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SUPERDIFFUSIVE SHOCK ACCELERATION

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ABSTRACT

The theory of diffusive shock acceleration is extended to the case of superdiffusive transport, i.e., when the mean square deviation grows proportionally to t^α, with $\alpha > 1$. Superdiffusion can be described by a statistical process called Lévy random walk, in which the propagator is not a Gaussian but it exhibits power-law tails. By using the propagator appropriate for Lévy random walk, it is found that the indices of energy spectra of particles are harder than those obtained where a normal diffusion is envisaged, with the spectral index decreasing with the increase of α. A new scaling for the acceleration time is also found, allowing substantially shorter times than in the case of normal diffusion. Within this framework we can explain a number of observations of flat spectra in various astrophysical and heliospheric contexts, for instance, for the Crab Nebula and the termination shock of the solar wind.

Key words: acceleration of particles – diffusion – shock waves – turbulence

Online-only material: color figure

1. INTRODUCTION

Accelerated particles are routinely observed in several astrophysical contexts, spanning from high-energy particles propagating in the interstellar medium to energetic populations observed in interplanetary space and in the planetary magnetospheres. The processes that lead to the acceleration of particles are not yet fully understood. In the interstellar medium, cosmic rays up to about 10^{15} eV are believed to be accelerated by shock waves at supernova remnants by a Fermi process called diffusive shock acceleration (DSA). For highly relativistic particles, this process is known to give rise to a power-law energy distribution $dN/dE \sim E^{-\gamma}$ with spectral index:

$$\gamma = \frac{r+2}{r-1} \tag{1}$$

(e.g., Hillas 2005), where $r = V_1/V_2$ is the *compression ratio* of the shock, namely the ratio between the unshocked plasma speed V_1 and the shocked plasma speed V_2 measured in the shock frame. Assuming a maximal compression ratio of four, Equation (1) yields the standard cosmic rays spectral index $\gamma = 2$. However, observations show that harder spectral indices can be obtained. For instance, radio observations of the Crab Nebula show that electrons below 10 GeV have a spectral index of $\gamma \sim 1.5$ (Bietenholz et al. 1997; Hester 2008; Sironi & Spitkovsky 2011) and radio hot spots in radio galaxies exhibit $\gamma < 2$ for 1 GeV electrons. In interplanetary space, recent observations of the solar wind termination shock (TS) by the LECP instrument on board *Voyager 2* show that, for non-relativistic ions accelerated at the shock, the measured spectral index is smaller than that corresponding to the observed compression ratio (i.e., $r = 2$), and it is rather consistent with $r = 3$ (Decker et al. 2008). Furthermore, the recent PAMELA experiment indicates that different spectral indices for hydrogen and helium ions in the 30–1000 GV rigidity range are observed (Adriani et al. 2011) at variance with the predictions of DSA, which links the spectral index only to the shock compression ratio (see Equation (1)), thus being the same for many different cosmic ray species. All the above quoted observations challenge DSA as the main acceleration mechanism for energetic particles.

Recently, experimental evidences of superdiffusive transport (i.e., the mean square deviation growing superlinearly in time) for electrons and protons upstream of interplanetary shocks have been found by studying the time profiles of energetic particles in the solar wind (Perri & Zimbardo 2007, 2008, 2009a; Sugiyama & Shiota 2011). During propagation in interplanetary space, particles can be pitch angle scattered when resonating with magnetic field fluctuations. If, at the resonant scales, the amplitude of magnetic field fluctuations is very low, the scattering is reduced and particles can propagate almost scatter-free, thus giving rise to a non-diffusive transport (e.g., Bieber et al. 1994; Zimbardo et al. 2012). In particular, Figure 1 of Sugiyama & Shiota (2011) shows that the magnetic fluctuations upstream of the shock are statistically independent of the shock distance, so that the observed power-law time profile of energetic ions cannot be due to a spatial variation of the pitch angle scattering rate.

