

Fermionic matter under the effects of high magnetic fields and its consequences in white dwarfs

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2015 J. Phys.: Conf. Ser. 630 012039

(<http://iopscience.iop.org/1742-6596/630/1/012039>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 80.82.77.83

This content was downloaded on 16/03/2017 at 15:53

Please note that [terms and conditions apply](#).

You may also be interested in:

[The \$A = 8\$ isotriplet in Fermionic Molecular Dynamics](#)

KR Henninger

[8B structure in Fermionic Molecular Dynamics](#)

K R Henninger, T Neff and H Feldmeier

[Quantum Gravity as a Deformed Topological Quantum Field Theory](#)

Aleksandar Mikovi

[Lagrangian formulation for fermionic and supersymmetric non conservative systems](#)

N E Martínez-Pérez and C Ramírez

[Quantum phases of AB2 fermionic chains](#)

L S Murcia-Correa, R Franco and J Silva-Valencia

[Fully degenerate fermionic dark matter in the cluster-center and black hole](#)

Tadashi Nakajima and Masahiro Morikawa

[Search for the Higgs boson in fermionic channels using ATLAS detector](#)

Kazuya Mochizuki and ATLAS collaboration

[On Fermionic Entangled State Representation and Fermionic Entangled Wigner Operator](#)

Fan Hong-Yi and Wang Ji-Suo

[On the generalized Einstein--Cartan action with fermions](#)

Meriem-Hadjer Lagraa and Mohammed Lagraa

Fermionic matter under the effects of high magnetic fields and its consequences in white dwarfs

E. Otoniel ^{a†} M. Malheiro ^a J. G. Coelho ^b

^aDepartment of Physics, Institute Technological of Aeronautics - ITA - DCTA, Vila das Acácias, São José dos Campos, SP, 12228-900, Brazil.

^b Department of Physics and ICRA, University of Rome - Sapienza P.le Aldo Moro 5, Rome, I-00185, Italy

E-mail: [†]otoniel@ita.br

Abstract. We investigate a recently proposed effect of strong magnetic fields in Fermionic matter that is important to the structure of magnetic white dwarfs. This work is highly relevant in view of the recent observations of magnetized white dwarfs ($B \sim 10^{8-9}$ G), and possible candidates for white dwarfs pulsars as an alternative descriptions for SGRs and AXPs. Here, we consider the matter inside white dwarfs composed by ions surrounded by an electron degenerate Fermi gas subject to a strong magnetic field. We investigate the effect of the Landau levels due to the huge magnetic field on the equation of state (EoS). We see that the behaviour of the equation of state as a function of the mass and energy density is much stiffer when only one Landau level is occupied. We also investigate the regime of lower magnetic fields where many Landau levels are occupied.

1. Introduction

The study of white dwarf allows to improve our understanding of nuclear matter under extreme densities and high magnetic fields [1] [2]. The interior of these stars offers an unique point of encounter among astrophysics and atomic physics, since the macroscopic properties of compact stars, such as mass, radius, rotation and thermal evolution depend on the microscopic composition of the stellar matter. This composition can change under the presence of strong magnetic fields, affecting the equation of state, and as a consequence the structure of the star.

Recently, Coelho et al. discussed some basic equilibrium properties of magnetized white dwarfs, in particular the condition for dynamical instability of the star in the presence of an extremely large magnetic field [3]. This analysis was done in the context of the virial theorem extended to include a magnetic term. Following the work of Chandrasekhar & Fermi of 1953, when the star magnetic energy W_B exceeds its gravitational potential energy $|W_G|$ ($W_B > |W_G|$), the system becomes dynamically unstable [4]. In light of this, the new mass limit for very magnetized and spherical white dwarf of $2.58M_{\odot}$, recently calculated by [5], should be considered carefully, since these objects are unstable and unbound. Furthermore, it was showed that the new mass limit was obtained neglecting several macro and micro physical aspects such as gravitational, dynamical stability, breaking of spherical symmetry, general relativity, inverse β decay, and pycnonuclear fusion reactions. These effects are relevant for the self-consistent description of the structure and assessment of stability of these objects. When accounted for,



they lead to the conclusion that the existence of such ultramagnetized white dwarfs in nature is very unlikely due to violation of minimal requirements of stability, and therefore the canonical Chandrasekhar mass limit of white dwarfs has to be still applied.

