gamma}$, for waiting times larger than a few hours, $\Delta t \gtrsim 10\,h$, with a power-index $\gamma \simeq 2.36 \pm 0.11$. This result is interesting due to the fact that this scaling is very close to that for solar flares in the same time period (Boffetta et al. 1999; Wheatland 2000; Aschwanden & McTiernan 2010). The observational evidence that flares and CMEs share the same waiting time statistics led these authors to suggest a common origin for the two phenomena. Furthermore, Wheatland (2003) has shown that the power-law distribution for CMEs varies with the solar cycle, being steeper during the period of higher activity (1999–2001). Yeh et al. (2005) have extended the data sample to the year 2003 and have investigated the WTDs of fast and slow CMEs separately, finding further clues in support of a scenario in which solar flares and both types of CMEs are different manifestations of the same eruptive process.

A possible interpretation for the origin of the power-law-like WTD of CMEs and for its variation with the solar cycle (Wheatland 2003), resides in the fact that the observed occurrence of CMEs might be interpreted as a time-dependent Poisson process, i.e., as a realization of renewal Poisson processes with a variable rate, $\lambda = \lambda(t)$. As a matter of fact, the WTD $P(\Delta t)$ for a piecewise-constant Poisson process involving a large number of rates may be easily derived as (Wheatland 2003)

$$P(\Delta t) = \frac{1}{\lambda_0} \int_0^\infty \lambda^2 f(\lambda) e^{-\lambda \Delta t} d\lambda, \qquad (1)$$

where λ_0 is the mean CME rate and $f(\lambda)$ is the time distribution, say the probability density function, of the CME rate. If the distribution of the CME rates follows a power-law function, $f(\lambda) \propto \lambda^\alpha$, Equation (1) predicts a power-law tail $\propto \Delta t^{-(3+\alpha)}$, which thus seems to reproduce the qualitative features of the observed WTD. However, it is worth noting that the CME power-law distribution for large Δt arises from two a priori assumptions, not tested for consistency with observations: (1) a local Poisson hypothesis (Lepreti et al. 2001), and (2) a power-law behavior, $\propto \lambda^\alpha$, of the rate probability density (whose index α is furthermore constrained to be larger than -3 to ensure that the result holds). Finally, concerning the variation of the slope of the WTDs for CMEs during the solar cycle, Wheatland (2003) attributed its origin simply to the modulation in time of the CME rate.

In this Letter the WTD for CMEs observed from *SOHO*/LASCO is reexamined as follows. Section 2 provides a statistical analysis to test whether or not the sequence of CMEs is consistent with a time-varying Poisson process: it is shown that the local Poisson hypothesis does not generally hold for the temporal distribution of CMEs. Rather, the observed CME WTD can be explained by simply assuming the presence of correlations in the CME time series, which leads to the introduction of the Weibull distribution to fit the WTD for CMEs, as discussed in Section 3. Section 4 presents the results of how the statistical properties of the CME distribution vary over the course of solar activity, outlining their physical relevance in light of the mechanisms driving the eruption of CMEs.

2. DEPARTURE FROM A LOCAL POISSON DISTRIBUTION

The CME data used in this Letter to analyze the WTD for CMEs come from the *SOHO*/LASCO catalog, which spans

