



12th International Conference on Hydroinformatics, HIC 2016

## Experimental evidence of the discharge law in private tanks connected to water distribution networks

Mauro De Marchis\*, Gabriele Freni, Barbara Milici

*Faculty of Engineering and Architecture, University of Enna "Kore", Enna, Italy*

---

### Abstract

In almost all the Mediterranean countries, users store water resources in private tanks usually located on the rooftops. These local reservoirs are usually connected to the water distribution network (WDN). In such cases, network-operating conditions can be far from design ones, thus, specific models have to be developed to correctly simulate the WDN. Here, a new mathematical model able to reproduce the tank emptying/filling cycles is proposed. Specifically, through experimental analysis a new mathematical formulation of the emitter law is proposed.

© 2016 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Peer-review under responsibility of the organizing committee of HIC 2016

*Keywords:* Tank modeling; Discharge law; Water distribution network; experimental pipe network

---

### 1. Introduction

Most of the models for Water Distribution Network (WDN), developed to design and analyse the hydraulic system for planning, usually account for customer-demands as water withdrawal concentrated in nodes and are designed assuming nodal demands fixed a priori in the model, independent of nodal heads, and fully supplied (demand driven analysis). In demand-driven simulation methods, in fact, it is assumed that available discharge in demand nodes is always equal to the required discharge in those nodes and under any circumstances the network is able to provide the required demand, due to the independence between flow and pressure in the node. However, as known, the discharge of any node is proportional to its head and demand driven analysis is valid just when the hydraulic pressures at the nodes are adequate to demands, whereas under pressure deficient conditions the actual flow supplied

---

\* Corresponding author. Tel.: +39-0935-536438; fax: +39-0935-536710.  
*E-mail address:* [Mauro.demarchis@unikore.it](mailto:Mauro.demarchis@unikore.it)

to the consumers might decrease and some nodes might not be able to supply any discharge. Therefore, even though a WDS may perform satisfactorily under normal working condition, it may become temporarily deficient under pressure deficient conditions (due to improper design of the network or insufficient water supply from water sources, unplanned pipe outages or unexpected pipe breaks, valve failure, pump breakdown or excessive consumption). In these conditions the demand driven formulation leads to unrealistic solutions for WDNs hydraulic analysis. A good method for calculating the flow actually delivered by water distribution systems with unsatisfactory pressure, therefore, has to consider the relationship between nodal head and discharge (head driven analysis). Over the last three decades, various relationships have been proposed to link the available discharge to the nodal pressure. These relationships are divided into two categories: discontinuous and continuous ones. The analysis of the hydraulic networks assuming the demands as dependent on head/pressure status started with Bhave [2] who defined the minimum required nodal head value for normal working conditions and suggested a relationship which falls in the former category. This relationship admits that full demand is available for heads higher than the minimum required value: a  $[0,1]$  concept is used to express the pressure-outflow relation. In such a way over the minimum required head the demand is fully supplied whereas for heads lower than the minimum required value there is no discharge. On the other hand continuous relations attempt to consider the relationship between pressure and outflow discharge for the entire variation domain continuously. Bhave [2] was probably the first to consider the nodal flows and heads simultaneously. Later, Reddy and Elango [14] suggested head-dependent analysis, assuming uncontrolled outlets wholly dependent on residual heads according to  $Q_j = S_j (H_j - H_{min})^{0.5}$ , where  $S_j$  is a node constant,  $H_j$  represents  $j$  the available total head at demand node  $j$ ;  $H_{min}$  represents the pressure head below which service at demand node  $j$  is unavailable. Wagner et al. [22] proposed a similar relation for a general orifice. This relationship is a continuous function, with upper and lower bounds consistent with WDN's real behaviour. Germanopoulos [10] proposed a formula to calculate the available outflows at demand nodes, having a constant flow rate for high level of pressure in the network, zero value for pressure lower than a minimum threshold and an exponential law for intermediate level of pressure at the node. Gupta and Bhave [12] presented a comprehensive comparison of the head-discharge formulations developed by various researchers (see [2], [22], [9], [3]) and modified by Germanopoulos [10]. Even though the proposed relation addressed some weaknesses of Germanopoulos equation, some deviations between the discharge request and calculated were observed, thus more recently several researchers proposed new expressions able to improve precision (see [1], [4], [18], [19], [20], [21]). Starting from Wagner et al. [22], Tabesh et al. [17] considered nodal discharge in four conditions.

