Research Article

Novel inerter-based absorbers for suppressing beams vibration under successive moving loads

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Abstract
Inerter-based vibration absorbers are new generations of tuned mass dampers. The property of the inerter is that the relative acceleration between its two terminals is proportional to produced force. This paper proposes the use of inerter-based vibration absorbers for suppressing the beam vibration under successive moving loads. The vibrational system possesses continuous beam model accompanying by inerter-based vibration absorbers. The effectiveness of this new type of vibration absorber compared with the ordinary linear vibration absorber. The partial differential governing equation converts to a system of ordinary differential equations using the Bubnov–Galerkin method. The optimal parameters of the inerter-based vibration absorbers are presented. The results show that this novel vibration absorber can reduce vibration 37% more than the optimized classical linear vibration absorbers. Some results give quasi-beating response for the beam possesses inerter-based vibration absorbers.

Keywords Inerter-based vibration absorbers · Tuned mass dampers · Successive moving loads

1 Introduction

For long bridges and other structures under high speed, moving loads, the vibration is an important issue. Many researchers worked on beams vibration under moving loads, Refs. [1–3]. Applying dynamic vibration absorbers (DVAs) is a method to reduce unwanted vibrations, Refs. [4–7]. The effectiveness of nonlinear DVAs applied to beams excited by successive moving loads was analyzed in [8, 9]. These papers showed that nonlinear dynamic dampers are more effective to reduce the maximum amplitude of vibrations; while linear DVAs are more effective in eliminating vibration sooner for the cases of transient excitation.

A new type of passive DVAs is the inerter-based vibration absorber (IBVA), which introduced first by Smith [10] in 2002. The inerter in IBVA has the property that the force generated by IBVA is proportional to the relative acceleration between two joints. The proportionality of inerter constant is called inertance which measures in kilograms. In 2005 the inerter was profitably used as a part of suspension in Formula One racing cars [11]. Chen et al. [12] investigated the effect of using an inerter as a part of vibration absorber on the natural frequencies of one, two and more degree of freedom systems. They showed that the natural frequencies of system can be decreased by increasing the inertance of inerter.

The advantages of using an inerter to reduce vibration of multi-storey buildings under earthquake vibrations excitation in comparison to TMD vibration absorbers have been studied by Lazar et al. [13]. In their subsequent study [14] they compared the application of inerter-based vibration absorbers with viscose vibration absorber to reduce vibrations of cables and they showed that the

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vibration absorber performance increases by adding an inerter. Hu and Chen [15] showed that attaching an inerter to a damper without spring provides no performance improvement; while, by attaching the inerter to a vibration absorber consisting damper and also spring increase the performance, significantly. Jin et al. [16] suggested two inerter-based vibration control configurations to suppress the vibration of beam-type structures. They proved that under harmonic loads, the IBVAs are more efficient than the traditional DVAs, especially if the mass ratio is small. Zhou et al. [17] replaced the damper in a traditional DVA by a grounded inerter-based mechanical network. They optimized this IBVA installed on an undamped single degree of freedom system, under harmonic and transient excitations. Their numerical results proved that optimum design of TMDs with inerter is more efficient than classical TMDs. However, all inerter-based devices have to be tuned precisely. Moreover, susceptibility to detuning is also a weakness of the inerter-based devices. To eliminate the mentioned weakness, Brzeski et al. [18] proposed taking advantage of a device possesses the inerter that enables accurate changes of inertance. They showed that such a property can be achieved by using a continuously variable transmission (CVT) with a gear-ratio control system. In their subsequent paper [19] they described the experimental model of this novel type of vibration absorber. They performed several experiments to characterize the device. Also, they performed numerical simulation and proved that the presented prototype of the IBVA with the CVT presents significant damping properties in a wide range of vibration frequencies.

Herein this study, the inerter-base vibration absorber is attached to a beam subjected to equidistant successive moving loads to reduce the beam vibration. The equation of motion of the beam and IBVA is written and solved applying numerical explicit Runge–Kutta method. The goal is to determine the optimal vibration absorber parameters, i.e., speed ratio, stiffness and damping of the absorber. The effectiveness of the proposed IBVA is compared with the ordinary linear TMD. Section 2 describes the model and presents the basic equations. These equations solved numerically considering the numeric data in Sect. 3. Section 4 explains and discusses the effectiveness and performance of the new IBVA comparing to traditional DVA. The conclusion of this paper restates in the last section of this manuscript.

2 Model description and the basic equations

Figure 1 presents the schematic of the vibrational system consists of a uniform simply supported beam subjected to infinite equidistant successive moving loads. The beam is connected to a one DOF IBVA in the middle of the beam. The IBVA consists of: inertial component (m_0), elastic connection (k), dashpot (c), inerter mechanism (I), and the element that demonstrates dry friction (D). The inerter mechanism possesses the CVT, which allows unceasingly adjustments of the IBVA inertance variation.

