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AGE OF COSMIC-RAY PROTONS COMPUTED USING SIMPLE CONFIGURATIONS OF THE GALACTIC MAGNETIC FIELD

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ABSTRACT

The influence of various magnetic field structures of the Galactic disk on some fundamental properties of cosmic-ray protons has been investigated with a simulation program, which generates cosmic-ray trajectories in the disk volume. The regular component of the Galactic magnetic field has been approximated by three different geometrical configurations (circular, elliptical, and spiral). Ages, residence times, and grammages of cosmic protons in various conditions are calculated and discussed. These calculations indicate that the proton age is strongly influenced by the magnetic field configuration but weakly affected by the field strength. The age of cosmic-ray protons in the spiral field turns out to be 6.7×10^6 yr, and the corresponding matter thickness 12 g cm^{-2}.

Subject headings: acceleration of particles — cosmic rays — ISM: magnetic fields

1. INTRODUCTION

The simplicity and the low number of assumptions in the numerous variants of the leaky-box models are probably the major motivation for their widespread use in the interpretation of experimental data of Galactic cosmic rays. More elaborate computational schemes such as the steady state diffusion models (Ginzburg & Ptuskin 1976) or the convective-diffusion models (Jokipii 1976; Jones 1979; Kota & Owens 1980), while removing some severe restrictions in the leaky-box models, still retain an excessive simplification of the many processes induced by cosmic rays in the Galaxy. In spite of the fair agreement between experimental and predicted results, some important properties of Galactic cosmic rays cannot be described in these computational schemes.

The direct simulation of cosmic-ray trajectories in the Galactic disk is a powerful method for evaluating the properties of cosmic rays and is a complementary tool to assess the basic results of the models mentioned above. Numerical simulation of a cosmic-ray trajectory for solving specific problems was made at very high energies (see, e.g., Berezinskii & Miklailov 1987).

The simulation code CORSA (cosmic-ray simulation algorithms) developed in the past years (Codino et al. 1995) calculates the trajectories of cosmic rays in the Galaxy. Preliminary results regarding the effect of the regular component of the Galactic magnetic field on the age of cosmic-ray protons in the disk have been described elsewhere (Brunetti & Codino 1997a).

The purpose of this study is to show the influence of the regular component of the Galactic magnetic field on the proton age. Proton ages, residence times, and grammages are explicitly given with the complexity of the simulation code CORSA maintained at a modest level. A higher complexity of the code implying the removal of some approximations in the cosmic-ray description is purposely avoided to facilitate the comprehension of how the major results of this study depend on a small set of critical parameters.

The reduced complexity of the code is obtained by the following approximations or assumptions:

1. Only protons have been considered in the present study in the energy range 1–100 GeV.
2. Interstellar gas consists of pure hydrogen.
3. Extragalactic cosmic rays, if any, have been neglected.
4. The Galactic disk rotation is ignored.
5. The convective transport of cosmic rays in the disk is absent.
6. Any effect of the Galactic wind in the disk volume is neglected.

Some of these approximations are quite common in many studies of Galactic cosmic rays.

The structure of the paper is as follows. The major parameters dominating the propagation of cosmic-ray protons in the Galactic disk are introduced in § 2. Plausible intervals for these parameters derived from observational data are extensively discussed in a forthcoming paper. The multiplicity and energy spectra of secondary protons generated by elastic and inelastic proton-proton collisions in the interstellar space are given in § 3.

Proton trajectories in the Galaxy have been subdivided into three populations, depending on the mode in which they disappear from the disk volume (nuclear death, extinction by ionization energy losses, disk escape). This classification of the proton populations, though rather arbitrary, facilitates the study of how the Galactic magnetic field and the primary proton energy affect the age, the residence time and other properties of cosmic rays.

In § 4 the relative fractions of the proton populations as a function of the energy have been studied in the three magnetic field structures. In § 5 the residence time of cosmic-ray protons in the whole Galactic disk is calculated.

All instruments performing cosmic-ray observations are positioned close to the Earth; the particular spherical volume of the Galactic disk concentric with the Earth is hereafter referred to as "local Galactic zone" or "local Galactic region". The space distribution of cosmic-ray sources feeding the local zone is studied in § 6.

In § 7 the age and grammage of cosmic-ray protons reaching the local Galactic region is calculated.

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Some important implications of the results of this study are given in § 8 and the conclusions in § 9.

2. CHARACTERISTICS OF THE GALACTIC DISK

The geometrical boundaries of the Galaxy, the interstellar gas density, and the magnetic field structure and strength are represented in the simulation program CORSA by a set of parameters. A brief summary on the values of these simulation parameters is given here. The geometrical boundaries of the Galactic disk and its form are displayed in Figure 1. The bulge is a symmetric ellipsoid with the major axis in the Galactic midplane 4 kpc long and a minor axis of 3 kpc. Thus, the radius of the bulge on the Galactic plane, ρ, is 2 kpc. Around the bulge there is a thin cylinder with a radius, R, of 15 kpc and half-height, d, of 250 pc. Cylindrical coordinates r, l, and z are used in the disk, where r is the Galactocentric radius, l is Galactic longitude, and z is the height from the midplane of the Galaxy.

The interstellar gas density is taken as pure hydrogen uniformly distributed in the Galactic disk. The mean hydrogen density is 1 atom cm^{-3} (Gaisser 1990).

The Galactic magnetic field in the disk is decomposed in a regular component and a chaotic component. The magnetic field strength of the regular component is 3.0 μG (Manchester 1974; Thomson & Nelson 1980). The geometrical shapes of the magnetic field lines have been chosen according to observational data, which favor spiral field structures. Three different patterns of magnetic field lines have been utilized in the calculations, denoted as spiral, elliptical, and circular fields. The spiral field (shown in Fig. 2*a*) is represented by the following expression:

$$r = R\left(\frac{R}{\rho}\right)^{t/3} e^{\alpha\theta} , \qquad (1)$$

where $\alpha = (2\pi/3)\ \ln\ (\rho/R) = -4.22$ and t is an appropriate numerical parameter. The elliptical field (shown in Fig. 2*b*)

