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Another approach to the in-medium chiral condensates

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Abstract

A new formalism to calculate the in-medium chiral condensate is presented. At lower densities, this approach leads to a linear condensate. If it is compatible with the famous model-independent result, the pion–nucleon sigma term could be six times the average current mass of light quarks. The modification due to QCD-like interactions may, compared with the linear extrapolation, slow the decreasing speed of the condensate with increasing densities.

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The investigation of chiral condensates in medium has been an interesting topic in nuclear physics [1]. One of the most popularly used methods to calculate the in-medium quark condensate is the Feynman–Helmann theorem [2]. The main difficulty in this formalism is that one has to make assumptions on the derivatives of model parameters with respect to the quark current mass. To bypass this difficulty, we will apply a similar idea as in the study of strange quark matter [3–6] by defining an equivalent mass. A differential equation which determines the equivalent mass will be derived in detail. At lower densities, the new formalism leads to a linear condensate. Comparing the corresponding expression with the famous model-independent result in nuclear matter, it is found that the pion–nucleon sigma term could be six times the average current mass of light quarks. At higher densities, the derived differential equation within a QCD-like inter-quark interaction, which indicates that the decreasing speed of the condensate with increasing densities is usually slowed, compared with the linear extrapolation, due to interactions.

Let us first write the QCD Hamiltonian density schematically as

$$H_{\mathrm{QCD}} = H_{\mathrm{k}} + \sum_i m_{i0}\bar{q}_i q_i + H_{\mathrm{I}}, \tag{1}$$

where H_{k} is the kinetic term, m_{i0} is the current mass of quark flavor i, and H_{I} is the interacting part of the

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Hamiltonian. The summation $\sum_i$ goes over all flavors considered.

The basic idea of the mass-density-dependent model of quark matter is that the system energy can be expressed in the same form as a proper noninteracting system. The strong interaction between quarks is included within the appropriate variation of quark masses with density. In order not to confuse with other mass concepts, let us refer to such a density-dependent mass as an equivalent mass. Such an equivalent mass can be separated into two parts, i.e.,

$$m_i = m_{i0} + m_{\rm I}, \tag{2}$$

the first term is the quark current mass, the second term is the interacting part which is the same for all flavors. Therefore, if we use the equivalent masses m_i, the system Hamiltonian density should be replaced by a Hamiltonian density of the form

$$H_{\rm eqv} = H_{\rm k} + \sum_i m_i \bar{q}_i q_i. \tag{3}$$

Obviously, we must require that the two Hamiltonian densities $H_{\rm eqv}$ and $H_{\rm QCD}$ have the same expectation value for any state $|\Psi\rangle$, i.e.,

$$\langle\Psi|H_{\rm eqv}|\Psi\rangle = \langle\Psi|H_{\rm QCD}|\Psi\rangle. \tag{4}$$

Applying this equality to the state $|n_{\rm B}\rangle$ with baryon number density $n_{\rm B}$ and to the vacuum state $|0\rangle$, respectively, taking then the difference, one has

$$\langle H_{\rm eqv}\rangle_{n_{\rm B}} - \langle H_{\rm eqv}\rangle_0 = \langle H_{\rm QCD}\rangle_{n_{\rm B}} - \langle H_{\rm QCD}\rangle_0. \tag{5}$$

Here and in the following, we use $\langle A\rangle_{n_{\rm B}} \equiv \langle n_{\rm B}|A|n_{\rm B}\rangle$ and $\langle A\rangle_0 \equiv \langle 0|A|0\rangle$ for an arbitrary operator A.

Since we are considering a system with uniformly distributed particles, or in other words, the density $n_{\rm B}$ has nothing to do with the space coordinates, we can write $\langle\Psi|m(n_{\rm B})\bar{q}q|\Psi\rangle = m(n_{\rm B})\langle\Psi|\bar{q}q|\Psi\rangle$. This equality is especially obvious if we consider it in terms of quantum mechanics: $|\Psi\rangle$ is a wave function with arguments $n_{\rm B}$ and coordinates, the expectation value is nothing but an integration with respect to the coordinates. Therefore, if $n_{\rm B}$ does not depend on coordinates, the function $m(n_{\rm B})$ is also a coordinate-independent c-number, and can naturally be taken out of the integration. However, if $n_{\rm B}$ is local, the case becomes much more complicated and it will not be considered here.