In addition, numerical simulations in the presence of magnetic turbulence have shown that particle transport in the direction parallel to the mean magnetic field can be superdiffusive (Zimbardo et al. 2006; Pommois et al. 2007; Shalchi & Kourakis 2007). When the level of fluctuations is particularly low, superdiffusion is also obtained in the perpendicular direction (Pommois et al. 2007) and perpendicular superdiffusion of magnetic field lines is obtained when enough turbulence power is found at larger scales (Shalchi & Weinhorst 2009), or when a nonlinear regime is considered analytically (Ragot 2011). Those findings point out that it is necessary to understand how superdiffusion modifies the scenario of standard DSA. While the case of subdiffusion has been considered by Duffy et al. (1995) and Kirk et al. (1996), here we develop a superdiffusive shock acceleration (SSA) model, and show how this may be a good candidate for explaining a variety of observations.

2. THE SPECTRAL INDEX FROM SUPERDIFFUSIVE SHOCK ACCELERATION

In a Fermi process, a test particle having initial energy E_0 can undergo a number of encounters with both inhomogeneities in the interstellar space and with planar shocks. The number of particles, which have gained an energy equal to or larger than

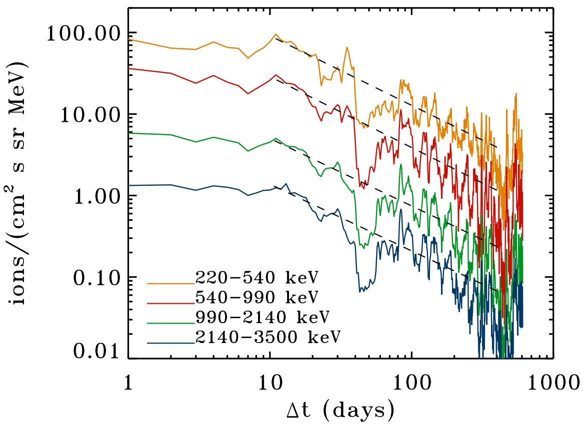

Figure 1. Time power-law profiles of energetic particles accelerated at the TS of the solar wind. The colors indicate four energy channels and the data have been "cleaned" from background contaminations. For clarity, ion fluxes have been displaced vertically. From Zimbardo & Perri (2010) (Copyright: 2010 American Institute of Physics).

(A color version of this figure is available in the online journal.)

E, is given by

$$N(\geqslant E) \propto \left(\frac{E}{E_0}\right)^{-\tilde{\gamma}}. \tag{2}$$

Equation (2) represents the particle integral energy spectrum and $\tilde{\gamma}$ is the corresponding spectral index. Considering the process of the acceleration of relativistic particles at planar shocks, the relative gain of energy for particles is $\zeta = \Delta E/E = (4/3)(V_1 - V_2)/v$. This is related to the slope of the power law in Equation (2) via (e.g., Gaisser 1990)

$$\tilde{\gamma} = \frac{P_{\mathrm{esc}}}{\zeta} = \frac{3}{V_1/V_2 - 1} \equiv \frac{3}{r-1}, \tag{3}$$

where P_{esc} is the escape probability (see below); Equation (3) yields Equation (1) when the differential energy spectrum is considered, that is $\gamma = \tilde{\gamma} + 1$.

Let us consider the standard definition of the probability for a particle to escape from the acceleration region (e.g., Gaisser 1990): this is given by the ratio of the particle flux Φ_2 exiting from downstream over the incident flux Φ_1 coming from upstream, $P_{\mathrm{esc}} = \Phi_2/\Phi_1$ in the shock frame, with the assumption that particles leave that region on the downstream side. The reference frame considered is the same as the one adopted by Perri & Zimbardo (2007, 2008), that is the planar shock is moving from $(x, t) = (-\infty, -\infty)$ according to $x' = V_{\mathrm{sh}}t'$, with the shock's speed V_{sh} fixed to a constant value in the downstream frame, and for $t < 0$ the origin of the reference frame is upstream of the shock. We note that V_{sh} in the downstream frame equals V_2. Anyway, here we consider V_2 (and V_1) to be the magnitude of the plasma speed in the shock frame.