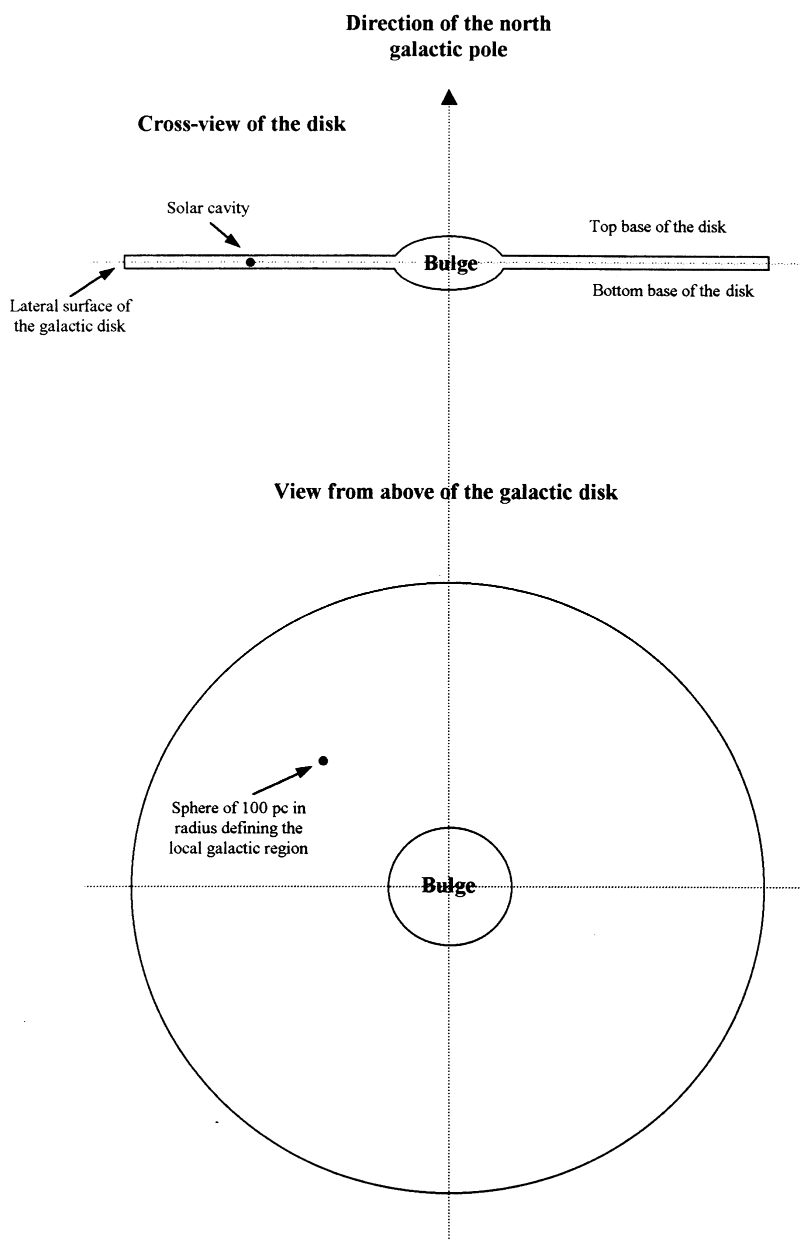

FIG. 1.—Dimensions of the Galactic disk incorporated in the simulation program CORSA. The bulge is reduced to a symmetric ellipsoid, and the disk is a cylinder with a half-height 250 pc and a radius 15 kpc.

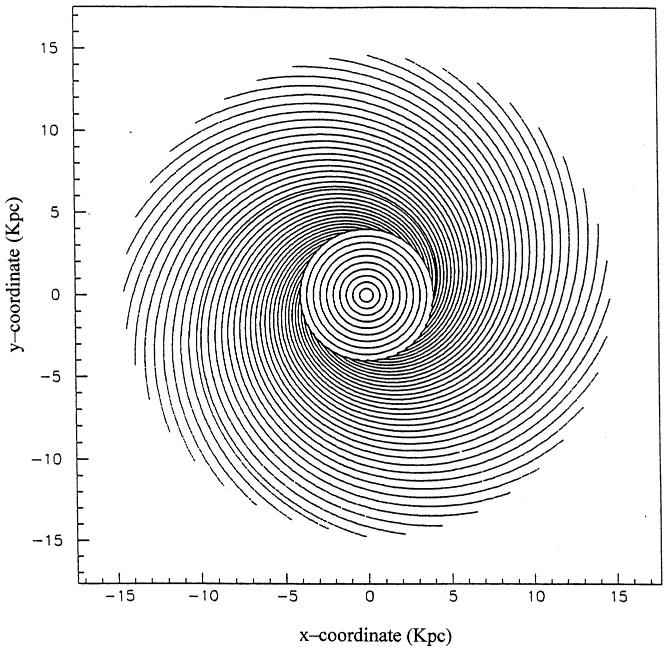

FIG. 2*a*

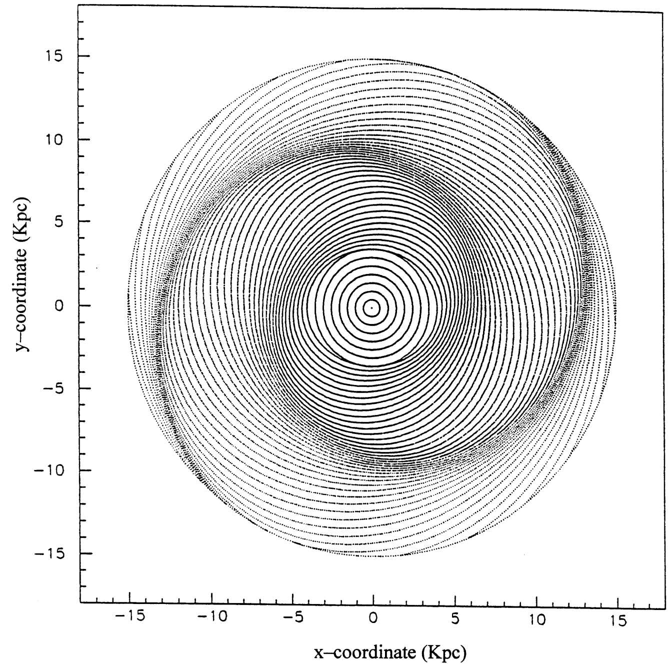

FIG. 2*b*

FIG. 2.—Patterns of the field lines of the regular component of the Galactic magnetic field for the spiral (2*a*) and elliptical configuration (2*b*)

is represented by the following expression

$$\frac{(x \cos \theta + y \sin \theta)^2}{a^2} + \frac{(y \cos \theta - y \sin \theta)^2}{b^2} = \left(\frac{\theta}{\pi}\right)^2 , \quad (2)$$

where $a = 4$ kpc, $b = 3.5$ kpc if $\theta = \pi/2$. The range of values for θ is $\pi/2 \leq \theta \leq (15/7)\pi$. The elliptic configuration is only used as computational test field since observational evidence on the large scale field of the Milky Way favors the spiral field (Sofue & Fujimoto 1982). Note, however, that the circular field is not conclusively excluded from the experimental data (Vallée 1983).

The magnetic field structure in the galactic halo is not used in this calculation and accordingly is not described here. The magnetic field in the halo is not a critical parameter of this calculation because, on average, the fraction of cosmic protons generated in the disk, propagated into the halo and reflected back to the disk is less than 0.5×10^{-3}.

The chaotic component consists of magnetic cloudlets with variable space dimensions and random field orientation. The probability of finding a magnetic cloudlet is governed by an exponential distribution with an average length of 125 pc (Osborne et al. 1973; Chi & Wolfendale 1990). The magnetic cloudlets are spheres with a mean radius of 2.5 pc that is sampled according to a normal distribution with a standard deviation of 1 pc (Bell et al. 1974). The magnetic field strength of the chaotic component has a mean value of 10 μG (Heiles 1976; Freier et al. 1977; Rand & Kulkarni 1989).

Various source distributions of cosmic-ray protons, $Q(r, l, z)$, have been used in this study.

A uniform distribution of sources is represented by the expression

$$Q(r, z, l) = C\Theta(r - R)N(\sigma, z) , \quad (3)$$

where C is an appropriate constant, $\Theta(r - R)$ is the radial distribution with a maximum radius $R = 15$ kpc, and $N(\sigma, z)$ is a normal distribution in the z direction with a standard deviation, σ, of 80 pc.