Now we can solve Eq. (5) for $m_{\rm I}$, and accordingly obtain

$$m_{\rm I} = \frac{\epsilon_{\rm I}}{\sum_i(\langle\bar{q}_i q_i\rangle_{n_{\rm B}} - \langle\bar{q}_i q_i\rangle_0)}, \tag{6}$$

where $\epsilon_{\rm I} \equiv \langle H_{\rm I}\rangle_{n_{\rm B}} - \langle H_{\rm I}\rangle_0$ is the interacting energy density.

Therefore, considering the quarks as a free system, i.e., without interactions, while keeping the system energy unchanged, quarks should acquire an equivalent mass of the corresponding current mass plus the common interacting part shown in Eq. (6). Obviously the equivalent mass is a function of both the quark current mass and the density. At finite temperature, it depends also on the temperature as well. Here we consider only zero temperature. Due to quark confinement, we have the following natural requirement:

$$\lim_{n_{\rm B}\to 0} m_{\rm I} = \infty. \tag{7}$$

Because the Hamiltonian density $H_{\rm eqv}$ has the same form as that of a system of free particles with equivalent masses m_i, the energy density of quark matter can be expressed as

$$\begin{aligned}\epsilon &= \sum_i \frac{g}{2\pi^2}\int_0^{k_{\rm f}} \sqrt{k^2+m_i^2}\,k^2\,dk \\ &= 3n_{\rm B}\sum_i m_i F\left(\frac{k_{\rm f}}{m_i}\right),\end{aligned} \tag{8}$$

where $g = 3(\text{colors}) \times 2(\text{spins}) = 6$ is the degeneracy factor, and

$$k_{\rm f} = \left(\frac{18}{g}\pi^2 n_{\rm B}\right)^{1/3} \tag{9}$$

is the Fermi momentum of quarks. The function $F(x)$ is

$$F(x) \equiv \frac{3}{8}\left[x\sqrt{x^2+1}(2x^2+1) - \operatorname{arcsinh}(x)\right]/x^3. \tag{10}$$

For convenience, let us define another function $f(x)$ to be

$$\begin{aligned}f(x) &\equiv -x^2 d\left[xF(x)/x\right]/dx \\ &= F(x) - x\,dF(x)/dx \\ &= \frac{3}{2}\left[x\sqrt{x^2+1} - \ln\left(x+\sqrt{x^2+1}\right)\right]/x^3.\end{aligned} \tag{11}$$

On the other hand, the total energy density can also be expressed as

$$\epsilon = \sum_i \frac{g}{2\pi^2} \int_0^{k_f} \sqrt{k^2 + m_{i0}^2}\, k^2\, dk + \epsilon_I$$
$$= 3n_B \sum_i m_{i0} F\left(\frac{k_f}{m_{i0}}\right) + \epsilon_I. \tag{12}$$

The first term is the energy density without interactions, the second term is the interacting part.

Combining Eqs. (12) and (8) gives

$$\frac{\epsilon_I}{3n_B} = \sum_i \left[m_i F\left(\frac{k_f}{m_i}\right) - m_{i0} F\left(\frac{k_f}{m_{i0}}\right)\right]. \tag{13}$$

From Hellmann–Feynman theorem [2], one has

$$\langle\Psi|\frac{\partial}{\partial\lambda} H(\lambda)|\Psi\rangle = \frac{\partial}{\partial\lambda}\langle\Psi|H(\lambda)|\Psi\rangle, \tag{14}$$

where $|\Psi\rangle$ is a normalized eigenvector of the Hamiltonian $H(\lambda)$ which depends on a parameter λ. The symbol $\partial/\partial\lambda$ has been used here to indicate keeping other independent variables (e.g., the bayron number density n_B of the system) fixed when taking the derivative.

