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A high-order nonlinear Schrödinger equation with the weak non-local nonlinearity and its optical solitons

K. Hosseini [a], K. Sadri [a], M. Mirzazadeh [b], Y.M. Chu [c,*], A. Ahmadian [d,e], B.A. Pansera [e], S. Salahshour [f,*]

[a] Department of Mathematics, Rasht Branch, Islamic Azad University, Rasht, Iran
[b] Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan, P.C. 44891-63157 Rudsar-Vajargah, Iran
[c] Department of Mathematics, Huzhou University, Huzhou 313000, PR China
[d] Institute of IR 4.0, The National University of Malaysia, Bangi, 43600 UKM, Selangor, Malaysia
[e] Department of Law, Economics and Human Sciences & Decisions Lab, Mediterranea University of Reggio Calabria, Reggio Calabria 89125, Italy
[f] Faculty of Engineering and Natural Sciences, Bahcesehir University, Istanbul, Turkey

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ABSTRACT

The present paper explores a high-order nonlinear Schrödinger equation in a non-Kerr law media with the weak non-local nonlinearity describing solitons' propagation through nonlinear optical fibers. To this end, the real and imaginary parts of the model are firstly extracted using a wave variable transformation. The modified Kudryashov method and symbolic computations are then adopted to successfully retrieve optical solitons of the model. The results presented in the current study demonstrate the great performance of the modified Kudryashov method in handling high-order nonlinear Schrödinger equations.

Introduction

The extent of published papers in the last two decades with the subject of ''Soliton Solutions of Nonlinear PDEs'' reveals the importance of such an important area of research. Particularly, the spread of symbolic computation packages led to emerging a group of systematic methods to retrieve soliton solutions of nonlinear PDEs. Among recently introduced methods, the new Kudryashov method [1–4] has gained a significant interest because of its potential in dealing with nonlinear PDEs. The new Kudryashov method considers the solution of a nonlinear ODE like

$$O(U(\epsilon), U^{'}(\epsilon), U^{''}(\epsilon), \dots) = 0, \qquad (1)$$

as the following finite series

$$U(\epsilon) = a_0 + \sum_{i=1}^{N} a_i K^i(\epsilon), a_N \neq 0, \qquad (2)$$

where $a_i (0 \le i \le N)$ are found later, N is derived throught applying the balance principle, and $K(\epsilon)$ satisfies the following equation

$$(K^{'}(\epsilon))^2 = K^2(\epsilon)(1 - \eta K^2(\epsilon)), K(\epsilon) = \frac{4A}{(4A^2 - \eta)\sinh(\epsilon) + (4A^2 + \eta)\cosh(\epsilon)}, \eta = 4AB,$$

where A and B are free parameters. The impressive performance of the new Kudryashov method has persuaded Hosseini and his collaborators to frequently adopt such a distinguished method in their works. More information can be found in [5–8].

There are many nonlinear PDEs to describe the pulse propagation in optical fibers. The Triki–Biswas equation [9–11], the Fokas–Lenells equation [12–14], the Biswas–Arshed equation [15–17], and the Kudryashov equation [18–20] are the most commonly used models that have been studied by a group of academic researchers. In this paper, the following high-order nonlinear Schrödinger equation with the weak non-local nonlinearity [21]

$$i\frac{\partial u}{\partial t} + ic_1\frac{\partial^3 u}{\partial x^3} + c_2\frac{\partial^4 u}{\partial x^4} + \left(c_3|u|^2 + c_4|u|^4 + c_5\frac{\partial^2|u|^2}{\partial x^2}\right)u - i\left(c_6\frac{\partial u}{\partial x} + c_7\frac{\partial|u|^2 u}{\partial x} + c_8 u\frac{\partial|u|^2}{\partial x}\right) = 0, \qquad (3)$$

* Corresponding authors.
E-mail addresses: chuyuming@zjhu.edu (Y.M. Chu), soheil.salahshour@eng.bau.edu.tr (S. Salahshour).