The supernova remnant distribution (Stecker & Jones 1977) is represented by the same expression 3 except that the radial distribution $\Theta(r - R)$ is replaced by the function

$$q(r) = (r/r_0)^A \exp\left[-B(r/r_0)\right] , \quad (4)$$

with $A = 1.20$, $B = 3.22$ and $r_0 = 10$ kpc.

A third distribution of cosmic-ray sources is represented by the expression

$$Q(r, l, z) = C\Theta(r - R)\Theta(z - z_0) , \quad (5)$$

which differs from the distribution in equation (3) by the $\Theta(z - z_0)$ term with $z_0 = 10$ pc. In this case the sources are uniformly concentrated in a thin disk quite close to the symmetry plane of the Galaxy The distribution in equation (5) is referred to as a "thin-source distribution" and is only used as a test distribution.

However, the results reported in this study do not depend on the specific form of the source distributions, as will be explained in § 6.

The simulation of proton trajectories is based on the following algorithm. A proton with initial energy E and space coordinates r, z, and l is located in the disk volume. Using the proton-proton cross sections, an interaction path length, s, is sampled from an exponential distribution. The trajectory of length s is a helix along the appropriate magnetic field line (regular component). The helix may terminate either in the disk volume or in the halo. An example of a cosmic-ray trajectory is shown in Figure 3.

The propagation of cosmic rays in a direction normal to the magnetic field lines takes place because of the presence of the chaotic component of the magnetic field. Should the chaotic component be removed from the simulation algorithms, cosmic rays will never escape from the disk for the circular field, and they are ultimately destroyed by nuclear interactions with the interstellar hydrogen. In this ideal cir-

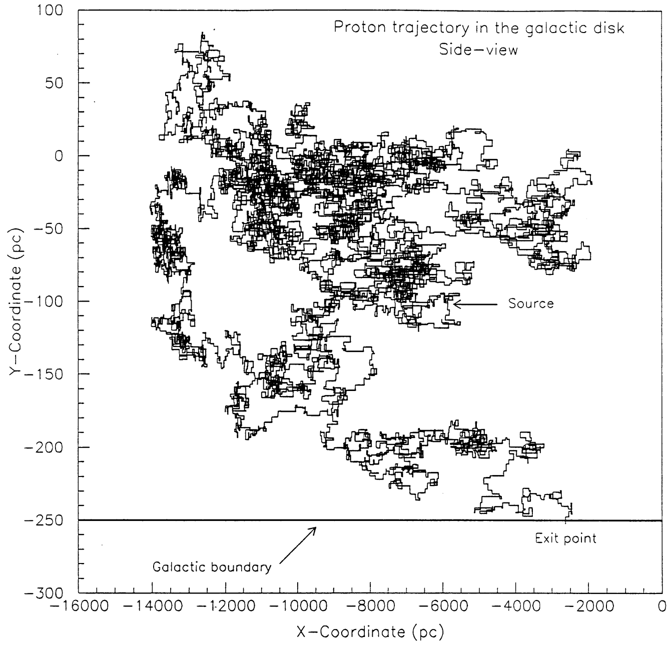

FIG. 3.—Trajectory of a cosmic-ray proton of 10 GeV generated in the Galactic disk and escaping toward the southern hemisphere. The source and exit point are indicated. The helix of the cosmic proton is represented by a series of straight-line segments coincident with the helix axis.

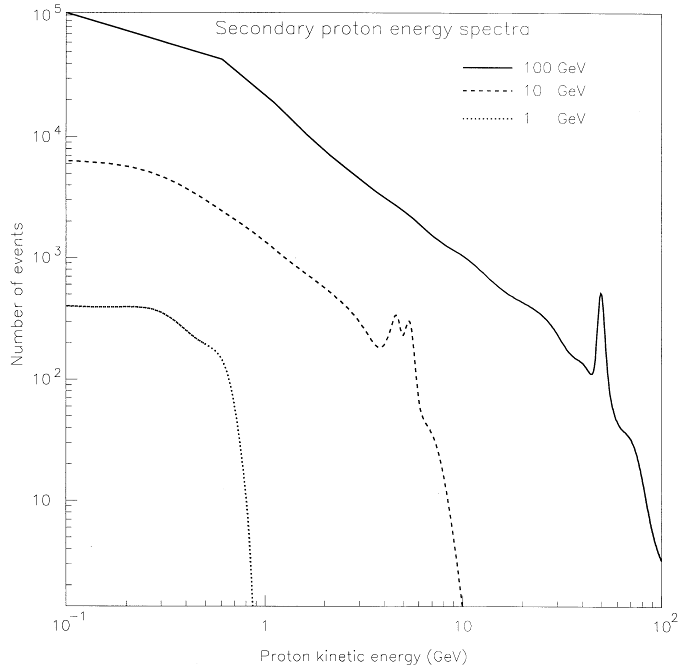

FIG. 4.—Distribution of secondary protons generated by collisions of primary protons of kinetic energy 1, 10, and 100 GeV/c with the interstellar hydrogen in a spiral magnetic field.

cumstance, the spiral field, unlike the circular field, would allow the leakage of cosmic rays across the lateral surface of the disk. The effect of the chaotic component on some properties of Galactic cosmic rays is reported elsewhere (Codino 1998).

The very simple parameterizations of the Galactic volume, interstellar matter density, and magnetic field shapes and field strength are appropriate for this calculation because the sources of cosmic rays feeding the local Galactic zone occupy a small fraction of the Galactic volume. They are mainly disseminated along the magnetic field lines (regular component) leading to the local zone (coronae). This is shown in § 6 and in another paper (Codino & Vocca 1999).

3. SECONDARY PROTONS GENERATED IN THE GALACTIC DISK

Some results regarding the production of secondary protons are anticipated in this section, since the age and grammage of cosmic-ray protons populating the Galactic disk are significantly affected by the presence of these slow secondaries. In a simulated event, a primary proton of 1.4 GeV (kinetic energy) collides with an interstellar proton, generating two secondary protons of 0.58 and 0.75 GeV. These two secondaries, still being in the disk volume, are processed by the simulation algorithms of CORSA and produce two other secondaries of 0.15 and 0.50 GeV. The resulting collision chain has generated four protons. This event represents a typical example for this energy. In Figure 4 are shown the energy spectra of secondary protons produced in the Galactic disk by primary protons of kinetic energy 1, 10, and 100 GeV. The two nearly symmetric peaks close to 5 GeV for the dashed curve (primary proton of 10 GeV) are due to kinematical properties of two-body scattering. The two same peaks for primaries of 1 GeV are hardly visible because at these low energies, the effect of the ionization energy losses is important and the peaks are smeared out. Note that the vast majority of proton secondaries populates low-energy intervals.

In Figure 5 is shown the mean number of secondary protons produced in the Galactic disk as a function of the energy for the three magnetic field structures. The average number of secondaries in the spiral field is less than that in the circular (or elliptical) field because a higher fraction of protons escapes from the disk in this configuration.