On application, in Eq. (14), of the substitutes $\lambda \to m_{i0}$ and $H(\lambda) \to \int d^3x\, H_{QCD}$, one gets $\langle\Psi|\int d^3x \times \bar{q}_i q_i|\Psi\rangle = \frac{\partial}{\partial m_{i0}}\langle\Psi|\int d^3x H_{QCD}|\Psi\rangle$ for each flavor i. Now applying this equality, respectively, to the state $|n_B\rangle$ (quark matter with baryon number density n_B) and the vacuum $|0\rangle$, and taking then the difference, one obtains

$$\langle\bar{q}_i q_i\rangle_{n_B} - \langle\bar{q}_i q_i\rangle_0 = \frac{\partial\epsilon}{\partial m_{i0}}, \tag{15}$$

where $\epsilon \equiv \langle H_{QCD}\rangle_{n_B} - \langle H_{QCD}\rangle_0$ is the total energy density. It has been expressed in Eq. (8) with the equivalent masses. Now let us substitute Eq. (8) into Eq. (15), carry out the corresponding derivative, and get

$$\langle\bar{q}_i q_i\rangle_{n_B} - \langle\bar{q}_i q_i\rangle_0$$
$$= 3n_B\left[f\left(\frac{k_f}{m_i}\right) + \sum_j f\left(\frac{k_f}{m_j}\right)\frac{\partial m_I}{\partial m_{i0}}\right]. \tag{16}$$

Summing over both sides, we have

$$\sum_i \left[\langle\bar{q}_i q_i\rangle_{n_B} - \langle\bar{q}_i q_i\rangle_0\right]$$
$$= 3n_B \sum_i f\left(\frac{k_f}{m_i}\right)[1 + \nabla m_I] \tag{17}$$

with $\nabla \equiv \sum_i \partial/\partial m_{i0}$. Please note, ∇ is a differential operator in mass space, rather than in coordinate space as it is in its normal definition.

Eq. (6) can be rewritten as

$$\sum_i \left[\langle\bar{q}_i q_i\rangle_{n_B} - \langle\bar{q}_i q_i\rangle_0\right] = \frac{\epsilon_I}{m_I}. \tag{18}$$

Comparing this equation with Eq. (17) gives

$$\nabla m_I = \frac{\epsilon_I/(3n_B)}{m_I \sum_i f(k_f/m_i)} - 1. \tag{19}$$

On application of Eq. (13), this equation leads to

$$\nabla_0 m_I = \frac{\sum_i [m_i F(k_f/m_i) - m_{i0} F(k_f/m_{i0})]}{m_I \sum_i f(k_f/m_i)} - 1 \tag{20}$$

which is a first order differential equation satisfied by the interacting equivalent mass. Such a mass really exists, and we can prove that it can be expressed in terms of the interacting energy density ϵ_I formly as

$$m_I = \frac{\epsilon_I/(3n_B)}{\sum_i f(\frac{k_f}{m_{i0}}) + \frac{\nabla\epsilon_I}{3n_B}}. \tag{21}$$

In fact, applying the operator ∇ at both sides of Eq. (13) gives

$$\frac{\nabla\epsilon_I}{3n_B} = \sum_i \left[f\left(\frac{k_f}{m_i}\right) - f\left(\frac{k_f}{m_{i0}}\right)\right]$$
$$+ \sum_i f\left(\frac{k_f}{m_i}\right)\nabla m_I. \tag{22}$$

Substituting Eq. (19) into this equation, solving then for m_I, we immediately get Eq. (21).

In the flavor symmetric case, i.e., $m_{u0} = m_{d0} = \cdots = m_0$, we have $m_u = m_d = \cdots = m$, $\langle\bar{q}_u q_u\rangle = \langle\bar{q}_d q_d\rangle = \cdots = \langle\bar{q}q\rangle$, and $\nabla = \partial/\partial m_0$. Accordingly, Eqs. (20), (18) and (13) become, respectively,

$$\frac{\partial m_I}{\partial m_0} = \frac{mF(k_f/m) - m_0 F(k_f/m_0)}{m_I f(k_f/m)} - 1, \tag{23}$$

$$\frac{\langle\bar{q}q\rangle_{n_B}}{\langle\bar{q}q\rangle_0} = 1 + \frac{1}{N_f\langle\bar{q}q\rangle_0}\frac{\epsilon_I}{m_I}, \tag{24}$$

$$mF = \left(\frac{k_f}{m}\right) - m_0 F\left(\frac{k_f}{m_0}\right) = \frac{\epsilon_I}{3N_f n_B}, \tag{25}$$

where N_f is the number of flavors.