With the assumption of isotropic particle distribution (Fisk & Lee 1980; Drury 1983), the flux of particles crossing the shock from upstream to downstream is $\Phi_1 = n_0 v/4$, where n_0 is the density of particles with velocity v at the shock in the shock frame. On the other hand, with the x-axis pointing upstream, the far downstream density is given by $n(-\infty) \equiv n_2$, thus $\Phi_2 = n_2 V_{\mathrm{sh}}$. To determine the escape probability, we have to compute the values of the particles' densities in those two regions. As suggested in Kirk et al. (1996), in stationary conditions far downstream, the particle density becomes homogeneous and proportional to the flux of particles Φ_0 at the injection, that is, $n_2 = \Phi_0(E)/V_{\mathrm{sh}}$. We can calculate the particle density at any other position x by means of the propagator $P(x, t)$:

$$n(x,t) = \int P(x - x', t - t') S_{\mathrm{sh}}(x', t') dx' dt', \tag{4}$$

with the source

$$S_{\mathrm{sh}}(x', t') = \Phi_0(E)\delta(x' - V_{\mathrm{sh}}t'); \tag{5}$$

above, $\Phi_0(E)$ is the flux of particles of energy E injected at the shock. The propagator in Equation (4) can be computed in the superdiffusive case from the Fourier inversion of (Zumofen & Klafter 1993)

$$P(k,t) \sim \exp -Ct|k|^{\mu-1}, \tag{6}$$

where C is a constant, k is the wavenumber, and μ is the scaling exponent of the particle jumps probability (see below).

Note that in the case of superdiffusion, the mean square deviation grows as

$$\langle \Delta x^2 \rangle \propto D_\alpha t^\alpha, \tag{7}$$

with $1 < \alpha < 2$, and with the anomalous diffusion coefficient D_α having dimensions $[D_\alpha] = l^2/t^\alpha$ (here the square brackets denote the physical dimensions). Superdiffusion is based on non-Gaussian statistics, modifying the probability of interaction between particles and waves, and can be described by means of a Lévy random walk. The latter is a random walk characterized by a space-time coupled power-law distribution of free paths ℓ, i.e., $\psi(\ell, t) \propto |\ell|^{-\mu}\delta(\ell - vt)$ (Klafter et al. 1987), valid for $\ell > \ell_0$; this power-law distribution leads to a non-negligible probability of having very long free paths. The anomalous diffusion exponent α is directly related to μ by $\alpha = 4 - \mu$ (Klafter et al. 1987; Zumofen & Klafter 1993), and superdiffusion is obtained for $2 < \mu < 3$. It can be easily shown that $P(x,t) = \int dk\, e^{ikx} P(k,t)$ has scaling properties such that

$$P(x,t) = \tilde{C}\frac{f(\xi)}{t^{1/(\mu-1)}}, \tag{8}$$

with the scaling variable $\xi = \hat{x}/\hat{t}^{1/(\mu-1)}$, with $2 < \mu < 3$. Here we have introduced the dimensionless quantities $\hat{x} = x/\ell_0$ and $\hat{t} = t/t_0$, with ℓ_0 and t_0 the normalized parameters related by $\ell_0 = vt_0$. The explicit inversion of the Fourier–Laplace transform of $P(x, t)$ is only possible in limiting cases: if $\xi \ll 1$ (close to the source) the transport can be described via the Gaussian-like propagator $P(x,t) = A/t^{1/(\mu-1)} \exp[-x^2/(k_\mu t^{2/(\mu-1)})]$ (Zumofen & Klafter 1993; Perri & Zimbardo 2008), where A is a constant and k_μ is an anomalous diffusion constant (see Zumofen & Klafter 1993); if $\xi \gg 1$ (far away from the source), then the propagator exhibits a power-law form $P(x,t) = (B/t^{1/(\mu-1)})(t^{1/(\mu-1)}/|x|)^\mu$ with the constant B having dimensions $[B] = l^{\mu-1}/t$.