2. Fermion matter

Our starting point will be the microscopic energy-momentum tensor obtained from the system Lagrangian. The Lagrangian density of a fermionic system in the presence of a magnetic field is given by

$$\mathcal{L} = \bar{\psi}(i \not{D} - m)\psi + \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad (1)$$

where we used the notation usual $\not{a} = \gamma^\mu a_\mu$ and $D_\mu = \frac{1}{2}(\overrightarrow{\partial} - \overleftarrow{\partial}) + \Gamma_\mu + i|q|A_\mu$ with Γ_μ being the spin connection which is zero in flat space, and $F^{\mu\nu}$ is the field strength tensor of the electromagnetic field. We have still the vector potential chosen as

$$A^\mu(x) = \delta_{\mu 2}x_1 B$$

that produces a constant magnetic field in the Z direction.

Solving the Dirac equation from the Lagrangian density we determined the dispersion relation:

$$E_\nu = \sqrt{p_z^2 c^2 + m_e^2 c^4 + 2\nu|q|\hbar B c}. \quad (2)$$

where p_z is the longitudinal momentum, q electric charge, m_e the electron mass, c the speed of light, and ν the Landau level given by

$$\nu = (l + \frac{1}{2} + s)$$

being $s = \pm 1$.

2.1. Zero Temperature

The integration in momentum for an electron gas with charge $|q|$ immersed in a magnetic field, restricted to discrete Landau levels of magnetic field is [6]:

$$\int_{p_z(\nu)} \rightarrow \frac{2eB}{(2\pi\hbar)^2 c} \sum_\nu g_{\nu 0} \int_{-\infty}^{\infty} f(E) dp_z(\nu)$$

where $f(E) = \frac{1}{e^{\beta(E - \epsilon_{F(0)})} + 1}$ is the Fermi-Dirac function, $\beta = 1/K_B T$, and $g_{\nu 0} = (2 - \delta_{\nu 0})$ is the degeneracy in each Landau level. At zero temperature the distribution function is given by a theta function to one-particle

$$f(E) = \Theta(\mu - E)$$

where μ is the chemical potential.

2.2. Basic Equations for Landau Level Systems of Degenerate Electrons

In terms of the chemical potential μ , the maximum p_z is defined

$$p_z = \sqrt{p_F^2 - \frac{2\nu|q|B\hbar}{c}} \quad (3)$$

Rewriting the equation above, introducing of the dimensionless parameter $x_F = p_F/m_e c$ we have still $x_F^2 = \mu^2 - 1$ where $\mu^2 = \epsilon_e/mc^2$ this leads to

$$p_z/m_e c = \chi(\nu) = \sqrt{\mu^2 - 1 - 2\nu\gamma}$$

being $B_c = \frac{|q|\hbar}{m_e^2 c^3} \sim 4.414 \times 10^{13} G$, $\gamma = B/B_c$, and ν_{max} is the maximum number of Landau level given by [7],

$$\nu \leq \nu_{max} = \frac{\mu^2 - 1}{2\gamma} \quad (4)$$

If the lowest Landau level $\nu = 0$ is occupied, $\nu_{max} = 1$. Similarly, for two level system, when the lowest $\nu = 0$, and first, $\nu = 1$, levels are occupied, $\nu_{max} = 2$, and so on [2].

3. Equation of State

3.1. Number Density Equation

The number density is given by

$$n_e = \frac{2\gamma}{(2\pi)^2 \lambda^3} \sum_{\nu} g_{\nu 0} \int d\chi(\nu)$$

where $\lambda = \frac{\hbar}{m_e c}$. Integrating the equation above we have:

$$n_e = \frac{2\gamma}{(2\pi)^2 \lambda^3} \sum_{\nu} g_{\nu 0} \sqrt{\mu^2 - 1 - 2\nu\gamma} \quad (5)$$

3.2. Mass Density

We can relate the matter density with the electron number density n_e from

$$\rho = m_n \frac{A}{Z} n_e$$

where Z is the atomic number, A the mass number, and m_n the neutron mass. The matter density can be rewritten in the following way,

$$\rho = K_1 \sum_{\nu} g_{\nu 0} \sqrt{\mu^2 - 1 - 2\nu\gamma} \quad (6)$$

being $K_1 = \frac{2\gamma m_n \mu_e}{(2\pi)^2 \lambda^3}$, However, we see that K_1 changes with the magnetic field due to the term γ .