It is not intent of the present analysis to enter within the several mathematical formulations, especially because all the formulations are valid for continuous supply. The problem rises when water supplies are inadequate to meet consumer's demands. In many situations, when the network does not operate in normal conditions, unequal distribution of water, due to pressure head distribution different from designed, may cause serious disadvantage in terms of user's supply. The problem is more severe when there is shortage of supply or intermittent distribution (see [5], [8]). When the user experiences water resources rationing due to water shortage, in many technical situations, the service pipe often fills a local private storage (e.g. a roof tank or a basement tank), interposed between the water meter and the users, from which the water is actually delivered to customers by gravity or pumping systems, according to the position of the water storage, i.e. on the roof or in the basement, respectively). Such local water storage volumes are actually very common in those countries (e.g. in the Mediterranean area) where water supply is not reliable and users are brought to collect water by building private tanks during distribution periods, in order to cover their needs when the service is unavailable. In this operative scheme, customer water demand is supplied by the tank, which is subjected to a filling/emptying, connected to the hydraulic system.

Therefore, private tanks modify the demand profile of normal domestic users. The replenishment is controlled by proportional float valves that open partially or totally as a function of water level (under the assumption that the node required pressure is available). During periods of low consumption the water level does not fall as much to induce the valve to open completely (when the tank is almost full) and it dampens the instantaneous water demand and consequently the flow rate passing through the meter is lower than the user's demand. In order to correctly model the user water consumption, when water roof tanks are adopted in the network, that is to estimate the actual flow spilled from the network which fills the tank, it is necessary to evaluate the pressure-consumption law taking into account the hydraulic behaviour of the reservoir ruled by the ball valve.

In WDN, characterized by the presence of several private tanks, where the classic head/discharge relationships cannot be employed, specific models need to be developed to correctly simulate the WDN operation, accounting for reservoirs located between the hydraulic network and users (see [11]). Modeling the inflow/outflow process of the

tank needs to consider the occurrence of a pressure dependent water demand at the node (classic head/discharge relationship) when the tank is not full and the valve completely open and the closure of the orifice when the tank is completely filled. Actually, the orifice feeding inline tanks are controlled by floating valves, which follow nonlinear behaviour depending on the valve type, therefore, in order to correctly analyse the effect of inline private tanks in WDN models an accurate discharge law is needed. A mathematical model able to reproduce the tank emptying/filling cycles has been developed by Criminisi et al. [4], which combines a tank continuity equation with a float valve emitter law. The tank model has been recently validated against experimental data [7], showing that even though the emitter law well reproduces the experimental data, some deviations are observed at the starting valve opening and closing. This work proposes a modified version of the emitter law of Criminisi et al. [4]. The new formulation is able to reproduce the whole experimental data set. The mathematical model of the new emitter law is validated for two different valve branches, thus achieving a formula that can be used irrespective of the tank and valve geometry. The model can be easily used in hydrodynamic models to take into account for the private tanks.