The beam under successive moving loads formulated by the linear thin beam theory; the equations of motion of the system with Euler–Bernoulli beam are given by:

$$EI_b y_{,xxxx}(x,t) + \rho A y_{,xx}(x,t) + \left[f(u) + D(u) + Cu_{,t} + Iu_{,tt}\right] \delta(x-d) = F(x,t), \quad x\epsilon(0,L), t>0 \tag{1a}$$

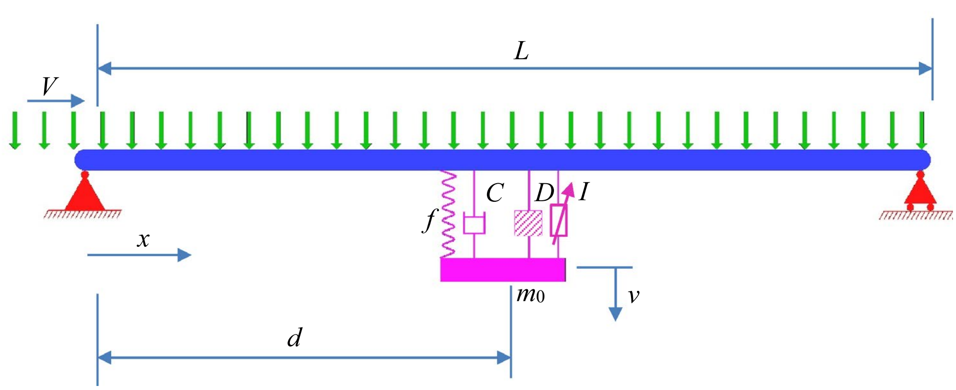
$$y(0,t)=0, \quad y(L,t)=0, \quad y_{,xx}(0,t)=0, \quad y_{,xx}(L,t)=0 \tag{1b}$$

$$y(x,0)=0, \quad y_{,t}(x,0)=0 \tag{1c}$$

$$F(x,t) = \sum_{i=-\infty}^{\infty} F_0(t)\delta[x-(Vt-i\Delta)]S(i,t), \quad x\epsilon(0,L), t>0 \tag{2a}$$

$$S(i,t) = \left[H\left(t-\frac{i\Delta}{V}\right)\left(\frac{L}{V}-\left(t-\frac{i\Delta}{V}\right)\right)\right] \tag{2b}$$

Fig. 1 The beam model with IBVA attachment



$y(x,t)$ is the displacement field of the beam, E is the modulus of elasticity, I_b is the moment of inertia of the beam section, $m=\rho A$ is the beam mass per unit length, ρ is the material density and A is the beam cross section area. Equation (1a) presents the dynamic of a uniform Euler–Bernoulli beam with simply supported boundary conditions (1b) and initial conditions (1c). $F(x,t)$ is the external force due to an equidistant infinite series of traveling loads by speed V which is presented in Eq. (2a). Δ is the loads distance; $\delta[x-(Vt-i\Delta)]$ defines the location of the ith load; $S(i,t)$ defines whether the ith load at time t lies on the beam or it is out of the beam boundaries, Eq. (2b). $\delta(t)$ and $H(t)$ are the Dirac and Heaviside functions, respectively:

$$\delta_{ij}=\begin{cases}1 & i=j\\ 0 & i\neq j\end{cases},\quad H(t)=\begin{cases}1 & t>0\\ 0 & t<0\end{cases} \tag{3}$$

Figure 2 presents the schematic of an inerter base vibration absorber (IBVA) which is equipped with a CVT. The body of the inerter is built of two parallel plates (P1) and (P2) positioned vertically and integrated with two horizontal parallel plates (P3) and (P4). The IBVA is attached to the beam by using the upper horizontal plate (P3) in a way that the axis of the IBVA is parallel to the beam vibration direction. Lower horizontal plate (P4) is attached to helical spring (1) by a connection and the other end of the spring (1) is fixed to plate (2). Massive plate (2) is connected to gear rack (3) and can move in direction of the axis of the IBVA. Plate (2) together with gear rack (3) function as a moving element of TMD. Gear rack (3) cooperates with pinion (4) that is affixed on the drive shaft (5) of continuously variable transmission (6).