It was shown by Kirk et al. (1996) that the ratio of densities n_2/n_0 depends only on the scaling properties of the propagator. However, the propagator appropriate for a Lévy random walk has scaling properties, expressed by the variable ξ, different from those considered in Kirk et al. (1996) and intimately related to the space-time coupling of the free path distribution $\psi(\ell, t)$ (Zumofen & Klafter 1993). Thus, we compute the particle density via the propagator for superdiffusive transport in Equation (8). From Equations (4) and (5) it can be obtained

$$n(x,t) = \Phi_0(E)\tilde{C}\int \frac{f(x-x', t-t')}{(t-t')^{1/(\mu-1)}}\delta(x'-V_{\rm sh}t')dx'dt'. \quad (9)$$

After exploiting the δ function in Equation (9), we introduce a new time variable $\tau = t - t'$,

$$n(x,t) = \Phi_0(E)\tilde{C}\int_0^\infty \frac{f(x,t,\tau)}{\tau^{1/(\mu-1)}}d\tau, \quad (10)$$

so that when $x = V_{\rm sh}t$ we are moving with the shock and $n(V_{\rm sh}t, t) \equiv n_0$. Using $\Phi_0(E) = n_2 V_{\rm sh}$ and the dimensionless variables $\hat{x} = x/\ell_0 \equiv (V_{\rm sh}\tau)/\ell_0$ and $\hat{\tau} = \tau/t_0$, we express the integration variable in terms of $\xi = \hat{x}/\hat{\tau}^{1/(\mu-1)}$. Thus, the expression in Equation (10) reduces to

$$n_0 = \tilde{C}\frac{\mu-1}{\mu-2}n_2\ell_0 t_0^{1/(1-\mu)}\int_0^\infty f(\xi)d\xi. \quad (11)$$

Note that $[\tilde{C}] = t^{1/(\mu-1)}/l$. Under general conditions, it is possible to set $\int_{-\infty}^{\infty} P(x,t)dx = 1$. Considering the symmetry properties of $P(x,t)$, we have $\int_{-\infty}^{\infty} P(x,t)dx \equiv 2\int_0^\infty P(x,t)dx$. Therefore, using the propagator in Equation (8) and exploiting all the quantities in term of the scaling variable ξ, we can easily obtain

$$\int_{-\infty}^{\infty} P(x,t)dx \sim 2\tilde{C}\ell_0 t_0^{1/(1-\mu)}\int_0^\infty f(\xi)d\xi = 1, \quad (12)$$

with $2 < \mu < 3$. From Equation (12), the constant $\tilde{C}$ can be derived, and inserting it in Equation (11) we have

$$\frac{n_2}{n_0} = 2\frac{\mu-2}{\mu-1}, \quad (13)$$

which links the density at the shock to its asymptotic value far downstream. If the Gaussian propagator of normal diffusion would be used, then $n_2 = n_0$ would be obtained. The above derivation allows us to compute the probability $P_{\rm esc} = \Phi_2/\Phi_1$ (Drury 1983; Gaisser 1990),

$$P_{\rm esc} = \frac{n_2 V_2}{n_0 v/4} = 8\ \frac{\mu-2}{\mu-1}\frac{V_2}{v}, \quad (14)$$

where v is the particle's speed. From Equation (3), considering a relativistic process, we can now obtain the exponent of the integral energy spectrum,

$$\tilde{\gamma} = \frac{P_{\rm esc}}{\zeta} = 8\ \frac{\mu-2}{\mu-1}\frac{V_2}{v}\frac{3v}{4(V_1-V_2)} = 6\ \frac{\mu-2}{\mu-1}\frac{1}{r-1}. \quad (15)$$

In the limit of diffusive motion $\alpha = 4 - \mu \rightarrow 1$, that is for $\mu = 3$, Equation (15) gives the classical result for DSA, i.e., $\tilde{\gamma} = 3/(r-1)$.

Let us now consider non-relativistic particles: in this case, the factor ζ is related to the relative gain of the particle's impulse, namely $\zeta = \Delta p/p$, and this leads to the corresponding spectral index $\tilde{\gamma_p} = 6[(\mu-2)/(\mu-1)](r-1)^{-1}$. Furthermore, we have $(dN/dE)dE = (dN/dp)dp$, being $dN/dE \sim E^{-\gamma}$; thus, exploiting all the relations in terms of the momentum p, we obtain

$$dN/dp \sim p^{-(2\gamma-1)} \equiv p^{-\gamma_p}. \quad (16)$$

Integrating Equation (16), the integral momentum spectrum is derived, i.e., $N(\geqslant p) \sim p^{-2(\gamma-1)} \equiv p^{-\tilde{\gamma_p}}$. Since $\tilde{\gamma_p} = 2(\gamma-1)$, the exponent of the differential energy spectrum is easily obtained:

$$\gamma = \frac{\tilde{\gamma_p}}{2} + 1 = \frac{1}{2}\left[2\ \frac{\mu-2}{\mu-1}\frac{3}{r-1}\right]+1. \quad (17)$$