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is considered. In Eq. (3), u is a complex-valued function, the first term is the linear temporal evolution, c_1 and c_2 are the coefficients of the third order and fourth order dispersions, c_3 and c_4 are the coefficients of the parabolic nonlinearity while c_5 is the coefficient of the non-local nonlinearity, c_6 is the coefficient of the inter-modal dispersion, c_7 is the coefficient of the self-steepening term, and c_8 is the coefficient of the nonlinear dispersion. To inform the readers, new solitons and other solutions of the model (3) have been extracted in [21] using the unified Riccati expansion method. Hosseini et al. [22] obtained soliton solutions of special cases of Eq. (3) using the new Kudryashov method.

The authors' concern in the present paper is employing the modified Kudryashov method to look for soliton solutions of the above high-order nonlinear Schrödinger equation with the weak non-local nonlinearity. The modified Kudryashov method benefits from utilizing the following finite series [23,24]

$$U(\epsilon)=a_0+\sum_{i=1}^{N}\left(\frac{K(\epsilon)}{1+K^2(\epsilon)}\right)^{i-1}\left(a_i\frac{K(\epsilon)}{1+K^2(\epsilon)}+b_i\frac{1-K^2(\epsilon)}{1+K^2(\epsilon)}\right),a_N\text{or}b_N\neq 0, \tag{4}$$

as the solution of Eq. (1). Such a representation provides a group of soliton solutions that are different from those obtained using its classical version. Recently, Hosseini et al. [24] used successfully the modified Kudryashov method to derive optical solitons of an integrable nonlinear Schrödinger equation in (2 + 1) dimensions. More papers have been listed in [25–37].

The remainder of this paper is as follows: In Section 2, a wave variable transformation is formally adopted to extract the real and imaginary parts of the model. In Section 3, the modified Kudryashov method and symbolic computations are utilized to retrieve optical solitons of the model. The last section presents a summary of the results.

The governing model and its reduced form

To obtain the reduced form of the governing model with the weak non-local nonlinearity, a wave variable transformation is considered as

$$u(x,t)=U(\epsilon)e^{i(-\kappa x+\mu t)},\ \epsilon=x-\upsilon t, \tag{5}$$

where υ, κ, and μ are the soliton velocity, the soliton wave number, and the soliton frequency. Substituting Eq. (5) into the model (3) and separating the real and imaginary parts results in

$$c_2\frac{d^4U(\epsilon)}{d\epsilon^4}+\left(3\kappa c_1-6\kappa^2c_2\right)\frac{d^2U(\epsilon)}{d\epsilon^2}+\left(\kappa^4c_2-\kappa^3c_1-\kappa c_6-\mu\right)U(\epsilon)+2c_5U(\epsilon)\left(\frac{dU(\epsilon)}{d\epsilon}\right)^2+2c_5\frac{d^2U(\epsilon)}{d\epsilon^2}U^2(\epsilon)+(c_3-\kappa c_7)U^3(\epsilon)+c_4U^5(\epsilon)=0, \tag{6}$$

$$(c_1-4\kappa c_2)\frac{d^3U(\epsilon)}{d\epsilon^3}+\left(4\kappa^3c_2-3\kappa^2c_1-\upsilon-c_6\right)\frac{dU(\epsilon)}{d\epsilon}-(3c_7+2c_8)\frac{dU(\epsilon)}{d\epsilon}U^2(\epsilon)=0. \tag{7}$$

The linearly independent principle is employed on Eq. (7) to obtain the soliton wave number, the soliton velocity, and the parameter c_8 as

$$\kappa=\frac{c_1}{4c_2},\upsilon=-\frac{c_1^3+8c_2^2c_6}{8c_2^2},c_8=-\frac{3c_7}{2}.$$

Now, one can rewrite Eq. (6) as follows

$$c_2\frac{d^4U(\epsilon)}{d\epsilon^4}+\frac{3c_1^2}{8c_2}\frac{d^2U(\epsilon)}{d\epsilon^2}-\left(\frac{3c_1^4}{256c_2^3}+\frac{c_1c_6}{4c_2}+\mu\right)U(\epsilon)+2c_5U(\epsilon)\left(\frac{dU(\epsilon)}{d\epsilon}\right)^2+2c_5\frac{d^2U(\epsilon)}{d\epsilon^2}U^2(\epsilon)-\left(\frac{c_1c_7}{4c_2}-c_3\right)U^3(\epsilon)+c_4U^5(\epsilon)=0. \tag{8}$$