Because the function $F(x)$ has the property $\lim_{x\to 0} F(x) = 1$, Eq. (25) becomes at lower densities

$$m = m_0 + \frac{\epsilon_I}{3N_f n_B}. \tag{26}$$

This means $m_I = \epsilon_I/(3N_f n_B)$, i.e., $\epsilon_I/m_I = 3N_f n_B$. Substituting this ratio into Eq. (24), we get

$$\frac{\langle\bar{q}q\rangle_{n_B}}{\langle\bar{q}q\rangle_0} = 1 - \frac{n_B}{n^*} \tag{27}$$

with

$$n^* \equiv \frac{1}{3}\langle\bar{q}q\rangle_0 = \frac{m_\pi^2 f_\pi^2}{6m_0}. \tag{28}$$

In obtaining the last equality of Eq. (28), we have used the Gell-Mann–Oakes–Renner relation $-2m_0\langle\bar{q}q\rangle_0 = m_\pi^2 f_\pi^2$ [7], where $m_\pi \approx 140$ MeV is the pion mass and $f_\pi \approx 93.2$ MeV is the pion decay constant.

It should be noted that we have said nothing about the form of the interacting energy density in deriving Eq. (27). The only requirement we used is the quark confinement which ensures that the equivalent mass becomes extremely large at lower densities. At this meaning, Eq. (27) is a direct consequence of quark confinement and thus independent of models. Recalling that there is another model-dependent result in nuclear matter, i.e.,

$$\frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_0} = 1 - \frac{\rho}{\rho^*} \quad \text{with } \rho^* \equiv \frac{M_\pi^2 F_\pi^2}{\sigma_N}, \tag{29}$$

which was first proposed by Drukarev et al. [8], and later re-justified by many authors [9–11], we get, from the requirement $n^* = \rho^*$, a very interesting relation

$$\sigma_N = 6m_0, \tag{30}$$

i.e., the pion–nucleon sigma term σ_N is six times the average current quark mass m_0. If one takes $\sigma_N = 45$ MeV [12–14] and $m_0 = (m_{u0} + m_{d0})/2 = (5 + 10)/2 = 7.5$ MeV [15], Eq. (30) is naturally satisfied.

In principle, one can also calculate the chiral condensate at higher densities from Eqs. (23)–(25) if one knows some information on the interacting energy density ϵ_I from a realistic quark model. The obvious advantage of this formalism is that one does not need to make further assumptions on the current mass derivatives of model parameters. In the following, we consider a simple model, as an example.

Denoting the average distance between quarks by $\bar{r}$, the interaction between quarks by $\mathrm{v}(m_0, n_B)$, and assuming each quark can only interact strongly with another N_0 nearest quarks at any moment, because of the saturation property of strong interactions, the interacting energy density ϵ_I to density by

$$\epsilon_I = \frac{3}{2}N_0 n_B \mathrm{v}(m_0, \bar{r}). \tag{31}$$

This is obtained as follows. Suppose the total particle number is N. The interacting energy due to particle i is $N_0\mathrm{v}(m_0, n_B)$, accordingly the total interacting energy is $\frac{1}{2}\sum_{i=1}^N N_0\mathrm{v}(m_0, n_B) = \frac{1}{2}NN_0\mathrm{v}(m_0, n_B)$ (the extra factor 1/2 is for the correction of double counting). Dividing this by the volume, we then have the interacting energy density in Eq. (31) where the average inter-quark distance $\bar{r}$ is linked to density through

$$\bar{r} = \frac{\xi}{n_B^{1/3}}. \tag{32}$$

The average volume occupied by one particle is $1/(3n_B)$. If this volume is considered as a spherical ball of diameter $\bar{r}$, one finds $\xi = (2/\pi)^{1/3}$. If, however, this volume is considered as a cubic box of side $\bar{r}$, one has $\xi = 1/3^{1/3}$. In the former case, one divides the system into sub balls and place a particle at the center of each one. In the latter case, the system is divided into sub cubic boxes. Obviously there are unoccupied spaces between balls in the former case, so we take the latter value, i.e., $\xi = 1/3^{1/3}$.