Consequently, $\tilde{\gamma} = \gamma - 1 = (1/2)[2(\mu-2)/(\mu-1)][3/(r-1)]$. We note that the departure from the energy spectral index of the standard DSA is given by $2(\mu-2)/(\mu-1)$, both in the relativistic (see Equation (15)) and in the non-relativistic (see Equation (17)) cases. This difference is entirely due to the ratio between the density far downstream and the density at the shock, as obtained from the form of the propagator. For $\mu < 3$ (superdiffusion), the spectral index is harder than the standard result ($\mu = 3$). In the limit of nearly ballistic transport, $\mu \rightarrow 2$, the differential energy spectrum $\gamma \rightarrow 1$ is in both the relativistic and the non-relativistic case; therefore, SSA allows us to explain a wide range of observations of flat spectra.

The harder spectral index is due to the fact that superdiffusing particles can move more efficiently against the average downstream flow with velocity V_2. In other words, downstream of the shock the particle motion will be composed of the advection and of the random superdiffusive motions: for long periods of time the advective motion prevails ($\Delta x \sim V_2 t$), but superdiffusive particles propagate like $\langle \Delta x^2\rangle \sim t^{4-\mu}$. For an isotropic velocity distribution, in the downstream frame there are as many particles going upwind as going downwind. The latter are lost in both the diffusive and in the superdiffusive case. Particles going upwind in the superdiffusive case, that is for $\mu < 3$, have more chances to propagate upwind and to cross the shock to be accelerated compared to diffusing particles. For upwind particles, the larger return probability is related to the power-law tails of the propagator given above. Therefore, a larger return probability means a smaller escape probability, $P_{\rm esc}$, and a harder spectral index (see Equation (3)). For the same reason, shorter acceleration times can be obtained for SSA, as shown in the following.

2.1. *Comparison with Spectral Indices from Observations*

For the Crab Nebula, as for other pulsar wind nebulae (Gaensler & Slane 2006; Hester 2008), a flat radio spectrum is observed (Bietenholz et al. 1997), namely an electron differential energy spectrum with $\gamma \sim 1.5$ below GeV energies (Stephens & Streitmatter 2003). The spectrum has been observed to become steeper toward $\gamma \sim 2.2$ at higher energies. Considering a compression ratio of $r = 4$, we obtain from Equation (15), $\gamma = \tilde{\gamma} + 1 = 2(\mu-2)/(\mu-1) + 1$. Equating the latter to the spectral index 1.5, this yields $\mu = 7/3$ and $\alpha = 4 - \mu = 5/3$. This means that the flat electron spectrum can be explained by SSA for electrons. The value $\alpha \sim 1.67$ is consistent with those reported for electrons in the heliosphere (Perri & Zimbardo 2007, 2008, 2009a, 2009b). The evidence that the spectrum gets steeper at higher energies may be explained with a faster pitch angle scattering of particles having larger Larmor radii. Indeed, they can resonate with magnetic field fluctuations (which usually exhibit a power-law, $k^{-5/3}$, spectrum) that have larger amplitude, thus leading to normal diffusion and to the usual spectral index $\gamma \geqslant 2$.

For the TS particles (Stone et al. 2005, 2008) observed by *Voyager 2*, Perri & Zimbardo (2009a) have found superdiffusion with $\alpha \sim 1.3$. The typical value of the exponent of the power law reproducing the proton's profiles, i.e., $J = K\Delta t^{-a}$, is