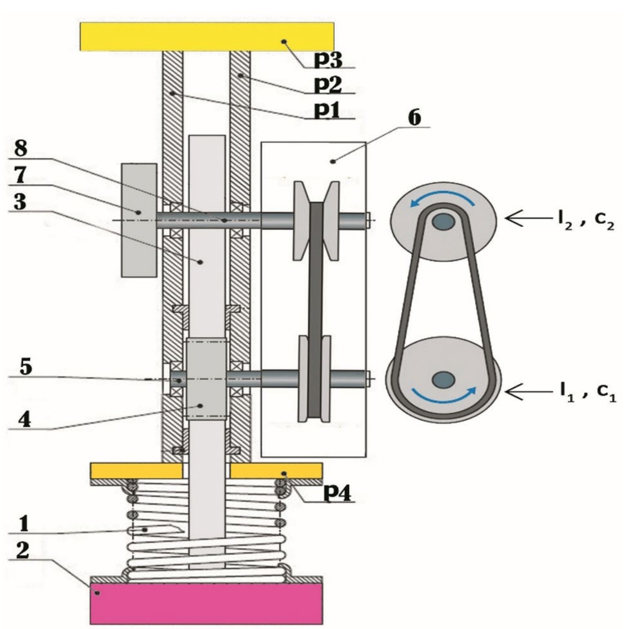


Fig. 2 Schematic of the novel TMD possesses inerter mechanism and CVT

In the presented mechanism, the usage of belt-driven CVT is assumed, but other types are also permissible. Therefore, through rack–pinion and CVT, reciprocating motion of moving elements is transferred into rotational motion of the flywheel (7) that is mounted on driven shaft (8) of CVT. If CVT ratio remains constant, the device works like a classical TMD equipped with the inerter. However, by manipulating CVT ratio (r) it is possible to change the ratio between linear velocity of moving mass (m_0) and rotational speed of the flywheel. The undamped natural frequency of the IBVA can be described by the following formula:

$$\omega_{\mathrm{IBVA}}(I)=\sqrt{\frac{k}{m_0+I(r)}} \tag{4}$$

In fact, by applying the CVT mechanism, it is possible to adjust the natural frequency of IBVA as the particular parameter, $I(r)$, can be modified as a function of transmission ratio (r). As, it is presented in Ref. [19], $I(r)$ is the inertance parameter of the inerter that can be estimated by the following formula:

$$I(r)=\frac{1}{d_p^2}I_1+\frac{r^2}{d_p^2}I_2 \tag{5}$$

where I_1 and I_2 are the inertia of drive and driven shafts of CVT and d_p is the pitch diameter of the pinion (4) that cooperates with the moving rack. The equations of motion of the IBVA are formulated as follow:

$$m_0v_{,tt}-\left(f(u)+D\left(u_{,t}\right)+C(r)u_{,t}+I(r)u_{,tt}\right)=0$$
$$v(0)=v_0,\quad v_{,t}(0)=v_1,\ t>0 \tag{6a}$$

$$u(t)=y(d,t)-v(t) \tag{6b}$$

$$f(u)=ku(t) \tag{6c}$$

$$D\left(u_{,t}\right)=d_f\frac{2}{\pi}arctan\left(10^5u_{,t}\right) \tag{6d}$$

$u(t)$ is the elongation of the spring, $y(d,t)$ is the transverse displacement of the beam midspan and $v(t)$ is the displacement of the IBVA mass (m_0). $f(u)$ represents the spring force, $D(u)$ is the dry friction force, generated by the sliding surfaces, and d_f is the amplitude of dry friction, see Ref. [19]. $C(r)$ represents the damping coefficient of IBVA and estimates by following formula:

$$C(r) = C_m + \frac{1}{d_p^2}C_1 + \frac{r^2}{d_p^2}C_2 \tag{7}$$

C_m is the damping coefficient of dashpot that connects mass (m_0) with the beam, C_1 and C_2 are the damping coefficients of drive and driven shafts of CVT. By considering small values for C_1 and C_2, $C(r) \cong C_m$.

By applying Bubnov–Galerkin method to solve the PDEs, the transverse displacement field of the beam obtains as follows:

$$y(x,t) = \sum_{q=1}^{\infty} a_q(t)\emptyset_q(x) \tag{8}$$

$a_r(t)$ is the modal coordinate which defines by solving the ordinary equations and $\emptyset_q(x)$ are the eigenfunctions, define as follow:

$$\emptyset_q(x) = \sqrt{\frac{2}{mL}}\sin\left(\frac{q\pi x}{L}\right), \quad \omega_q = (q\pi)^2\sqrt{\frac{EI}{mL^4}}, \quad q = 1,2,\ldots. \tag{9a}$$

$$\int_0^L m\emptyset_i(x)\emptyset_j(x)\mathrm{d}x = \delta_{ij}, \quad \int_0^L \emptyset_i(x)\frac{\mathrm{d}^2\left(EI\frac{\mathrm{d}^2\emptyset_j(x)}{\mathrm{d}x^2}\right)}{\mathrm{d}x^2}\mathrm{d}x = \omega_j^2\delta_{ij}, \quad i,j = 1,2,\ldots. \tag{9b}$$