Eq. (31) may not be an absolutely exact expression for the interacting energy density. Some other components, for example, the possible three-body interactions have not been included. However, they should be of higher orders in density, and will not be considered here. To compensate for these ignorances, to some extent, one may regard the N_0 as a free parameter. However, we will take $N_0 = 2$ due to the fact that the quark has a trend to interact strongly with other two quarks to form a baryon. It is found that the concrete value does not influence the density behavior of the chiral condensate significantly.

Substituting Eq. (31) into Eqs. (24) and (25), we have, respectively,

$$\frac{\langle\bar{q}q\rangle_{n_B}}{\langle\bar{q}q\rangle_0} = 1 - \frac{N_0}{2N_f}\frac{n_B}{n^*}\frac{\mathrm{v}}{m_I}, \tag{33}$$

$$mF\left(\frac{k_f}{m}\right) - m_0 F\left(\frac{k_f}{m_0}\right) = \frac{N_0}{2N_f}\mathrm{v}(m_0, n_B). \tag{34}$$

Keeping in mind the function property $\lim_{x\to\infty} F(x) \to 3x/4$, we have

$$\lim_{n_B\to\infty} \mathrm{v}(m_0, \bar{r}) = 0, \tag{35}$$

which is consistent with asymptotic freedom. Note that we have said nothing about the form of the interaction $\mathrm{v}(m_0, n_\mathrm{B})$.

Because Eq. (23) is a first order differential equation, we need an initial condition at $m_0 = m_0^*$ to get a definite solution. Let us suppose it to be

$$m\big(m_0^*, n_\mathrm{B}\big) = m(n_\mathrm{B}). \tag{36}$$

Usually, we will have

$$\mathrm{v}(m_0, n_\mathrm{B})|_{m_0=m_0^*} = \mathrm{v}(\bar{r}), \tag{37}$$

where $\mathrm{v}(\bar{r})$ is the inter-quark interaction for the special value m_0^* of the quark current mass m_0.

Eq. (23) is difficult to solve analytically. However, this can be done at lower densities. Let us rewrite Eq. (21) as

$$m_\mathrm{I} = \frac{\mathrm{v}(m_0, \bar{r})}{\frac{2N_\mathrm{f}}{N_0} f(\frac{k_\mathrm{f}}{m_0}) + \frac{\partial V(m_0,\bar{r})}{\partial m_0}}. \tag{38}$$

At lower densities, the Fermi momentum k_f is small, so the function $F(x)$ approaches to 1. Accordingly, Eq. (34) leads to

$$m = m_0 + \frac{N_0}{2N_\mathrm{f}} \mathrm{v}(m_0, \bar{r}) \tag{39}$$

which means $m_\mathrm{I} = \frac{2N_0}{2N_\mathrm{f}} \mathrm{v}(m_0, n_\mathrm{B})$. Replacing the left-hand side of Eq. (38) with this expression, we get

$$\frac{N_0}{2N_\mathrm{f}} \frac{\partial \mathrm{v}}{\partial m_0} + f\left(\frac{k_\mathrm{f}}{m_0}\right) = 1. \tag{40}$$

Integrating this equation under the initial condition given in Eq. (37), we have

$$\mathrm{v}(m_0, \bar{r}) = \mathrm{v}(\bar{r}) + \int_{m_0^*}^{m_0} \frac{2N_\mathrm{f}}{N_0}\left[1 - f\left(\frac{k_\mathrm{f}}{m_0}\right)\right] dm_0. \tag{41}$$

Then the interacting part of the equivalent mass is

$$m_\mathrm{I}(m_0, n_\mathrm{B}) = \frac{N_0}{2N_\mathrm{f}} \mathrm{v}(\bar{r}) + \int_{m_0^*}^{m_0} \left[1 - f\left(\frac{k_\mathrm{f}}{m_0}\right)\right] dm_0. \tag{42}$$

It is interesting to note that Eq. (42) can, because of the dominant linear confining interaction, lead to a quark mass scaling of the form $m_\mathrm{I} \propto 1/n_\mathrm{B}^{1/3}$ which is consistent with the result in Ref. [3] (PRC61:015201).