3.3. Energy Equation and Longitudinal Pressure

The total energy density to zero temperature is given by

$$\epsilon = \rho c^2 + \epsilon_e$$

where

$$\epsilon_e = K_2 \sum_{\nu} g_{\nu 0} \int E_{\nu} d\chi(\nu)$$

is the electron energy density of the magnetized fermi gas and $k_2 = \frac{\gamma m_e c^2}{(2\pi)^2 \chi^3}$. Then, by integration in the zero temperature limit

$$\epsilon_e = K_2 \sum_{\nu} g_{\nu 0} (1 + 2\nu\gamma) \xi_+ \left[\frac{\chi(x)}{\sqrt{1 + 2\nu\gamma}} \right] \quad (7)$$

As the previous case, we obtain the equation of state for the pressure

$$P_{\parallel} = K_2 \sum_{\nu} g_{\nu 0} (1 + 2\nu\gamma) \xi_- \left[\frac{\chi(x)}{\sqrt{1 + 2\nu\gamma}} \right] \quad (8)$$

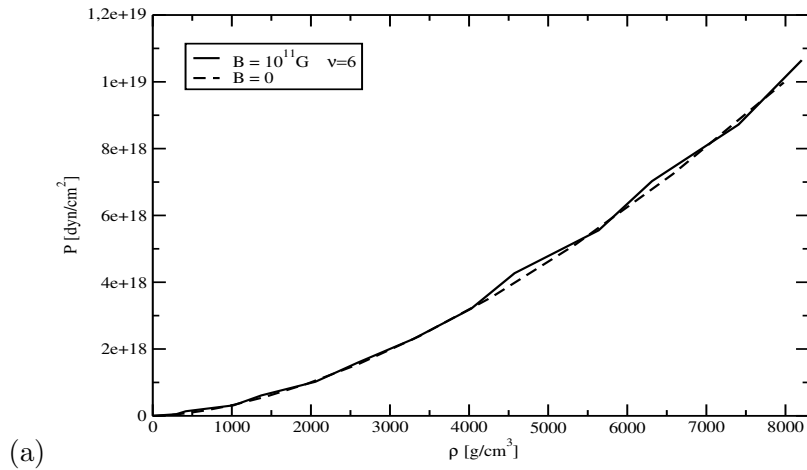
where the function ξ is given by

$$\xi_{\pm} = \mu \sqrt{1 + \mu^2} \pm \ln \left(\mu + \sqrt{1 + \mu^2} \right) \quad (9)$$

4. Results

In this section we present the numerical values for the longitudinal pressure, mass density and the mass-radius relations.

In Figure 1. we see the longitudinal pressure as a function of the density, we observe in graphs (a), (b), and (c) that for magnetic fields up to $B = 10^{13}G$ the equation of state with magnetic field behaves like non-magnetic equation of state, with no significant effect caused by Landau levels.