The governing model and its optical solitons

Due to the terms $\frac{d^4U(\epsilon)}{d\epsilon^4}$ and $U^5(\epsilon)$, one can find the balance number as $N=1$. Consequently, the solution of Eq. (8) can be written as

$$U(\epsilon)=a_0+a_1\frac{K(\epsilon)}{1+K^2(\epsilon)}+a_2\frac{1-K^2(\epsilon)}{1+K^2(\epsilon)},a_1\text{or}a_2\neq 0, \tag{9}$$

where a_0, a_1, and a_2 are found later. By substituting Eq. (9) into Eq. (8), we find a system of nonlinear algebraic equations that its solution yields the following cases:

Case 1.. $B=0$,

$$a_0=0,$$

$$a_2=0,$$

$$c_2=-\frac{1}{384}a_1^4c_4+\frac{1}{16}a_1^2c_5,$$

$$c_3=-\frac{5a_1^8c_4^2-144a_1^6c_4c_5+576a_1^4c_5^2+2304a_1^2c_1c_7+27648c_1^2}{24a_1^4(a_1^2c_4-24c_5)},$$

$$\mu=-\left(a_1^{16}c_4^4-96a_1^{14}c_4^3c_5+3456a_1^{12}c_4^2c_5^2-55296a_1^{10}c_4c_5^3+55296a_1^8c_1^2c_4^2-36864a_1^8c_1c_4^2c_6+331776a_1^8c_5^4-2654208a_1^6c_1^2c_4c_5+1769472a_1^6c_1c_4c_5c_6+31850496a_1^4c_1^2c_5^2-21233664a_1^4c_1c_5^2c_6-254803968c_1^4\right)/\left(384a_1^6\left(a_1^2c_4-24c_5\right)^3\right)$$

Now, based on the Case 1, the following soliton solution to the model (3) with the weak non-local nonlinearity is derived

$$u_1(x,t)=\frac{a_1A\left(\sinh\left(x+\left(\frac{c_1^3+8c_2^2c_6}{8c_2^2}\right)t\right)+\cosh\left(x+\left(\frac{c_1^3+8c_2^2c_6}{8c_2^2}\right)t\right)\right)}{2A^2\sinh\left(x+\left(\frac{c_1^3+8c_2^2c_6}{8c_2^2}\right)t\right)\cosh\left(x+\left(\frac{c_1^3+8c_2^2c_6}{8c_2^2}\right)t\right)+2A^2\left(\cosh\left(x+\left(\frac{c_1^3+8c_2^2c_6}{8c_2^2}\right)t\right)\right)^2-A^2+1}\times e^{i\left(-\frac{c_1}{4c_2}x+\mu t\right)}, \tag{10}$$

where

$$c_2=-\frac{1}{384}a_1^4c_4+\frac{1}{16}a_1^2c_5,$$

$$c_3=-\frac{5a_1^8c_4^2-144a_1^6c_4c_5+576a_1^4c_5^2+2304a_1^2c_1c_7+27648c_1^2}{24a_1^4(a_1^2c_4-24c_5)},$$