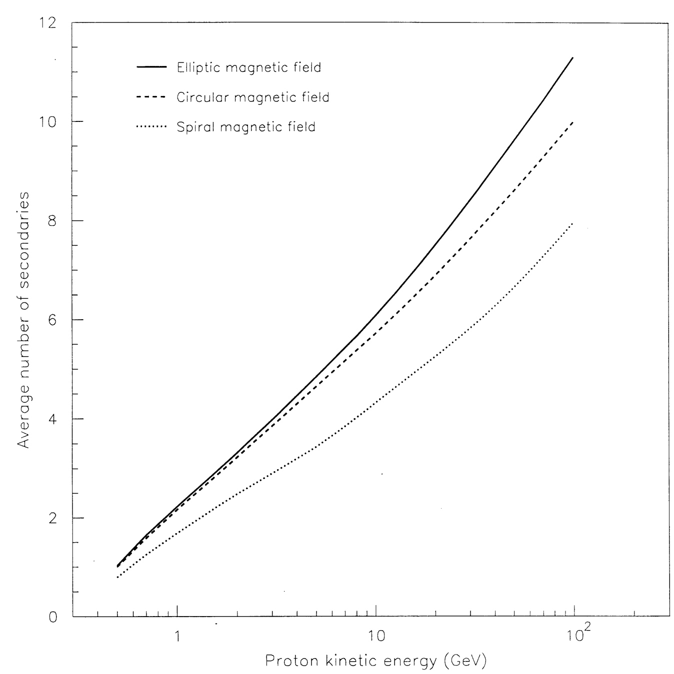

FIG. 5.—Mean number of secondary protons generated in the disk volume as a function of the primary proton energy for the three magnetic field configurations.

One should note that the mean elastic collision length for proton-proton interaction is 172 g cm^{-2} at a momentum of 100 GeV/c and decreases to the value of 4 g cm^{-2} at 100 MeV/c, where the secondary spectra shown in figure 4 are mostly populated. As a consequence, a cascade of secondary protons develops in the interstellar medium at low energy, unless the primary proton escapes from the disk boundaries before suffering elastic interactions. Note also that inelastic proton-proton collisions also produce secondary neutrons, which decay into protons.

An instrument positioned in the local Galactic zone will not register the presence of these slow secondaries, which cannot propagate at large distances because they are extincted by ionization energy losses before reaching the instrument. For example, a proton of 100 MeV/c has a range in hydrogen of 1.7 mg cm^{-2} which turns out to be 1 part per thousand of the total interstellar gas thickness traversed by cosmic rays according to the simple leaky-box model.

4. PROTON POPULATION IN THE GALACTIC DISK

The source distribution in space represented by equation (3) is used for the results of this section. A birth point, a trajectory, and a death point characterize the cosmic-ray life-cycle. The Galactic sites where quiescent protons in the interstellar medium are accelerated to high energy represent the birth points. It is generally assumed that the acceleration time is negligible compared to the cosmic-ray lifetime, and, accordingly, the acceleration sites or sources may be represented by space points. The trajectory is the curve in space tracked by the cosmic ray and either may be confined to the Galaxy or may eventually extend into the intergalactic space as shown in Figure 6. The death of a cosmic-ray proton takes place in three modes: (1) by elastic or inelastic nuclear collisions; (2) by ionization energy losses; and (3) by overflowing from the disk boundaries. In the three cases, the original proton disappears from the disk as a cosmic-ray particle. Since the elastic nuclear scattering is a quantum-mechanical process, the original proton loses its identity and the two secondary protons emerging from the collision are regarded as new protons. The space point where the collision occurs is the death point of the original proton and the birth point for the two secondary protons. The ionization death is due to ionization energy losses leading the cosmic-ray proton to a complete stop in the interstellar medium. Since our study is limited to the disk volume, the overflow from the disk boundaries also represents a type of death.

The fractions of cosmic-ray protons that terminate their life-cycle in the three modes described above are given in Figure 7 as a function of the initial kinetic energy for the circular and spiral magnetic fields and in Figure 8 for the elliptical magnetic field. Two regions in energy are distinguishable where two different regimes for proton destruction operate. The former is below 0.8 GeV, where ionization energy losses are important, and the latter is above 5 GeV, where nuclear death and disk overflow dominate.

At energy greater than 1 GeV, the probability of nuclear death is nearly constant, amounting to 0.63 for the circular magnetic field and to 0.50 for the spiral field. The difference of 21% is due to a greater probability of escaping from the spiral field. Escape probability from the disk boundaries is 0.36 for the circular field and 0.50 for the spiral one. The results for the elliptical field differ from those of the circular field by less than 3%.

Almost all protons leaving the Galactic disk cross the top and the bottom base of the cylinder, and only a small fraction escapes from its lateral surface. In the energy interval 10–100 GeV, the populations of cosmic-ray protons that escape from the disk or experience nuclear death are constant.

The three fractions of cosmic-ray protons populating the Galactic disk are heavily modified when secondary protons are included in the results. The probabilities of the three

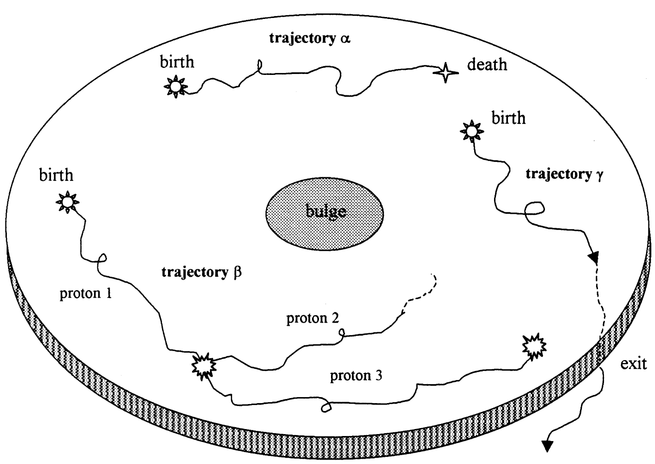

FIG. 6.—Qualitative illustration of proton trajectories in the Galactic disk. Trajectories have been subdivided into three categories depending on the mode in which protons disappear from the disk. The trajectory α represents a proton stopped in the interstellar hydrogen by ionization energy losses. The trajectory β represents a proton (proton 1) undergoing an elastic collision with an interstellar hydrogen and generating two protons after the collision (protons 2 and 3). The trajectory γ represents a proton escaping from the disk. A trajectory as it results from computer simulation is shown in Fig. 3.

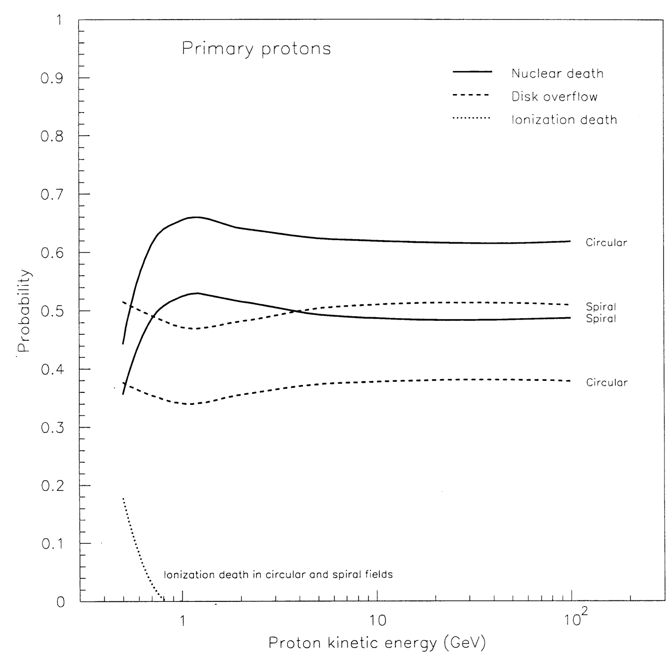

FIG. 7.—Fractions of cosmic-ray protons that experience a nuclear collision, extinction by ionization energy losses, or leave the disk boundaries as a function of the kinetic energy in the circular and spiral field. Only primary protons are included in this calculation.