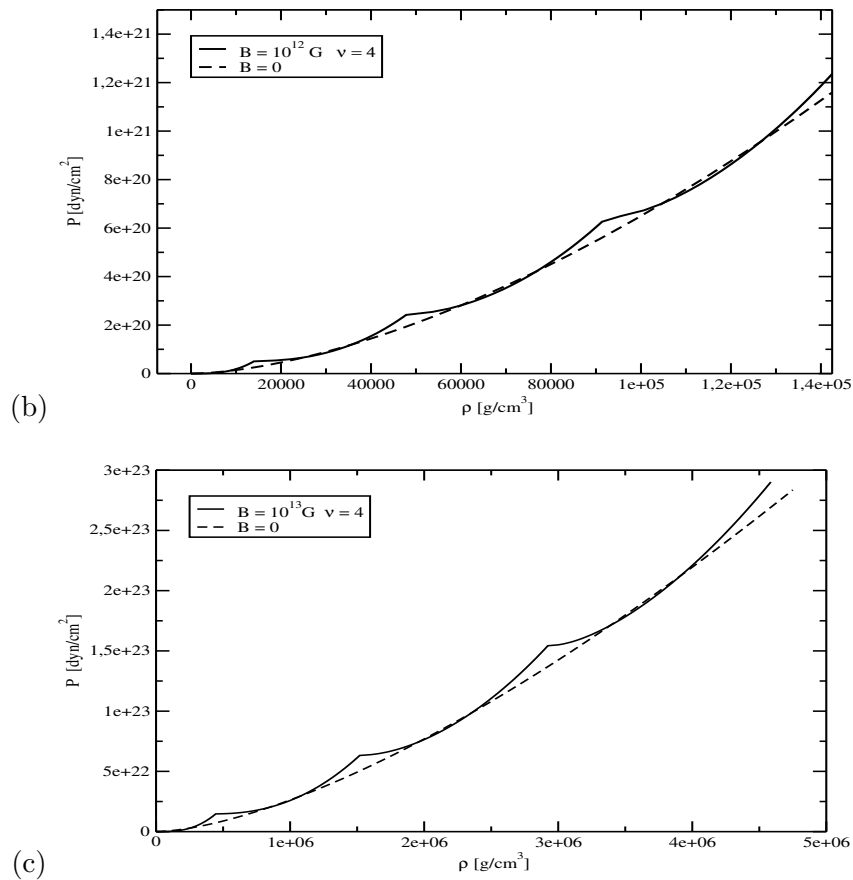


Figure 1. The figures above show the calculation of the longitudinal pressure versus mass density for a gas of strongly magnetized electron. All cases are plotted with solid line for magnetic fields below $B_c = 4.414 \times 10^{13} G$ (a) $B = 10^{11} G$, (b) $B = 10^{12} G$ and (c) $B = 10^{13} G$. The dashed line represent the EOS without magnetic field ($B = 0$).

The next figure shows the effects caused by the Landau levels in the equations of state with magnetic fields up to $8.8 \times 10^{15} G$. We see that for fields of this magnitude the EOS becomes stiffer than when we have only the the lowest state occupied. As we increase the number of occupied levels, as in Figure 2., the equation of state approaches the nonmagnetic case as can be seen in Figure 1..

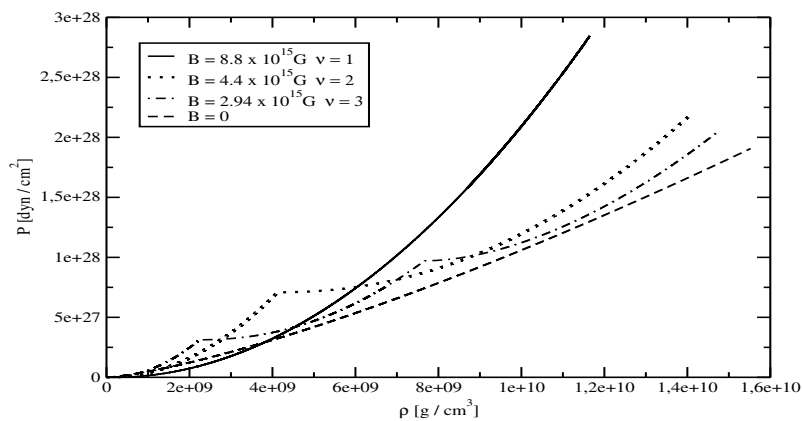


Figure 2. The graph 2 shows the pressure versus mass density for the Fermi energy $E_F = 20m_e c^2$, The black line $B = 8.8 \times 10^{15}$, the dash point line for $B = 4.4 \times 10^{15}$, and the dashed and point line $B = 2.94 \times 10^{15}$. Each of these curves are for one, two, and three Landau levels, respectively. The dashed line is the EOS without magnetic field ($B = 0$).

In the Fig. 3 show the solution of a Tolman-Oppenheimer-Volkoff equation (TOV), In this figure we observe that for magnetic fields of values up to $B_c = 4.414 \times 10^{13} G$ the radius-mass curve behaves similarly to non-magnetic case, but for value $B = 8.8 \times 10^{15}$ the mass becomes independent of the radius reaching values of $M = 2.58M_\odot$.