$$c_8=-\frac{3}{2}c_7,$$

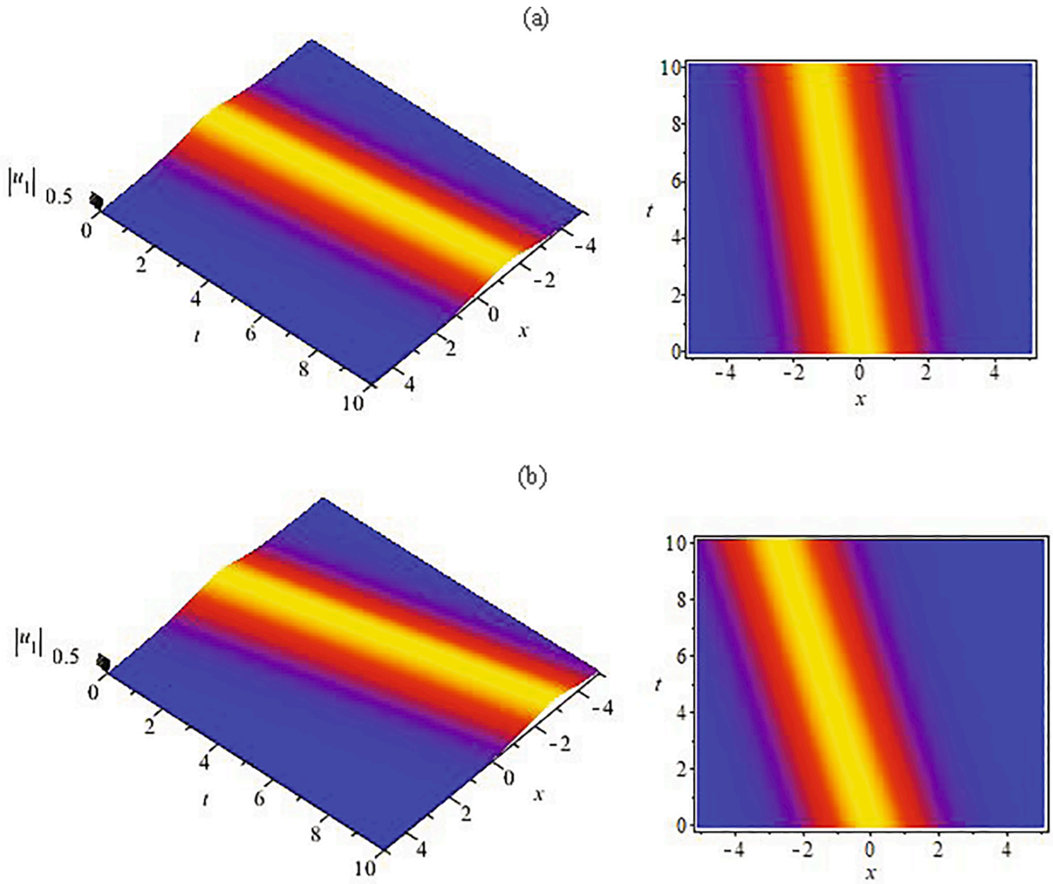

Fig. 1. Three-dimensional and density graphs of $|u_1(x,t)|$ (namely Eq. (10)) when $A = 1$, $c_4 = 0.1$, $c_5 = 1$, $c_6 = 0.1$, $a_1 = 1$ and (a) $c_1 = 0.1$; (b) $c_1 = 0.175$.

$$\mu = -(a_1^{16}c_4^4 - 96a_1^{14}c_4^3c_5 + 3456a_1^{12}c_4^2c_5^2 - 55296a_1^{10}c_4c_5^3 + 55296a_1^8c_1^2c_4^2 - 36864a_1^8c_1c_4^2c_6 + 331776a_1^8c_5^4 - 2654208a_1^6c_1^2c_4c_5 + 1769472a_1^6c_1c_4c_5c_6 + 31850496a_1^4c_1^2c_5^2 - 21233664a_1^4c_1c_5^2c_6 - 254803968c_1^4)/\left(384a_1^6\left(a_1^2c_4 - 24c_5\right)^3\right)$$

Case 2.. $B = 0$,

$$a_0 = 0,$$

$$a_1 = 0,$$

$$c_2 = -\frac{1}{24}a_2^4c_4 - \frac{1}{4}a_2^2c_5,$$

$$c_3 = -\frac{5a_2^8c_4^2 + 36a_2^6c_4c_5 + 36a_2^4c_5^2 + 18a_2^2c_1c_7 - 54c_1^2}{3a_2^4(a_2^2c_4 + 6c_5)},$$

$$\mu = -2\left(a_2^{16}c_4^4 + 21a_2^{14}c_4^3c_5 + 162a_2^{12}c_4^2c_5^2 + 540a_2^{10}c_4c_5^3 - 27a_2^8c_1^2c_4^2 - 9a_2^8c_1c_4^2c_6 + 648a_2^8c_5^4 - 324a_2^6c_1^2c_4c_5 - 108a_2^6c_1c_4c_5c_6 - 972a_2^4c_1^2c_5^2 - 324a_2^4c_1c_5^2c_6 - 243c_1^4\right)/\left(3a_2^6\left(a_2^2c_4 + 6c_5\right)^3\right)$$