**2. The mathematical models to estimate the flow rate in inline tanks**

The pressure-discharge relation used in WDN continuity equations is one of the most important components of hydraulic models based on head-driven simulation methods. In the last few years, various relationships have been proposed to link the available discharge to the nodal pressure (see [2], [10], [22], [14], [3], [13], [12], [21], [1], [18], [19], [20], [4] among many). This work proposes the demand modeling accounting for local storages actually supplying water to customers (by gravity or pumping systems, according to the position of the water storage, i.e. on the roof or in the basement, respectively), describing the effect of the floating valve on the demand profile. A modified version of Criminisi et al. [4] is described below. The model is able to represent tank inflow process taking into account float valve characteristics as a function of tank water level. The model is based on the combination of the tank continuity equation:

$$Q_{up} - D = \frac{dV}{dt} = A \frac{dh}{dt} \tag{1}$$

and the float valve emitter law, consistent with the Torricelli law (the kinetic component is considered negligible):

$$Q_{up} = C_v a \sqrt{2g(H - z_r)} \tag{2}$$

where  $D$  and  $Q_{up}$  are the user water demand and the discharge, respectively,  $V$  is the storage volume having area  $A$  and variable water depth  $h$ .  $C_v$  is the float valve emitter coefficient,  $a$  is the valve discharge area,  $H$  the hydraulic head over the distribution network,  $z_r$  is the height of the floating valve and  $g$  the gravity acceleration. Both the float valve emitter coefficient and the discharge area depend on the floater position, that is on the water level in the tank. Here the valve discharge area was kept constant, whereas the emitter coefficient was calculated according to the following empirical laws:

$$c_v = f(h) = \begin{cases} C_v = C_v^* \rightarrow h < h_{min} \\ C_v = C_v^* \left( \frac{h_{max} - h}{h_{max} - h_{min}} \right)^m \rightarrow h > h_{min} \end{cases} \tag{3}$$

where  $h_{min}$  and  $h_{max}$  are respectively the water depths when the valve is fully open and fully closed, respectively.  $C_v^*$  is the emitter coefficient and discharge area of the fully open valve and  $m$  is a shape coefficient usually ranging between 0.5 and 2, to be experimentally estimated

$$c_v = f(h) = \begin{cases} C_v = C_v^* \rightarrow h < h_{\min} \\ C_v = C_v^* \cdot \tanh\left(m \frac{h_{\max} - h}{h_{\max} - h_{\min}}\right) \rightarrow h > h_{\min} \end{cases} \quad (4)$$

The formulation is based on a laboratory set able to simulate the real operation a part of a real water distribution network where a roof reservoir is installed. The equation 4 was obtained through the best fit to the laboratory measurements, used to compare the proposed pressure-discharge relationship proposed for demands when a tank is present.

### 3. Experimental setup

Several experiments were carried out at the Environmental Hydraulic Laboratory of the University of Enna (Italy) to analyse the law of the flow valve. The flow facility is a water distribution network having high-density polyethylene (HDPE 100 PN16) pipes. The network is designed with three main loops and is characterised by nine nodes and eleven pipes having a diameter DN 63 mm. Each pipe is about 45 m long. The network has a modular behaviour; thank to three ball valves located in three different pipes, in fact, it is possible to simulate networks having one, two or three loops as well as inline pipe of about 500 m. In Fig. 1 it is plotted a schematic representation of the water distribution network. A pump station, composed by four pumps (P), working in a parallel way, supplies the network from the recycling reservoirs. All the pumps are equipped with inverter thus to change the speed rotation and vary the water head in a range between 10 to 60 m. The system is monitored by 6 electromagnetic flow meters. Pressure cells and multi-jet water meters are distributed over the whole network at each node position. The network is designed to model the effect of real losses as well as apparent losses. The apparent loss are analysed through a private tank, located in the roof of the laboratory at about 17.5 m above the network level, and connected to the system by means of high-density polyethylene (HDPE 100 PN16) pipe 30 m long, with a diameter of (1/2)". The tank filling process is governed by a float ball valve. Fig. 2 plots a scheme of water tank and float valve. Details on the water distribution system can be found in De Marchis et al. [7].