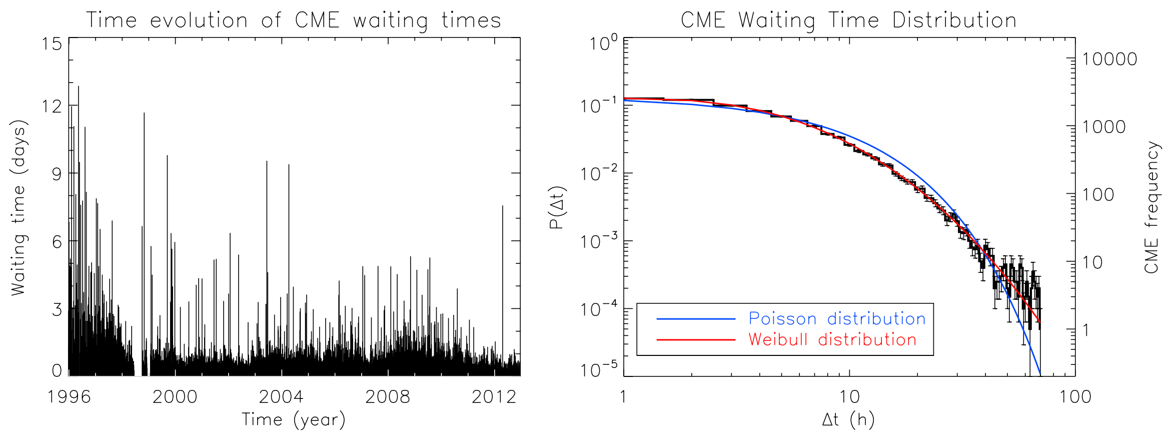

Figure 1. Left: temporal evolution of the waiting times of CMEs observed with *SOHO*/LASCO from the beginning of 1996 to the end of 2012, as a function of CME occurrence time. Right: WTD $P(\Delta t)$ for the *SOHO*/LASCO CMEs, fitted by a exponentiated Weibull function (red curve), which accounts for the presence of correlations among the waiting times; the theoretical distribution function expected under Poisson statistics is also shown (blue curve).

(A color version of this figure is available in the online journal.)

17 yr from 1996 to 2012, i.e., from the minimum of solar cycle 23 to the rise to the maximum phase of solar cycle 24. According to Wheatland (2003), the waiting times Δt, shown in the left panel of Figure 1 as a function of the time occurrence of CMEs, are estimated as the differences between the times of the first appearance of successive CMEs in the field of view of the LASCO coronagraph. The distribution $P(\Delta t)$ of the CME waiting times, sampled in intervals of 1 hr, is shown in the right-hand panel of Figure 1.

A power-law tail $\propto \Delta t^{-\gamma}$ with scaling exponent $\gamma \simeq 2.97 \pm 0.10$ (the corresponding χ^2 is 0.03), is more or less recovered for a small range for Δt larger than a few hours, thus pointing to a significant departure of the process from Poisson statistics, which is further suggested by the unsuitability of the Poisson distribution function (given by $P(\Delta t) = \bar{\lambda} e^{-\bar{\lambda}\Delta t}$, where $\bar{\lambda} = 0.13\,h^{-1}$ is the average CME occurrence rate over the period 1996–2012) to reproduce the observed WTD for $\Delta t \gtrsim 10\,h$ in the hypothesis of a random, say Gaussian, occurrence of CMEs. It is indeed readily seen that the CME WTD is systematically smaller than the theoretical distribution expected under Poisson statistics (blue line in the right panel of Figure 1), thus indicating that long waiting times rarely occur in the CME catalog, namely that non-stochastic, correlated clusters are present in the CME time sequence.

In order to prove the nonreliability of the Poisson hypothesis and to investigate the statistics of persistence times between CME events, thus characterizing their temporal clustering and quantifying their degree of correlation, a statistical analysis based on the Kolmogorov–Smirnov (K–S) test (Bi et al. 1989; Lepreti et al. 2001) is required. The K–S test is indeed a nonparametric and distribution-free test usually used to compare a data sample with a reference probability distribution, by quantifying the maximum distance D between the empirical and the reference distribution functions. It is based on the study of the statistical properties of the stochastic variable h defined as (Bi et al. 1989; Lepreti et al. 2001)

$$h_i(\delta t_i, \delta\tau_i) = \frac{2\delta t_i}{2\delta t_i + \delta\tau_i}, \qquad (2)$$

where δt_i and $\delta\tau_i$ are the waiting times between a CME occurring at t_i and the two (either following or preceding) nearest events:

$$\delta t_i = \min\{t_{i+1} - t_i, t_i - t_{i-1}\}, \qquad (3)$$

$$\delta\tau_i = \begin{cases} t_{i-1} - t_{i-2} & \text{if} \quad \delta t_i = t_i - t_{i-1} \\ t_{i+2} - t_{i+1} & \text{if} \quad \delta t_i = t_{i+1} - t_i. \end{cases} \qquad (4)$$