Now, based on the Case 2, the following soliton solution to the model (3) with the weak non-local nonlinearity is obtained

$$u_2(x,t) = \frac{a_2\left(A^4 + 4A^2\sinh\left(x + \left(\frac{c_1^3 + 8c_2^2c_6}{8c_2^2}\right)t\right)\cosh\left(x + \left(\frac{c_1^3 + 8c_2^2c_6}{8c_2^2}\right)t\right) - 1\right)}{A^4 + 4A^2\left(\cosh\left(x + \left(\frac{c_1^3 + 8c_2^2c_6}{8c_2^2}\right)t\right)\right)^2 - 2A^2 + 1} \times e^{i}\left(-\frac{c_1}{4c_2}x + \mu t\right), \quad (11)$$

where

$$c_2 = -\frac{1}{24}a_2 4c_4 - \frac{1}{4}a_2^2c_5,$$

$$c_3 = -\frac{5a_2^8c_4^2 + 36a_2^6c_4c_5 + 36a_2^4c_5^2 + 18a_2^2c_1c_7 - 54c_1^2}{3a_2^4(a_2^2c_4 + 6c_5)},$$

$$c_8 = -\frac{3}{2}c_7,$$

$$\mu = -2\left(a_2^{16}c_4^4 + 21a_2^{14}c_4^3c_5 + 162a_2^{12}c_4^2c_5^2 + 540a_2^{10}c_4c_5^3 - 27a_2^8c_1^2c_4^2 - 9a_2^8c_1c_4^2c_6 + 648a_2^8c_5^4 - 324a_2^6c_1^2c_4c_5 - 108a_2^6c_1c_4c_5c_6 - 972a_2^4c_1^2c_5^2 - 324a_2^4c_1c_5^2c_6 - 243c_1^4\right)/\left(3a_2^6\left(a_2^2c_4 + 6c_5\right)^3\right)$$