In order to investigate on the optimal empirical law to calculate the flow discharging into the local reservoir, two sets of experiments were carried out, modifying the length of float valve branch. Specifically, in the first set of simulation we installed a branch about 40 cm long (hereafter referred as test case 1, TC1), whereas in the second set of experiments the float valve was equipped with a branch of about 20 cm (hereafter indicated with test case 2, TC2), see Fig. 2. In the first test case the float valve starts to close when the water level inside the tank reaches depth  $h_{\min}=0.53$  cm, whereas when the water depth is  $h_{\max}=0.65$  cm, the valve can be considered completely closed. On the other hand, in the TC2  $h_{\min}=0.60$  cm. In all cases, the experiments have been carried out considering that at the beginning of each test the water tank was empty and successively filled considering the daily water supply of the user (350 l per inhabitant per day); this condition simulates a daily intermittent network in which the tank is filled every two days and it is emptied in the day in which water supply is not guaranteed (see [5], [6]). For each test case, four scenarios have been simulated changing the pressure from 2 bar to 5 bar, with step of 1 bar. The water volume flowing into the tank has been estimated using the electromagnetic flow meter (having an accuracy of 0.4%) installed in the inlet pipe, downstream the pump station. The hydraulic head  $H$  used in eq. 5 to estimate  $Q_{up}$  has been measured by means of a piezoresistive pressure transducer (having an accuracy of 0.1%) located at the node 9 (see Fig. 1). The data have been collected with a sampling rate of 1 Hz. Each experiment has been repeated twice and the average value has been considered.

### 4. Results

The experiments have been carried out to find a mathematical law able to reproduce the flow discharging into private tanks, especially during the closure phase of the float valve. In Fig. 3 the data collected with the set of experiment TC1 have compared with the law proposed by Criminisi et al. [4] and the hyperbolic law here proposed. Four different pressures have been considered. Overall as expected, the more the hydraulic head grows the more the

flow rate  $Q$  increases. Furthermore, the filling process becomes more and more short and the front of the curve, representative of the closure phase of the valve, progressively increases its slope. This increase is coherent with a more rapid closure of the valve. Comparing the Fig. 3(a) and 3(d), the filling process is completed in about 45 minutes, with a water head of 2 bar, whereas increasing the network pressure ( $P = 5$  bar), the tank is completely filled in about 30 minutes. The comparison between the experimental and hyperbolic laws clearly shows that even though both equations are able to reproduce the experimental data, the hyperbolic tangent law, Eq. 4, perfectly reproduces the filling process, whereas the exponential law fails when the valve closure begins and during the final phase of closure. Obviously Eqs. 3 and 4 overlap each other until  $h < h_{min}$ . When  $h > h_{min}$  only the proposed hyperbolic tangent law perfectly reproduces the experimental flow rate during all the filling process. The collected data reported in Figs. 3 and 4 have been averaged thus to achieve one datum per minute. The proposed results demonstrate the ability of the hyperbolic tangent law to estimate the flow discharging into a private tank where the discharge is governed by a float valve. The hyperbolic tangent law can be, thus, used in numerical codes to improve the node demand models in all cases, typical of the Mediterranean countries, where the users cope the water scarcity condition with local reservoirs. In Table 1 is reported the values of the coefficients  $C_v^*$  and  $m$  achieved fitting the experimental data, both for the experimental and hyperbolic laws.