ω_q is the natural circular frequency of the rth, δ_{ij} is the Kronecker delta. By replacing Eq. (8) into Eqs. (1a) and (6a), propelling on the pth eigenfunction obtain as follow:

$$\frac{\mathrm{d}^2a_p(t)}{\mathrm{d}t^2} + 2\xi_p\omega_p\frac{\mathrm{d}a_p(t)}{\mathrm{d}t} + \omega_p^2a_p(t) + \left\{\overline{K}(t) + \overline{C}(t) + \overline{I}(t) + \overline{D}(t)\right\}\emptyset_p(d) = \overline{F}(t), \quad p = 1,2,\ldots \tag{10a}$$

$$m_0\frac{\mathrm{d}^2v}{\mathrm{d}t^2} - \left\{\overline{K}(t) + \overline{C}(t) + \overline{I}(t) + \overline{D}(t)\right\} = 0 \quad v(0) = v_0, \quad v_{,t}(0) = v_1, \quad t > 0 \tag{10b}$$

$$\overline{K}(t) = k\left[\sum_{q=1}^{\infty} a_q(t)\emptyset_q(d) - v(t)\right] \tag{10c}$$

$$\overline{C}(t) = C\left[\sum_{q=1}^{\infty}\left(\frac{\mathrm{d}a_q}{\mathrm{d}t}\right)\emptyset_q(d) - \left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)\right] \tag{10d}$$

$$\overline{I} = I\left[\sum_{q=1}^{\infty}\left(\frac{\mathrm{d}^2a_q}{\mathrm{d}t^2}\right)\emptyset_q(d) - \left(\frac{\mathrm{d}^2v}{\mathrm{d}t^2}\right)\right] \tag{10e}$$

The Fourier transform decomposes the periodic force excitation into its constituent frequencies as follow:

$$\overline{F}(t) = \alpha_p^0 + \sum_{j=1}^{\infty}\alpha_p^j\cos\left(\frac{2j\pi t}{T}\right) + \beta_p^j\sin\left(\frac{2j\pi t}{T}\right), \quad p = 1,2,\ldots \tag{10f}$$

$$\alpha_p^0 = \frac{2}{T}\sum_{i=-n_L}^{0}\int_0^T F_0S(i,t)\emptyset_p(Vt - i\Delta)\mathrm{d}t, \quad p = 1,2,\ldots \tag{10g}$$

$$\alpha_p^j = \frac{2}{T}\sum_{i=-n_L}^{0}\int_0^T F_0S(i,t)\emptyset_p(Vt - i\Delta)\cos\left(\frac{2j\pi t}{T}\right)\mathrm{d}t, \quad p = 1,2,\ldots, \ j = 1,2,\ldots \tag{10h}$$

$$\beta_p^j = \frac{2}{T}\sum_{i=-n_L}^{0}\int_0^T F_0S(i,t)\emptyset_p(Vt - i\Delta)\sin\left(\frac{2j\pi t}{T}\right)\mathrm{d}t, \quad p = 1,2,\ldots, \ j = 1,2,\ldots \tag{10i}$$

These equations solved numerically using explicit Runge–Kutta method with Wolfram Mathematica computing system.

The optimal parameters for the linear DVA obtain from Eq. (11), [8]. The linear DVA obtained by this relation suppresses the first resonance of the beam. This relation depends on the location of the DVA on the beam; which in this paper is in the middle of the beam. In this equation k is the stiffness coefficient of spring of linear DVA and c is its damping.

$$k = m_0\left(\frac{\omega_1}{1+\mu}\right)^2 \tag{11a}$$

$$c = 2m_0\omega_1\sqrt{\frac{3\mu}{8(1+\mu)^3}} \quad (11b)$$

ω_1 is the first natural frequency of the beam, see the Eq. 9(a). $\mu = m_0/m_e$ is the mass ratio; m_e is the beam equivalent mass:

$$m_e = \frac{mL}{2\sin^2\left(\frac{\pi d}{L}\right)} \quad (12)$$

3 Numerical results

In this section, the performance of the proposed vibration absorber (IBVA) is examined using a numerical case study. The ability of the IBVA in vibration reduction of the beam under successive moving loads compared with the performance of the traditional linear DVA. The numerical parameters of the presented case study are listed in Table 1, Ref. [8]. Truncation for the Bubnov–Galerkin terms, Eq. 8, and the constituent frequencies Fourier transform, Eq. 10, both are selected equal to 3. The error percentage with these truncations approximately calculated equal to 1%. Ref. [8] possess the validation of the present results; in this manuscript, details are omitted for the sake of brevity.

There are several design variables for optimal parameters set; C_1, C_2, C_m, k, m_0, I_1, I_2 and r. While the equations solved numerically, there is no analytical formulation for defining the optimal parameters of the IBVA. The IBVA mass, m_0, considered as 0.05 of the beam mass; this is equivalent to the mass of the traditional linear DVA. The combination of C_1, C_2 and C_m makes $C(r)$. By considering small values for C_1 and C_2, $C(r)$ remain unchanged and one needs to optimize only $C(r)$ which is almost equal to C_m. Consequently, in order to seek the optimized parameters, the optimal values for C_m, k, I_1, I_2 and r have to be calculated. The mass inertia I_1 and I_2 estimated by mass scaling respect to the results of the Ref. [19] damping and stiffness parameters, C_m and k, optimized by means of brute-force method. Finally, transmission ratio,r, optimized for the overall velocity.