Case 3.. $B = 0$,

$$a_0 = 0,$$

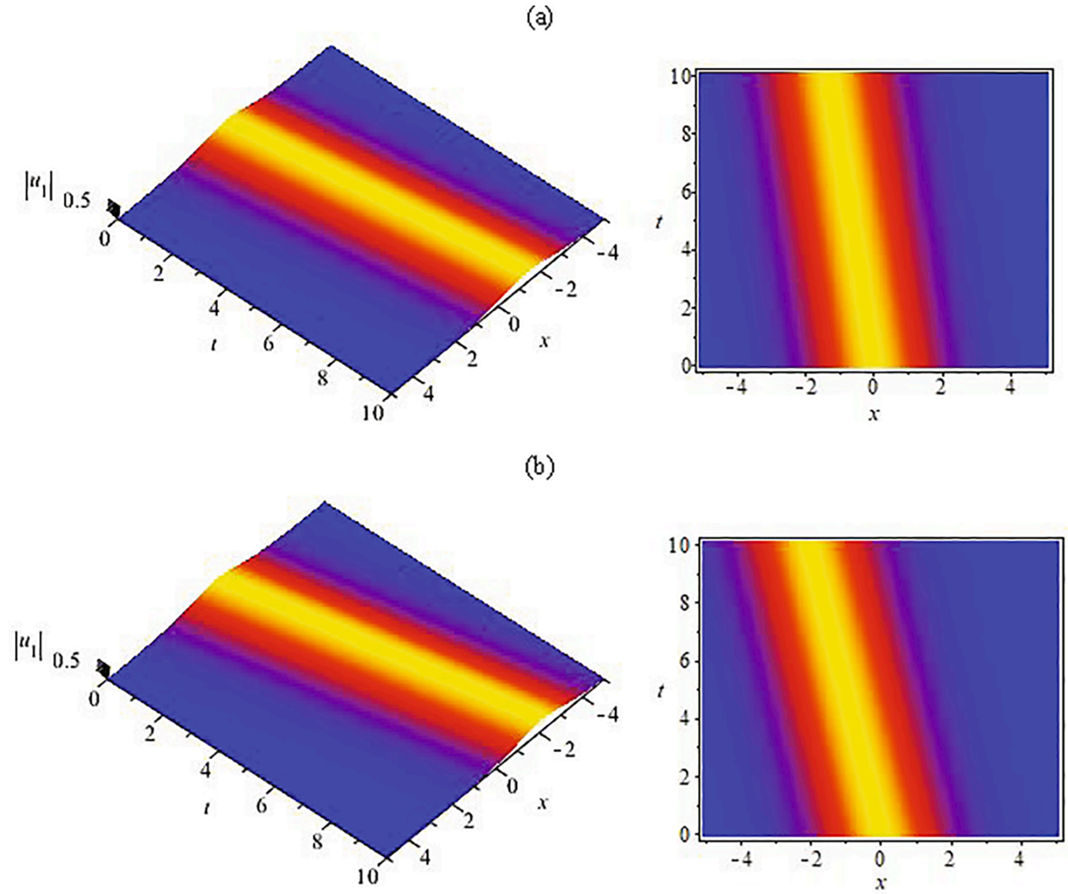

Fig. 2. Three-dimensional and density graphs of $|u_1(x,t)|$ (namely Eq. (10)) when $A = 1$, $c_1 = 0.1$, $c_4 = 0.1$, $c_5 = 1$, $a_1 = 1$ and (a) $c_6 = 0.1$; (b) $c_6 = 0.175$.

$$a_1 = \pm\sqrt{-\frac{12{c_1}^2 - 352{c_2}^2}{3{c_1}^2 + 8{c_2}^2}}a_2,$$

$$c_3 = \frac{72{a_2}^2c_1{c_2}^2c_7 - 9{c_1}^4 - 174{c_1}^2{c_2}^2 - 400{c_2}^4}{288{a_2}^2{c_2}^3},$$

$$c_4 = \frac{(3{c_1}^2 + 8{c_2}^2)^2}{384{a_2}^4{c_2}^3},$$

$$c_5 = -\frac{(3{c_1}^2 - 40{c_2}^2)(3{c_1}^2 + 8{c_2}^2)}{576{a_2}^2{c_2}^3},$$

$$\mu = -\frac{45{c_1}^4 + 1104{c_1}^2{c_2}^2 + 576c_1{c_2}^2c_6 + 2816{c_2}^4}{2304{c_2}^3}.$$

Now, based on the Case 3, the following exact solutions to the model (3) with the weak non-local nonlinearity are derived

$$u_{3,4}(x,t) = \frac{a_2\left((A^2+1)\sinh\left(x + \left(\frac{{c_1}^3 + 8{c_2}^2c_6}{8{c_2}^2}\right)t\right) + (A^2-1)\cosh\left(x + \left(\frac{{c_1}^3 + 8{c_2}^2c_6}{8{c_2}^2}\right)t\right) \pm 2\sqrt{-\frac{3{c_1}^2 - 88{c_2}^2}{3{c_1}^2 + 8{c_2}^2}}A\right)}{(A^2-1)\sinh\left(x + \left(\frac{{c_1}^3 + 8{c_2}^2c_6}{8{c_2}^2}\right)t\right) + (A^2+1)\cosh\left(x + \left(\frac{{c_1}^3 + 8{c_2}^2c_6}{8{c_2}^2}\right)t\right)}$$

$$\times e^{i\left(-\frac{c_1}{4c_2}x - \frac{45{c_1}^4 + 1104{c_1}^2{c_2}^2 + 576c_1{c_2}^2c_6 + 2816{c_2}^4}{2304{c_2}^3}t\right)},$$

where

$$c_3 = \frac{72{a_2}^2c_1{c_2}^2c_7 - 9{c_1}^4 - 174{c_1}^2{c_2}^2 - 400{c_2}^4}{288{a_2}^2{c_2}^3},$$

$$c_4 = \frac{(3{c_1}^2 + 8{c_2}^2)^2}{384{a_2}^4{c_2}^3},$$

$$c_5 = -\frac{(3{c_1}^2 - 40{c_2}^2)(3{c_1}^2 + 8{c_2}^2)}{576{a_2}^2{c_2}^3},$$

$$c_8 = -\frac{3}{2}c_7.$$

To illustrate the dynamical behavior of Eqs. (10) and (11), the authors have considered Figs. 1-4. Eq. (10) has been plotted in Fig. 1 for $A = 1$, $c_4 = 0.1$, $c_5 = 1$, $c_6 = 0.1$, $a_1 = 1$ and different values of the parameter c_1, namely (a) $c_1 = 0.1$; (b) $c_1 = 0.175$. This equation also is