$a \sim 0.70 \pm 0.07$ (see Figure 1). Since particle profiles, as computed from Equation (4) with the propagator for superdiffusion (Perri & Zimbardo 2007, 2008), are power-law decays with a typical exponent of $a = \mu - 2$, we have $\mu \sim 2.70 \pm 0.07$. From Equation (17) for non-relativistic particles, and imposing the observed compression factor $r = 2$ (Decker et al. 2008), an exponent for the energy spectrum of $\gamma \sim 2.2 \pm 0.2$ is obtained. This value has to be compared with the spectral index of the particle differential intensity, j, which exhibits a power-law trend $j \sim E^{-\gamma_j}$ (Decker et al. 2005; Fisk et al. 2006). Since, from dimensional considerations $j \sim E^{-\gamma+(1/2)}$, the relation $\gamma_j = \gamma - 1/2$ holds; therefore, for $\gamma \sim 2.2$ a $\gamma_j \sim 1.7$ is obtained. We note that such a value is smaller than the one expected for normal diffusion, namely $\gamma_j \sim 2$. The γ_j found is closer to the $\gamma_j \sim 1.6$ reported by *Voyager 1* (Decker et al. 2005) than to the 1.25 value obtained from the *Voyager 2* observations (Decker et al. 2008). Indeed, the latter is substantially smaller than 1.7; this could be due to the fact that the theory of superdiffusive acceleration assumes isotropy for particles. In reality this is not the case; in fact, spacecraft data show a very strong anisotropy of particles along the direction of the magnetic field. This means that the superdiffusive transport of particles yields spectral indices closer to the observations, although the inclusion of additional effects, for example, the adiabatic cooling and the strong anisotropy of particles, is required for building up a complete description of the phenomenon.

3. ESTIMATION OF THE ACCELERATION TIME

The acceleration time can be obtained from the time $t_{\rm cycle}$ needed for particles to complete a cycle from upstream to downstream and then return upstream. For normal diffusion, this time can be estimated as $t_{\rm cycle} = (4/v)[(D_1/V_1)+(D_2/V_2)]$ (Drury 1983; Gaisser 1990), where D_1 and D_2 are the upstream and the downstream diffusion coefficients, respectively. We point out that in the case of superdiffusion, D_1 and D_2 are diverging (e.g., Zimbardo et al. 2012). However, the time for cycling can be estimated as follows: equate the distance due to advection $\Delta x = V_2 \Delta t$ to that covered by superdiffusive motion, $\langle \Delta x^2 \rangle \sim D_\alpha \Delta t^\alpha$. Within this distance, downstream particles have a reasonable probability of returning upstream (Duffy et al. 1995). By eliminating time, we find

$$\Delta x = \frac{(D_\alpha)^{1/(2-\alpha)}}{V_2^{\alpha/(2-\alpha)}}. \tag{18}$$

By dividing the number of particles in a sheath of thickness Δx, i.e., $n\Delta x$, by the flux of particles $\Phi = nv/4$ and considering a similar situation upstream, we find

$$t_{\rm cycle} = \frac{4}{v}\left[\left(\frac{D_{\alpha 1}}{V_1^\alpha}\right)^{1/(2-\alpha)} + \left(\frac{D_{\alpha 2}}{V_2^\alpha}\right)^{1/(2-\alpha)}\right]. \tag{19}$$

The corresponding acceleration time is $t_{\rm acc} = t_{\rm cycle}/\zeta$ (Drury 1983; Gaisser 1990); therefore, we obtain an anomalous scaling of the acceleration time on the anomalous diffusion constant and on the plasma velocity. We can compare the acceleration time with that obtained from the DSA, considering the standard estimate for the diffusion coefficient $D \sim \lambda v$, where λ is the diffusive mean free path. This yields

$$t^D_{\rm acc} = \frac{3}{V_1 - V_2}\left(\lambda_1 \frac{v}{V_1} + \lambda_2 \frac{v}{V_2}\right). \tag{20}$$

The corresponding expression for SSA is obtained by estimating the anomalous diffusion coefficient as $D_\alpha \sim \ell_0^2/t_0^\alpha = \ell_0^{2-\alpha} v^\alpha$, where ℓ_0 is a scale parameter (not to be identified with λ, since the latter is diverging in the case of superdiffusion). Hence, the superdiffusive acceleration time is

$$t^S_{\rm acc} = \frac{3}{V_1 - V_2}\left[\ell_{01}\left(\frac{v}{V_1}\right)^{\alpha/(2-\alpha)} + \ell_{02}\left(\frac{v}{V_2}\right)^{\alpha/(2-\alpha)}\right]. \tag{21}$$