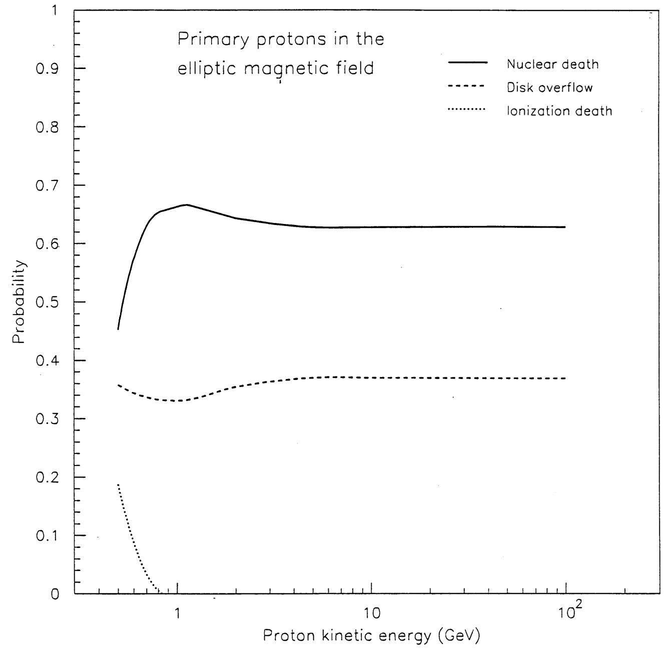

FIG. 8.—Fraction of cosmic-ray protons that suffer a nuclear collision, extinction by ionization energy losses, or leave the disk boundaries as a function of the kinetic energy in the elliptical magnetic field. Only primary protons are included in this calculation.

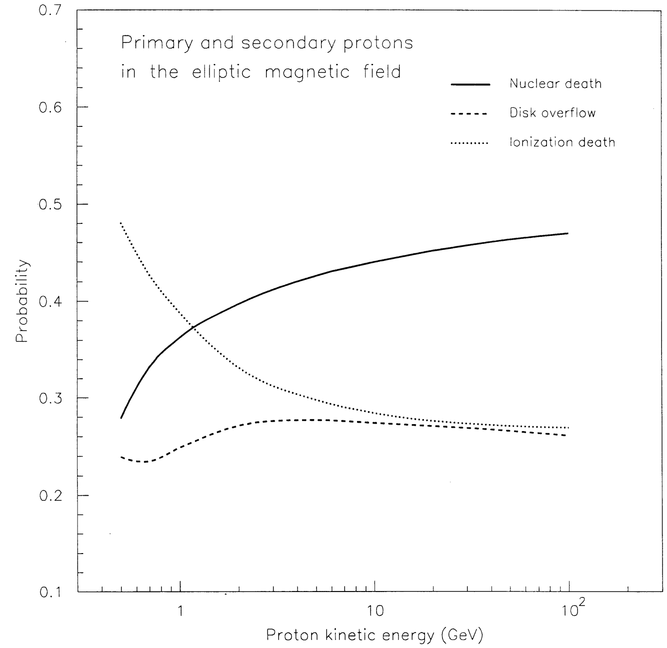

FIG. 10.—Fractions of cosmic-ray protons that experience a nuclear collision, extinction by ionization energy losses, or leave the disk boundaries as a function of the kinetic energy in the elliptical field. Primary and secondary protons are included in this calculation.

populations, which take into account primary and secondary protons, are shown in Figure 9 for the circular and spiral fields and in Figure 10 for the elliptical field.

The fraction of protons stopped by ionization energy losses levels off at about 0.25 in the energy interval 5–100 GeV for the three field configurations, while it is zero for primary protons above 0.8 GeV. The bulk of the low-energy secondary protons provides this large fraction of cosmic rays extinguished by ionization energy losses. The fraction of protons suffering nuclear death has a nearly constant increase ranging, on average, from 0.38 to 0.46 in the energy interval 3–100 GeV. This trend differs from that of primary protons, which is constant in the same energy interval. The computed increase is due to the corresponding decrease in the number of protons stopped by ionization energy losses.

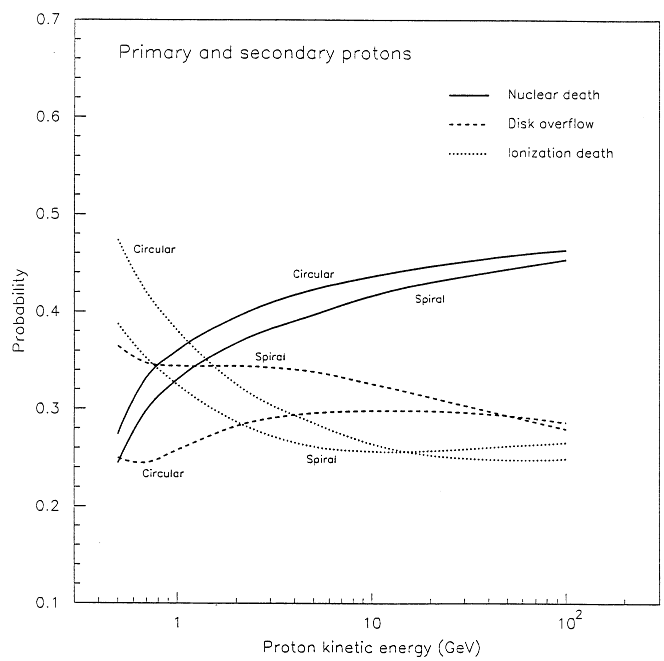

FIG. 9.—Fractions of cosmic-ray protons that experience a nuclear collision or extinction by ionization energy losses, or leave the disk boundaries as a function of the kinetic energy in the circular and spiral field. Primary and secondary protons are included in this calculation.

5. RESIDENCE TIME OF COSMIC-RAY PROTONS IN THE GALACTIC DISK

The residence time of cosmic-ray protons is defined as the time interval elapsed between the birth and death in the disk via nuclear collision or extinction by ionization energy losses and between the birth and the crossing of the disk boundaries. The mean residence time of a proton with energy E in the Galactic disk, $\tau_D(E)$, depends on the various fractions of proton populations that occupy the disk. It is calculated by the equation

$$\tau_D(E) = f_I\, T_I + f_N\, T_N + f_E\, T_E \;, \tag{6}$$

where f_I, f_N, and f_E are the fractions of cosmic-ray protons that terminate their lifecycle, respectively, by ionization, nuclear death, and disk leakage, and T_I, T_N, and T_E are the corresponding mean residence times.

A summary of the residence times and grammages calculated at three energies is given in Table 1. The fraction of proton populations extinguished by ionization energy losses for primary proton energies of 10 and 100 GeV is less than 10^{-3}. If secondary protons are included in the calculation, the grammage and residence times suffer a systematic decrease of 36% (grammage) and 14% (residence time) with respect to the values reported in Table 1, which refers to only primary protons. This decrement is explained by comparing the results shown in Figures 7 and 8 (regarding only primaries) with those in Figures 9 and 10 (regarding primaries and secondaries).