For the parameters listed in Table 1, the maximum steady-state beam deflection without DVA obtained as y_{max} = 25.3 mm at the velocity equal to V = 25.5 m/s. The application of linear DVA with c = 14 N.s/m and k = 870 N/m presents the maximum deflection of y_{max} = 2.56 mm, Ref. [8]. Figure 2 presents the maximum steady-state beam deflection vs. velocity. The optimal stiffness for IBVA specified as k = 1400 N/m. The range of variation of the r, IBVA frequency ratio, considered as 0 to 3 inspired by Ref. [18]. The results of Fig. 3 show that neither r = 0 nor r = 3 gives any improvement for vibration absorber performance; i.e., IBVA with constant internal frequency ratio is not beneficial comparing with the classical linear DVA, which presented in Ref. [8]. To achieve the desired results, the IBVA with a tunable frequency ratio considered. The tuned r is determined utilizing brute force optimization. Results in Fig. 3 show that, particularly, for the velocity range of (17–29 m/s), the IBVA with tunable r, presents much lower maximum deflection. Note that, in this graph $C(r)$ = 0 N s/m is invariant. For the IBVA with tunable frequency ratio, the optimized r within the range of (0–3) presented in Fig. 4.

Similar to Fig. 3, Fig. 5 presents the maximum steady-state deflection vs. velocity for $C(r)$ = 10 N s/m. Even though r is varying, $C(r)$ remains constant. The variation of the adjusted internal frequency ratio is similar to the case of $C(r)$ = 0. This adjustable IBVA presents the maximum deflection equal to 2.02 mm, which means a 21% reduction respect to the optimized linear DVA. Although

Table 1 Numerical parameters of the considered case study

Parameter	Description	Value	Unit
E	Modulus of elasticity	206,800	MPa
ρ	Density	7820	kg/m^3
A	Cross section	0.03 × 0.03	m^2
L	Beam length	4	m
F_0	Load	9.8	N
ξ	Damping ratio	0.01	–
m	Beam mass per unit length	7.038	kg/m
d	IBVA location	2	m
m_0	IBVA mass	1.41	kg
v	Loads velocities	20	m/s
Δ	Loads distances	5.84	m

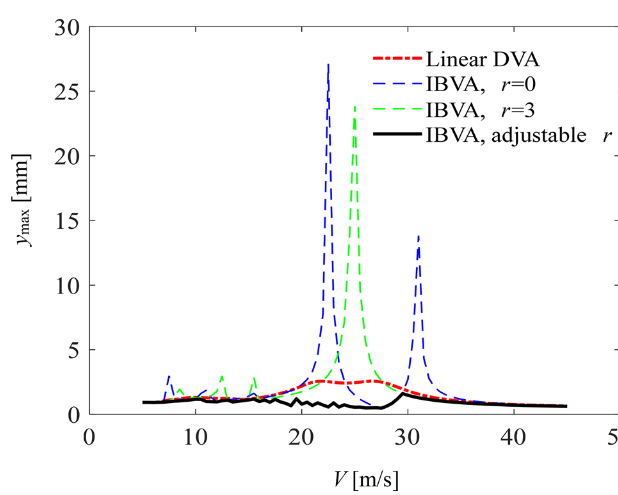


Fig. 3 Comparison of maximum deflection vs. velocity for linear DVA (Ref. [8]) and IBVA, $C(r)$ = 0 N s/m

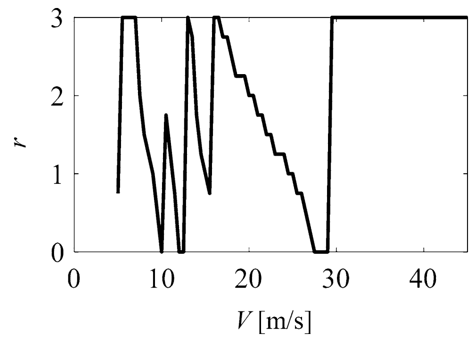


Fig. 4 Variation of *r* versus velocity for the IBVA with adjustable *r*, *C*(*r*) = 0 N s/m