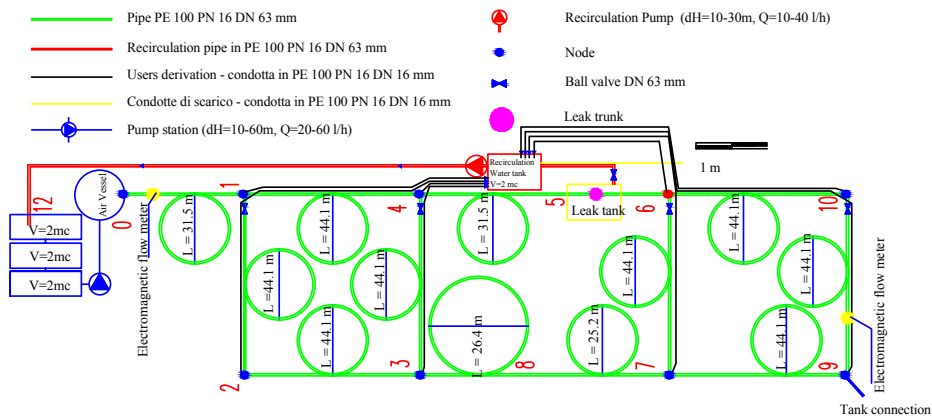
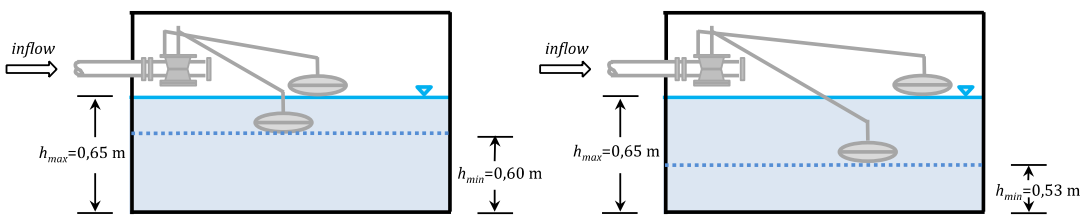


Fig. 1. Layout of the water distribution network.



In order to verify the validity of the proposed hyperbolic tangent law, the mathematical model has been compared with a second set of laboratory experiments, TC2. In Fig. 4 the data of flow rate collected using a short branch are plotted for the same values of network pressure as in the test TC1. Until the water inside the tank is lower than  $h_{min}$  the data overlap the results of the test TC1, whereas for  $h > h_{min}$  the valve closures are clearly more rapid in TC2 than those observed in TC1.

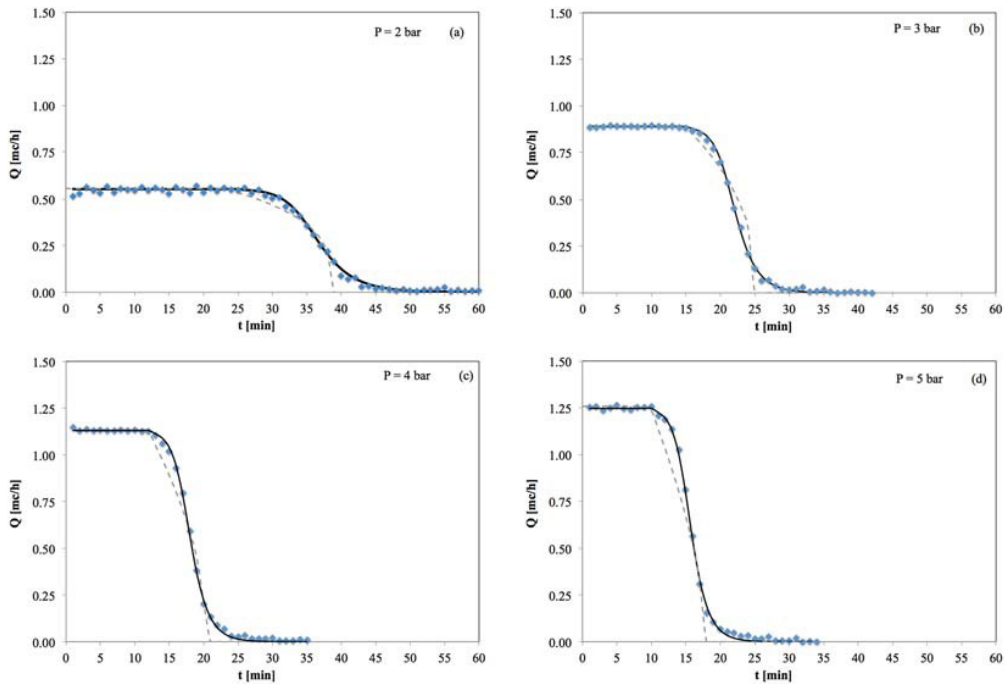


Fig. 3. Test case 1. Flow rate during the tank filling process. Symbols: Experiments; Bold line: hyperbolic law; dashed line: exponential law. (a) Pressure 2 bar; (b) Pressure 3 bar; (c) Pressure 4 bar; (d) Pressure 5 bar.