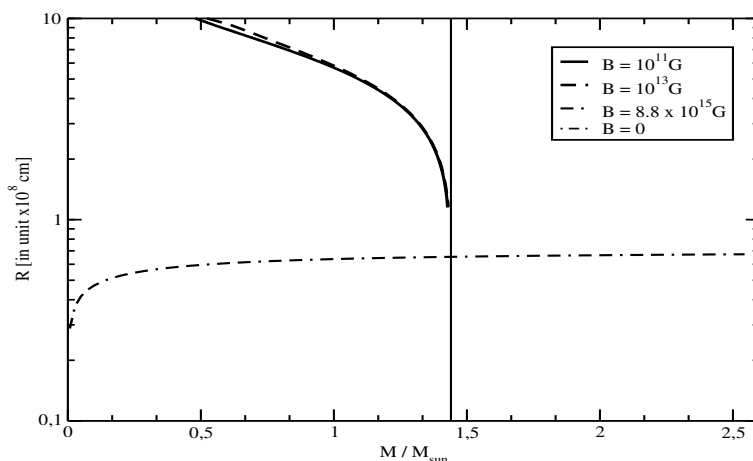


Figure 3. The figure shows the star radius R as a function of the mass with $E_F = 20m_e c^2$ as the maximum Fermi energy. The vertical solid line marks the $1.44M_\odot$ Chandrasekhar limit. The solid line $B = 10^{11}G$, and dashed $B = 10^{13}G$ are obtained with the EOS magnetized. The pointed-dashed $B = 0$ is obtained with the EOS non-magnetized. The dotted-dashed represent the one-level to magnetic field $B = 8.8 \times 10^{15}G$.

5. Conclusions

In this work we solve the equations of state for an degenerate electron Fermi gas under the presence of strong magnetic fields. We investigate the effect of the Landau levels due to a strong magnetic field in the equation of state (EOS) for several values of the magnetic field. We conclude that for strong magnetic fields the separation among Landau levels is large, so electrons with lower energy (non-relativistic) can only occupy the ground state. As the magnetic field decreases, the separation between Landau levels decreases, as shown in Fig. 2. Hence it becomes energetically favorable for electrons to jump to a higher level, so the number of occupied Landau levels increases accordingly. Likewise, in the case of relativistic electrons, if the magnetic field is low, the separation of Landau levels is comparable to the rest energy of the electrons and hence the electrons can pass freely between the highest Landau levels making the EOS similar to nonmagnetic case as we can see in the Fig. 1. This case is also shown in Fig. 1, which for low values of the magnetic field the Landau levels behave almost continuously and it is no longer possible to see the peaks shown in Fig. 2, that represent precisely the critical density for each Landau level.

We also conclude that for magnetic fields below B_c the mass radius relation of the star is similar to the nonmagnetic with $M = 1.44M_\odot$ as the mass limit case. Furthermore, for fields of the order $B \sim 10^{15}G$ the mass exceeds the well-known limit for a white dwarf, may reach

masses $M = 2.58M_{\odot}$. However, as has been discussed in [3], these white dwarfs are unstable and not reliable.

6. Acknowledgments

EO acknowledges the support by CAPES - Brazilian Federal Agency for Support and Evaluation of Graduate Education within the Ministry of Education of Brazil and MM acknowledges the financial support of CNPq and FAPESP (São Paulo state agency, thematic project #2013/26258-4).

References

- [1] Das U and Mukhopadhyay B 2012 *Physical Review D* **86** 042001
- [2] Kundu A and Mukhopadhyay B 2012 18 (*Preprint arXiv:1204.146v1 [astro-ph.SR]* 6 Apr 2012)
- [3] Coelho J G, Marinho R M, Malheiro M, Negreiros R, Cáceres D L, Rueda J A and Ruffini R 2014 *The Astrophysical Journal* **794** 86
- [4] Chandrasekhar S and Fermi E 1953 *Astrophysical Journal* **118** 116
- [5] Das U and Mukhopadhyay B 2013 *Phys. Rev. Lett.* **110** 071102
- [6] Strickland M, Dexheimer V and Menezes D P 2012 *Phys. Rev. D* **86** 125032
- [7] Lai D and Shapiro S L 1991 *The Astrophysical Journal* **383** 745–751