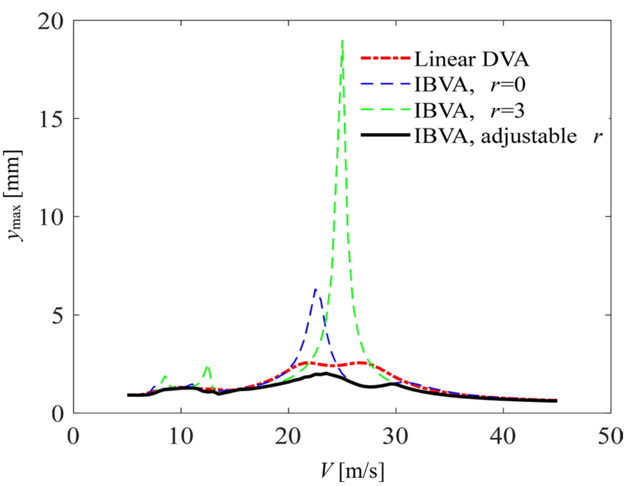


Fig. 5 Comparison of maximum deflection vs. velocity for linear DVA (Ref. [8]) and IBVA, *C*(*r*) = 10 N s/m

the situation of the IBVA without damping, i.e., *C*(*r*) = 0, gives better consequence, the situation with damping, i.e., *C*(*r*) = 10, represent less fluctuation. In other words, with increasing the IBVA damping, the sensitivity of the maximum deflection to the velocity variation decreases.

Figure 6 presents the transient response for some significant velocities with different types of linear DVA and IBVAs. The mentioned y_{max} is the maximum steady-state beam deflection for the considered velocity with the vibration absorber characterized in the first column. The steady-state response of the beam with optimized linear DVA achieved much sooner respect to the cases with IBVA. Time response of the beam with linear DVA becomes steady after 2 s, while for the cases with IBVAs it takes 30 or 60 s for arriving at the steady-state response. The time response results provided for three velocities: 20, 25 and 30 m/s.

4 Discussion

In this section, a review on the numerical outcomes of the previous section is provided. The noticeable results of this paper illustrated in Figs. 3 and 5. These figures show the ability of the new proposed vibration absorber (IBVA) to reduce maximum steady-state deflection of the beam under successive moving loads. From these two figures, it is recognizable that the new vibration absorber (IBVA) with adjustable internal frequency ratio can be much more beneficial respect to the linear classical vibration absorber. Besides, it is shown that the IBVA with constant internal frequency ratio is not suitable; for the moving loads velocity range of (17–29 m/s), the beam deflection unfavorably increases tangibly. Thus, the IBVA should use with a tunable internal frequency ratio.

The new tuning parameter for the proposed IBVA is the internal frequency ratio (*r*). The optimized values for the internal frequency ratio presented in Fig. 4. This graph is for *C*(*r*) = 0; for the case of *C*(*r*) = 10 a similar figure achieved, which does not mention in this article. This variation shows that for the velocity range of (17–29 m/s), the adjustment of IBVA internal frequency ratio becomes more important. The equivalent excitation frequency of this velocity range is (2.91 – 4.97 Hz), which is calculated by the following formula: $f = V/\Delta$. This frequency range is around the fundamental natural frequency of the beam, i.e., 4.37 Hz.

The IBVA with adjustable *r* and *C*(*r*) = 0, presents the maximum deflection equal to 1.60 mm, which is 37% less than the maximum beam deflection with an optimized linear DVA. Similarly, this IBVA with *C*(*r*) = 10 presents maximum deflection equal to 2.02 mm, which means a 21% decrease respect to linear DVA. Note that in all cases, the maximum deflection of the vibration reduction is more than 90% respect to the bare beam (beam without DVA); in this article, the advantage of the new proposed DVA (IBVA) is compared with classical linear DVA.

For the case of the beam possessing IBVA with tunable internal frequency ratio (thick black line in Fig. 3), *C*(*r*) = 0, fluctuations appear due to the discrete variation of the IBVA frequency ratio. While in Fig. 5, for the IBVA with higher damping, *C*(*r*) = 10, the fluctuations are not observable. Damping cause smoothness of all alterations. As a result, by increasing the damping of the IBVA, the sensitivity of the IBVA performance to the internal frequency ratio decreases.

Time responses for selected velocities with different types of vibration absorbers illustrates in Fig. 6. In the graphs in Fig. 5, for the cases possessing classical linear DVA, the transient segment is very shorter respect to the cases possessing IBVAs (first row). Moreover, for the IBVA without damping (*C*(*r*) = 0), the transient response is longer