TABLE 1

SOME COMPUTED PROPERTIES OF PRIMARY PROTONS OF 1, 10, AND 100 GeV IN THE GALACTIC DISK VOLUME FOR CIRCULAR, ELLIPTICAL, AND SPIRAL FIELDS

			FRACTIONS OF PROTON POPULATIONS		
MAGNETIC FIELD STRUCTURE	RESIDENCE TIME (10^6 yr)	GRAMMAGE (g cm^{-2})	Extinction by Ionization	Disk Overflow	Nuclear Death
Proton Kinetic Energy 1 GeV					
Circular	15.3	26.6	0.002	0.34	0.65
Elliptical	15.1	26.7	0.003	0.33	0.66
Spiral	12.1	21.2	0.002	0.47	0.52
Proton Kinetic Energy 10 GeV					
Circular	14.7	30.2	0.00	0.37	0.61
Elliptical	14.9	30.5	0.00	0.38	0.61
Spiral	11.6	23.1	0.00	0.51	0.48
Proton Kinetic Energy 100 GeV					
Circular	14.5	29.8	0.00	0.37	0.61
Elliptical	14.7	30.2	0.00	0.37	0.62
Spiral	11.8	24.1	0.00	0.50	0.48

It turns out that protons in the circular and elliptical fields have nearly the same residence time that is independent of the energy in the range I–100 GeV. This result reflects the similarity of the two fractions of proton populations versus energy reported in Figures 7 and 8.

In order to study the dependence of the residence time and grammage on the interstellar gas density, the cloudlet density has been varied from 1 to 10 atoms cm^{-3}, and that of the interstellar medium from 0.6 to 1 atoms cm^{-3}. The grammage traversed remains unaltered for those proton populations suffering ionization and nuclear death. On the contrary, for proton populations escaping from the disk, the grammage changes from a minimum of 17.5 g cm^{-2} for the minimum gas density (clouds of 2 atoms cm^{-3} and interstellar gas 0.6 atoms cm^{-3}) to the maximum value of 22.3 g cm^{-2} for the maximum density (clouds of 10 atoms cm^{-3} and interstellar gas of 1 atom cm^{-3}). In general, the residence time decreases as the matter density increases. For the minimum density the residence time is 13×10^6 yr, while that of the maximum is 16×10^6 yr. The age and grammage are also insensitive (less than $\pm$ 1.5%) to magnetic field gradients.

6. COSMIC-RAY PROTONS FEEDING THE LOCAL GALACTIC ZONE

In the previous section, the age and other properties of cosmic protons populating the entire disk volume have been calculated. Instruments, however, are placed in the

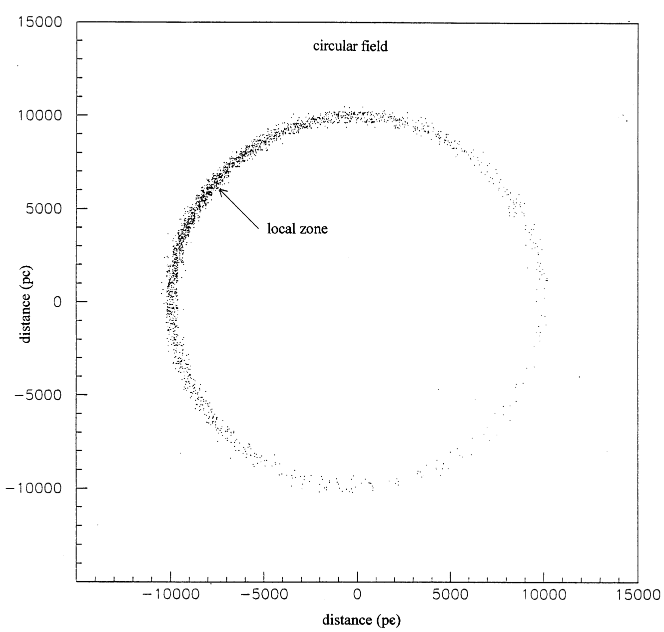

FIG. 11*a*

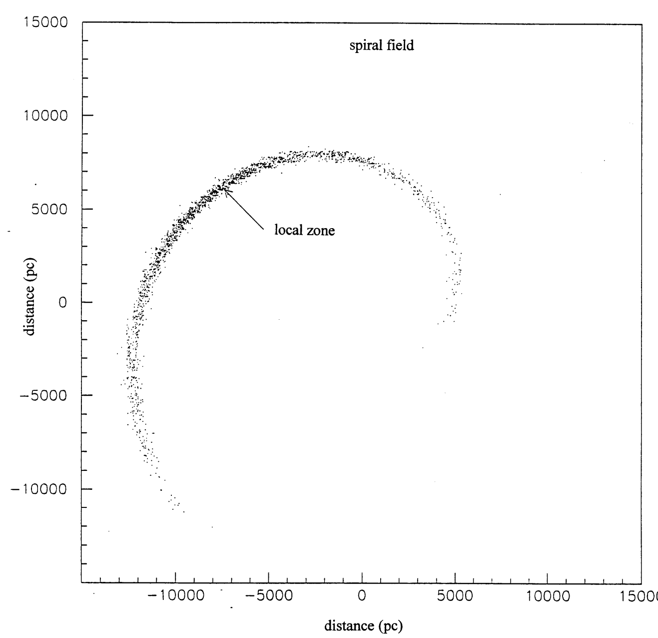

FIG. 11*b*

FIG. 11.—Source distributions projected in the Galactic plane for primary cosmic-ray protons entering the local Galactic region for (*a*) the circular and (*b*) the spiral fields.

solar cavity, and the related observations are necessarily limited to those cosmic rays reaching the local Galactic region. Thus, it is important to calculate the proton age also in the local Galactic zone the results being more manageable for a comparison with a variety of observations.

Since the ratio of this volume to that of the Galactic disk (see Fig. 1) is 0.9×10^{-5}, the number of proton trajectories intercepting the local zone is expected to be exceedingly small.

Because of the long time taken by the presently available computers for simulating several millions of proton trajectories, the necessity for an appropriate averaging volume is apparent.

In order to calculate the age of cosmic-ray protons arriving to the local Galactic zone, a sphere with a radius of 100 pc is taken as averaging volume. For this study the sphere has been positioned concentric with the solar cavity, with coordinates $l = 0$, $r = 10$ kpc, and $z = +14$ pc.

The source distribution in space feeding the local Galactic region differs greatly from those of the entire disk given in § 1 because only a small fraction of the primary protons emanating from the sources reaches the local zone. In Figure 11 are shown the source distributions projected onto the Galactic plane for those cosmic-ray protons intercepting the local Galactic zone. This result is obtained with a uniform source distribution.

Most of the protons reaching the local region come from sources located along the regular component of the Galactic magnetic field. The characteristic form of the source distribution along the magnetic field lines is here referred to as "corona". The thin distribution and that of supernovae generate coronae similar to those of Figure 11. The width of the corona projected onto the Galactic midplane is related both to the radius of the local zone and to the parameters of the chaotic component of the magnetic field. A more quantitative description of the source distributions in space feeding the local Galactic zone, including various parameterizations, is given elsewhere (Codino & Vocca 1999).