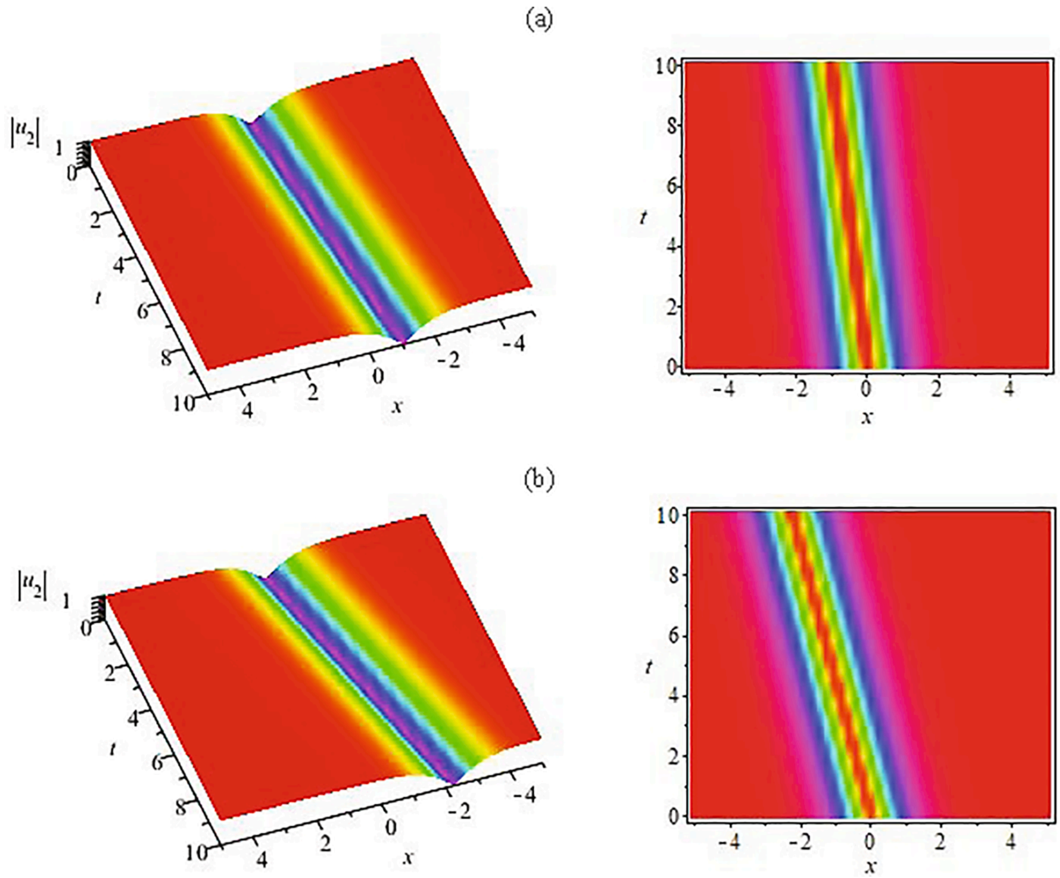

Fig. 3. Three-dimensional and density graphs of $|u_2(x,t)|$ (namely Eq. (11)) when $A = 1$, $c_4 = 0.1$, $c_5 = 1$, $c_6 = 0.1$, $a_2 = 1$ and (a) $c_1 = 0.1$; (b) $c_1 = 0.4$.