The stochastic variable h thus simply represents the suitably normalized time between CME events. Under the null hypothesis that the data sample is drawn from the Poisson reference distribution, that is in the hypothesis that δt_i and $\delta\tau_i$ are independently distributed with exponential probability densities given by $P(\delta t_i) = 2\lambda_i \exp(-2\lambda_i \delta t_i)$ and $P(\delta\tau_i) = \lambda_i \exp(-\lambda_i \delta\tau_i)$, where λ_i is the local (not constant) event rate, it can easily be shown that the cumulative distribution function (CDF) of h, $P(h < H)$, is simply $P(h < H) = H$, where $P(h)$ is the probability distribution function (PDF) of h. Hence, if the local Poisson hypothesis holds, the stochastic variable h is uniformly distributed in [0:1]. The maximum deviation D of the empirical CDF of the CME events from the reference relation $P(h < H) = H$, quantifies the degree of departure of the process from the Poisson statistics or, in other words, the level of correlation among the CME eruptions.

The PDF $P(h)$ of the normalized waiting time h between CMEs, and the corresponding CDF $P(h < H)$, are shown in the left- and right-hand panels of Figure 2, respectively, where they are compared with the theoretical Poisson probability (dotted lines).

A significant deviation from the uniform distributions expected if the Poisson hypothesis holds can clearly be observed (being the significance level of the K–S test smaller than 10^{-9}%). In particular, the probability function $P(h)$, which roughly flattens in the range $0.2 \lesssim h \lesssim 0.8$ (though at a value somewhat larger than that expected under Poisson statistics), significantly decreases toward $h \approx 0$ and $h \approx 1$. Similarly, the cumulative function $P(h < H)$ strongly departs from the Poisson theoretical distribution $P(h < H) = H$, being mostly lower than the latter for $H < 1/2$. This observational evidence represents unambiguous indications of temporal clustering of the CME events, thus fully confirming the results outlined by looking at the WTD of CMEs.

It turns out that the results presented here clearly show that, at least when looking at the whole solar cycle, Poisson statistics cannot account for the WTD of CMEs, thus fully contradicting the conclusions by Wheatland (2003) that imply that the power law does not have fundamental significance arising from the statistics of independent CMEs with time-varying rates; rather,

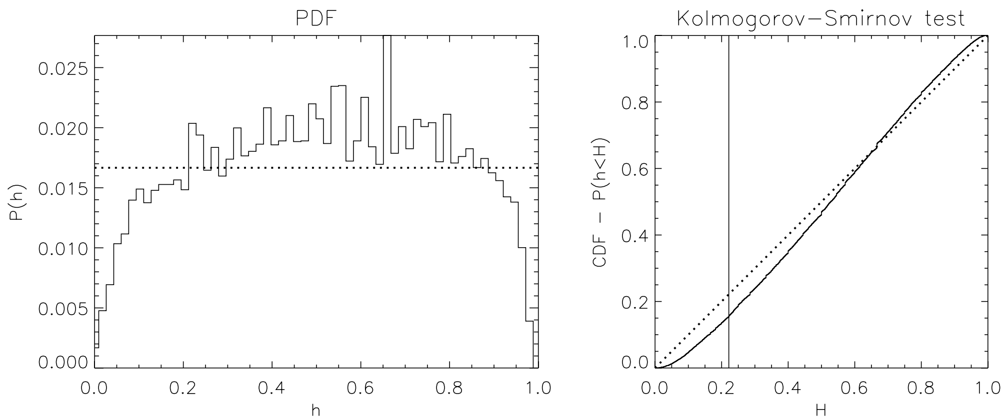

Figure 2. PDF $P(h)$ (left) and corresponding CDF $P(h < H)$ (right) of the normalized waiting time h between CMEs; the dotted lines represent the theoretical probability expected under Poisson statistics; the vertical line indicates where the CDF mostly deviates from it.

the observed power-law WTD is due to the presence of long-term correlations between events, which implies that the process underlying the origin of CMEs is characterized by a significant degree of persistence, i.e., memory.