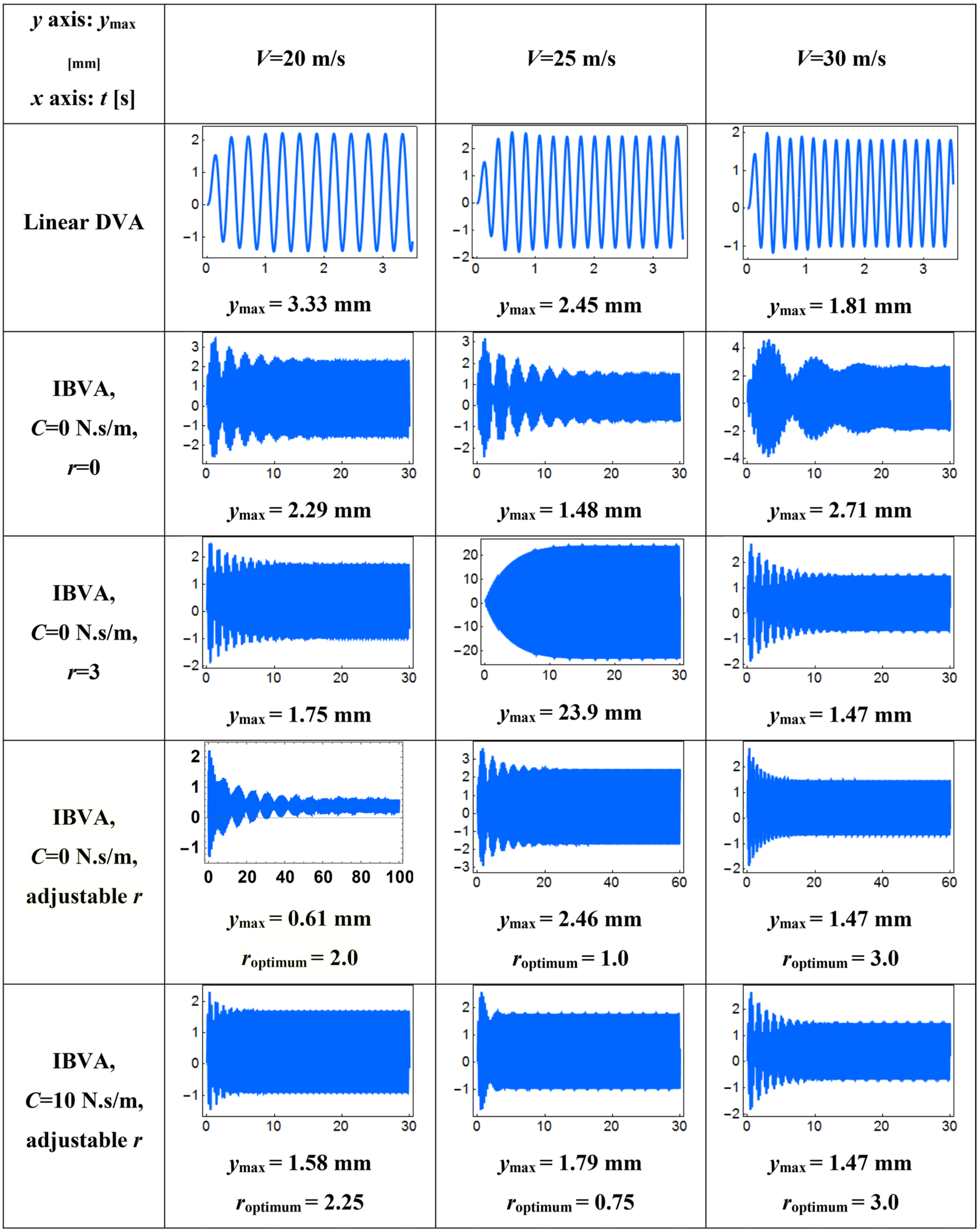


Fig. 6 Time responses of the beam middle point with different types of the DVA and IBVA

respect to the cases with more damping, i.e., $C(r) = 10$. Although for the steady-state excitation, the transient segment of the response cannot be determinant, the maximum steady-state deflection response is the critical goal. More deflection causes more significant strain and consequently, greater stress on the beam. More stress on the beam structure reduces structure lifetime.

Time responses provided in Fig. 6 have conformity with the data presented in Figs. 3 and 5. For each case, the maximum steady-state response of Fig. 6, is demonstrated in Figs. 3 or 5 for the intended velocities. As mentioned before, the steady-state response for the beams with linear DVA is achievable swift; accordingly, just a few seconds of the time response provided. It is evident that all responses for the ceases of the beam with linear classical DVA (first line in Fig. 6) are periodic. In this case, no superharmonic, subharmonic, jump, and other similar phenomena observed; since no nonlinearity introduce to the system. For the cases of the beams with IBVAs, much more fluctuations arise. In particular conditions such as $V = 20$ m/s and IBVA with $C = 0$ N s/m with adjustable r (the last row and left column), the response is quasi-beating; the energy interchange between the beam and the IBVA. The energy pumps to the IBVA and gets back to the beam continually and repeatedly.

5 Conclusions and future works

This paper investigates the performance of the novel type of vibration absorbers, which is called inerter-based vibration absorber (IBVA) for the beams under successive moving loads. The considered IBVA consists of the mass, spring, dashpot, inerter mechanism and dry friction. Internal frequency ratio of the inerter mechanism is tunable; i.e., it can adjust the frequency ratio for each velocity. The IBVA performance is distinguished with a numerical case study. The efficiency of the proposed IBVA is compared with efficiency of the traditional linear dynamic vibration absorber.