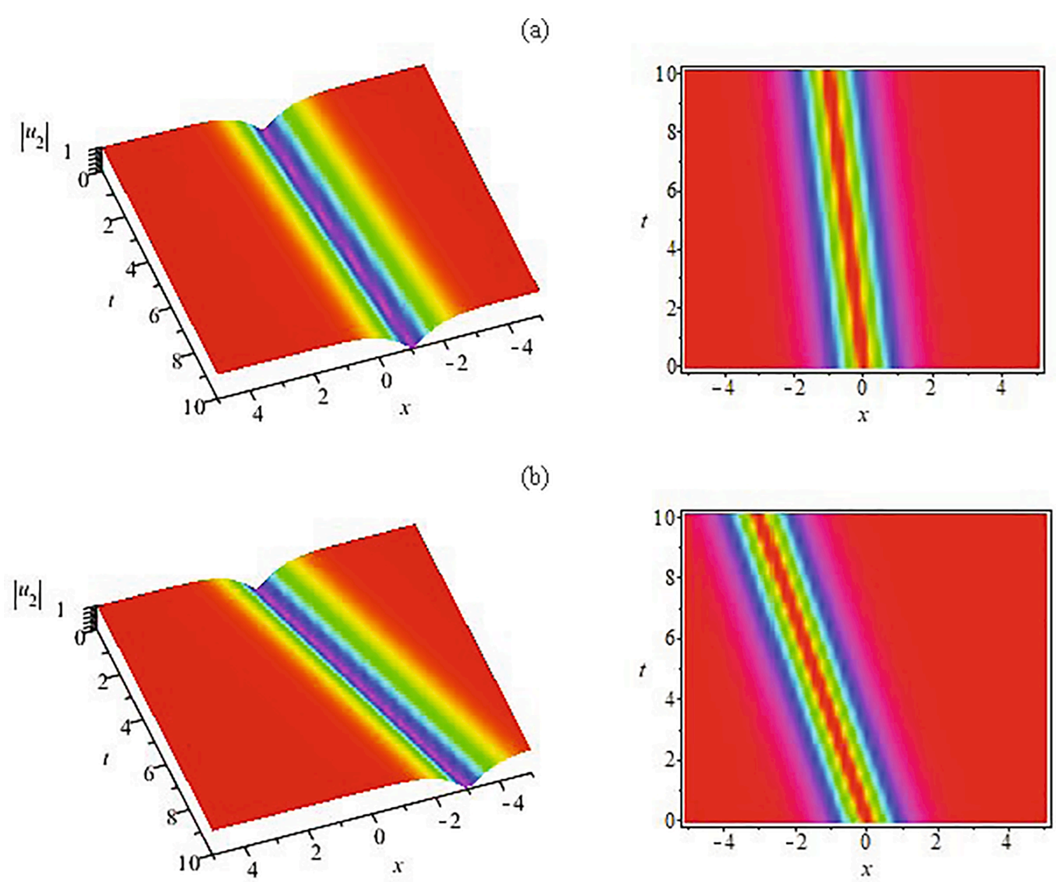

Fig. 4. Three-dimensional and density graphs of $|u_2(x,t)|$ (namely Eq. (11)) when $A = 1$, $c_1 = 0.1$, $c_4 = 0.1$, $c_5 = 1$, $a_2 = 1$ and (a) $c_6 = 0.1$; (b) $c_6 = 0.3$.

portrayed in Fig. 2 for $A = 1$, $c_1 = 0.1$, $c_4 = 0.1$, $c_5 = 1$, $a_1 = 1$ and different values of the parameter c_6, namely (a) $c_6 = 0.1$; (b) $c_6 = 0.175$. Fig. 3 shows three-dimensional and density graphs of Eq. (11) for $A = 1$, $c_4 = 0.1$, $c_5 = 1$, $c_6 = 0.1$, $a_2 = 1$ and different values of the parameter c_1, namely (a) $c_1 = 0.1$; (b) $c_1 = 0.4$. Three-dimensional and density graphs of this equation are also plotted in Fig. 4 for $A = 1$, $c_1 = 0.1$, $c_4 = 0.1$, $c_5 = 1$, $a_2 = 1$ and different values of the parameter c_6, namely (a) $c_6 = 0.1$; (b) $c_6 = 0.3$. Obviously, the first two figures show bright solitons while the last two figures indicate dark solitons. A bright soliton is featured as a localized intensity peak above a continuous wave background while a dark soliton is characterized as a localized intensity dip below a continuous wave background [38].

Remark: It is noted that soliton solutions given in this study are different from those presented in [21].

Conclusion

The authors' concern of this paper was to analytically explore a high-order nonlinear Schrödinger equation in a non-Kerr law media with the weak non-local nonlinearity. Such a goal was formally carried out by considering a wave variable transformation and the modified Kudryashov method to successfully retrieve optical solitons of the model. The results presented in the current study demonstrated the great performance of the modified Kudryashov method in handling high-order nonlinear Schrödinger equations.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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