Table 1. Coefficients of the float valve laws achieved with the test case 1.

Pressure	2 bar	3 bar	4 bar	5 bar
$C_v^*$	0.61	0.39	0.36	0.33
m hyper law	2.94	2.80	2.80	2.42
m expon. law	0.23	0.30	0.36	0.40

Once again, the relation of Criminisi et al. [4] and the proposed hyperbolic law reproduce well the experiments, even though also in this case Eq. 4 shows a better agreement than Eq. 3. Table 2 collects the value of the coefficient  $C_v^*$  and  $m$  achieved fitting the experimental data, both for the experimental and hyperbolic laws. Comparing the data fitting of the hyperbolic tangent law shown in Figs. 3 and 4 it can be observed that a worst agreement is verified for the TC2. This is necessary if an universal law for the coefficient  $m$  needs to be obtained. As plotted in Fig. 5, in fact, the values of the coefficient  $m$  are clearly aligned for both series of experiments TC1 and TC2. To predict the flow rate, thus, the system of Eqs. 5 and 4 can be completed using the linear dependence of  $m$  with the water head through the equation:

$$m = -0.018H + 3.35 \tag{5}$$

The emitter parameter  $C_v^*$  of the open valve decreases with pressure depending on the increase of local head loss due to the valve and, as reported in the preliminary research of De Marchis et al. [7], the variation is not linear and in the analysed case is well interpolated by the following power law: □

$$C_v^* = C_{v0} + b \cdot (P - P_0)^{-n} \tag{6}$$

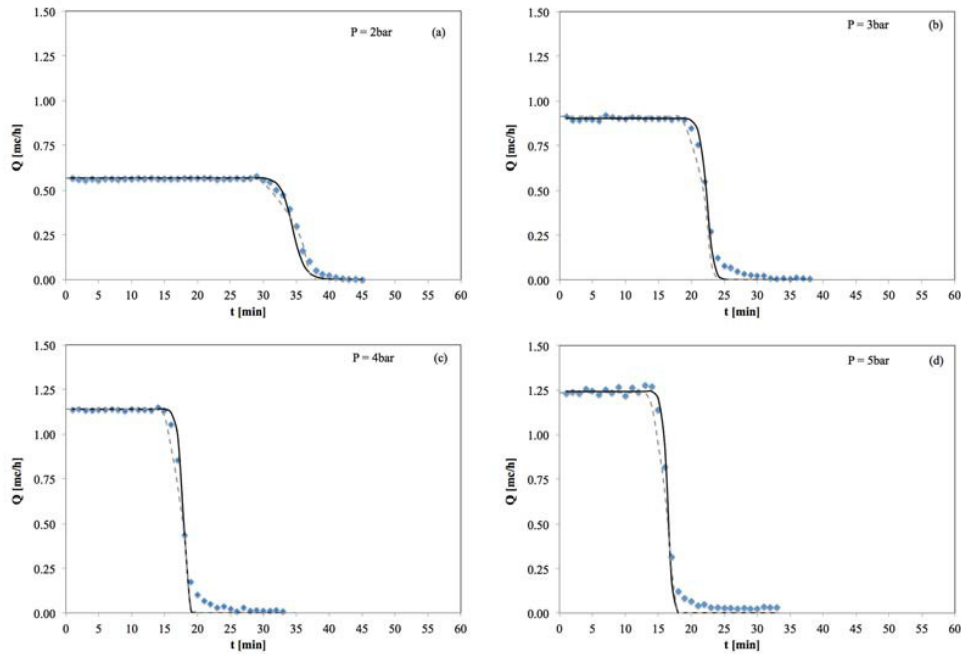


Fig. 4. Test case 2. Flow rate during the tank filling process. Symbols: Experiments; Bold line: hyperbolic law; dashed line: exponential law. (a) Pressure 2 bar; (b) Pressure 3 bar; (c) Pressure 4 bar; (d) Pressure 5 bar.