3. THE ORIGIN OF THE POWER-LAW DISTRIBUTION

The observed form of the WTD can be simply understood in terms of a process characterized by a certain amount of memory. In order to outline the necessary theory, it is worthwhile to introduce the function $z(\Delta t)$ as

$$z(\Delta t) = \frac{P(\Delta t)}{P(\Delta t \geqslant \Delta T)}, \tag{5}$$

where $P(\Delta t)$ and $P(\Delta t \geqslant \Delta T)$ are the PDF and the surviving distribution function for the CME waiting times, respectively. That is, $z(\Delta t)$ represents the local CME rate given that a time interval Δt has elapsed, with a probability of $P(\Delta t \geqslant \Delta T)$, between two consecutive CMEs. This easily gives

$$P(\Delta t) = z(\Delta t) e^{-\int_0^{\Delta t} z(x)dx}. \tag{6}$$

In a memoryless stochastic process, the probability of the occurrence of an event remains constant (Feller 1968), i.e., $z(\Delta t) = \lambda$, thus obtaining the Poisson distribution $P(\Delta t) = \lambda e^{-\lambda \Delta t}$. On the other hand, if the probability of occurrence changes with time (namely, the process has memory), $z(\Delta t)$ can be expressed as

$$z(\Delta t) = \lambda^k k \Delta t^{k-1}, \tag{7}$$

where k is the key parameter describing the statistical properties of the process: $k \lessgtr 1$ indicates that the probability of occurrence decreases (increases) over time, hence clusters (voids) are present in the system, whilst for $k = 1$ Equation (7) trivially reduces to $z(\Delta t) = \lambda$, thus indicating, as discussed above, random (i.e., uncorrelated) events. Substituting Equation (7) into Equation (6), the PDF $P(\Delta t)$ can be easily rewritten as

$$P(\Delta t) = \frac{k}{\beta} \left(\frac{\Delta t}{\beta}\right)^{k-1} e^{-\left(\frac{\Delta t}{\beta}\right)^k}, \tag{8}$$

which is the probability function for the Weibull distribution (Weibull 1951), where $\beta = 1/\lambda$ is the reciprocal of the occurrence rate of the events.

The Weibull distribution is a special case of the four parameter exponentiated Weibull distribution:

$$P(\Delta t) = \alpha \frac{k}{\beta} \left(\frac{\Delta t - \theta}{\beta}\right)^{k-1} \left[1 - e^{-\left(\frac{\Delta t - \theta}{\beta}\right)^k}\right]^{\alpha - 1} e^{-\left(\frac{\Delta t - \theta}{\beta}\right)^k}, \tag{9}$$

where $\theta < 0$ accounts for the realistic possibility that the probability of occurrence of an event is not zero and finite for $\Delta t \to 0$, i.e., it accounts for simultaneous events (a limitation of Weibull's equation for $z(\Delta t)$: $z(0)$ is indeed either zero or infinite in Equation 7): the number of events occurring at the same time is greater and greater with a decreasing value of θ. The additional parameter α ($\alpha = 1$ gives the Weibull distribution) accounts for the possibility that the statistics change during time: for instance, assuming the presence of clusters in the system (that is $k < 1$) for $k\alpha \lessgtr 1$, the level of clustering is not constant and there exists a time period when this is minimum (maximum). It can be easily seen that for large waiting times the exponentiated Weibull function displays a power-law tail, $P(\Delta t) \sim \Delta t^{-\mu}$, where $\mu = k\alpha - 1$.