Let us now estimate the length ℓ_0 present in Equation (21) for the TS case, assuming that $\ell_{01} = \ell_{02} = \ell_0$. This can be obtained from the observed position x of the knee in the particle profile; that is, when the shape of the profile at a given distance from the shock's position, becomes a power law (Perri & Zimbardo 2007, 2008, 2009a; see Figure 1, from Zimbardo & Perri 2010). The change in the shape of the particle profiles happens roughly 10 days from the shock front and the power-law decay, observed at times $t > 10$ days, and persists for about one year; the latter can be considered an asymptotic time with respect to the rapid change in the particle profile close to the shock event. We can compute that length from the condition that the knee is located at $\xi = 1$: theoretically, this indicates that the shape of the propagator changes from $\xi < 1$ to $\xi > 1$. Indeed, the power-law part of the profiles in Figure 1 is obtained for $\xi > 1$. Considering that the spacecraft motion is along the x direction while particles move along the magnetic field, we have $v_x = v\cos\theta$ with θ being the magnetic field-shock normal angle. The condition $\xi = 1$ leads to

$$\ell_{0x} = \frac{x^{(\mu-1)/(\mu-2)}}{(v\cos\theta\ t)^{1/(\mu-2)}}, \tag{22}$$

where ℓ_{0x} is the projection of ℓ_0 along the x direction and v is the particle's speed along the magnetic field direction. The latter can be evaluated, in the case of the non-relativistic TS particles, from the value of the energy channel considered. For 2×10^3 keV protons, a value of $v = 2 \times 10^4$ km s^{-1} is obtained. We further consider that the change in the particle time profile is observed roughly 10 days upstream of the shock position (see Figure 1) and that the relation $x' = V'_{\rm sh} t'$ in the spacecraft reference frame holds (Perri & Zimbardo 2007, 2009a). The velocity of the shock that is moving inward toward the satellite in the reference frame of the spacecraft is given by $|V'_{\rm sh}| = V_{\rm sh} + V_{\rm sc} \sim 27$ km s^{-1}, where $V_{\rm sh} \sim 12$ km s^{-1} is the TS's speed with respect to the Sun and $V_{\rm sc} \sim 15$ km s^{-1} is the spacecraft's speed (Wang & Belcher 1999; Washimi et al. 2007). This implies that the knee in the particle profile happens at a distance x from the shock of 0.15 AU. For angle θ, we note that Richardson et al. (2008) report $\theta \sim 74^\circ$–82°, depending on the specific TS crossing. As a typical value, we assume $\theta = 80^\circ$ so that the computation of ℓ_{0x} from Equation (22), assuming $\mu \sim 2.7$, gives $\ell_{0x} \sim 2.6 \times 10^4$ km and $\ell_0 \sim 1.5 \times 10^5$ km. Because in the case of diffusive transport, the value of the mean free path at the TS has been suggested to be ~ 30 AU (Giacalone 2011), the very large ratio $\lambda/\ell_0 \sim 3 \times 10^4$ implies that the comparison between Equations (20) and (21) yields $t^D_{\rm acc} \gg t^S_{\rm acc}$, even if $(v_x/V_1)^{\alpha/(2-\alpha)} \sim (v_x/V_2)^{\alpha/(2-\alpha)} \sim 10^2$, where $\alpha \sim 1.3$, is larger than one. This finding shows that superdiffusive particles can reach a given energy in a shorter time compared to DSA, i.e., energies larger than 10^4 GeV can be obtained for supernova shocks within their lifetime (Lagage & Cesarsky 1983; Gaisser 1990; Ptuskin et al. 2010).

4. CONCLUSIONS

In this paper, we have computed the expression for the energy spectral index of particles accelerated at shock fronts in the framework of superdiffusive propagation. The values of the spectral indices found are smaller than those predicted by DSA and allow us to explain the observed spectral indices of $E < 10$ GeV electrons in the Crab Nebula and of ions accelerated at the TS. In addition, the expressions for the spectral index γ, shown in Equations (15) and (17), depend on the regime of particle transport via μ, at variance with the result coming from DSA, which links γ only to the compression ratio. Therefore, this model allows us to have different spectral indices for different ion species, which may exhibit different transport properties. Also, shorter acceleration times can be obtained with SSA, thus allowing us to solve the problem of the maximum energy of supernova accelerated cosmic rays.

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