Substituting the above results for v and m_I into Eq. (33), we will get the same equality shown in Eq. (27).

In general, an explicit analytical solution for the condensate is not available, and we have to perform numerical calculations. For a given inter-quark interaction $\mathrm{v}(\bar{r})$, we can first solve Eq. (34) to obtain the initial condition in Eq. (36) for the equivalent mass, then solve the differential Eq. (23), and finally calculate the quark condensate through Eq. (33).

There are various expressions for $\mathrm{v}(\bar{r})$ in literature, e.g., the Cornell potential [16], the Richardson potential [17], the so-called QCD potentials [18, 19], and purely phenomenological potentials [20,21], etc. The common feature is that they are all flavor-independent. This independence is supported in a model-independent way by applying the inverse scattering approach to extract a potential from the measured spectra [22]. Let us take a QCD-like interaction of the form

$$\mathrm{v}(\bar{r}) = \sigma(\bar{r}) - \frac{4}{3}\frac{\alpha_s(\bar{r})}{\bar{r}}. \tag{43}$$

The linear term $\sigma\bar{r}$ is the long-range confining part. It is consistent with modern lattice simulations [23] and string investigations [24]. The second term is to incorporate perturbative effect. To second order in perturbation theory, one has [18,19]

$$\alpha_s(\bar{r}) = \frac{4\pi}{b_0\lambda(\bar{r})}\left[1 - \frac{b_1}{b_0^2}\frac{\ln\lambda(\bar{r})}{\lambda(\bar{r})} + \frac{b_2}{\lambda(\bar{r})}\right], \tag{44}$$

where [25]

$$\lambda(\bar{r}) \equiv \ln\big[(\bar{r}\Lambda_{\overline{ms}})^{-2} + b\big] \tag{45}$$

with $b_0 = (11N_\mathrm{c} - 2N_\mathrm{f})/3$, $b_1 = [34N_\mathrm{c}^2 - N_\mathrm{f}(13N_\mathrm{c}^2 - 3)/N_\mathrm{c}]/3$, and $b_3 = (31N_\mathrm{c} - 10N_\mathrm{f})/(9b_0)$ for $SU(N_\mathrm{c})$ and N_f flavors.

Besides these constants, there are three parameters, i.e., σ, $\Lambda_{\overline{ms}}$, and b. The QCD scale parameter is usually taken to be $\Lambda_{\overline{ms}} = 300$ MeV. The value for the string tension σ from potential models varies in the range 0.18–0.22 GeV^2 [26], and we here take $\sigma = 0.2\ \mathrm{GeV}^2$. As for the parameter b, we take three values, i.e., 10, 20 and 30, in the reasonable range [25]. Because these parameters are determined from heavy quark experimental data, the initial value m_0^* is taken