7. PROTON AGE AND GRAMMAGE IN THE LOCAL GALACTIC ZONE

The proton age is defined as the time elapsed between the birth of the proton and its arrival in the local Galactic zone. As a general rule, the age differs from the residence time. The difference could be quite large if all cosmic-ray sources were located at great distances from the observing site (in our case, the local Galactic zone). For example, in the implausible case that all cosmic-ray sources were located in the Galactic bulge and not in the disk, as assumed in this study and in many others, the computed proton age in the local Galactic zone would have been much longer.

The age of cosmic-ray protons reaching the local Galactic region has been calculated for the two magnetic field structures as a function of the proton energy, and the results are displayed in Figure 12. The source distribution represented by equation (3) has been used for these results. Note the large difference in the proton age between the spiral and the circular field. In the kinetic energy range 1–100 GeV, an increase in the age of about 15% is observed for the two fields. Thus, the age is rather insensitive to the proton energy.

The proton age has been also calculated, in various parameter ranges, as a function of the radius of the local zone for the same disk thickness (see Fig. 13) and as a function of the disk thickness for a constant radius of the local zone (see Fig. 14). In spite of the crude approximations involved in this calculation, the computed age turns out to be independent, to within $\pm 7\%$, of the radius of the sphere and to within $\pm 17\%$ of the disk thickness for plausible values (i.e., 400–600 pc).

One should notice that the ages in the local zone are smaller than the residence times calculated for the entire disk volume and are approximately half for the same field

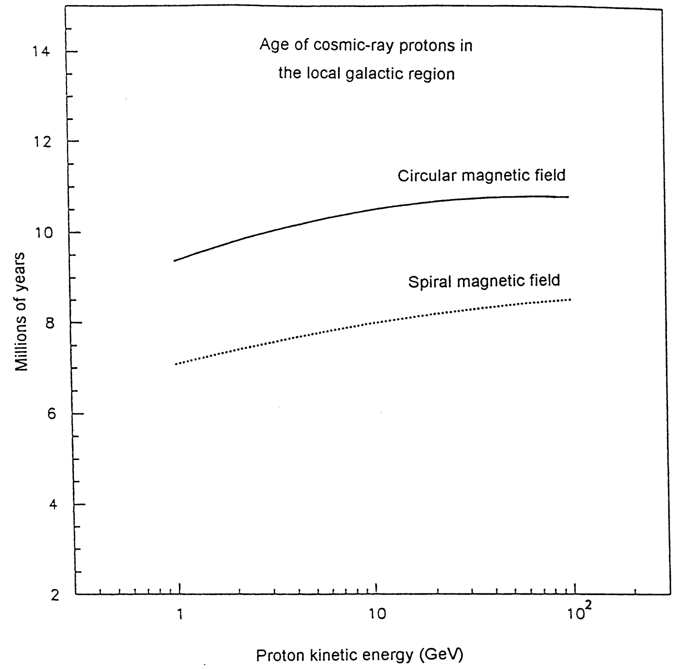

FIG. 12.—Mean age of secondary and primary protons intercepting the local Galactic region as a function of the kinetic energy for the circular (*solid curve*) and spiral (*dotted curve*) field. The age of the protons in the elliptical field is close to that of the circular field.

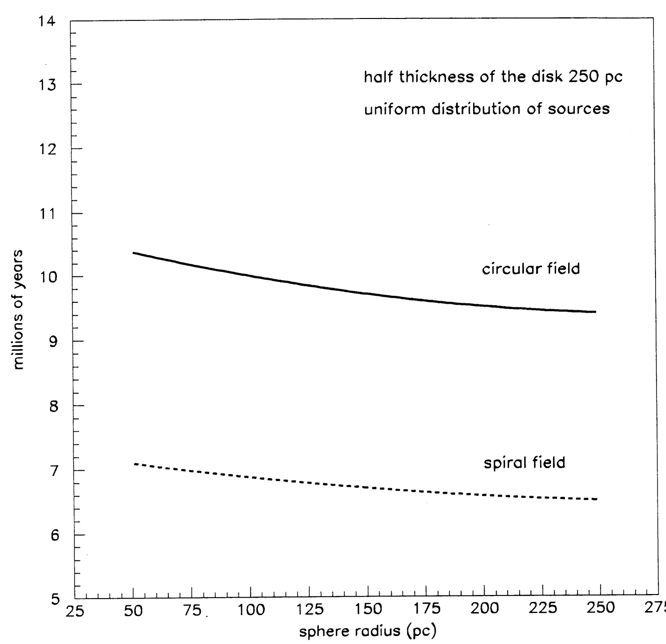

FIG. 13.—Age of cosmic-ray protons in the circular field as a function of the radius of the local Galactic zone represented by a sphere concentric to the solar cavity. The disk thickness is 500 pc, and a uniform distribution of sources is used.

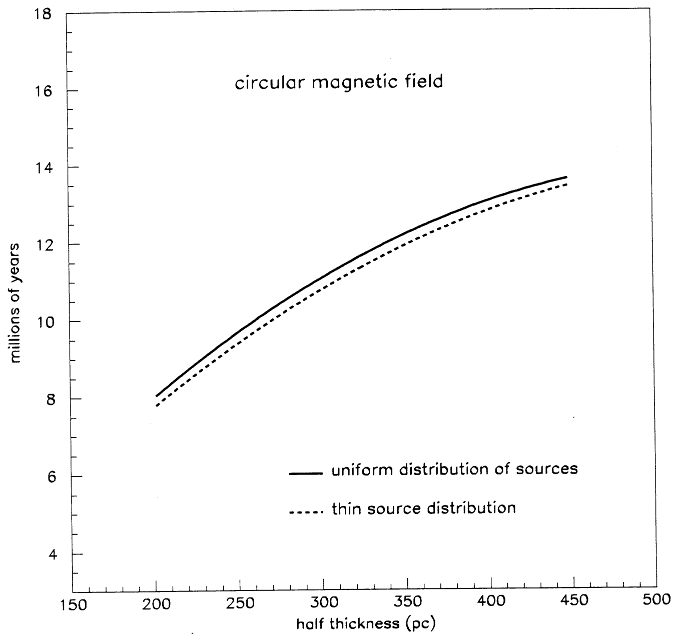

FIG. 14.—Age of cosmic-ray protons as a function of the half-thickness of the disk in the circular magnetic field for the uniform and the thin source distribution. The local Galactic zone is a sphere of constant radius 100 pc.

configuration. This is simply explained by considering the helix length distribution of cosmic protons in the whole disk volume in a circular magnetic field. Suppose that the average value is λ, and assume further that the disk boundaries are so large as to contain all proton trajectories. In this circumstances, an unbiased observer (e.g., an observer confined in the local Galactic region) randomly intercepting the proton trajectories will find an average helix length of $\lambda/2$. As a consequence, $\tau_D(E) = 2\tau_L(E)$ if all proton velocities were equal to c (light velocity). For a finite volume of the disk such as that contemplated in Figure 1, and for typical proton fractions crossing the disk boundaries shown in Figures 7, 8, 9, and 10, $\tau_D(E)$ should result in a value less than but approximately equal to $2\tau_L(E)$.

The simulation code provides us, while calculating the ages, with the matter thickness traversed by cosmic protons (often referred to as grammage). For example, in the spiral field, for a kinetic energy of 1 GeV, the grammage is 11.7 g cm^{-2} and becomes 14.6 g cm^{-2} at 10 GeV; in the circular field the grammage is 14.9 g cm^{-2} for 1 GeV and 19.1 g cm^{-2} for 10 GeV.