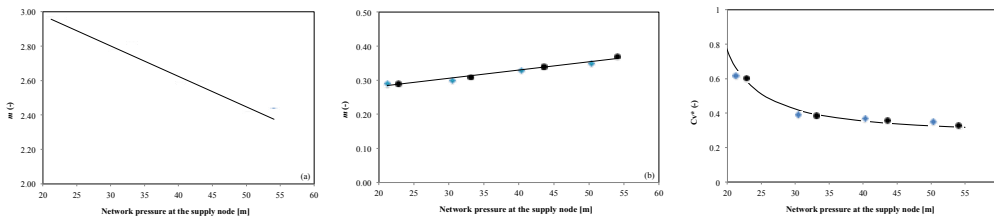


Fig. 5. Variation of the coefficient  $m$  with the pressure  $P$ . ♦: Test case 1; •: Test case 2; Thin line: interpolation curve. (a) hyperbolic law. Thin line Eq.:  $m=-0.018H + 3.35$ ; (b) exponential law. Thin line Eq.:  $m=0.0027H + 0.23$  [7]. (c) Bold line: interpolation law:  $C_v^* = C_{v0} + b \cdot (P - P_0)^{-n}$

where  $C_{v0}$  is the horizontal asymptote equal to 0.276,  $P_0$  is a vertical asymptote equal to 11.1;  $b$  and  $n$  are shape coefficients calibrated to 6.24 and 1.27, respectively.

Table 2. Coefficients of the float valve laws achieved with the test case 2.

Pressure	2 bar	3 bar	4 bar	5 bar
$C_v^+$	0.62	0.39	0.37	0.35
$m$ hyper law	2.95	2.76	2.57	2.40
$m$ expon. law	0.20	0.28	0.34	0.40

### 5. Conclusions

The private rooftop tanks greatly affect the hydraulic behaviour of the network, modifying the demand pattern of the users. User demand is in fact much higher than normal at the beginning of the service period reducing the pressure level on the network and presenting some disadvantaged users to be supplied. In such conditions the



formulation of the specific law accounting for the existence of inline tanks must be used. The existing formulations fail during the closure phase of the float valve. Experiments have been carried out to find a new head-discharge formulation able to reproduce the filling process of private water tanks, directly connected to the water distribution network. Specifically, an hyperbolic tangent law is proposed for the emitter coefficient  $C_v$  of the head-discharge relationship. The mathematical model has been compared with existing float valve law and with laboratory experiments, carried out for different values of pressure and for two branches having different length. The results demonstrate the ability of the hyperbolic tangent law to reproduce the filling process both when the valve is completely open and during the valve closure period. The mathematic system can be used in the steady state as well as in transient model to correctly estimate the supplied demand in presence of private tanks equipped with float valve.