The observed WTD displayed in the right panel of Figure 1 has been fitted with the exponentiated Weibull function (by applying a Levenberg–Marquardt least-squares minimization method), inferring as best values for the parameters $k = 0.477 \pm 0.006$, $\beta = 0.70 \pm 0.04$, $\theta = -1.31 \pm 0.02$, and $\alpha = 8.8 \pm 0.4$. The probability function $P(\Delta t)$ obtained by substituting this set of parameters in Equation (9) is shown (as a red line in the right panel of Figure 1) to reproduce the observed WTD for CMEs very well. It is worth noting some important results concerning the use of Equation (9) to fit the distribution of the CME waiting times and their physical relevance in light of the statistical properties of the CME time sequence: (1) the corresponding χ^2 is very low, $\chi^2 = 3.4 \times 10^{-5}$, hence the fit is very robust; (2) k is lower than 1, which represents the evidence for the presence of clusters in the CME time sequence; (3) θ is negative, which indicates that there exists a not-null probability that two or more CMEs occur simultaneously, further evidence for the temporal clustering of the CME events; (4) $k\alpha$ is larger than 1, which reveals that the clustering of CME events is not constant during the solar cycle, but there exists a period where the CME eruptions are more clustered; (5) asymptotically, the exponentiated Weibull function reproduces well the power-law tail of the observed WTD. In particular, the estimate of the power-law index inferred in the previous section, $\gamma = 2.97 \pm 0.10$, is in striking agreement with the