The calculations have been repeated by using a differential energy spectrum at the source proportional to $E^{-\gamma}$, where E is the proton kinetic energy in the range 1 GeV $\leq E \leq$ 100 GeV and γ is the spectral index of 2.2 (Berezinskii et al. 1990). The resulting ages are 6.7×10^6 yr (spiral) and 9.5×10^6 yr (circular) while the grammages are 12.0 g cm^{-2} (spiral) and 16.0 g cm^{-2} (circular).

8. SOME CONSEQUENCES OF THIS CALCULATION

In this section we consider some implications of this study, regarding fundamental properties of Galactic cosmic rays in the approximations given in § 1. These implications are based on the computed difference between age in the local Galactic zone and the residence time in the whole disk.

1. The average matter thickness swept out by cosmic rays (see, for example, Webber 1996) while wandering about the disk, t_D, is given by

$$t_D = n_H m L_h , \quad (7)$$

where n_H is the average hydrogen density, m is the hydrogen mass, and L_h is the average helix length of the various proton populations in the disk. Depending on the observing site, L_h is proportional to either the residence time $\tau_D(E)$ or the age $\tau_L(E)$. As a consequence, the grammage is crucially related to the observing site in the disk. Note that the grammage resulting from a variety of observations such as the boron-to-carbon ratio interpreted in the leaky-box models is of the same order of magnitude (Garcia-Munoz et al. 1987) of our calculations if the local Galactic zone is taken as averaging volume. Finally, one should note that the grammage estimated with heavy ions may differ from that calculated with protons because of the different role played by ionization energy losses of protons and heavy ions in the disk.

2. The total power transported by cosmic rays in the disk is

$$P = P_D + P_H + P_E , \quad (8)$$

where P_D, P_H, and P_E are the partial powers of the cosmic rays generated in the disk, halo, and extragalactic regions, respectively. Neglecting the contributions from halo and extragalactic regions, i.e., $P_H = 0$ and $P_E = 0$, we have the result $P = P_D$. A simple expression for P_D is

$$P_D = u_D V_D/\tau_D , \quad (9)$$

where u_D is the average cosmic-ray energy density in the disk, V_D is the disk volume, and τ_D is the average residence time in the disk. Typical values of u_D are in the range 0.5–1 eV cm^{-2} (Berezinskii et al. 1990). Because the residence time observed in the local zone, regardless of the magnetic field configuration, is approximately $\tau_D/2$, the total power observable in the local zone is nearly halved compared to that calculated in the whole disk.

3. The residence times of the Galactic protons are nearly independent of the energy in the interval 1–100 GeV, and the age as computed by using the notion of the local Galactic zone does not depend strongly on the energy.

On the contrary, we cannot fail to mention that residence times and ages of beryllium isotopes evaluated with this simulation code (Brunetti & Codino 1997b) have a peculiar energy dependence because of the different importance of ionization energy losses in the disk between protons and heavy ions. The study of the variation of the residence time with energy and the comparison with experimental data is beyond the scope of this paper.

9. CONCLUSIONS

The influence of the magnetic field in the residence time of cosmic rays in the Galactic disk has been investigated by using three different shapes of its regular component. It has been found that the geometrical structure of the magnetic field lines has a strong influence both on the age and on the grammage of cosmic-ray protons; on the other hand, the influence of the magnetic field strength, regardless of its shape (circular, spiral, elliptical), is totally negligible in the range of plausible values resulting from measurements. For example, if the magnetic field strength in the circular configuration is changed from the value adopted in this study of 3 μG to the value 6 μG (i.e., + 100%) the resulting proton age remains unaltered within $\pm 1.5\%$. This independence holds also for the elliptical and spiral fields.

A slight dependence on the age and grammage on the proton energy has also been found as reported in Figure 12, Table 1, and § 7.

Important results emerging from this study indicate that (1) the mean age of cosmic-ray protons intercepting the local Galactic region is nearly half of the residence time calculated in the entire disk, and (2) the grammage experienced by cosmic-ray protons intercepting the local Galactic region is a factor of about 2 that in the whole disk.

These results are explained by the source distributions in space of cosmic-ray protons feeding the local Galactic region that have the form of coronae of Galactocentric radius 10 kpc (see Figs. 11*a* and 11*b*). The existence of coronae is anchored to the observational evidence that spiral galaxies possess large-scale magnetic field patterns that are spiral or circular.

Although three different patterns of the magnetic field lines have been used in the present calculations to delimit the influence of the magnetic field on some cosmic-ray properties, the actual magnetic field in the disk certainly differs from any one of the three field structures used here. In fact, the detailed maps of the Galactic magnetic field as derived from radio observations and other measurements clearly exhibit a complexity irreducible to the simple shapes (circular, elliptical, and spiral) of this study. Nevertheless, we believe that it is interesting to compare any future results of more sophisticated calculations (presently unavailable), utilizing detailed maps of the Galactic magnetic field with those given here obtained with simple geometrical field structures.

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REFERENCES

Bell, M. C., et al. 1974, J. Phys. G., 7, 1409
Berezinskii, V. S., & Miklailov 1987, Proc. 20th Int. Cosmic Ray Conf. (Moscow), 2, 54
Berezinskii, V. S., et al. 1990, Astrophysics of Cosmic Rays (Amsterdam: Elsevier)
Brunetti, M. T., & Codino, A. 1997a, Proc. 26th Int. Cosmic Ray Conf. (Durban), 4, 273
———. 1997b, Proc. 26th Int. Cosmic Ray Conf. (Durban), 4, 277
Chi, X., & Wolfendale, A. W. 1990, J. Phys. G., 16, 1409
Codino, A. 1998, Numerical Simulation of Some Fundamental Properties of Galactic Cosmic Rays (Vulcano Workshop), (Bologna: SIF), in press
Codino, A., et al. 1995, Proc. 25th Int. Cosmic Ray Conf. (Rome), 3, 100
Codino, A., & Vocca, H. 1999, ApJ, submitted
Freier, P. S., et al. 1977, ApJ, 213, 588
Gaisser, T., 1990 Cosmic Rays and Particle Physics, (Cambridge: Cambridge Univ. Press)
Garcia-Munoz, M., et al. 1987, ApJJ, 64, 269
Ginzburg, W. L., & Ptuskin, V. S. 1976, Rev. Mod. Phys., 48, 161
Heiles, C. 1976, ARA&A, 14, 1
Jokipii, J. R. 1976, ApJ, 208, 900
Jones, F. C. 1979, ApJ, 229, 747
Kota, J., & Owens, A. J. 1980, ApJ, 237, 814
Manchester, R. N. 1974, ApJ, 188, 637
Osborne, J. L., et al. 1973, J. Phys. A., 6, 421
Rand, R. J., & Kulkarni, S. R. 1989, ApJ, 343, 760
Sofue, T., & Fujimoto, M. 1983, ApJ, 265, 722
Stecker, S. W., & Jones, F. C. 1977, Proc. 12th ESLAB Symp., 171
Thomson, C. R., & Nelson, A. H. 1980, MNRAS, 191, 863
Vallée, J. P. 1983, A&A, 23, 85
Webber, W. R. 1996, ApJ, 457, 435