## References

- [1] J.R.L. Ackley, T.T. Tanyimboh, B. Tahar and A.B. Templeman. Head-driven analysis of water distribution systems, Proc. Computer and Control in Water Industry, Water software systems: theory and applications. Ulanicki, B. (ed.). Research Studies Press, UK, (2001), 183-192.
- [2] P.R. Bhawe, Node flow analysis of water distribution systems, J. Transp. Eng., 107, (1981), 457-467.
- [3] J. Chandapillai, Realistic simulation of water distribution system, Journal of Transportation Engineering, 117(2), (1991), 258-263.
- [4] A. Criminisi, C.M. Fontanazza, G. Freni, and G. La Loggia, Evaluation of the apparent losses caused by water meter under-registration in intermittent water supply, Water Sci. Technol. 60(9), (2009), 2373-2382.
- [5] M. De Marchis, C.M. Fontanazza, G. Freni, G. La Loggia, E. Napoli and V. Notaro. A model of the filling process of an intermittent distribution network. Urban Water J. 7(6), (2010) 321–333.
- [6] M. De Marchis, C.M. Fontanazza, G. Freni, G. La Loggia, E. Napoli and V. Notaro. Analysis of the impact of intermittent distribution by modelling the network-filling process. Journal of Hydroinformatics 13(3), (2011) 358–373.
- [7] M. De Marchis C.M. Fontanazza, G. Freni, B. Milici and V. Puleo, Experimental investigation for local tank inflow model, Procedia Engineering, 89, (2014) 656–663.
- [8] G. Freni, M. De Marchis and E. Napoli. Implementation of pressure reduction valves in a dynamic water distribution numerical model to control the inequality in water supply. Journal of Hydroinformatics 16(1), (2014) 207–217.
- [9] O. Fujiwara and T. Ganesharajah. Reliability assessment of water supply systems with storage and distribution networks. Water Resour. Res., 29(8), (1993), 2917-2924.
- [10] G. Germanopoulos, A technical note on the inclusion of pressure dependent demand and leakage terms in water supply network models. Civil Engineering Systems, 2 (September), (1985), 171-179.
- [11] M. De Marchis, G. Freni and B. Milici, Pressure-discharge law of local tanks connected to a water distribution network: Experimental and mathematical results. Water, 7(9), (2015) 4701–4723.
- [12] R. Gupta, P.R., Bhawe, Comparison of methods for predicting deficient-network performance, Journal of Water Resources Planning and Management, 122(3), (1996), 214-217.
- [13] P. W. Jowitt, C. Xu, Predicting pipe failure effects in water distribution networks, Journal of Water Resources Planning and Management, 119(1), (1993), 18-31.
- [14] L.S. Reddy, K. Elango, Analysis of water distribution networks with head dependent outlets, Civil Engineering Systems 6(3), (1989), 102-110.
- [15] Shirzad, A., Tabesh, M., Farmani, R., and Mohammadi, M. Pressure-Discharge Relations with Application to Head-Driven Simulation of Water Distribution Networks. J. Water Resour. Plann. Manage., 139(6), (2013), 660-670.
- [16] M. Tabesh, T.T. Tanyimboh and R. Burrows, Head driven simulation of water supply networks. Engineering, Transactions A: Basics, 15 (1), (2001), 11-22.
- [17] M. Tabesh, A. Shirzad, V. Arefkhani and A. Mani. A comparative study between the modified and available demand driven based models for head driven analysis of water distribution networks, Urban Water Journal 11, no. 3 (2014), 221-230.
- [18] T.T. Tanyimboh, M. Tabesh, and R. Burrows. Appraisal of source head methods for calculating reliability of water distribution networks. J. Water Res. Plan. Manage. 127, (2001), 206-213.
- [19] T.T. Tanyimboh, and A.B. Templeman, A new nodal outflow function for water distribution networks. Proc. 4th International Conference on Eng. Computation Technology (CD-ROM), Civil-Comp Press, Stirling, UK, paper 64 (2004).
- [20] T.T. Tanyimboh and A.B. Templeman. Seamless pressure deficient water distribution system model. Water Manag., 163(8), (2010), 389-396.
- [21] T. Tucciarelli, A. Criminisi and D. Termini, Leak analysis in pipeline systems by means of optimal valve regulation. J. Hydr. Engrg., 125, (1999), 277-285.
- [22] J. M. Wagner, U. Shamir and D. H. Marks Water distribution reliability: simulation methods. J. Water Res. Plan. Manage. 114, (1988), 276-294.
- [23] Wu, Z.Y., Wang, P.H., Walski, T.M., Yang, S.Y., Bowdler, D. and Baggett, C.C., 2009. Extended global-gradient algorithm for pressure-dependent water distribution analysis. Journal of Water Resources Planning and Management, ASCE, 135 (1), (2009), 13-22.