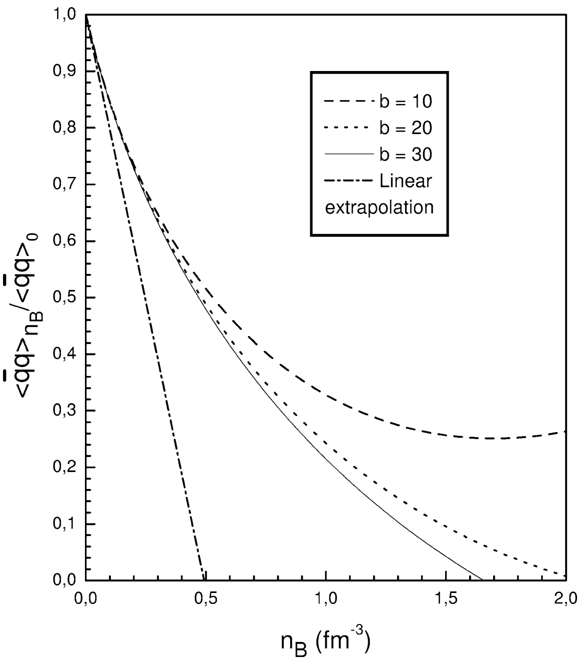

Fig. 1. Density dependence of the quark condensate in quark matter. The straight line is the linear extrapolation of Eq. (27). The other three lines are for $m_0 = 7.5$ MeV, but for different b values, as indicated in the legend.

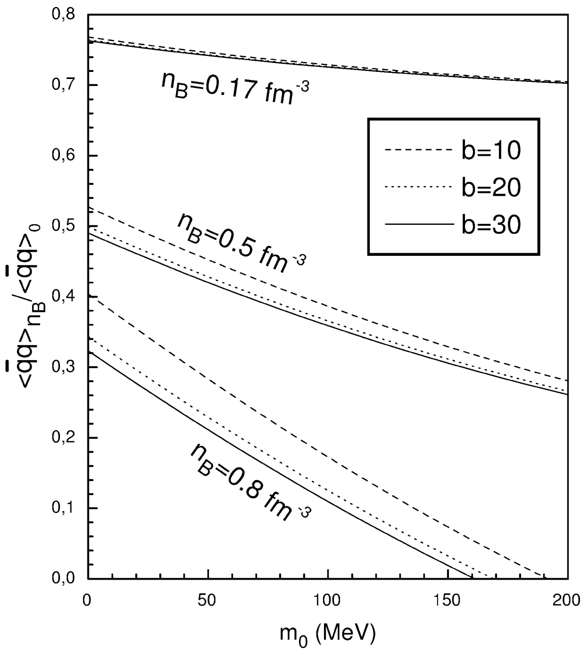

Fig. 2. Current mass dependence of the quark condensate in quark matter.

to be 1500 MeV which is compatible with the mass of heavy quarks. The numerical results are plotted in Figs. 1 and 2.

Fig. 1 shows the density behavior of the chiral condensate in quark matter. The straight line is the linear extrapolation of Eq. (27). It does not depend on the form of the inter-quark interaction $\mathrm{v}(\bar{r})$, and so, at this meaning, 'model-independent'. The other three lines are for $m_0 = 7.5$ MeV, but for different b values, as indicated in the legend. At lower densities, the chiral condensate decreases linearly with increasing densities. When the density becomes higher, the decreasing speed is slowed. For a bigger value of b, the condensate finally approaches to zero while for smaller values, it will saturate.

Fig. 2 shows the current mass dependence of the chiral condensate in quark matter. In general, the condensate drops if the current mass become large. This effect is especially obvious at higher densities. At lower densities, the condensate changes little with increasing current mass. This is understandable: because we have in fact assumed that the vacuum condensate or the quantity n^* in Eq. (27) is constant, the lower density condensate is nearly only a function of the density.

It should be noted that if the Fermi momentum in the first equality of Eqs. (8) and (12) is expressed, instead of Eq. (9), as $k_\mathrm{f} = [(18/g)\pi^2 n_\mathrm{B}/N_\mathrm{f}]^{1/3}$ because $3n_\mathrm{B}/N_\mathrm{f}$ is the flavor-averaged number density, the main Eqs. (23) and (24) are still valid while the flavor number N_f in Eq. (25) and accordingly in Eqs. (26), (34) and (38)–(42), will disappear. However, the gross feathers of the chiral condensate do not change significantly.

In summary, a new formalism to calculate the in-medium chiral condensate has been presented. The great advantage of this approach is that one does not need to make further assumptions on the current mass derivatives of model parameters. Lower density condensate from this method is liner and independent of the concrete form of inter-quark interactions. If this linear result is compared with the previous model independent result, the pion–nucleon sigma term could be 6 times the average current mass of light quarks. The decreasing speed of the condensate with increasing densities may be slowed by the QCD-like interactions.

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