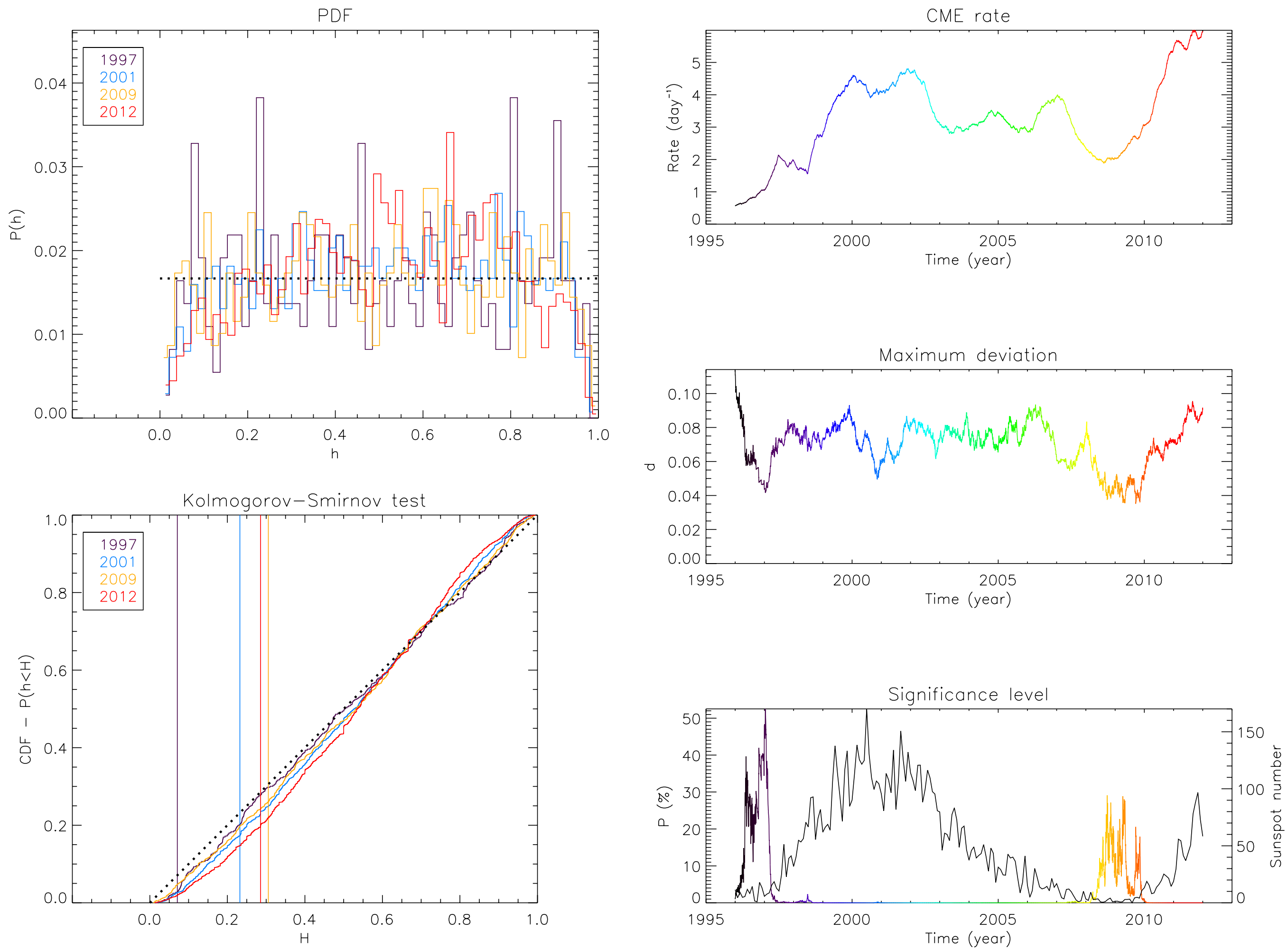

Figure 3. Left: PDFs $P(h)$ (top) and corresponding CDFs $P(h < H)$ (bottom) of the normalized waiting time h between CME events sampled in the years 1997, 2001, 2009, and 2012 (violet, light blue, orange and red curves, respectively); the dotted lines represent the theoretical probability expected under Poisson statistics; the vertical lines indicate where the CDFs mostly deviate from it (the different colors refer to the different years as shown in the legend). Right: occurrence rate λ of CME eruptions (top), maximum deviation D between the empirical and the reference (i.e., Poisson) CDFs (middle), and significance level P of the K–S test compared with the monthly sunspot number (bottom), as functions of the time during the period 1996–2012 encompassed by the *SOHO*/LASCO catalog; colors, in the rainbow scale, are the same as used for the left panels.

(A color version of this figure is available in the online journal.)

exponent scaling predicted by Equation (9) for large waiting times, $\mu = k\alpha - 1 = 3.20 \pm 0.20$: the indices are the same within the statistical uncertainties. Hence, it can be concluded that the Weibull distribution can satisfactorily account for the observed WTD for CMEs, by simply supposing that they are correlated to some degree. That is, the eruptions of CMEs are regulated by a persistent process, namely characterized by some memory, which generates correlations among the CME events.

4. THE NATURE OF THE PHYSICAL PROCESSES UNDERLYING THE CME ORIGIN

The results outlined in the previous sections refer to the *SOHO*/LASCO CME dataset as a whole (encompassing one and a half solar cycles). In order to investigate the variation with the solar cycle of the CME waiting time statistics and to localize when the CMEs are more clustered, thus looking for the periods where the process underlying the CME eruptions is characterized by a higher level of persistency, the K–S test is applied to 5845 year-long-windows running day-by-day along the period 1996–2012. The results are shown in Figure 3.