Results show that the IBVA with tunable frequency ratio (r) is more performant respect to the ordinary linear DVA. The tunable IBVA reduces the maximum deflection sensibly for the load velocity, which is close to the resonance. It can reduce the maximum deflection of the beam by 37% more than the optimized ordinary linear DVA. The main disadvantage of the proposed IBVA is that the beam with IBVA attachment takes more time to achieve steady-state response with respect to the linear DVA. Nonetheless, steady-state time response deflection of the beam with proposed IBVA is smaller than the beam with linear DVA. In some conditions with low damping IBVA, the quasi-beating phenomenon observe for the beam possessing IBVA with particular parameters.

The results of this paper show that IBVA can be a suitable substitute for the classical linear DVAs. With proper tuning of the parameters, the IBVA is more performant for reducing beam deflection under periodic excitation. Nevertheless, specifying the optimal parameters for each problem is an open research. Innovating a new method for determination of IBVA suitable parameters is a future research in this field.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Xiao F, Chen G, Hulsey J, Zatar W (2018) Characterization of nonlinear dynamics for a highway bridge in Alaska. J Vib Eng Technol 6(5):379–386. https://doi.org/10.1007/s42417-018-0048-x
2. Parandeh Afshar N, Samani FS, Molaie M (2015) Application of linear and nonlinear vibration absorbers for the nonlinear beam under moving load. J Comput Appl Res Mech Eng (JCARME) 5(1):51–60. https://doi.org/10.22061/jcarme.2015.344
3. Wang Y, Zhu X, Lou Z (2019) Dynamic response of beams under moving loads with finite deformation. Nonlinear Dyn 98(1):167–184. https://doi.org/10.1007/s11071-019-05180-6
4. Abbasi M, Moradi H (2020) Optimum design of tuned mass damper via PSO algorithm for the passive control of forced oscillations in power transmission lines. SN Appl Sci. https://doi.org/10.1007/s42452-020-2677-4
5. Zhao S, Chen Q, Yao B (2018) Damped vibration absorbers for multi-mode longitudinal vibration control of a hollow shaft. J Vib Eng Technol 6(1):1–12. https://doi.org/10.1007/s42417-018-0002-y
6. Shakeri S, Samani FS (2017) Application of linear and nonlinear vibration absorbers in micro-milling process in order to suppress regenerative chatter. Nonlinear Dyn 89(2):851–862. https://doi.org/10.1007/s11071-017-3488-z
7. Zeighami F, Palermo A, Marzani A (2019) Inertial amplified resonators for tunable metasurfaces. Meccanica 54(13):2053–2065. https://doi.org/10.1007/s11012-019-01020-4
8. Samani F, Pellicano F (2012) Vibration reduction of beams under successive traveling loads by means of linear and nonlinear dynamic absorbers. J Sound Vib 331(10):2272–2290. https://doi.org/10.1016/j.jsv.2012.01.002
9. Samani F, Pellicano F, Masoumi A (2013) Performances of dynamic vibration absorbers for beams subjected to moving loads. Nonlinear Dyn 73(1–2):1065–1079. https://doi.org/10.1007/s11071-013-0853-4
10. Smith M (2002) Synthesis of mechanical networks: the inerter. IEEE Trans Autom Control 47(10):1648–1662. https://doi.org/10.1109/tac.2002.803532
11. Rosenthal J (2017) Statistics using just one formula. Teach Stat 40(1):7–11. https://doi.org/10.1111/test.12142
12. Chen M, Hu Y, Huang L, Chen G (2014) Influence of inerter on natural frequencies of vibration systems. J Sound Vib 333(7):1874–1887. https://doi.org/10.1016/j.jsv.2013.11.025
13. Lazar I, Neild S, Wagg D (2013) Using an inerter-based device for structural vibration suppression. Earthq Eng Struct Dyn 43(8):1129–1147. https://doi.org/10.1002/eqe.2390
14. Lazar I, Neild S, Wagg D (2016) Vibration suppression of cables using tuned inerter dampers. Eng Struct 122:62–71. https://doi.org/10.1016/j.engstruct.2016.04.017
15. Hu Y, Chen M (2015) Performance evaluation for inerter-based dynamic vibration absorbers. Int J Mech Sci 99:297–307. https://doi.org/10.1016/j.ijmecsci.2015.06.003

16. Jin X, Chen M, Huang Z (2019) Suppressing random response of a regular structure by an inerter-based dynamic vibration absorber. J Vib Acoust. https://doi.org/10.1115/1.4042934
17. Zhou S, Jean-Mistral C, Chesne S (2019) Optimal design of an inerter-based dynamic vibration absorber connected to ground. J Vib Acoust. https://doi.org/10.1115/1.4043945
18. Brzeski P, Kapitaniak T, Perlikowski P (2015) Novel type of tuned mass damper with inerter which enables changes of inertance. J Sound Vib 349:56–66. https://doi.org/10.1016/j.jsv.2015.03.035
19. Brzeski P, Lazarek M, Perlikowski P (2017) Experimental study of the novel tuned mass damper with inerter which enables changes of inertance. J Sound Vib 404:47–57. https://doi.org/10.1016/j.jsv.2017.05.034