At a first glance, the behaviors of $P(h)$ and $P(h < H)$ corresponding to the minimum years (1997 and 2009) of solar cycles 23 and 24, to the maximum year (2001) of solar cycle 23, and to the year 2012, that is to the rise to the maximum phase of solar cycle 24 (top and bottom left panels of Figure 3, respectively) seem to be similar to the typical shapes of the probability and cumulative functions obtained by considering the entire *SOHO*/LASCO catalog of CMEs (left and right panels of Figure 2, respectively), thus appearing to deviate from the uniform, say Poisson, distribution (dotted lines). However, by looking at the significance level P of the K–S test as a function of time during the period 1996–2012 (bottom right panel of Figure 3) and by comparing it with the monthly sunspot number compiled by the Solar Influences Data Analysis Center in Belgium (black curve), it is readily seen that the CME temporal distribution clearly departs from the Poisson statistics in the years 1998–2007 and 2010–2012, namely just outside the minimum activity phases of solar cycles 23 and 24, where instead the Poisson hypothesis holds at a high degree of confidence (the maximum distance D between the empirical and the reference distributions is shown as a function of time in the middle right panel of the same figure). That is, the correlations shown in Sections 2 and 3 to exist among the CME events are actually present solely during the periods of higher solar activity: at the minimum of the solar cycle the CMEs are stochastically distributed in time. Furthermore, those correlations are suggested to be so numerous and so strong, with respect to the uncorrelated, i.e., random, waiting times observed during the minimum phases of solar activity, as to be revealed when investigating the global waiting time statistics in the years 1996–2012 (Figures 1 and 2). In this respect, it is fair to note that during the solar minimum, when the distribution of the CME events is consistent with the Poisson statistics, the power-law behavior of the WTD (see Figure 3 in Wheatland 2003) might likely be due to a non-stationary Poisson process. On the contrary, a piecewise-constant Poisson process cannot account for the observed WTD for the CMEs which occurred during the periods of higher activity when the analysis presented here clearly represents observational evidence of the departure of CME statistics from a local Poisson distribution.

The different degrees of departure of the CME WTD from the Poisson distribution point to different physical conditions during the different phases of the solar cycle, that is either to a temporal evolution of the efficiency of the mechanism underlying the origin of CMEs or, more likely, to the existence of two or more physical processes driving their eruptions. In the latter case, it can be suggested that a first stochastic mechanism might act during the entire solar cycle, generating uncorrelated CME events. This might likely be related to global magnetic rearrangements of the solar corona at large scales to account for the CMEs originated at times of solar minimum, when they often form primarily in the coronal streamer belt near the solar magnetic equator. Hence, the absence of correlations in the CME sequence during the intervals of minimum activity might be thus understood. A second persistent mechanism, superposed to the former, might be instead initiated during the periods of higher solar activity driving the formation and eruption of temporally correlated CMEs. In this context, by comparing the top and the bottom panels of Figure 3, it is worth noting that, quite interestingly, the occurrence rate of CMEs is not correlated with the significance level of the K–S test, namely with the degree of correlation of the CME waiting times. Hence, it looks as if the mechanism generating long correlation times among the CME events during solar maximum was related to the increased emergence of magnetic flux within active regions, rather than to their effective number, which is instead strictly associated to the activity phase of the solar cycle and with the frequency of CMEs (Webb & Howard 1994). Since during solar maximum CMEs mostly originate from active regions, these results seem to depict a scenario in which the correlations present among the CME events are related to solar surface perturbations characterizing the activity in the area where the CMEs form, somehow similar to what is observed for the temporal distribution of earthquakes, where major seismic events often mark the beginning of a series of (temporally and spatially correlated) earthquakes (aftershocks), leading to their clustering. As a matter of fact, spatial correlations among CMEs have already been the subject of scrutiny: Moon et al. (2003) have indeed found a sympathetic CME pair, of which the second CME may have been initiated by the eruption of the first one, thus providing clues supporting the existence of interdependent CMEs occurring sequentially in different locations and with a certain physical connection. Hence, the existence of a high degree of correlation at times of higher solar activity can thus be explained, as long as the consequent CME clustering is strong enough so as to not be destroyed by the declustering effect of the random CME events due to the underlying Gaussian process, which is continuously acting during the entire solar cycle.

To summarize, the increase of the level of correlation among CME events during the solar cycle from the minimum to the maximum provides some clues on the physical processes at the basis of the origin of CMEs, indicating that at least two mechanisms, in some sense opposite, should be working at the same time, one related to stochastic large-scale reconfigurations of the coronal magnetic field and the other to the persistent spatiotemporal variability of solar surface activity.

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